Self-induced neutrino flavor transitions in Supernovae

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based on arXiv:0707.1998 [hep-ph]
in collaboration with G.L. Fogli, E. Lisi, and A. Mirizzi

Plan of the talk

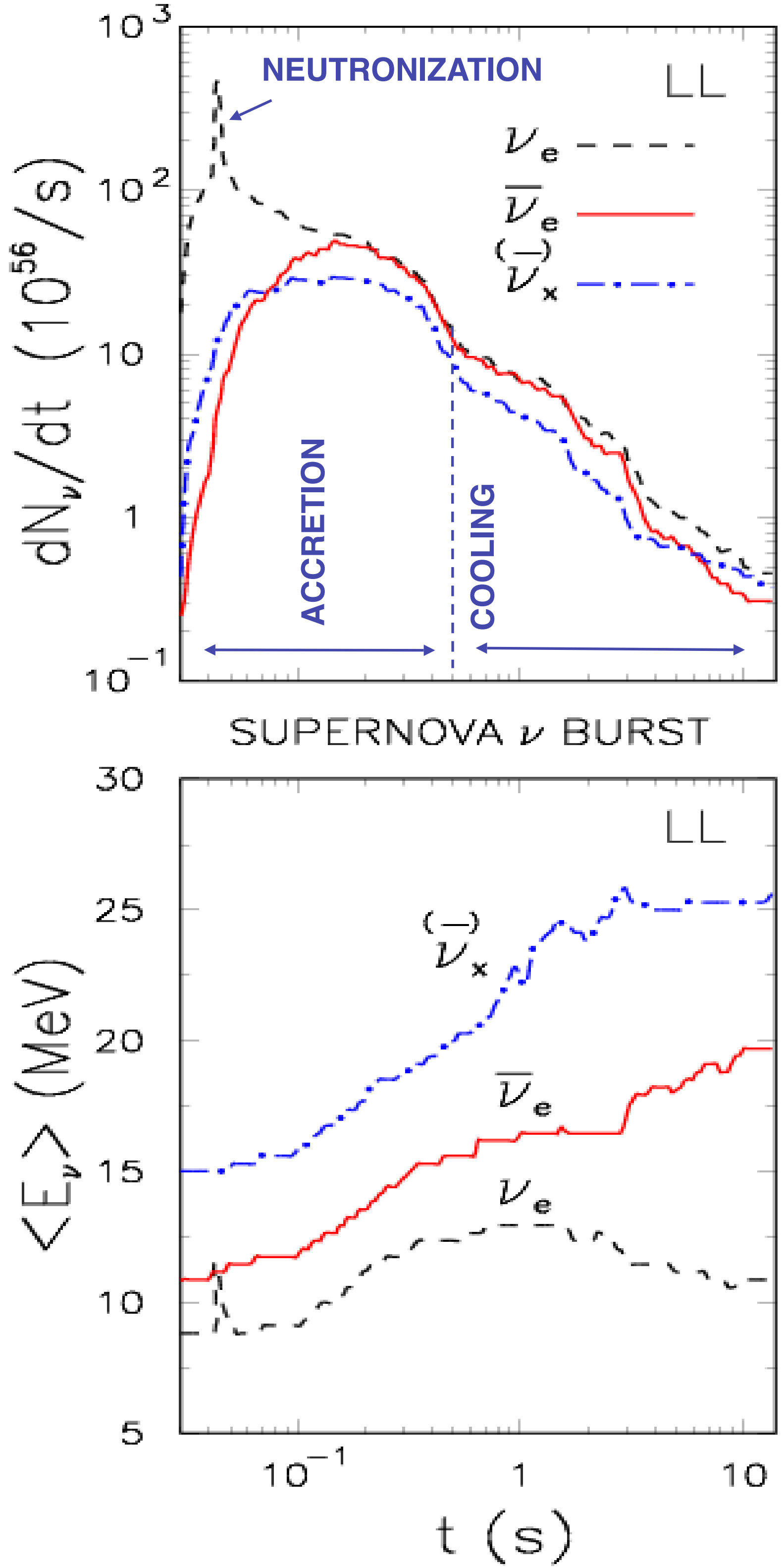
Introduction

Neutrino propagating in the SN core:
- the bulb model
- input spectra and potential
- polarization vectors
- the pendulum analogy

Numerical simulations:
- single-angle vs multi-angle

Conclusions

Supernova Neutrinos



$\sim 0.5 \times 10^{53}$ erg

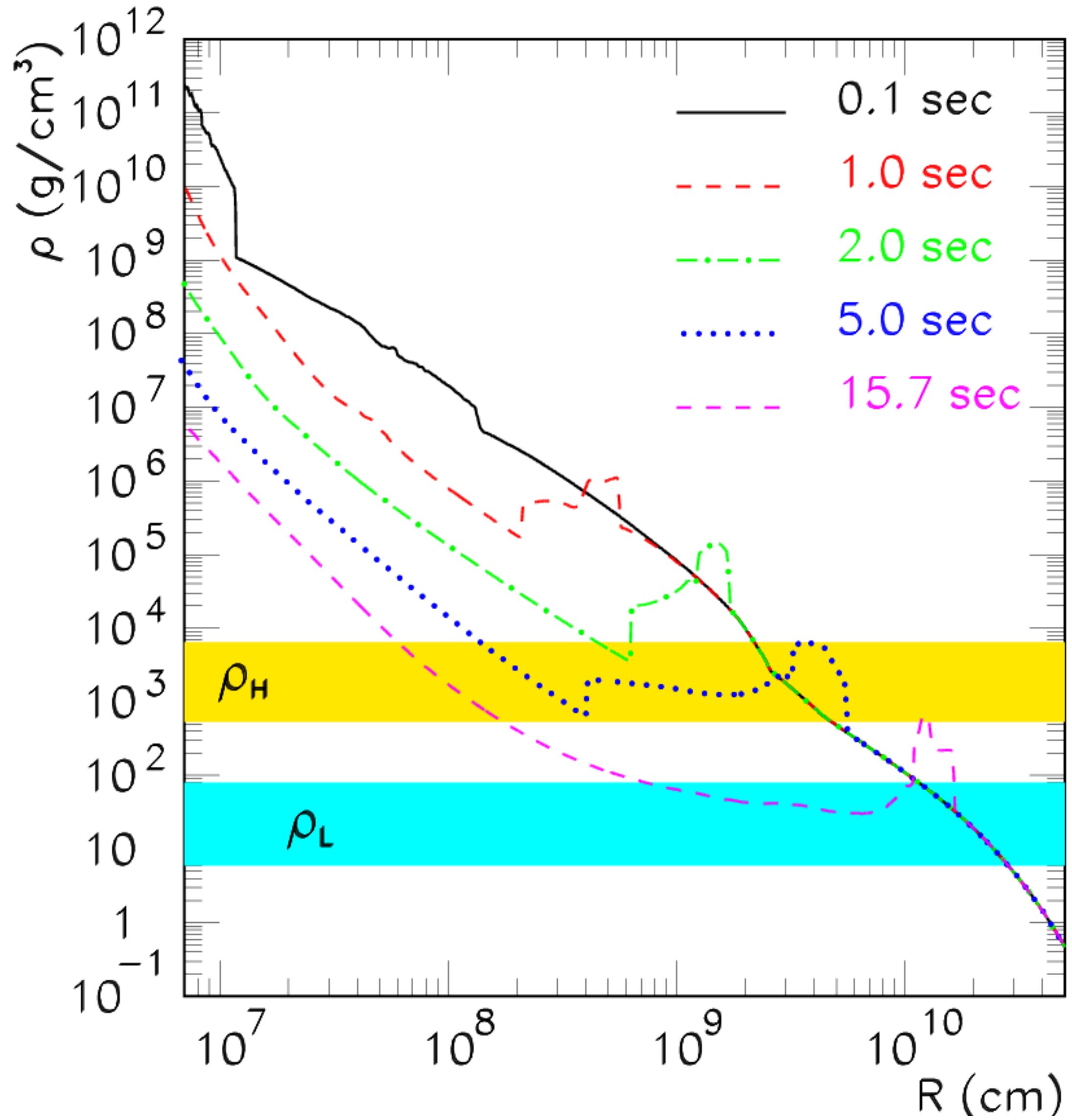
in each neutrino d.o.f.

