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Status of $\Delta\alpha_{\rm had}$ and $g-2$

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I. Motivation/Introduction

II. Input for dispersion integrals: $\sigma^0_{\rm had}$

III. Recent developments in $g-2$

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For details and Refs: Hagiwara+Martin+Nomura+T: PRD 69(2004)093003; PLB 649(2007)173; + to appear soon

I. Introduction/Motivation

- **Why $g-2$?** $\quad a_\mu = (g-2)/2 = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}} + a_\mu^{\text{BSM}}$
  - Testing QFT at highest precision; all sectors of the SM relevant.
  - Discrepancy $(a_\mu^{\text{exp}} - a_\mu^{SM})$ — if clearly established — signals existence of BSM.

- **Why $\Delta\alpha_{\text{had}}$?** $\quad \alpha(q^2) = \alpha \,/\, \left(1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2)\right)$

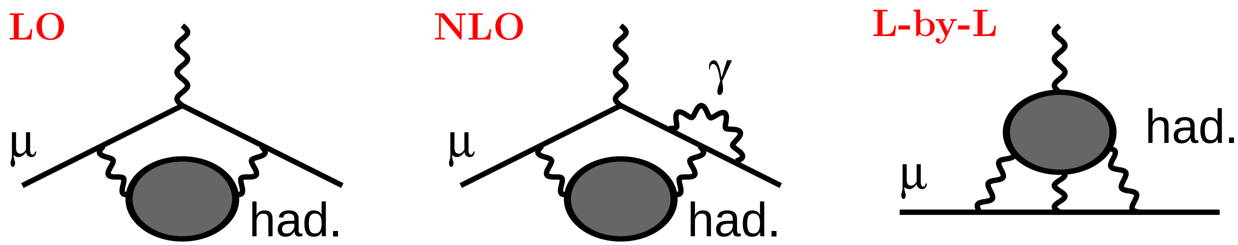
  Precise $\alpha(q^2)$ needed for:
  - Corrections for data used as input for $g-2$.
  - Determination of α_s and quark masses from total hadronic cross section R_{had} at low energies and of resonance parameters.
  - Ingredient in MC generators for many processes.

  $\longrightarrow$ $\boldsymbol{\alpha(M_Z^2)}$ a fundamental parameter at the Z scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits.

▶ Uncertainties in running α_{QED} and $g-2$ totally dominated by hadronic contributions.

II. Input for dispersion integrals: σ^0_{had}

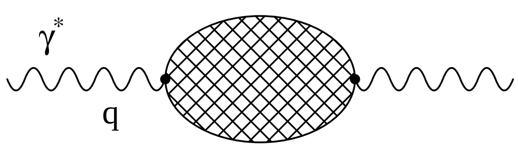
- $a_\mu^{\mathrm{had}} \quad = \quad a_\mu^{\mathrm{had,LO}} \quad + \quad a_\mu^{\mathrm{had,NLO}} \quad + \quad a_\mu^{\mathrm{had,Light-by-Light}}$



▶ Hadronic contributions from low virtualities s not calculable with perturbative QCD

▶ rely instead on *dispersion relations*, using experimental data for $\sigma^0_{\mathrm{had}}(s)$:

$$a_\mu^{\mathrm{had,LO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^\infty \mathrm{d}s\, \sigma^0_{\mathrm{had}}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s} \cdot (0.63 \ldots 1)$$

- 

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) \; = \; -\frac{q^2}{4\pi^2\alpha}\, P \int_{m_\pi^2}^\infty \frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s-q^2}$$

▶ Weighting of K extremely towards small s, less so for $\Delta\alpha$.

▶ σ^0 means without running $\boldsymbol{\alpha}$ ⇝ iteration needed.

→ Data input for $\sigma^0_{\rm had}(s)$ from the experiment CMD-2 at Novosibirsk:

(They provide the most precise $e^+e^- \to \pi^+\pi^-$ data with only 0.6% sys. error)

