Duality Based Vector and Axial Form Factors-Improved Modeling of Quasielastic Neutrino Cross Sections at all Energies

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BBBA2007 Vector and Axial Form Factors

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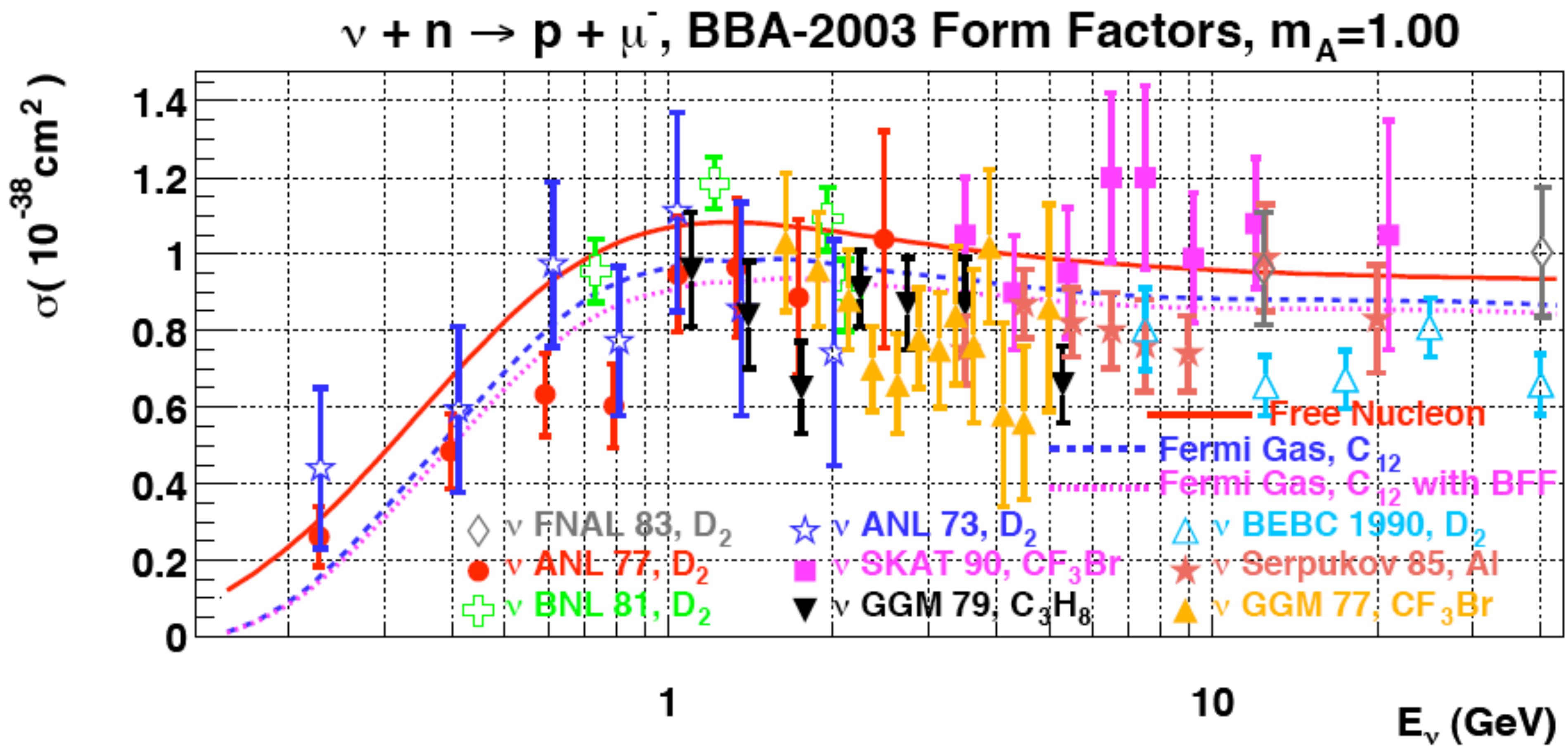


Neutrino Oscillations experiments need to know the precise energy dependence of low energy neutrino interactions: Need to Understand both vector and axial form factors, and nuclear corrections

The hadronic current for QE neutrino scattering is given by [2]

Axial form factor F_A

$$< p(p_2)|J^{+}_{\lambda}|n(p_1) >=$$

$$\overline{u}(p_2)\left[\gamma_\lambda F_V^1(q^2) + \frac{i\sigma_{\lambda\nu}q^{\nu}\xi F_V^2(q^2)}{2M} + \gamma_\lambda\gamma_5 F_A(q^2) + \frac{q_\lambda\gamma_5 F_P(q^2)}{M}\right] u(p_1),$$

Vector F_V

$$\frac{d\sigma^{\nu,\,\overline{\nu}}}{dq^2} = \frac{M^2 G_F^2 cos^2\theta_c}{8\pi E_\nu^2} \times \left[A(q^2) \mp \frac{(s-u)B(q^2)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4}\right],$$

where

Axial form factor

$$A(q^2) = \frac{m^2 - q^2}{4M^2}\left[\left(4 - \frac{q^2}{M^2}\right)|F_A|^2\right.$$

$$\left. - \left(4 + \frac{q^2}{M^2}\right)|F_V^1|^2 - \frac{q^2}{M^2}|\xi F_V^2|^2\left(1 + \frac{q^2}{4M^2}\right) - \frac{4q^2 ReF_V^{1*}\xi F_V^2}{M^2}\right],$$

$$B(q^2) = -\frac{q^2}{M^2}ReF_A^*(F_V^1 + \xi F_V^2),\;\; C(q^2) = \frac{1}{4}\left(|F_A|^2 + |F_V^1|^2 - \frac{q^2}{M^2}\left|\frac{\xi F_V^2}{2}\right|^2\right).$$

Vector form factor F_V

Vector from e-p e-n

$$F_V^1(q^2) = \frac{G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)}{1 - \frac{q^2}{4M^2}}, \quad \xi F_V^2(q^2) = \frac{G_M^V(q^2) - G_E^V(q^2)}{1 - \frac{q^2}{4M^2}}.$$

We use the CVC to determine $G_E^V(q^2)$ and $G_M^V(q^2)$ from the electron scattering form factors $G_E^p(q^2)$, $G_E^n(q^2)$, $G_M^p(q^2)$, and $G_M^n(q^2)$:

$$G_E^V(q^2) = G_E^p(q^2) - G_E^n(q^2), \quad G_M^V(q^2) = G_M^p(q^2) - G_M^n(q^2).$$

The axial form factor F_A and the pseudoscalar form factor F_P (related to F_A by PCAC) are given by

In 1970's assumed

$$F_A(q^2) = \frac{g_A}{\left(1 - \frac{q^2}{M_A^2}\right)^2}, \quad F_P(q^2) = \frac{2M^2 F_A(q^2)}{M_\pi^2 - q^2}.$$

Axial dipole form

dipole approximation.

2007: All those assumptions are wrong

Vector dipole form

$$G_D(q^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2}, \quad M_V^2 = 0.71\ GeV^2$$

$$G_E^p = G_D(q^2), \quad G_E^n = 0, \quad G_M^p = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2).$$

We refer to the above combination of form factors as 'Dipole Form Factors'.