Neutrinos emitted over a time scale of few seconds

Tipical neutrino energy of order several 10 MeV

MSW matter effect during the shock-wave propagation

Recent core-collapse SN simulations have calculated the propagation of the shock-wave in a range of time of about 20 sec after the core bounce



R.Tomas et al., astro-ph/0407132

MSW effect on supernova neutrino could be sensitive in principle to the mass hierarchy and to the mixing angle θ_{13}

Δm^2_{atm} resonance

δm^2_{sol} resonance

Not studied here

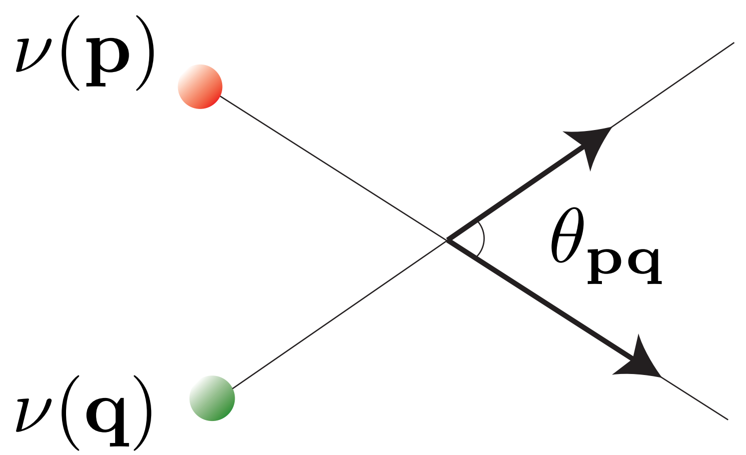
Collective neutrino oscillations

In the Supernova core neutrinos are so dense that they can be background matter to themselves

In analogy to ordinary matter the contribution to the Hamiltonian is proportional to $\sqrt{2}G_F n_\nu$ but

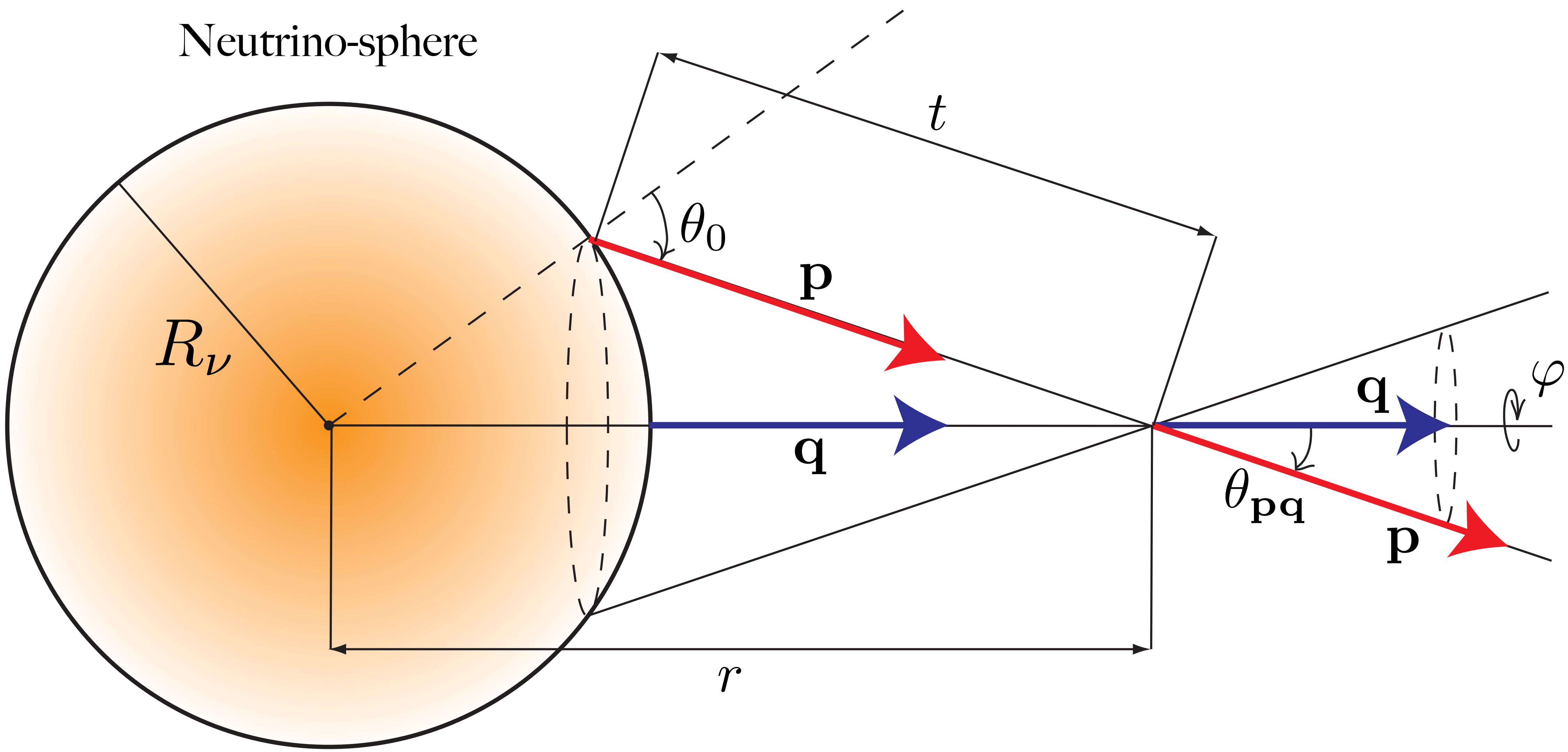
$\mathbf{p} \parallel \mathbf{q}$ collinear neutrinos no $\nu\nu$ scattering

$\mathbf{p} \nparallel \mathbf{q}$ the $\nu\nu$ cross section is maximal



The cross section interaction $\propto \sqrt{2}G_F n_\nu (1 - \cos\theta_{\mathbf{pq}})$