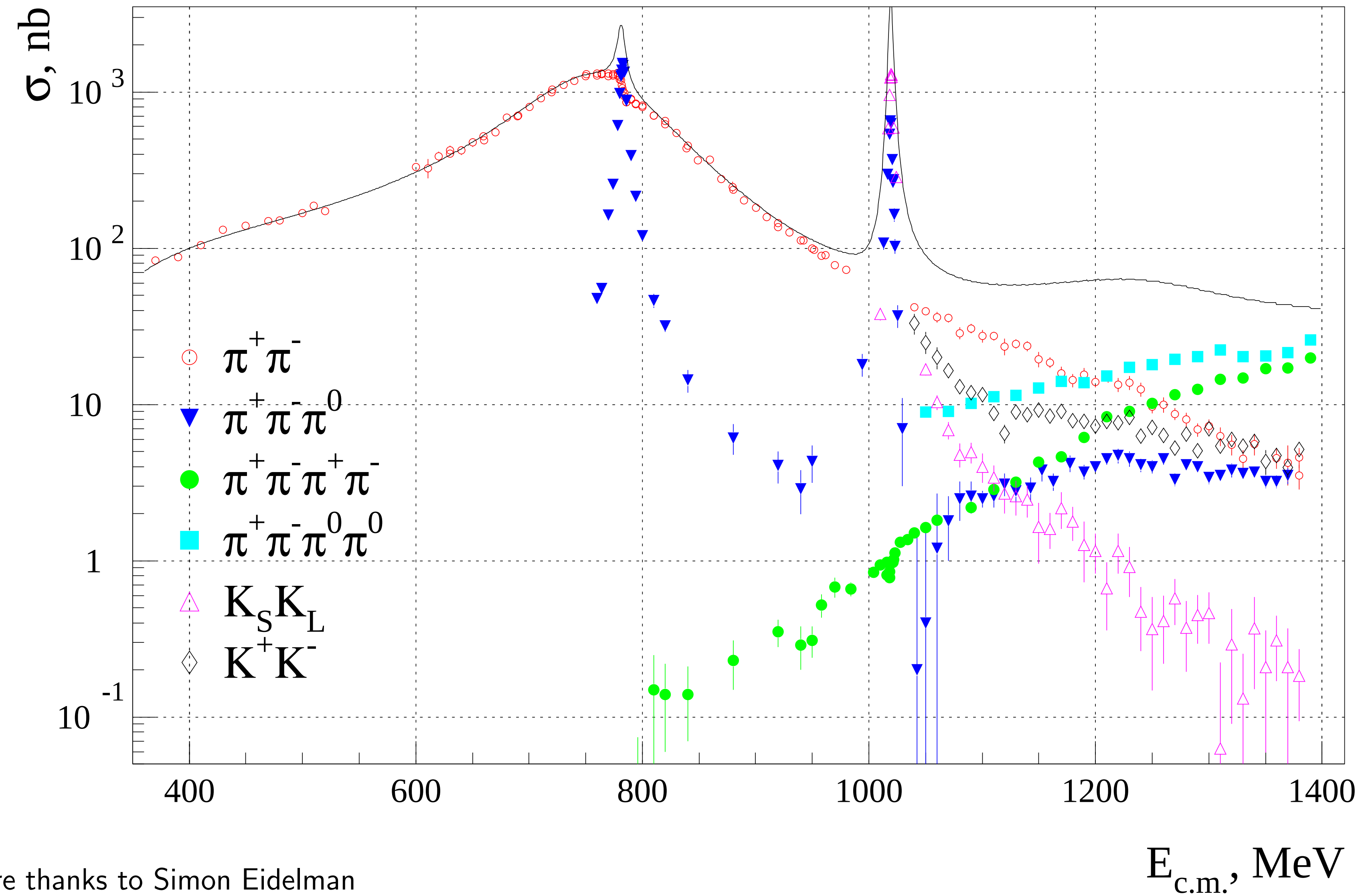


Figure thanks to Simon Eidelman

How to get the hadronic vac.-pol. contributions with precision:

- Lowest energies very important, i.e. the hadronic channels 2π, 3π, KK, 4π, 5π, etc. Have to sum ~ 24 exclusive channels and inclusive data for $\sqrt{s}$ above $1.43 - 2$ GeV to get total $\sigma_{\rm had}$ with high precision.

  $\longrightarrow$ Use of *state-of-the-art* perturbative QCD only above 11.09 GeV.

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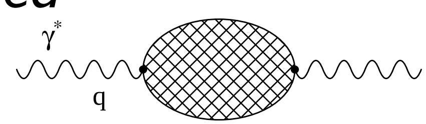
- Before averaging and $\sum$: Check Radiative Corrections of each data set:

$\longrightarrow$ Additional final state photons must be fully *included/estimated*

$\longrightarrow$ For σ^0, running coupling $\alpha(q^2)$ effects must be *subtracted*

(otherwise double-counting with $a_\mu^{\rm had,NLO}$/resum.)



$\longrightarrow$ but effects can cancel in $\sigma_{\rm had}/\sigma_{\rm norm}$, and corrections often done already partly... MANY COMPLICATIONS

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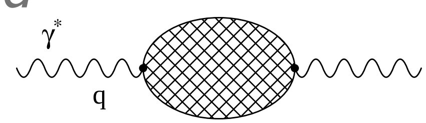


$\rightarrow$ but effects can cancel in $\sigma_{\rm had}/\sigma_{\rm norm}$, and corrections often done already partly.