Zero Gen

Form factor scaling

BBBA2007: Start with vector form factors

New Vector Form Factors new fit to (Gep, Gmp, Gen, Gmn) From Electron Scattering data

1. Incorporation of the recent BLAST results- Gep Gmp- C.B. Crawford et al, Phys. Rev. Lett 98, 052301 (2007).

2. Improved functional form that builds on the Kelly form [J. Kelly, Phys. Rev. C 70, 068202 (2004)]

3. Multiply by a modulating functions using the Nachtman scaling variable ξ to relate elastic and inelastic (vector and axial nucleon form factors);

4. Excellent low Q^2 description of the spatial structure of the nucleon by constraining the fit to yield the same values as Arrington and Sick $Q^2 < 0.64\ (GeV/c)^2$. J. Arrington I.Sick, nucl-th/0612079 (Submitted to Phys.Rev.C.)

5. Extend to satisfy quark-hadron duality constraints on the ratio of form factors at high- Q^2 **(ξ = 1)**: (a) Gmn/Gmp; (b) (Gen/Gmn)/(Gep/Gmp)

Start with Kelly (2004) form for Gep,Gmp

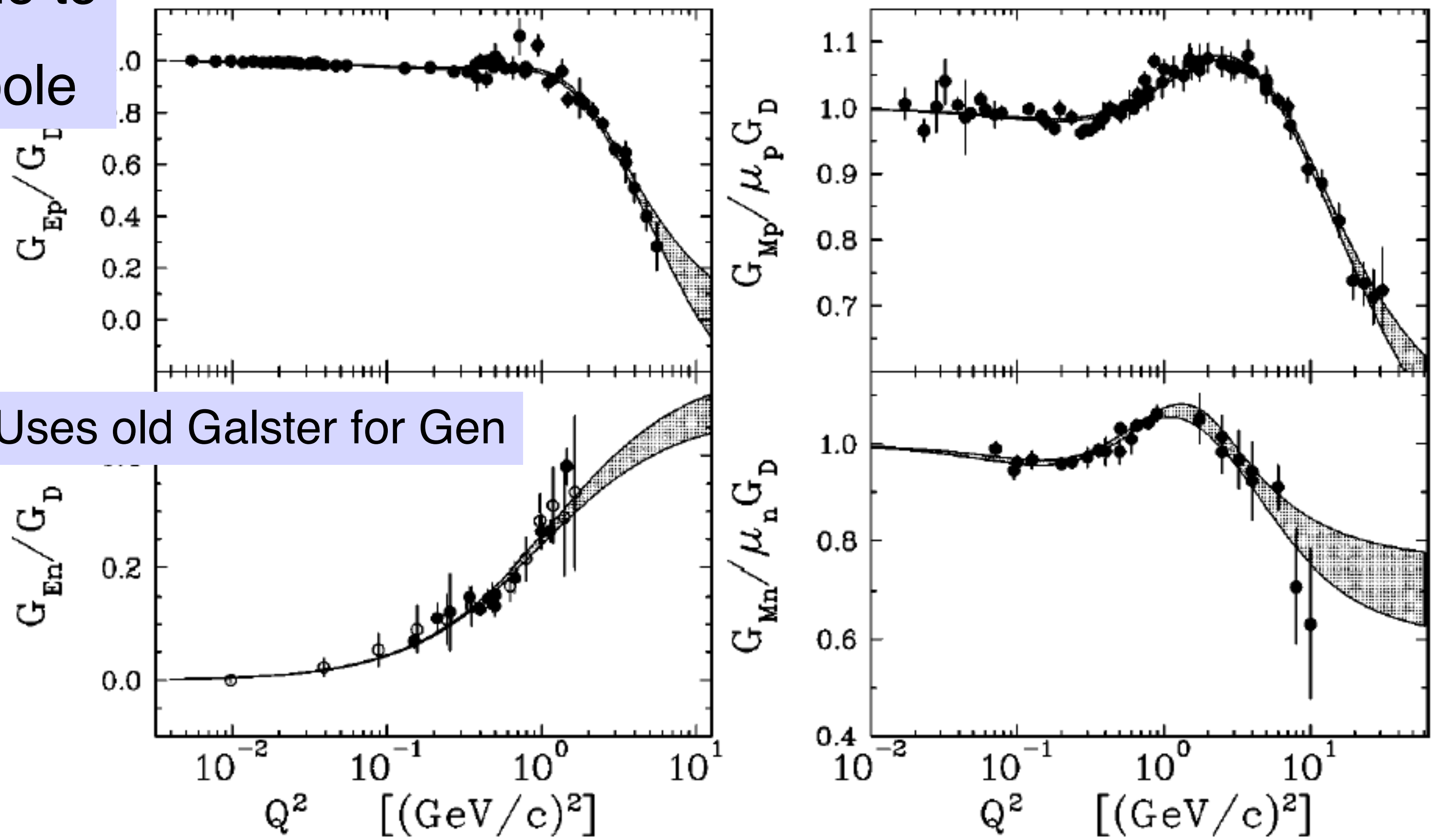
Kelly 2004 very well constrained at high Q^2, but satisfies QCD behavior for Gep,Gmp,Gmn at high Q^2

$\tau = Q^2/4M_N^2$.

$$G^{Kelly}(Q^2) = \frac{\sum_{k=0}^{m} a_k \tau^k}{1 + \sum_{k=1}^{m+2} b_k \tau^k},$$

Ratio to dipole

Kelly Uses old Galster for Gen



Source: J.J. Kelly, PRC **70** 068202 (2004).

Constraint 0:

Get excellent low Q^2 description of the spatial structure of the nucleon by constraining the fit to yield the same values as Arrington and Sick $Q^2 < 0.64\ (GeV/c)^2$. "*Precise determination of low-Q nucleon electromagnetic form factors and their impact on parity-violating e-p elastic scattering*" John Arrington (Argonne, PHY) , Ingo Sick (Basel U.) . Dec 2006. Submitted to Phys.Rev.C e-Print: nucl-th/0612079

Arrington and Sick fit elastic differential cross sections and polarization data and include corrections for.