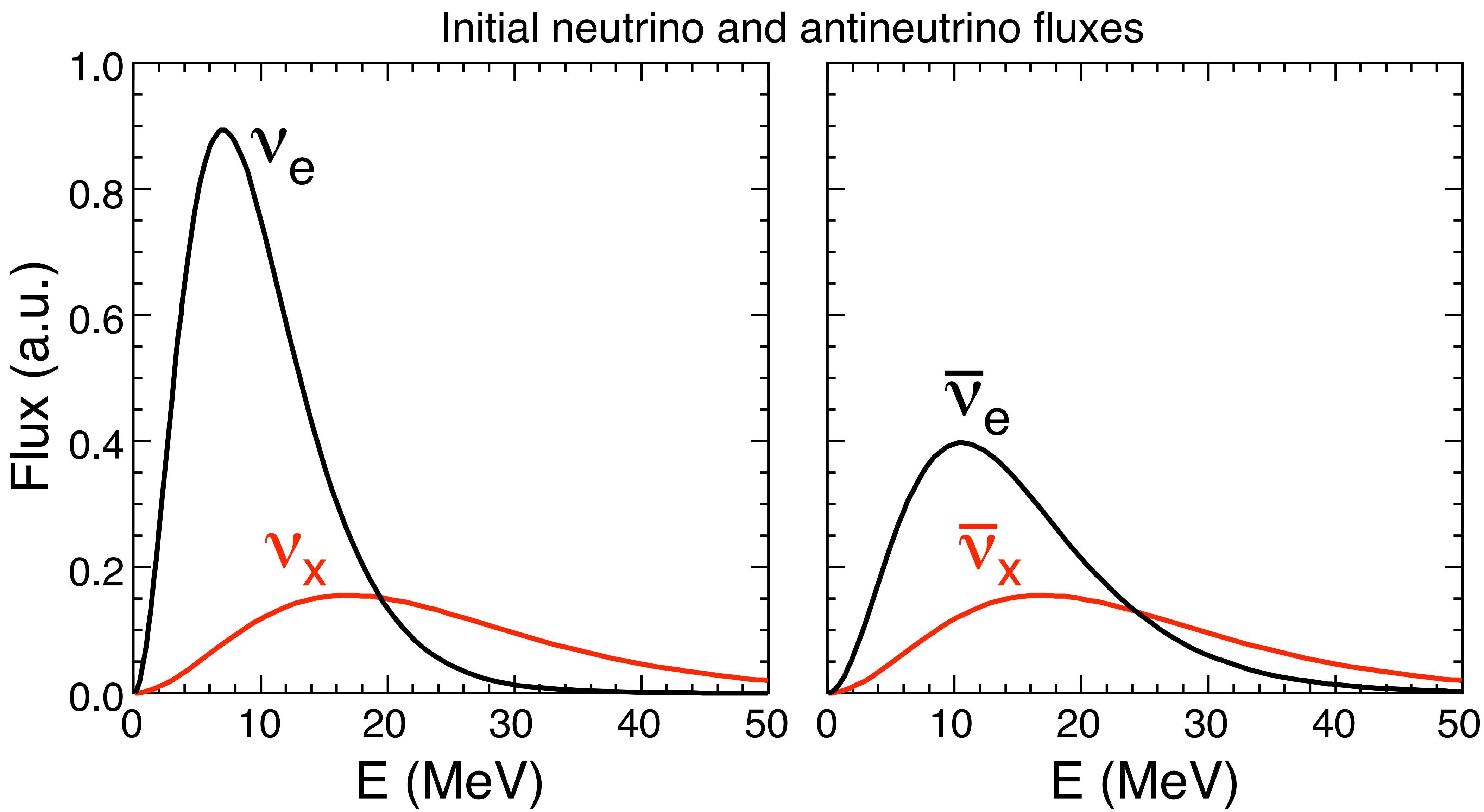
Geometry: the neutrino bulb model



$$r \sin\theta_{\mathbf{pq}} = R_\nu \sin\theta_0 \qquad t = \sqrt{r^2 - R_\nu^2 \sin^2\theta_0} - R_\nu \cos\theta_0$$

H. Duan et al. PRD 74,105014 (2006)

Input: spectra at the neutrino-sphere



Two-neutrino scenario

$$\Delta m^2 = \Delta m^2_{atm} = 2 \times 10^{-3}\ \mathrm{eV}^2$$

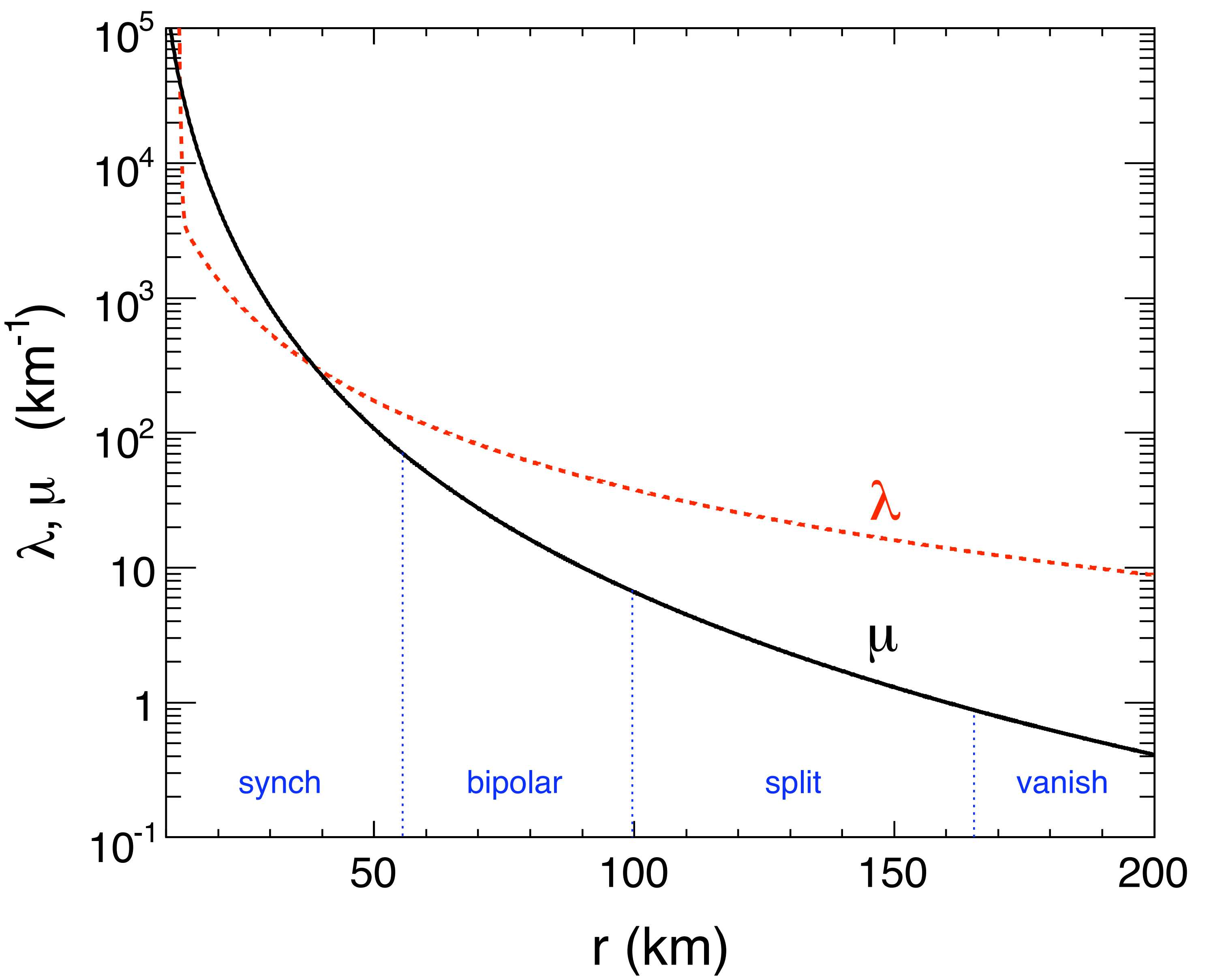
$$\sin^2\theta_{13} = 10^{-2}$$

$$\langle E_{\nu_e} \rangle = 10\ \mathrm{MeV}$$

$$\langle E_{\bar{\nu}_e} \rangle = 15\ \mathrm{MeV}$$

$$\langle E_{\nu_x} \rangle = \langle E_{\bar{\nu}_x} \rangle = 24\ \mathrm{MeV}$$