$\longrightarrow$ PRECISION ONLY FROM TH + MC + EXP

→ Important detail: Use of time-like running of $\alpha(s)$:

$$\alpha(s) = \alpha \,/ \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\right)$$

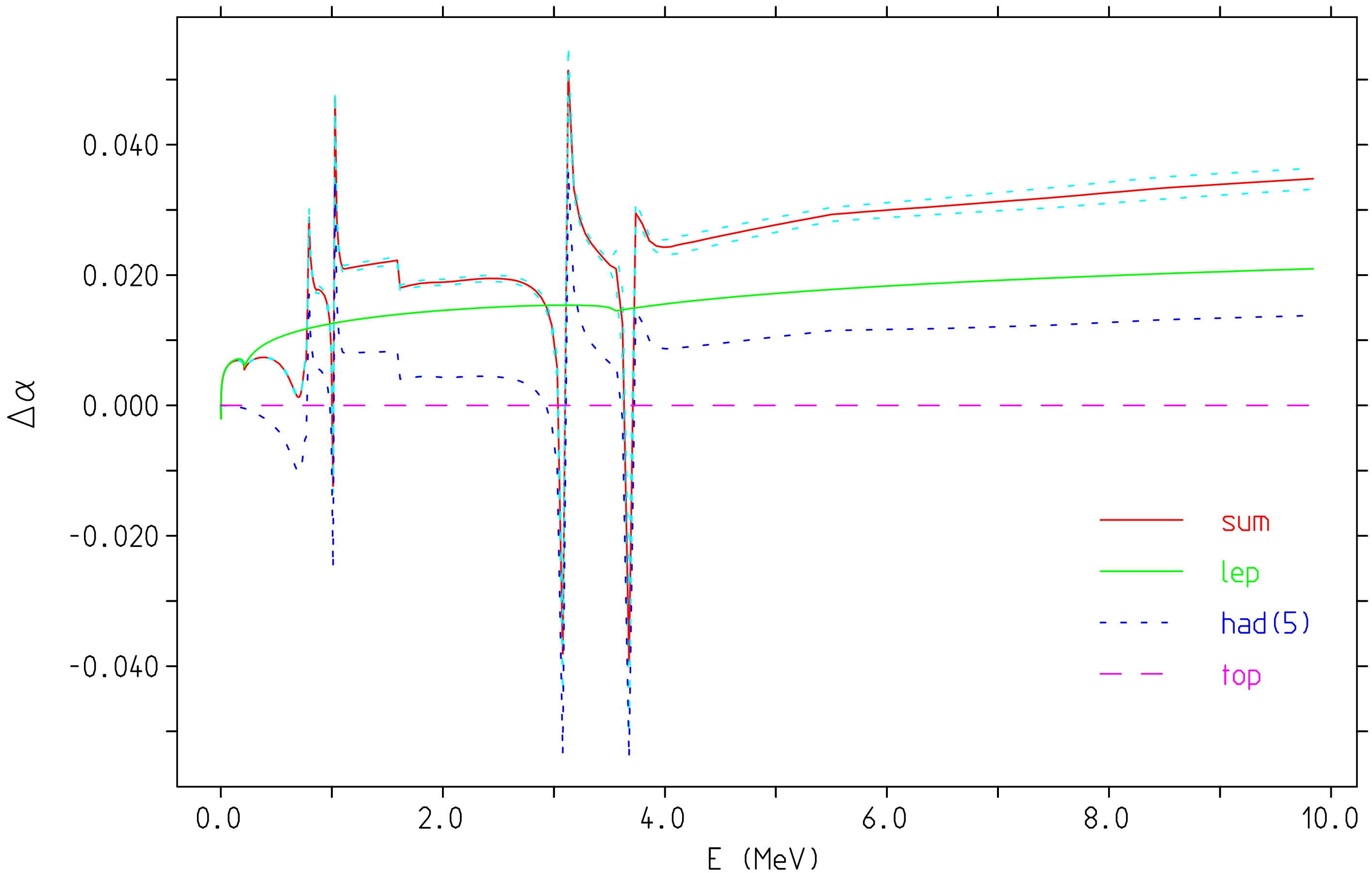


Figure from F. Jegerlehner

→ In total these radiative corrections lead to an additional uncertainty of

$$\delta a_\mu^{\rm had,VP+FSR} \simeq 1.8 \times 10^{-10} \quad [\sim\ 10 \cdot \Delta a_\mu^{\rm EW}\ ]$$