1. Two photon exchange effects
2. Nucleon coulomb field corrections on incoming and outgoing lepton

Since we fit form factors instaed of differential cross section we include these corrections by requiring our fits to agree with Arrington and Sick exactly for $Q^2 < 0.64\ (GeV/c)^2$

ξ in Elastic Scattering -for quark hadron duality

We use for x=1 elastic scattering (with $m_F = m_I = 0$ and Pt =0) ξ becomes)

$$\xi = \frac{2}{(1+\sqrt{1+1/\tau})}$$

$$\tau = Q^2/4M_N^2.$$

We use the above for ξ elastic

* The most general derivation of fractional momentum carried by quark of initial Pt, initial mass m_I and final mass m_F (A and B included for higher order QCD effects) yields (Bodek, Yang):

$$\xi = \frac{Q'^2 + B}{M\nu \left[1+(1+Q^2/\nu^2) \right]^{1/2} + A}$$

Where: $2Q'^2 = [Q^2 + m_F^2 - m_I^2] + \{ (Q^2 + m_F^2 - m_I^2)^2 + 4Q^2 (m_I^2 + P^2t) \}^{1/2}$

For Pt =0 one gets the Barbieri variable ξ [R. Barbieri et al Phys. Lett. 64B, 1717 (1976); Nucl. Phys. B117, 50 (1976)]

For $m_F = m_I = 0$ and Pt =0 - one gets the Nachtman or Georgi Politzer variable ξ. H. Georgi and H. D. Politzer, Phys. Rev. D14, 1829 (1976)

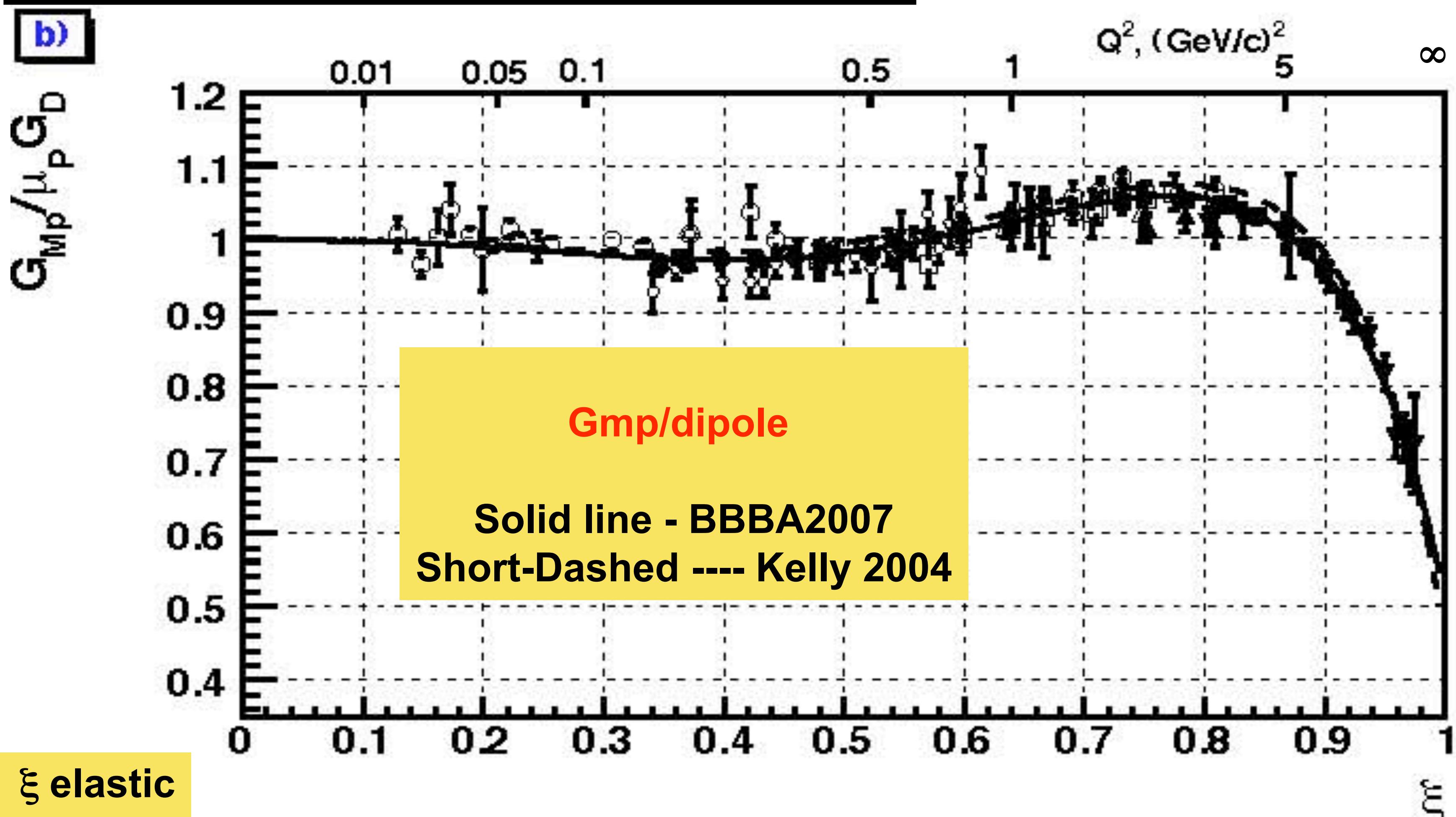
ξ in Elastic Scattering -for quark hadron duality Gmp

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1 + \frac{Q^2}{M_{V,A}^2}\right)^2}.$$

Here $C^{V,A} = (1, g_A)$, g_A = -1.267, M_V^2 = 0.71 $(GeV/c)^2$,

$$\xi = \frac{2}{(1+\sqrt{1+1/\tau})}$$

$$\tau = Q^2/4M_N^2.$$



ξ elastic

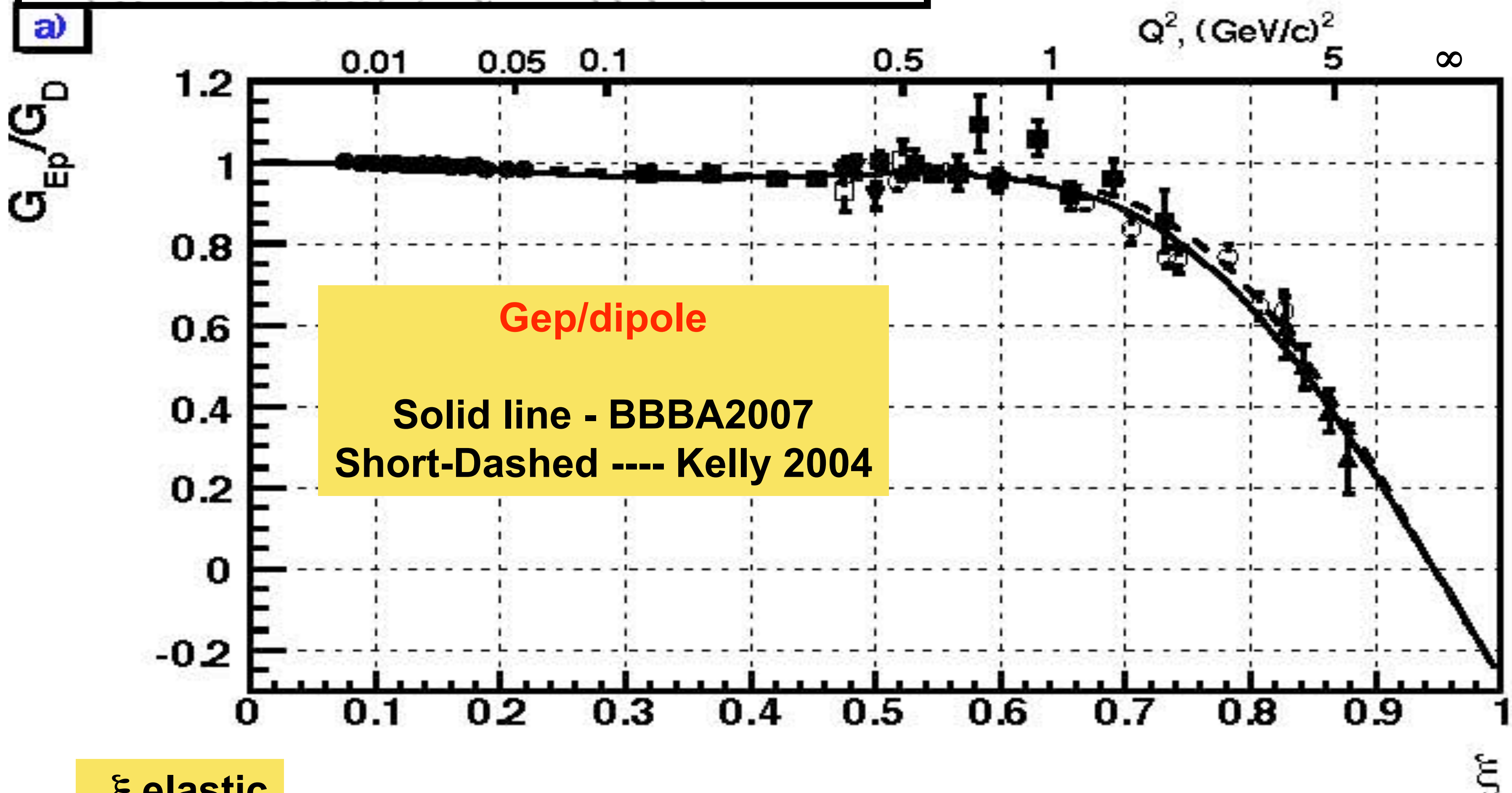
ξ in Elastic Scattering -for quark hadron duality Gep