Input: matter and self-interaction potential



$$\lambda(r) = \sqrt{2} G_F N_{e^-}(r)$$

Matter potential profile from numerical SN simulation at t=5 sec after the bounce. With this kind of potential MSW effects are effective well after the region studied here $(r \lesssim 200 \text{ Km})$

$$\mu(r) = \sqrt{2}\, G_F \left[N(r) + \overline{N}(r)\right]$$

Total (i.e. integrated over the energy) number density of all neutrino and antineutrino species

The self-interaction potential decreases as the fourth power of the distance, for large r

Density matrix formalism and polarization vector

$$(\rho_{\mathbf{p}})_{ij} = \langle a_i^{+} a_j \rangle_{\mathbf{p}}$$

Occupation number for the momentum mode $\mathbf{p}$

Pauli matrices

(momentum index omitted)

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix} = \begin{pmatrix} |\nu_e|^2 & \nu_e \nu_x^* \\ \nu_e^* \nu_x & |\nu_x|^2 \end{pmatrix} = \frac{n}{2}\left(\mathbf{1} + \mathbf{P} \cdot \boldsymbol{\sigma}\right)$$

Analogous equation for the antineutrinos

Polarization vector

$$P_{ee} = P(\nu_e^i \to \nu_e^f) = \frac{1}{2}\left(1 + \frac{P_z^f}{P_z^i}\right)$$

The mixing angle enters the equations through the “magnetic field” vector

$$\mathbf{B} = \sin 2\theta_{13}\, \mathbf{x} \mp \cos 2\theta_{13}\, \mathbf{z}$$

The multi-angle simulation consists of $6 \times N_E \times N_{\vartheta_0}$ differential equations

Single angle approximation

Average over the interaction angle between neutrinos: consider only propagation over radial direction

$$\mathbf{P}(E,\theta_0) \to \mathbf{P}(E)$$

Advantages

Numerically much easier to solve

Physical understanding through the pendulum analogy

$$\omega = \frac{\Delta m^2}{2E}$$

Vacuum oscillation frequency

Define some integral quantity

$$\mathbf{J} = \frac{1}{N+\overline{N}} \int dE\, n\, \mathbf{P} \qquad \mathbf{W} = \frac{1}{N+\overline{N}} \int dE\, \omega\, n\, \mathbf{P} \qquad \mathbf{S} = \mathbf{J} + \overline{\mathbf{J}}$$

$$\overline{\mathbf{J}} = \frac{1}{N+\overline{N}} \int dE\, \overline{n}\, \overline{\mathbf{P}} \qquad \overline{\mathbf{W}} = \frac{1}{N+\overline{N}} \int dE\, \omega\, \overline{n}\, \overline{\mathbf{P}} \qquad \mathbf{D} = \mathbf{J} - \overline{\mathbf{J}}$$

Gyroscopic pendulum in flavor space

$$\dot{\mathbf{S}} = \mathbf{B} \times (\mathbf{W} - \overline{\mathbf{W}}) + \mu \mathbf{D} \times \mathbf{S}$$
$$\dot{\mathbf{D}} = \mathbf{B} \times (\mathbf{W} + \overline{\mathbf{W}})$$

Consider only adiabatic variation of the self-interaction potential

$$\dot{\mu} \sim 0$$

When $\mu \mid \mathbf{D} \mid \gg \omega$

$$\mathbf{W} \simeq w\,\mathbf{J}$$
$$\overline{\mathbf{W}} \simeq \overline{w}\,\overline{\mathbf{J}}$$

It can be shown that all polarization vectors $\mathbf{P}, \bar{\mathbf{P}}, \mathbf{J}$ and $\bar{\mathbf{J}}$ have the same dynamics; they remain closely aligned to each other, and to the z-axis, as they are at the start. As μ decreases, the vacuum terms start to be non-negligible, and neutrino and antineutrino polarization vectors develop different precession histories

If one defines

$$\mathbf{Q} = \mathbf{S} - (\omega_{\rm ave}/\mu)\mathbf{B} \quad \text{with} \quad \omega_{\rm ave} = (w + \overline{w})/2$$

$$\dot{\mathbf{Q}} = \mu \mathbf{D} \times \mathbf{Q}$$
$$\dot{\mathbf{D}} = \omega_{\rm ave} \mathbf{B} \times \mathbf{Q}$$

Hannestad, Raffelt, Sigl, Wong, astro-ph/0608695; Duan, Carlson, Fuller, Qian, astro-ph/0703776

$$\begin{aligned} \mathbf{Q}/Q &\equiv \mathbf{r} \text{ (unit length vector)} \\ \mathbf{D} &\equiv \mathbf{L} \text{ (total angular momentum)} \\ \mu^{-1} &\equiv m \text{ (mass)} \\ \mathbf{D}\cdot\mathbf{Q}/Q &\equiv \sigma \text{ (spin)} \\ \omega_{\text{ave}}\,\mu\,Q\,\mathbf{B} &\equiv -\mathbf{g} \text{ (gravity field)} \end{aligned}$$

$$\begin{aligned} \mathbf{L} &= m\mathbf{r}\times\dot{\mathbf{r}} + \sigma\mathbf{r} \\ \dot{\mathbf{L}} &= m\mathbf{r}\times\mathbf{g} \end{aligned}$$

