Dalitz analyses of three body D^+ decays

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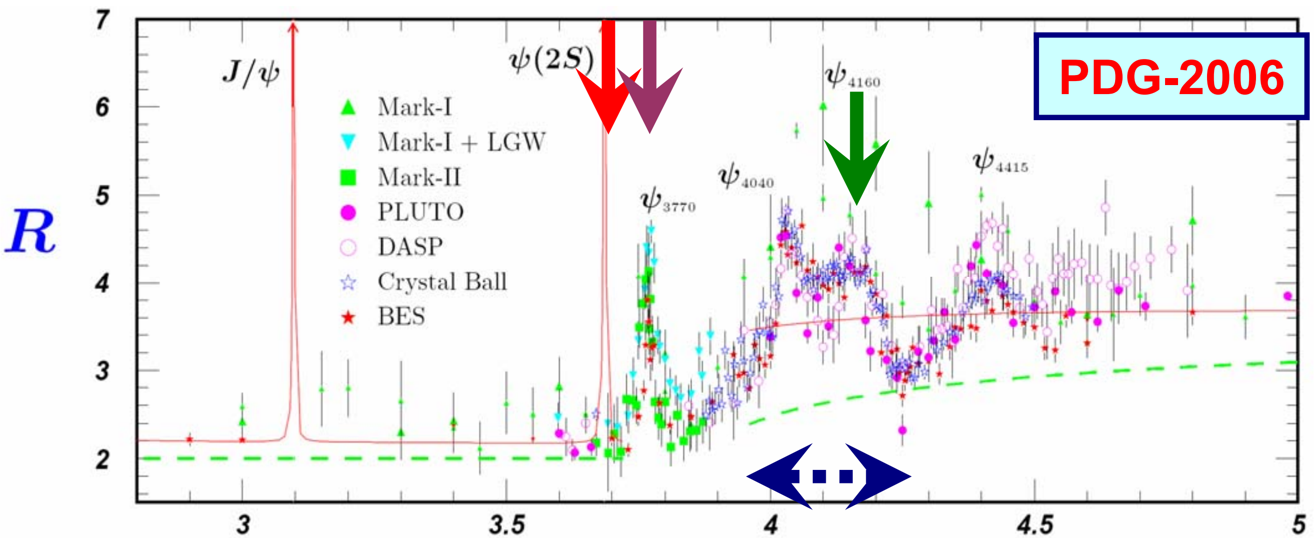
CLEO-c

- CLEO-c is a Charm Factory that is now in the final year of data taking with a scheduled shutdown date of April 1st, 2008. The main focus has been, and is, on precision measurements of D and D_s decays and the

Ψ(2S) 27 million events

Ψ(3770) 575 pb^{-1} ($\rightarrow$ 800pb^{-1}(estimate)) just above $D\bar{D}$ threshold

(4170) 314 pb^{-1} ($\rightarrow$ 630pb^{-1}(estimate)) maximize $D_s\bar{D}_s$ production



Outline

I will present CLEO-c results on Dalitz analysis of

- **$D^+ \rightarrow \pi^- \pi^+ \pi^+$ using $281 pb^{-1}$** PRD 76, 012001 (2007)
- **$D^+ \rightarrow K^- \pi^+ \pi^+$ using $281 pb^{-1}$** (Conference paper)

- **Current and future analyses**

Recent Review by Gianluca Cavoto
Charm Dalitz Analyses FPCP (2007) arXiv:0707,1242v1 [hep-ex]

Dalitz analysis of $D^+ \rightarrow \pi^- \pi^+ \pi^+$

Previous results from fixed target experiments

E791 1172±61 events PRL 86,770 (2001) hep-ex/0007028
Uses the isobar technique where each resonant contribution is modeled as a Breit Wigner amplitude with a complex phase.