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1+\frac{Q^2}{M^2_{V,A}}\right)^2}.$$

Here $C^{V,A} = (1, g_A)$, $g_A = -1.267$, $M_V^2 = 0.71\ (GeV/c)^2$,

$$\xi = \frac{2}{(1+\sqrt{1+1/\tau})}$$

$$\tau = Q^2/4M_N^2.$$



ξ elastic

Constraint 1 Gmn: From local duality:

F_{2n}/F_{2p} for Inelastic and Elastic scattering should be the same at high Q^2

$\xi \to 1$

- In the limit of $\nu \to \infty$, $Q^2 \to \infty$:

$$F_2 = x \sum_i e_i^2 f_i(x)$$

- In the elastic limit: $(F_{2n}/F_{2p}) \to (G_{mn}/G_{mp})^2$

$$\Rightarrow \left(\frac{G_{mn}}{G_{mp}} \right)^2 \approx \left(\frac{F_{2n}}{F_{2p}} \right)^2 \approx \frac{1 + 4\frac{d}{u}}{4 + \frac{d}{u}}$$

We **do fits with d/u=0**

$(F_{2n}/F_{2p}) \to (G_{mn}/G_{mp})^2$

--> 0.25 $\xi \to 1$

We do fits with d/u=0.2

F_{2n}/F_{2p}) $(F_{2n}/F_{2p}) \to (G_{mn}/G_{mp})^2$

--> 0.43 $\xi \to 1$

Note, F2inelastic=F2respnance appears to be valid for the average of the resonance region (global duality). Local duality sates that it may be valid for the sum of elastic peak and first resonance, and possibly also in the limit of the elastic peak only. We only assume that any violations of local duality will cancel in this ratio

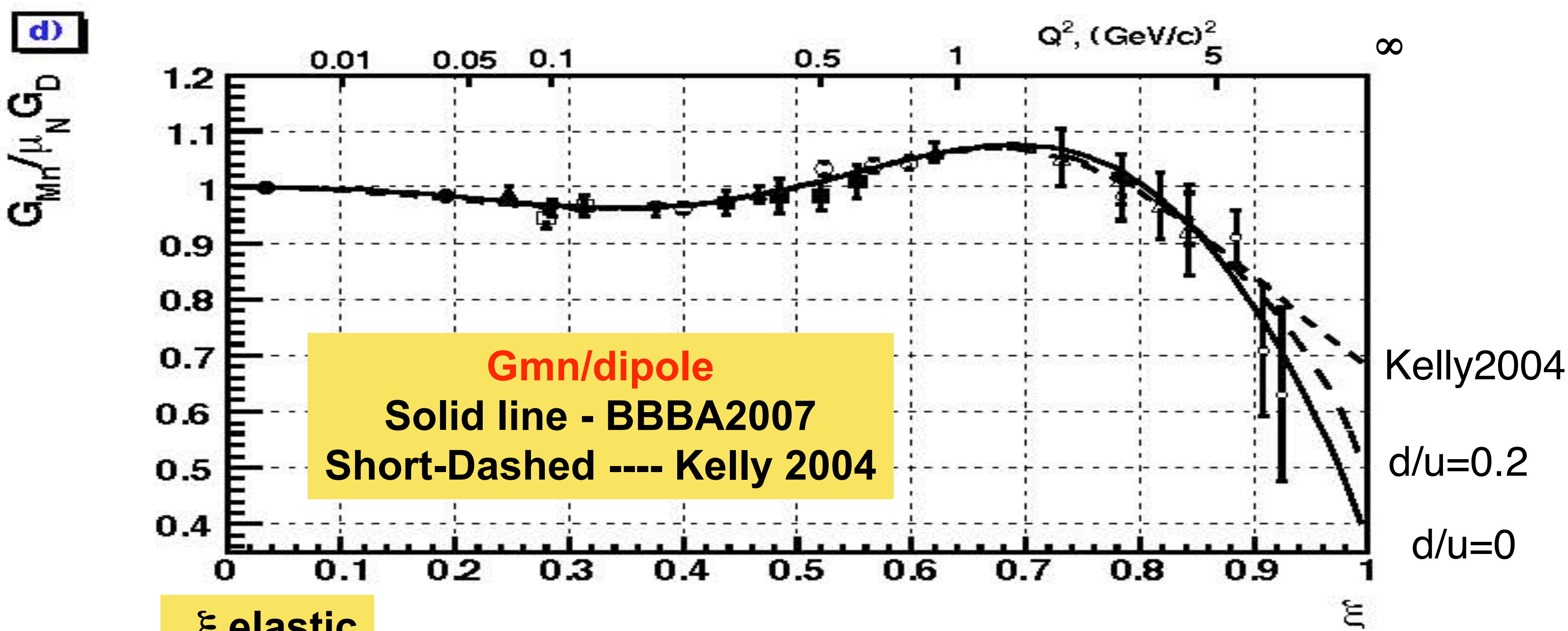
Constraint 1 Gmn: From local duality: F_{2n}/F_{2p} for Inelastic and Elastic scattering should be the same at high Q^2

$\xi \rightarrow 1$

Short Dashed ------ = Kelly

Long Dashed : **d/u=0.2** F_{2n}/F_{2p}) $(F_{2n}/F_{2p}) \rightarrow (G_{mn}/G_{mp})^2$ --> 0.43 $\xi \rightarrow 1$