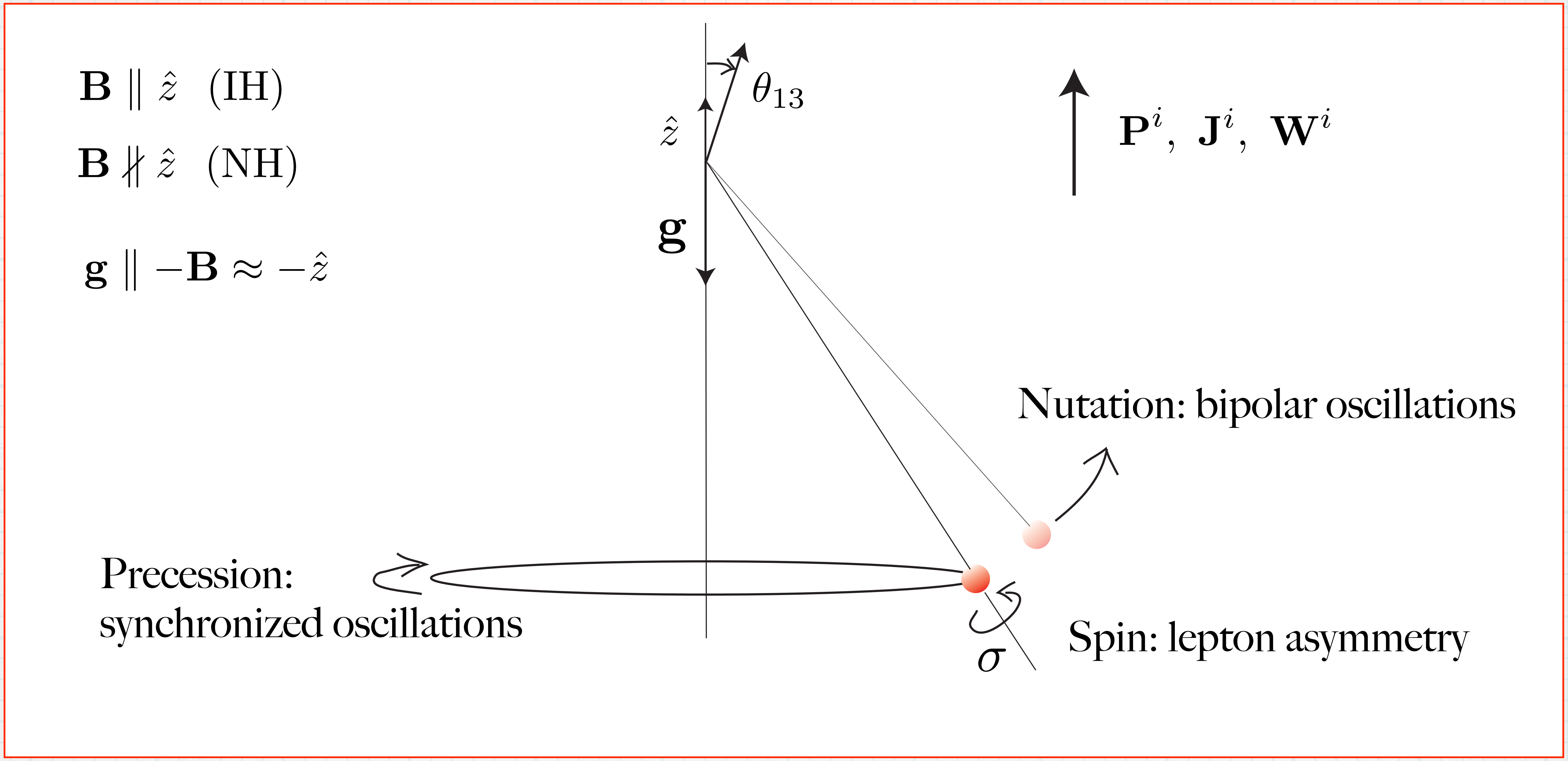
$$\mathcal{E} = -m\,\mathbf{g}\cdot\mathbf{r} + \left(\frac{m}{2}\,\dot{\mathbf{r}}^2 + \frac{\sigma^2}{2m}\right)$$

Two conserved quantity

$$\mathbf{L}\cdot\mathbf{g}/|\mathbf{g}| = \mathbf{D}\cdot\mathbf{B} = \text{const} = \mathbf{D}^i\cdot\mathbf{B} = \mp\frac{N_e - \overline{N_e}}{N_e + \overline{N_e}}$$

Conservation of the electron lepton number through transitions of the kind $\nu_e\overline{\nu}_e \longrightarrow \nu_x\overline{\nu}_x$

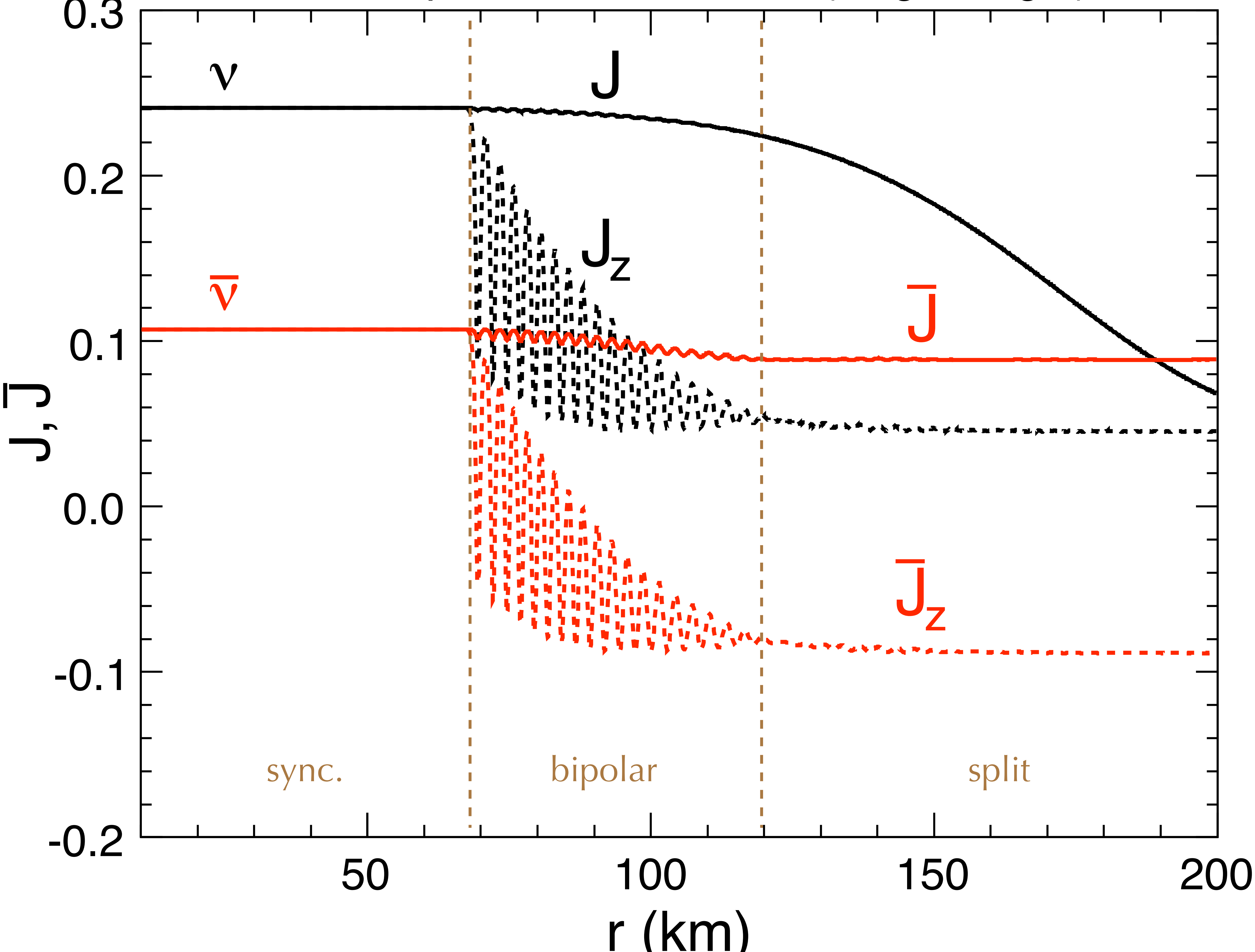
$$\mathcal{E} = \mathbf{B}\cdot(\mathbf{W} + \overline{\mathbf{W}}) + \frac{1}{2}\mu\mathbf{D}^2 = \mathcal{V} + \mathcal{T}$$