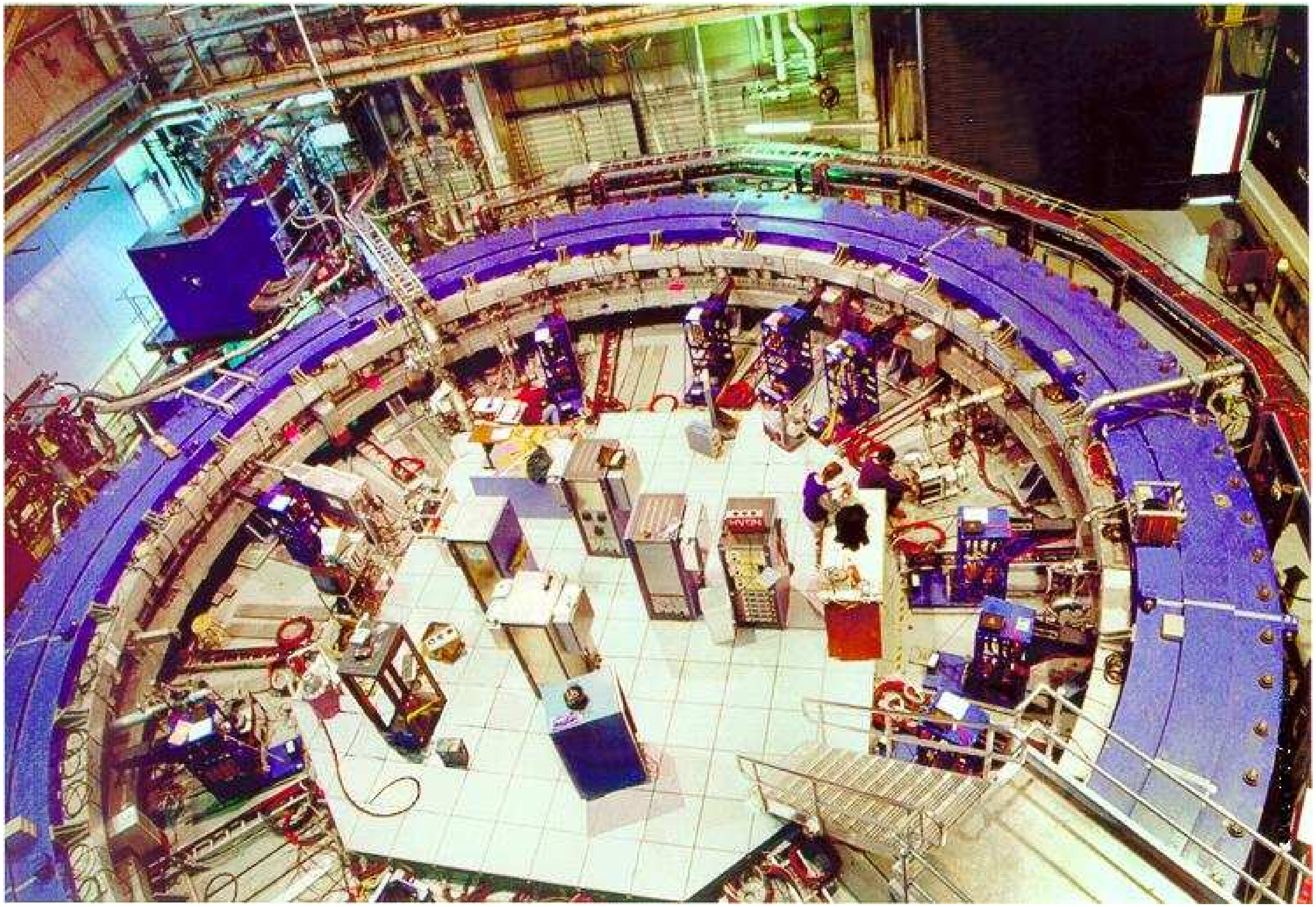
III. $g-2$: recent developments

Contributions numerically

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\,584\,718.09 \pm 0.16$	up to 5-loop! (Kinoshita+Nio, Passera)
	(was $116\,584\,719.35 \pm 1.43$)	▶ incl. recent updates of α
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		(agrees very well with Knecht+Peris+Perrottet+deRafael)
LO hadr.	$7110 \pm 50 \pm 8 \pm 28$	Davier+Eidelman+Hoecker+Zhang '03b (τ)
	$6963 \pm 62 \pm 36$	Davier+Eidelman+Hoecker+Zhang '03b (e^+e^-)
	$\mathbf{6924 \pm 59 \pm 24}$	Hagiwara+Martin+Nomura+T 03
new data:	$\mathbf{6894 \pm 42 \pm 18}$	HMNT06, incl. new CMD-2, SND, KLOE data
NLO hadr.	$\mathbf{-97.9 \pm 0.9}$	HMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	136 ± 25	▶ Melnikov+Vainshtein
< Dec. 2003:	80 ± 40	compilation from Nyffeler, `hep-ph/0203243`
		$\sim$ agrees (num.) w. Bijnens+Pallante+Prades
		and Hayakawa+Kinoshita *after*
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$\mathbf{116591804 \pm 51}$	with HMNT06 (e^+e^-)