FOCUS 1527±51 events PLB 585, 200 (2004) hep-ex/0312040
Uses the K matrix formalism and poles which provides a description of the s wave ππ resonances the σ or $f_0(600)$ and the $f_0(980)$

Results in this talk
CLEO-c ~2600 events PRD 76, 012001 (2007) arXiv:0704.3954

Analysis of $D^+ \rightarrow \pi^- \pi^+ \pi^+$

- **Possible contributions**

  $\rho(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1370)$, $f_0(1500)$, $\rho(1450)$
  plus $\sigma(600)$, $f_0(1710)$, $f_0(1790)$ and s wave I = 2 and NR

Three models

- **Isobar model with the σ and the Flatte parameterization for the threshold effects on the $f_0(980)$**

- **S wave description by Schechter (for the σ and $f_0(980)$.**

  (PRD 64, 014031 (2001) and Int.J.Mod.Phys A20,6149 (2005))

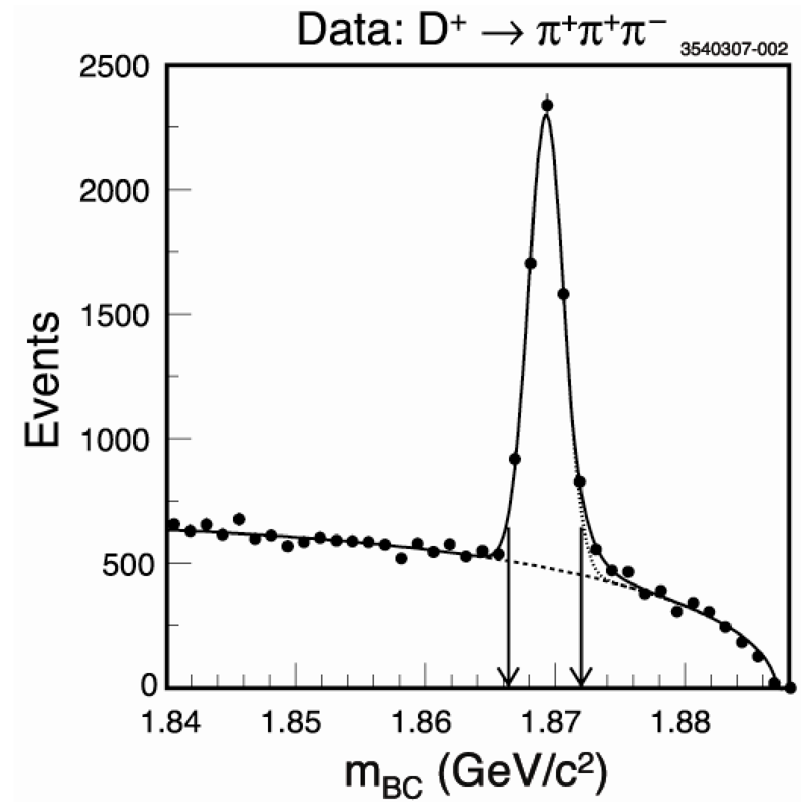
- **Model of Achasov et al**
- (PRD 67, 114018 (2003), Sov.J.Nucl.Phys 32,566(1980), PRD.D55,2663(1997), PRD70,111901 (2004), PRD 67,114018(2003), PRD 73,054029(2006)