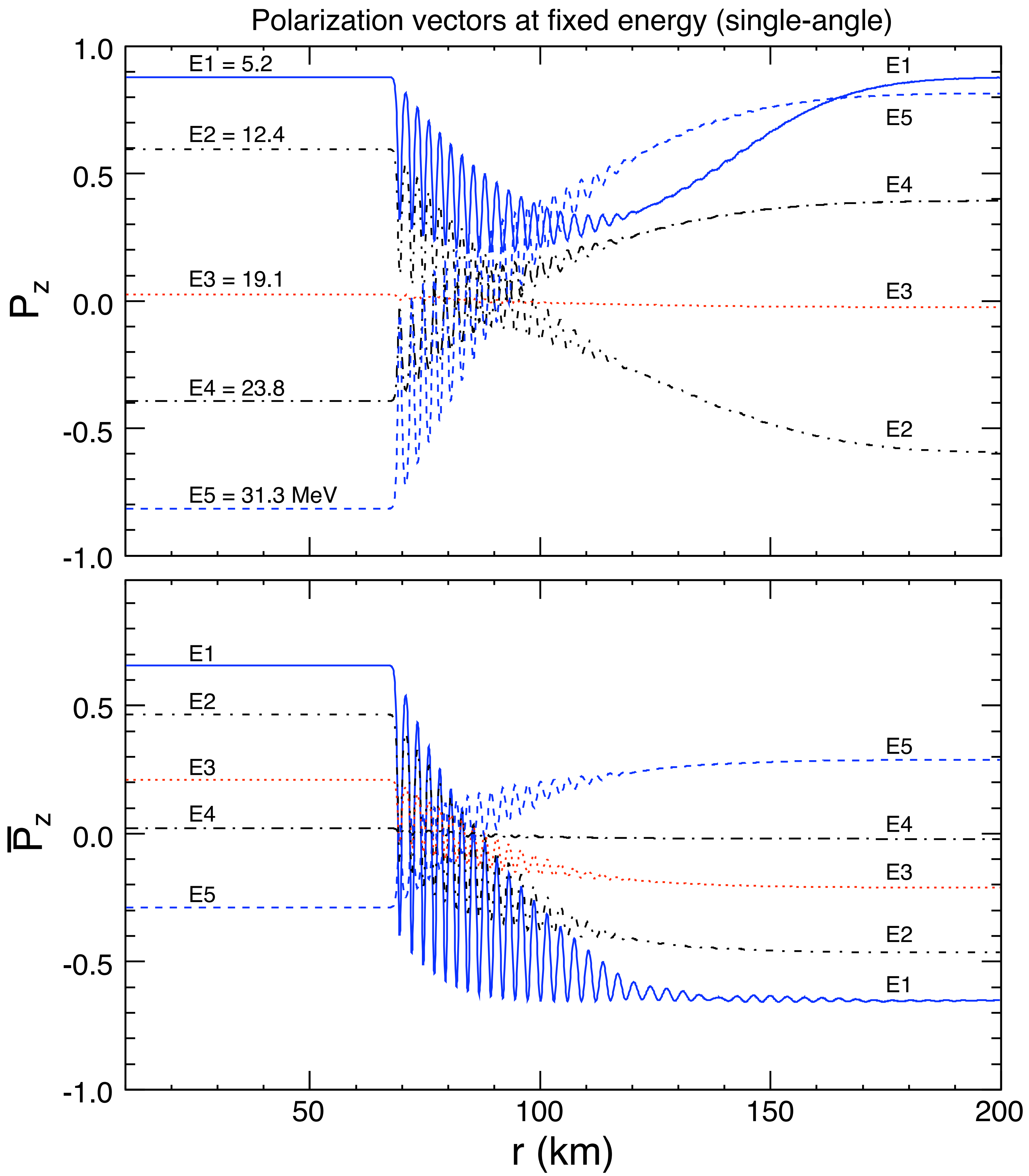


Normal hierarchy: the polarization vectors start aligned with the z-axis and they end up in the same position staying close to the potential energy minimum

Inverted hierarchy: the polarization vectors start antialigned with the z-axis. To conserve the electron lepton number only $\overline{\mathbf{W}}$ (the smallest) can completely reverse while only a partial reversal is possible for $\mathbf{W}$ $\longrightarrow$ spectral split

Global polarization vectors (single-angle)





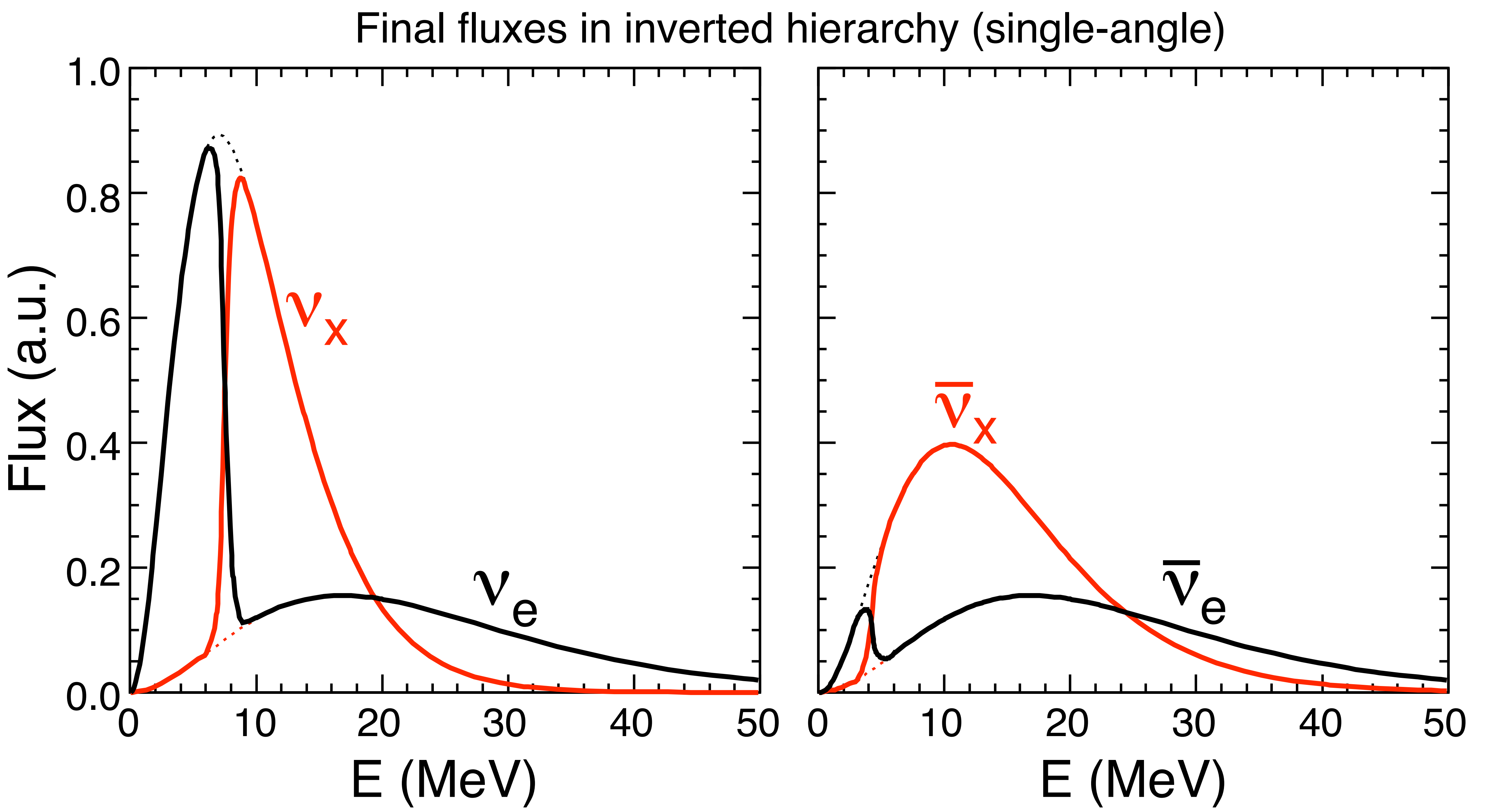
Low energy neutrino polarization vectors do not reverse themselves but the z component turn back to its initial value so that $P_{ee} = 1$