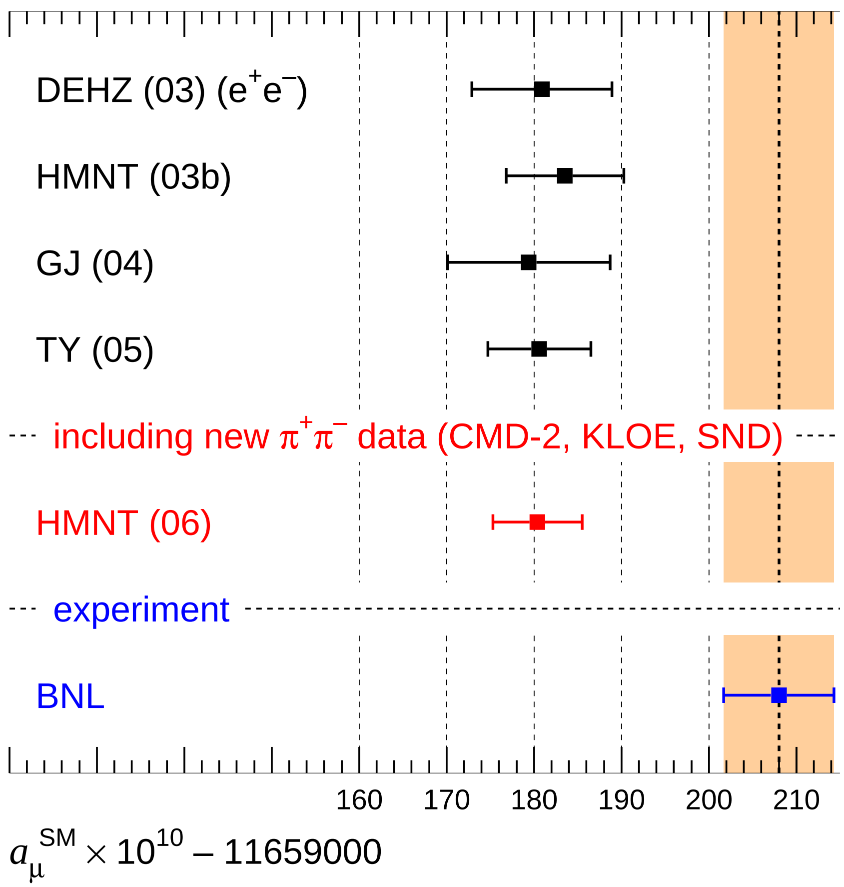
SM vs. BNL: A sign of Physics beyond the SM?

The experiment E821 at Brookhaven (Picture of the storage ring with three scientists)



For the first time TH is (slightly) more precise than EXP:

$a_\mu^{\rm SM}$ compared to BNL 04 world av.



DEHZ 06: 180.5 ± 5.6 $[3.3\,\sigma]$

Jegerlehner 06: 179.3 ± 6.8 $[3.2\,\sigma]$

Recent changes

TH: Update of QED, up to 5–loop, new α:

was: $(116\ 584\ 719.35 \pm 1.43) \cdot 10^{-11}$

$\longrightarrow$ is now: $(116\ 584\ 718.09 \pm 0.16) \cdot 10^{-11}$

TH: Improved LO hadr. (from e^+e^-):

Now, with new CMD-2, SND, KLOE:

$(6924 \pm 64) \cdot 10^{-11} \longrightarrow (6894 \pm 46) \cdot 10^{-11}$

EXP: BNL's '01 μ^- data [PRL92(2004)161802]:

$a_{\mu^-} = 11\ 659\ 214(8)(3) \times 10^{-10}\ (0.7ppm)$

$\longrightarrow a_\mu = 116\ 592\ 080(63) \times 10^{-11}\ (0.5ppm)$

▶ With this input we (HMNT) get:

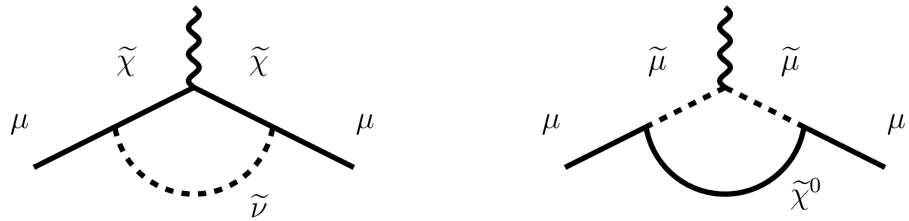
$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (27.6 \pm 8.1) \cdot 10^{-10}$, $\sim \mathbf{3.4\sigma}$

.. Discrepancy increased .. still not fully conclusive .. constrain SUSY ..

SUSY contributions in a_μ ?