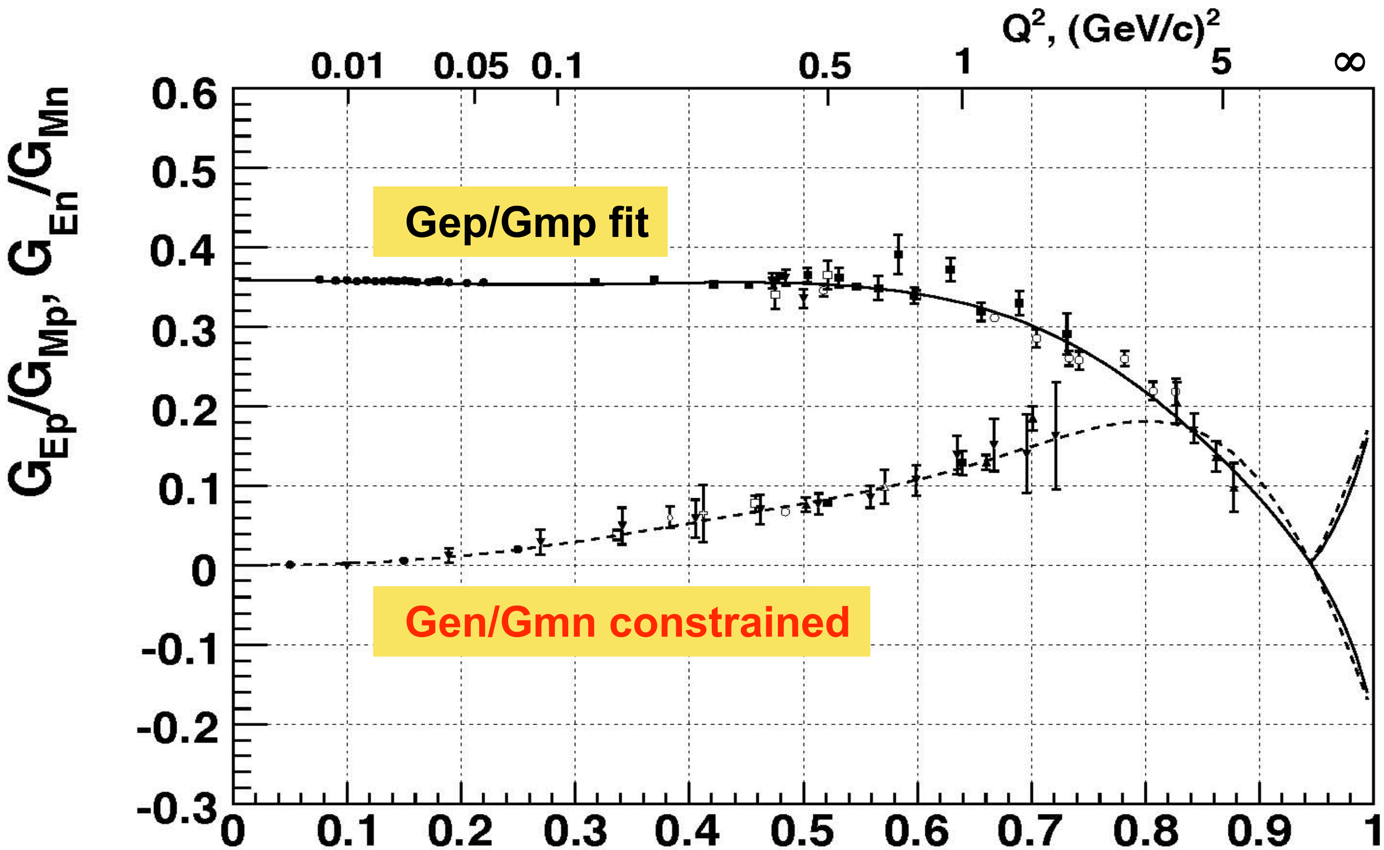
Solid: **d/u=0** $(F_{2n}/F_{2p}) \rightarrow (G_{mn}/G_{mp})^2$ --> 0.25 $\xi \rightarrow 1$



ξ elastic

Constraint 2: Gen Rp=Rn (from QCD) From local duality R for inelastic, and R for elastic should be the same at high Q2:

$$\left(\frac{G_{ep}}{G_{mp}}\right)^2 = \left(\frac{G_{en}}{G_{mn}}\right)^2 \quad \text{at high } Q^2,\ \xi \to 1$$



ξ elastic

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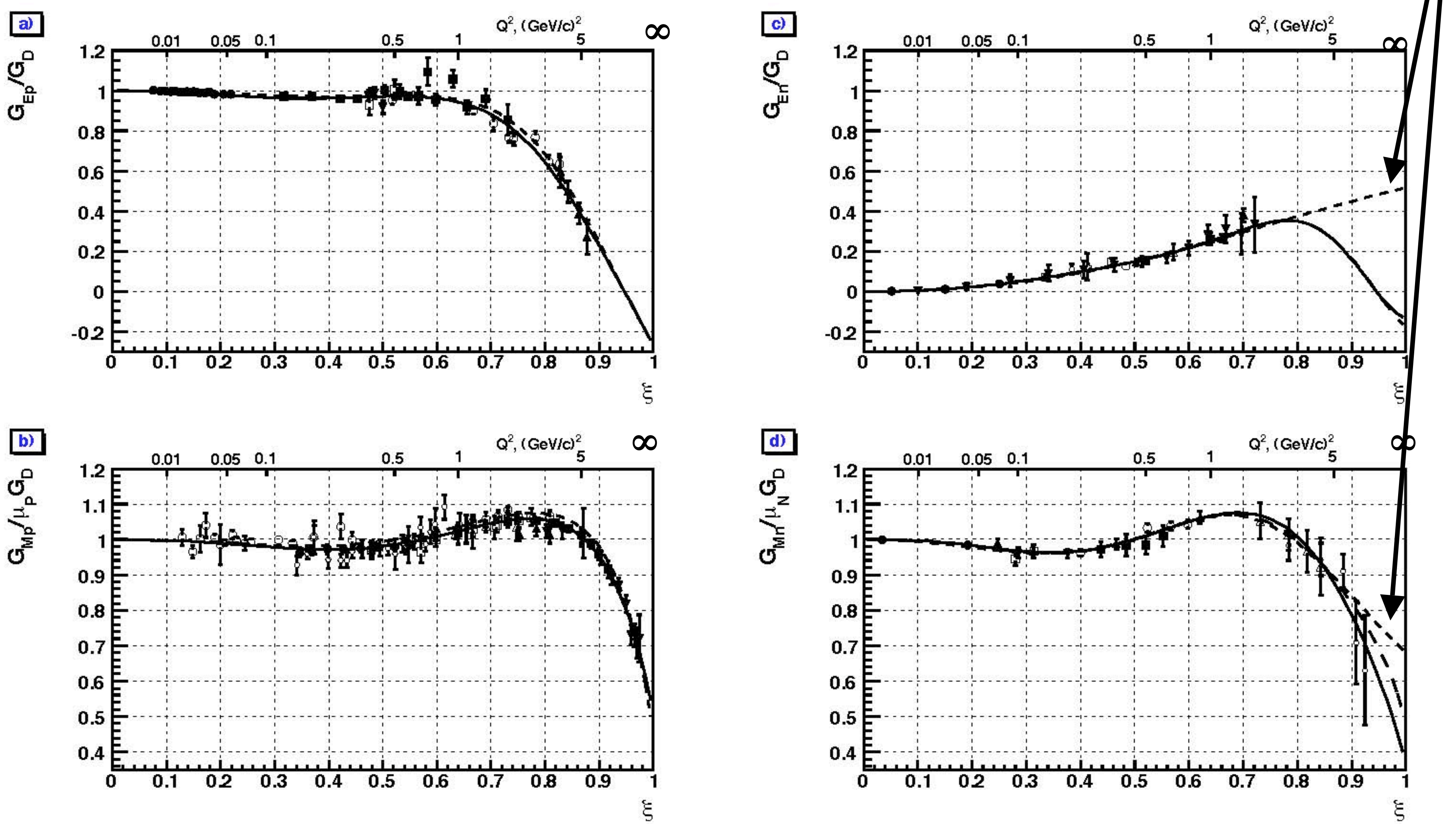
BBA2007-All form Factor fits - ratio to Dipole