For antineutrinos all P_z are inverted and there is complete spectral swap $(P_{ee} = 0)$

Actually, there is a lack of $\overline{P_z}$ reversal at very low energy $(\lesssim 4 \text{ MeV})$, related to the non conservation of $\overline{J}$ during the bipolar regime

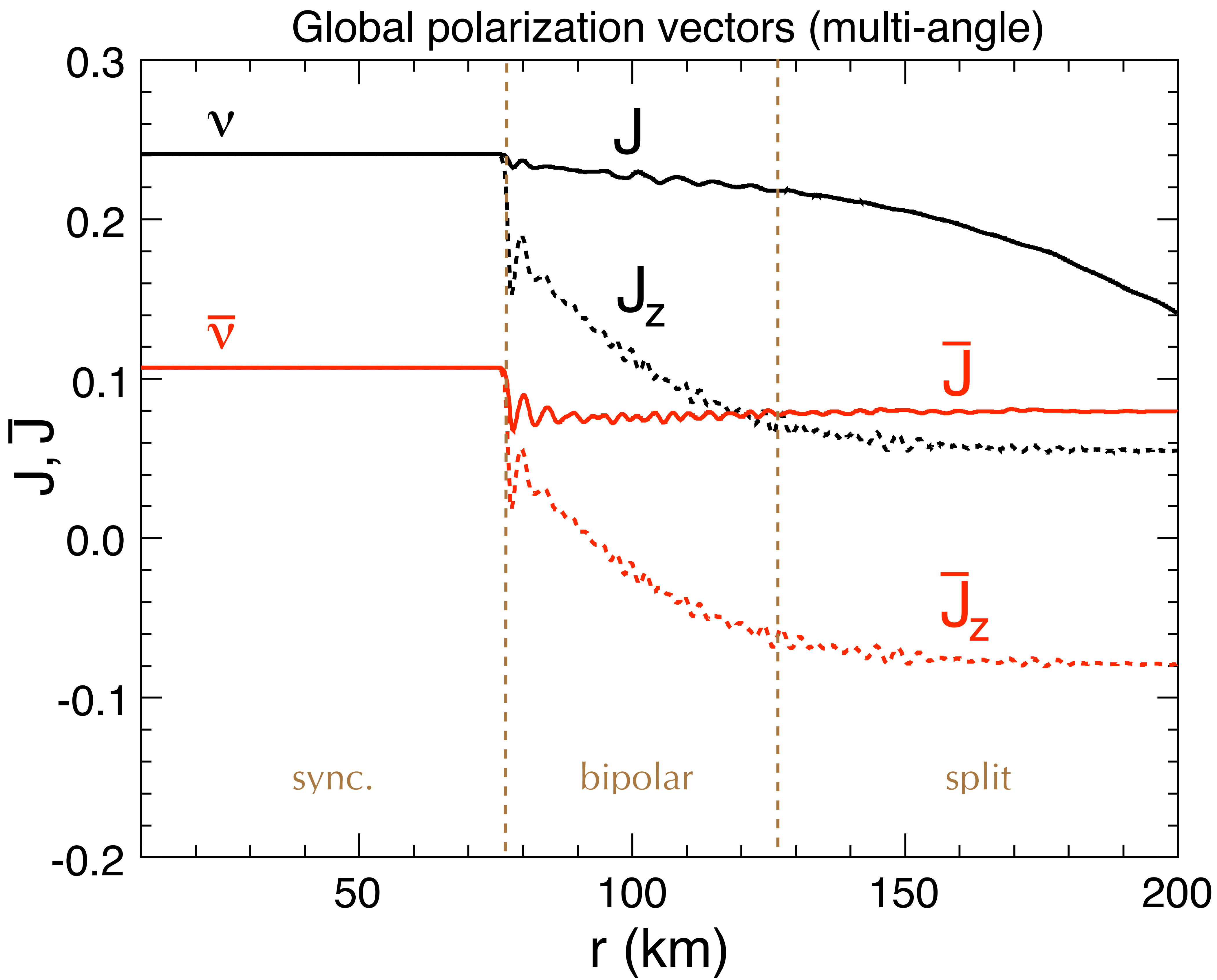
The critical energy $(\sim 7 \text{ MeV})$ above which there is complete spectral swapping can be determined from the following equation expressing the electron lepton number conservation

$$\int_{E_c}^{\infty} dE(n_e - n_x) = \int_0^{\infty} dE(\overline{n}_e - \overline{n}_x)$$



Spectral split for neutrinos above ~ 7 MeV

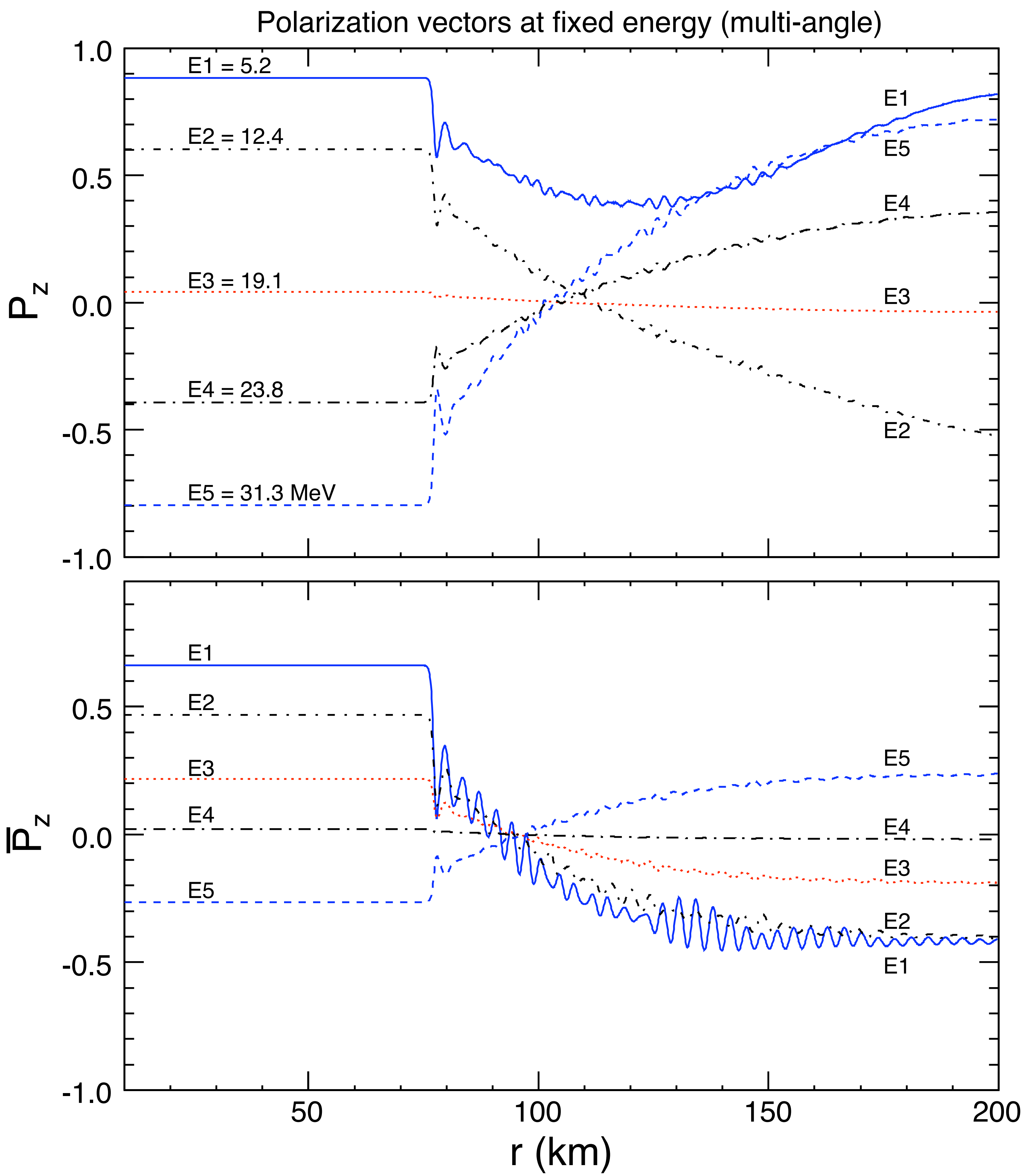
(Nearly) complete spectral swap for antineutrinos (low energy effect almost washed out in the multi-angle simulation)