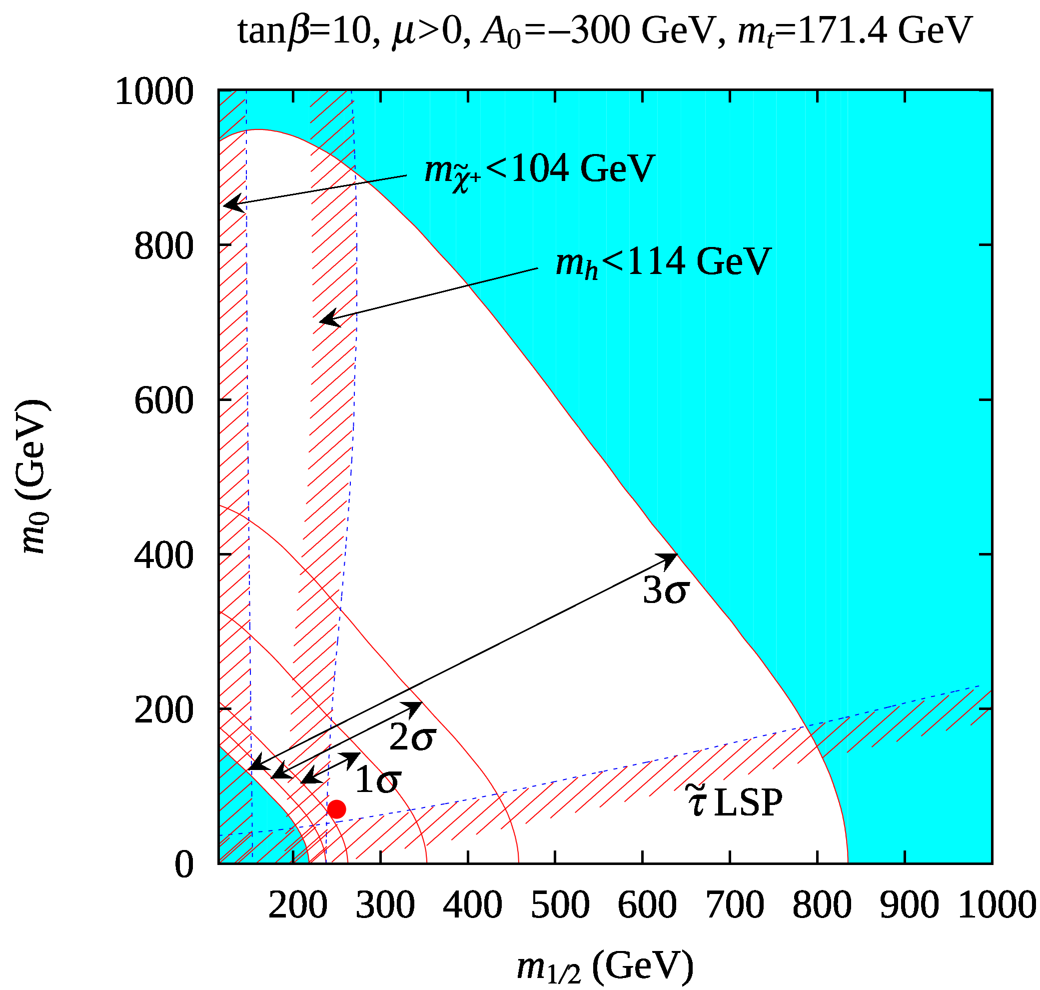
$$a_\mu^{\mathrm{SUSY,1-loop}} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \tan\beta \; \mathrm{sign}(\mu) \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

They mainly come from:



SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$, but

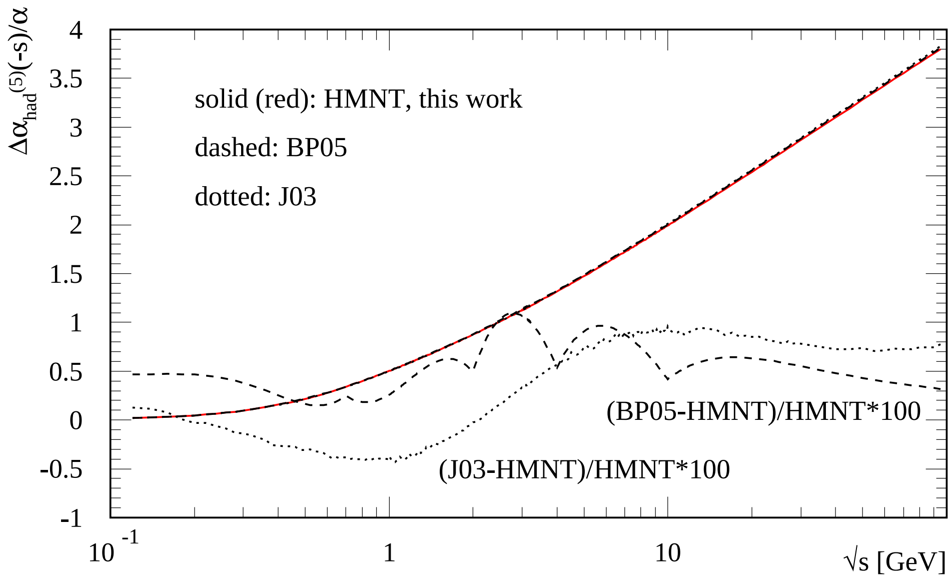
- no chargino at LEP
- so far no light Higgs
- $\tilde{\tau}$ prob. not LSP
- $+$ limits from direct searches
- SPS 1a′ in 1σ band from $g-2$