Kelly 2004



$$\tau = Q^2/4M_N^2.$$

$$, \xi = \frac{2}{(1+\sqrt{1+1/\tau})}$$

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1+\frac{Q^2}{M_{V,A}^2}\right)^2}.$$

Here $C^{V,A} = (1, g_A)$, $g_A = -1.267$, $M_V^2 = 0.71\ (GeV/c)^2$,

BBBA2007 (Bodek, Budd, Bradford, Avvakumov 2007)

Start with Functional form similar to that used by J. Kelly for Gep and Gmp only (satisfies correct power behavior at high Q^2)

Our fits to the form factors are:

$$G_{Mp}(Q^2)/\mu_p = A_{Mp}(\xi) \times G_{Mp}^{Kelly}(Q^2)$$
$$G_{Ep}(Q^2) = A_{Ep}(\xi) \times G_{Ep}^{Kelly}(Q^2)$$
$$G_{Mn}(Q^2)/\mu_n = A_{Mn}^{a,b}(\xi) \times G_{Mp}(Q^2)/\mu_p$$
$$G_{En}(Q^2) = A_{En}^{a,b}(\xi) \times G_{Ep}(Q^2) \times \left(\frac{a\tau}{1+b\tau}\right),$$

$$G^{Kelly}(Q^2) = \frac{\sum_{k=0}^{m} a_k \tau^k}{1+\sum_{k=1}^{m+2} b_k \tau^k},$$

Lagrange modulating function

$$A_N(\xi) = \sum_{j=1}^{n} P_j(\xi)$$
$$P_j(\xi) = p_j \prod_{k=1,k\neq j}^{n} \frac{\xi-\xi_k}{\xi_j-\xi_k}.$$

Each P_j is a LaGrange polynomial in the Nachtman variable, $\xi = \frac{2}{(1+\sqrt{1+1/\tau})}$. The ξ_j are equidistant "nodes" on an interval [0, 1] and p_j are the fit parameters that have an additional property $A_N(\xi_j) = p_j$. The functional form $A_N(\xi)$ (for G_{Ep}, G_{Mp}, G_{En}, G_{Mn}) is used with seven p_j parameters at ξ_j=0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1.0. In

1. We update the Kelly 2004 parameters for Gep and Gmp

2. The Lagrange modulations are small for Gep and Gmp and include Arrington-Sick corrections for two photon and Coulomb field

3. Gen and Gmn are expressed in terms of Gep and Gmp using duality contrainets

Updated Kelly form parameters for Gep and Gmp

	a_1	b_1	b_2	b_3	χ^2/ndf
G_{Ep}^{Kelly}	−0.24	10.98	12.82	21.97	0.78
G_{Mp}^{Kelly}	0.17195	11.2595	19.3219	8.33346	1.03

TABLE I: Parameters for G_{Ep}^{Kelly} and G_{Mp}^{Kelly}. Our parameterization employs the as-published Kelly parameterization to G_{Ep}^{Kelly} and an updated set of parameters for $G_{MP}^{Kelly}(Q^2)$ that includes the recent BLAST[8] results.

Lagrange modulating function

Gep,Gmp Ratio to updated Kelly

Gmn,Gen parameters

Gep,Gmpl ratio to Dipole for convenience

FA ratio to dipole axial parameters

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
ξ, Q^2	0, 0	0.167, 0.029	0.333, 0.147	0.500, 0.440	0.667, 1.174,	0.833, 3.668	1.0, ∞
A_{Ep}	1.	0.992707	0.989825	0.997507	0.981319	0.934137	1.
A_{Mp}	1.	1.001060	0.999111	0.997339	1.000996	1.000214	1.
$A_{Ep-dipole}$	1.	0.983874	0.963178	0.974797	0.913645	0.544722	−0.26820
$A_{Mp-dipole}$	1.	0.991586	0.977073	0.980147	1.032083	1.042908	0.508400
A_{Mn}^{25}	1.	0.995531	0.986748	1.017259	1.034998	0.911895	0.729953
A_{Mn}^{43}	1.	0.995911	0.985066	1.018644	1.030693	0.907969	0.955653
A_{En}^{25}	1.	1.101871	1.137845	1.019028	1.103693	1.522403	0.970600
A_{En}^{43}	1.	1.101871	1.137338	1.022130	1.098976	1.518870	1.270800
$A_{FA}^{25-dipole}$	1.0000	0.913266	0.995466	1.104324	1.175318	1.391203	0.744317

TABLE II: Fit parameters for $A_N(\xi)$, the LaGrange portion of the new parameterization. Note A_{Mn}^{25}, A_{En}^{25}, and A_{FA}^{25} are constrained to have $\frac{d}{u} = 0$ at $\xi = 1$, and A_{Mn}^{43}, A_{En}^{43}, are constrained to have $\frac{d}{u} = 0.2$.

BBBA2007…Axial

New Axial Form Factor (F_A)

- We perform new extractions of. M_A and F_A from previous neutrino Deuterium data, using the updated vector form factors, and updated constants.

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1+\frac{Q^2}{M_{V,A}^2}\right)^2}.$$

Here $C^{V,A}$= $(1,g_A)$, g_A = -1.267, M_V^2 = 0.71 $(GeV/c)^2$, and M_A = 1.015 GeV/c (as discussed below).

g_A	-1.267
G_F	1.1803×10^{-5} GeV^{-2}
$\cos\theta_c$	0.9740
μ_p	2.793 μ_N
μ_n	-1.913 μ_N
ξ	3.706 μ_N
M_V^2	0.71 GeV^2

Table 1
The most recent values of the parameters used in our calculations (Unless stated otherwise).