In multi-angle simulations, neutrino-neutrino angles can be larger than the (single-angle) average one, leading to somewhat stronger self-interaction effects

Two effects:

- Bipolar regime starts later
- More pronounced depolarization of $\bar{\mathbf{J}}$ and prolonged coherence of $\mathbf{J}$

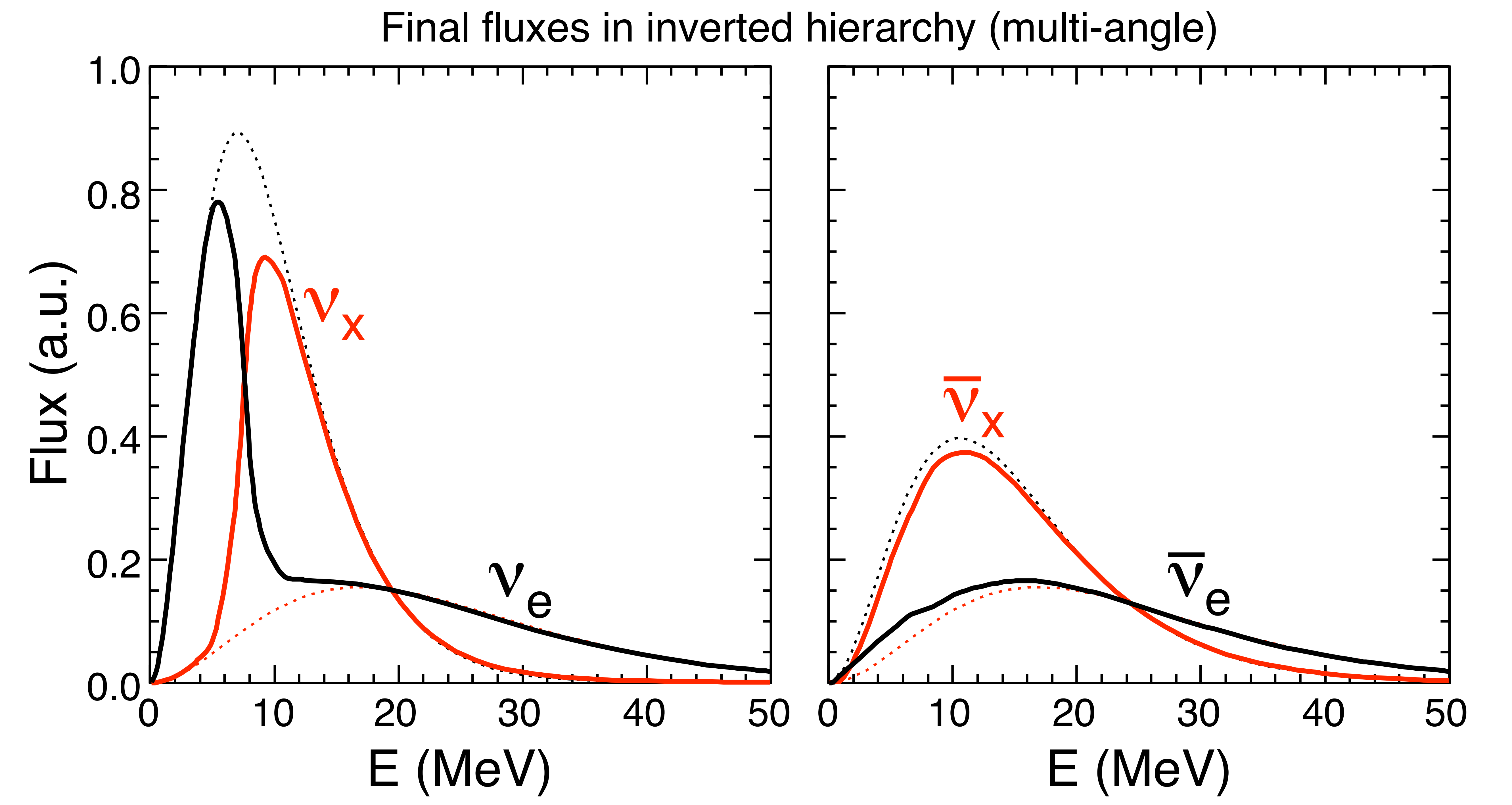


For the multi-angle case

$$\mathbf{P}(E) = \frac{\int dc_{\vartheta_0}\, c_{\vartheta_0}\, \mathbf{P}(E,\vartheta_0)}{\int dc_{\vartheta_0}\, c_{\vartheta_0}}$$

As for the single-angle case, low energy neutrino polarization vectors do not reverse themselves, while this appends for antineutrinos

Bipolar oscillations are smeared out by the angle averaging



The neutrino spectral split is evident, although less sharp than in the single-angle case. Antineutrino split largely washed out

The spectral split is a robust effect: variations of the mixing angle θ_{13} lead to (unobservable) effects in the bipolar regime (starting point and depth of the bipolar oscillations)

Conclusions

Neutrino-neutrino interactions near a supernova core produce very interesting collective effects

The interaction strength depends on the intersection angle of the neutrino trajectories. Averaging the radial trajectory allows analytical approximations and much easier calculations

Analogy with a gyroscopic pendulum in flavor space. For inverted hierarchy, swap of energy spectra above a critical energy (lepton number conservation)

In the multi-angle simulation "fine structure" details are smeared out but the spectral swap remains a robust feature

The swapping of the $\overline{\nu}_\mu$ and $\overline{\nu}_e$ (as well as of the ν_μ and ν_e) fluxes could have an impact on r-process nucleosynthesis, on the energy transfer to the stalling shock wave, and on the possibility to observe shock-wave propagation effects in neutrinos