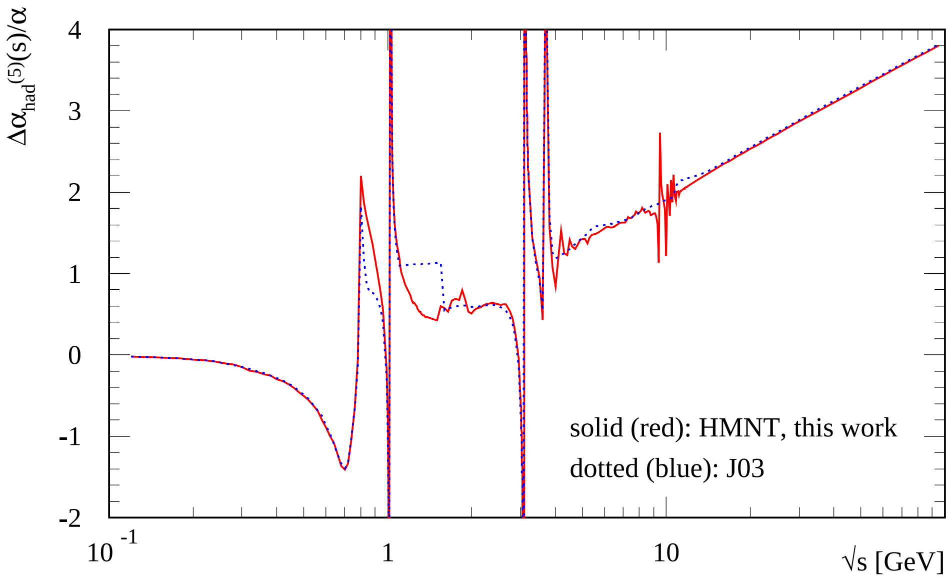
IV. $\alpha_{\rm QED}(q^2)$ at low and high energies

MNHT's evaluation of $\alpha_{\rm QED}(q^2)$ compared to other parametrizations: HMNT avail. soon

Spacelike (smooth $\alpha(q^2 < 0)$)



Timelike ($\alpha(q^2 > 0)$ follows resonance structure)



- Differences between parametrizations significant
- Slight shift in $\alpha(M_Z^2)$, see below
- What is in the MCs used by the experiments?
- $g-2$ and EW precision fits need better control / smaller error

What about $\Delta\alpha(M_Z^2)$?

- With the same compil. of $\sigma_{\rm had}$ as for $g-2$ we find:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02768 \pm 0.00022$$

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$ (HMNT '06)

Other compilations:

Group	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '06	0.02761 ± 0.00023	data driven/pQCD
$(s_0^2 = (10\mathrm{GeV})^2)$	0.02759 ± 0.00017	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

Adler function: $D(-s) \;=\; \dfrac{3\pi}{\alpha} s \dfrac{\mathrm{d}}{\mathrm{d}s}\Delta\alpha(s) \;=\; -(12\pi^2) s \dfrac{\mathrm{d}\Pi(s)}{\mathrm{d}s}$

allows use of pQCD and minimizes dependence on data.

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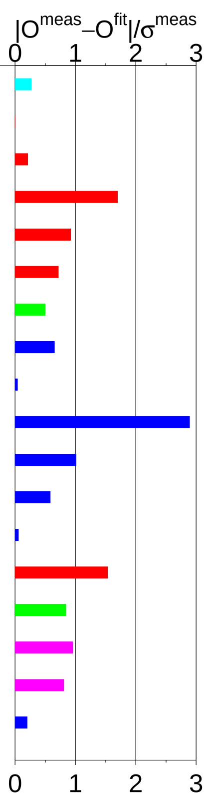
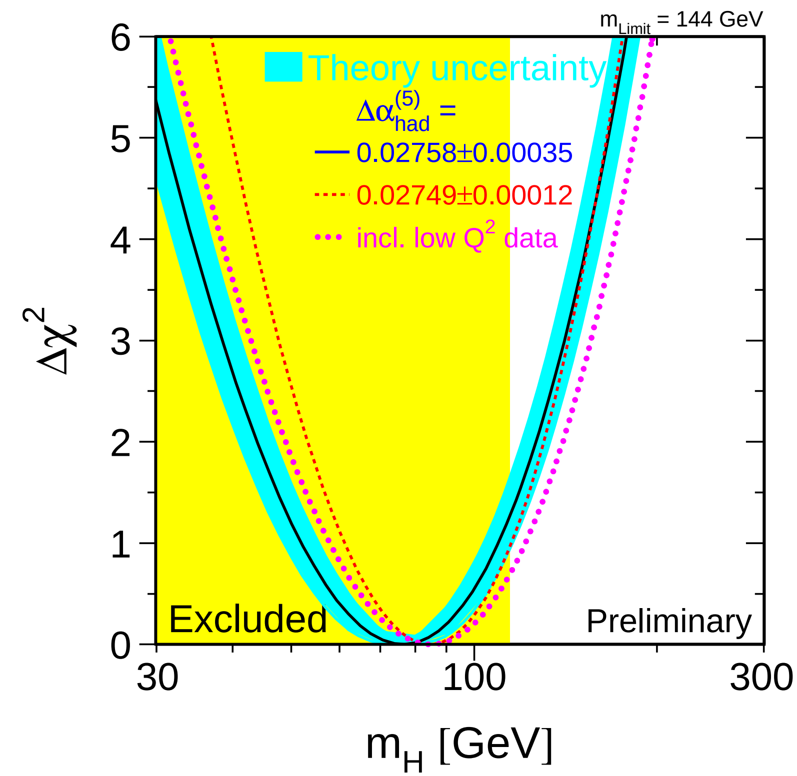
LEP EWWG 07:

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3



Fit of the SM Higgs mass: EWWG 07



$\longrightarrow$ preferred m_H moves down w. higher $\Delta\alpha$

$\longrightarrow$ lower error lowers excl. limit

V. The next round

Where is improvement needed most urgently?