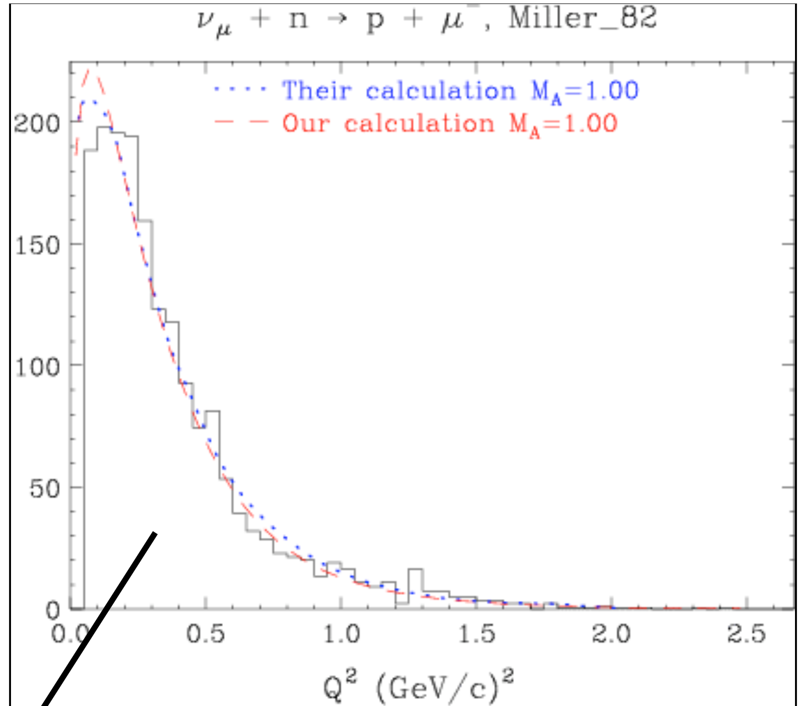
Experiment	M_A (published)	ΔM_A new–old
$Miller-D-ANL_{82,77,73}$	1.00 ± 0.05	-0.035
$Baker-D-BNL_{81}$	1.07 ± 0.06	-0.032
$Kitagaki-D-FNAL_{83}$	$1.05^{+0.12}_{-0.16}$	-0.024
$Kitagaki-D-BNL_{90}$	$1.070^{+0.040}_{-0.045}$	-0.039

We find new world average (Neutrino D2 data And pion electro-production data) for M_A

average values is 1.0155 ± 0.0136.

Miller 1982- ANL deuterium

- Miller is an updated version of Barish with 3 times the data
- They used Ga=-1.23 and Ollson Form factors
- 0.035 GeV should be subtracted from their fit value for modern form factors and Ga=-1.267



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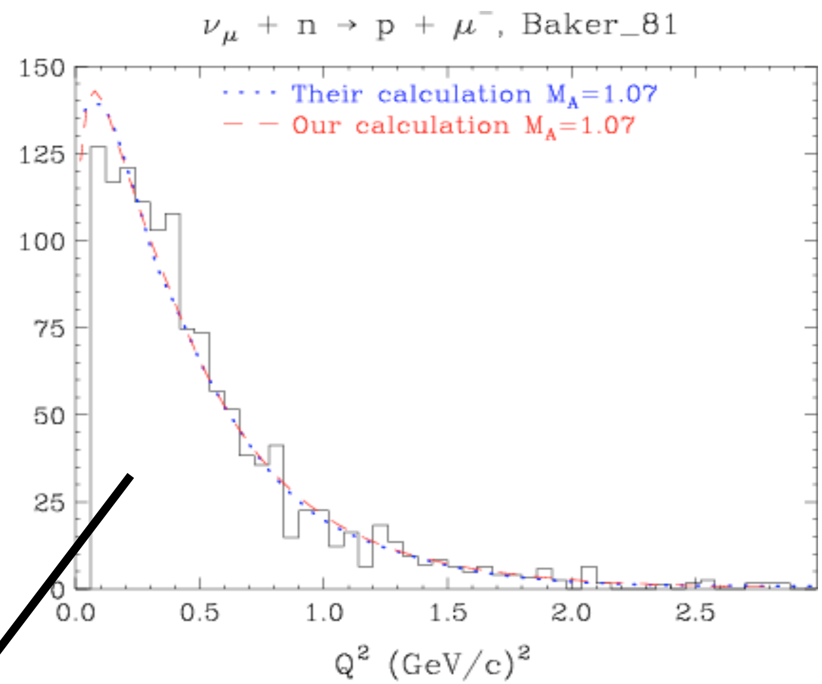
Determining m_A , Baker et al. – 1981 BNL deuterium

- The dotted curve shows their calculation using their fit value of 1.07 GeV
- Baker used Ga=-1.23 and Ollson Form factors
- 0.032 should be subtracted from their fit value for modern form factors and Ga=-1.267



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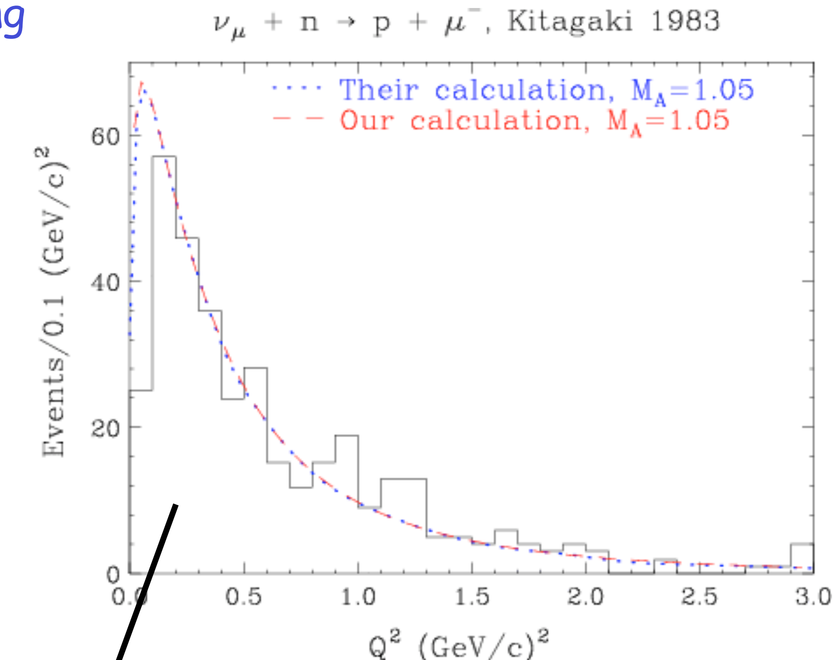
Kitagaki et al. 1983 FNAL deuterium

- The dotted curve shows their calculation using their fit value of M_A=1.05 GeV
- They used Ga=-1.23 and Ollson Form factors
- 0.024 should be subtracted from their fit value for modern form factors and Ga=-1.267



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Last Kitagaki BNL D2 1990

They used Ollson, Mv=0.84 and Ga=-1.254. We get that Ma should be corrected by -0.039 GeV