Pie diagrams of contributions to a_μ and $\alpha(M_Z^2)$ and their errors^2:

Critical regions:

$\rightarrow a_\mu$:

low(est) energy regime,

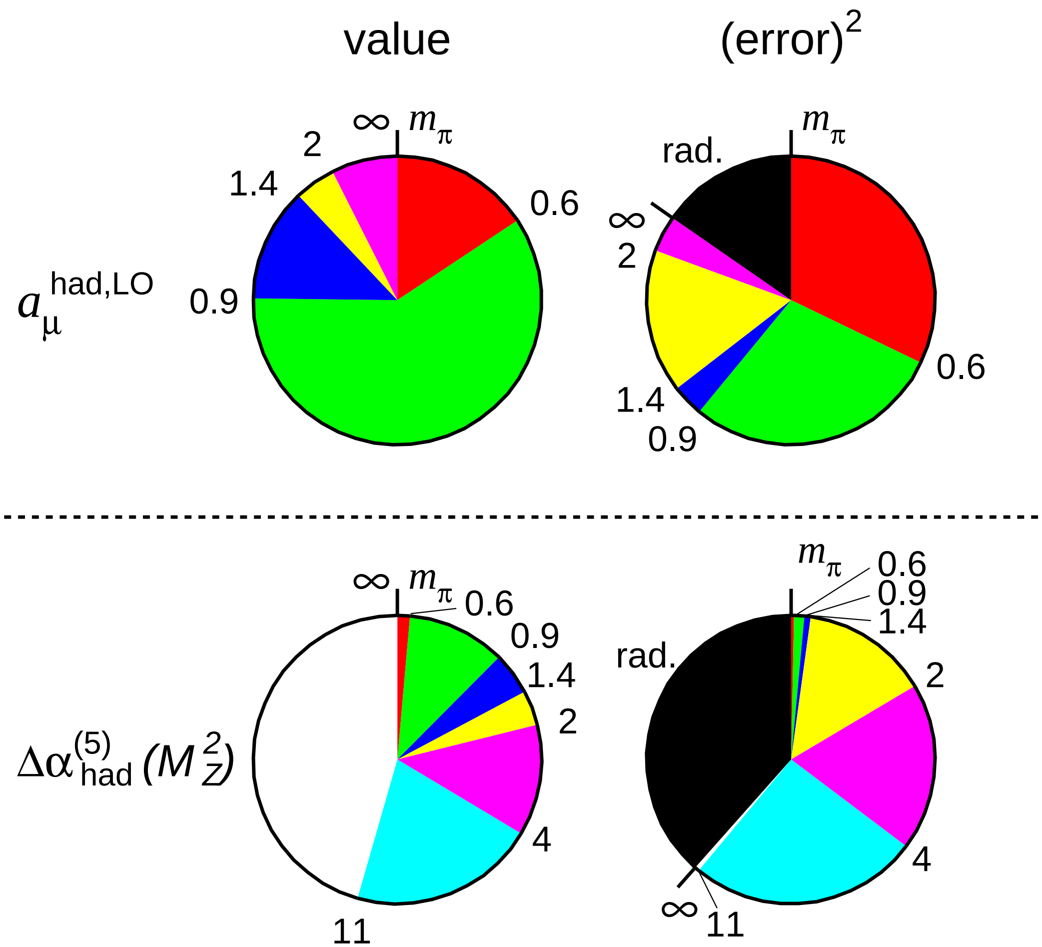
region below 2 GeV

$\rightarrow \alpha(M_Z^2)$:

again below 2 GeV,

$+$ intermed./large energies,

w. *better* radcors!



Summary/Outlook:

- **$g-2$**: at present a **$3.4\,\sigma$** deviation persists, possibly solved by SUSY.
  - At BNL: proposed upgrade of E821, E969, designed to achieve 0.2ppm.
  - J-PARC, a new high intensity proton accelerator under construction near KEK could host a radically new $g-2$ exp. Improvement by a factor 5-10 possible.

$\longrightarrow$ For further improvements, both $g-2$ and $\Delta\alpha$ require more accurate data and TH!

The prospects are good:

- Further Radiative Return analyses from KLOE, Frascati are in progress/reported; will check $\pi\pi$ down to the threshold and hopefully squeeze the error.
- BaBar is very successful with Rad. Ret. for higher multiplicity final states. Opportunities for BELLE?
- Even better prospects (factor 2–3 possible) with VEPP2000, possibly DANAE/KLOE-2.
- At higher energies, analyses from CLEO-III at Cornell and BES-II at BEPC in Beijing progressing; soon BEPCII with BES-III will start.