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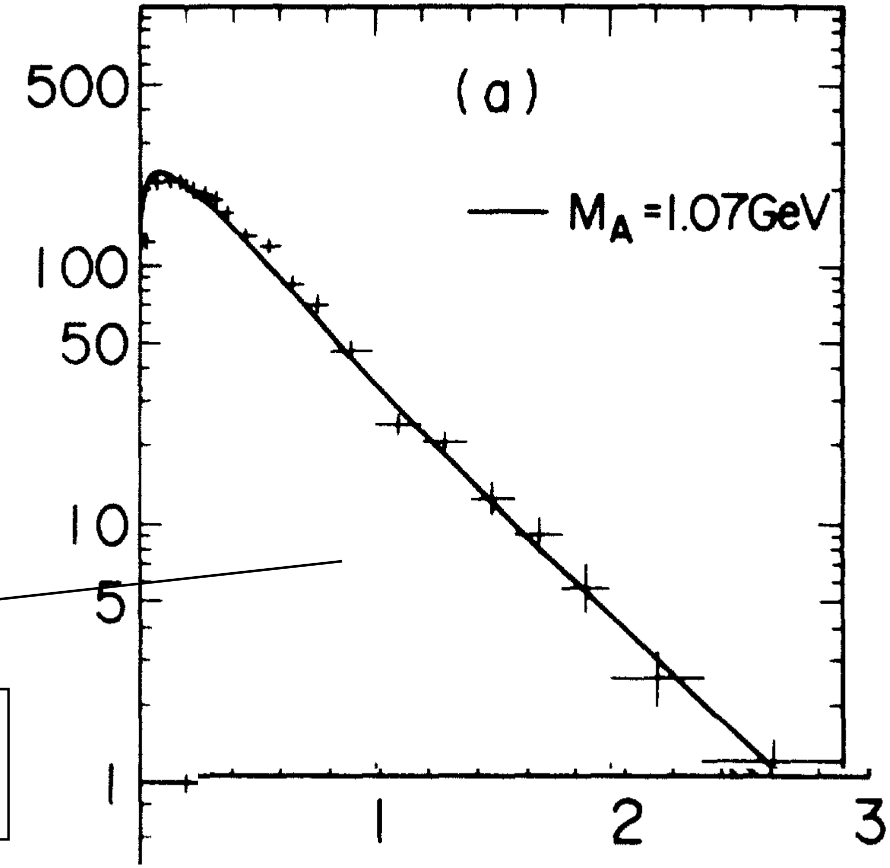

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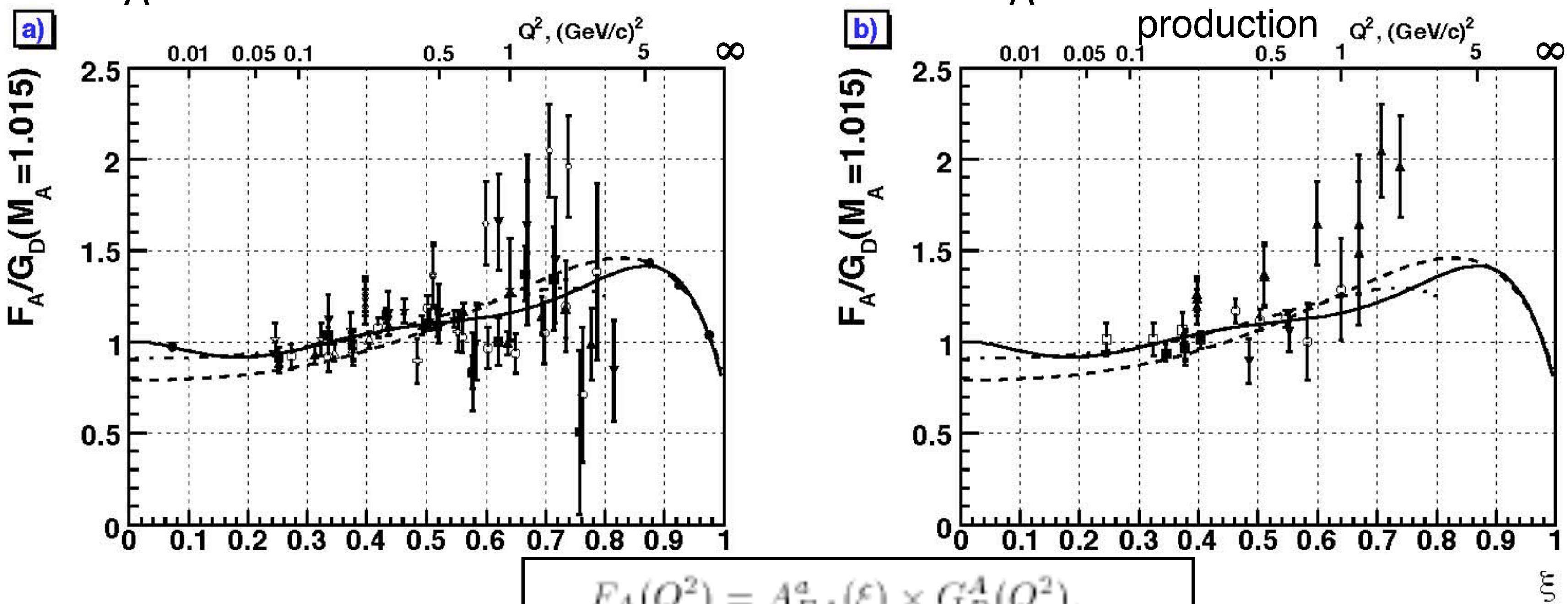
Next-> we look for deviations from the Dipole form - It is not Perfect for Vector, why should it be correct for axial

F_A/Dipole from neutrino data

F_A/Dipole from pion electro-production



$$F_A(Q^2) = A^a_{FA}(\xi) \times G^A_D(Q^2).$$

FIG. 3: (a) $F_A(Q^2)$ re-extracted from neutrino-deuterium data divided by $G^A_D(Q^2)$ (with M_A = 1.015). (b) $F_A(Q^2)$ from pion electroproduction divided by $G^A_D(Q^2)$ (corrected for for hadronic effects[12]). The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2=V2}$. The dashed-dot line is a prediction from a constituent quark model. The values of ξ and the corresponding values of Q^2 are shown on the bottom and top axis.

Fit F_A- and constrain that high Q^2, Vector=Axial as expected from duality

$$[F_A(Q^2)_{A2=V2}]^2 = \frac{(G_E^V)^2(Q^2) + \tau(G_M^V(Q^2))^2}{(1+\tau)},$$

Conclusions

With our BBBA 2007 vector form factor parameterization, our new extractions of Ma from neutrino data, and our fits to the updated values of $F_{A:}$ --> form factor uncertainties are no longer an issue in the modeling of quasi-elastic neutrino interactions.

The current uncertainties in the quasielastic cross section lie in the realm of nuclear effects.

These nuclear effects will be measured in the next generation neutrino experiment at Fermilab MINERvA

At high Q^2, our new duality based predictions for F_A and Gen can be tested in MINERvA at Fermilab and the next generation Gen electron scattering experiments at Jlab.