Dalitz analysis of $D^+ \to \pi^- \pi^+ \pi^+$

$$M_{BC} = \sqrt{E_{\text{beam}}^2 - |p(D)|^2}$$

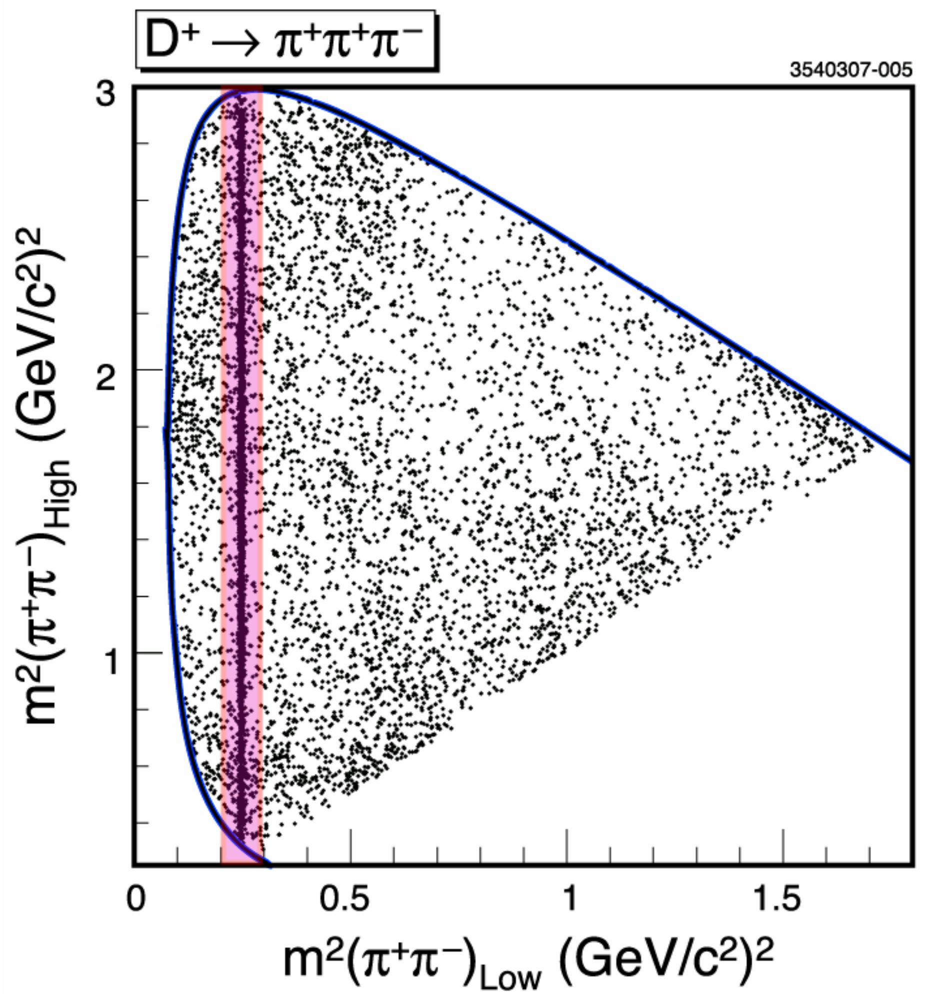
$$\Delta E = E(D) - E_{\text{beam}}$$

Since we are just above threshold for $D\bar{D}$ production each D is produced with the beam energy so we use the beam constrained mass and ΔE to select the events



6991 signal region
2159 background
2239 K^0_s decays
~2600 $D^+ \to \pi^- \pi^+ \pi^+$

$D^+ \to \pi^- \pi^+ \pi^+$ Dalitz plot



The Dalitz plot shown has been folded because of the symmetry of the two π^+.

The K^0_s band is clearly seen.

Basic formalism

- **Log likelihood**
- **PDF**

$$\mathcal{L} = -2\sum_{n=1}^{N} \log PDF(x_n, y_n)$$

$$PDF(x,y) = \begin{cases} \varepsilon(x,y) \\ B(x,y) \\ fN_S|\mathcal{M}(x,y)|^2\varepsilon(x,y) + (1-f)N_B B(x,y) \end{cases}$$

efficiency
background
signal

- **Matrix element**

$$\mathcal{M} = \sum_R c_R A_R \Omega_R F_R$$

(c_R is the complex factor amplitude and phase, A_R is a BW or Flatte, Ω_R is an angular distribution, F_R barrier penetration)

- **I=2 $\pi^+\pi^+$ S-wave**
- **$\pi^+\pi^-$ S-waves:**
  - **Oller**
  - **Flatte**

$$A^{I=2}_{J=0}(m) = \frac{\eta_0^2(m)e^{i\delta_0^2(m)} - 1}{2i}$$

$$A_\sigma(s) = \frac{1}{s_\sigma - s} \qquad s_\sigma = (0.47 - i0.22)^2 \text{ GeV}^2 \text{ for } \pi\pi \text{ S-wave.}$$

$$A_{f_0(980)}(m) = \frac{1}{m_{f_0}^2 - m^2 - i(g_{\pi\pi}^2\rho_{\pi\pi} + g_{K\bar{K}}^2\rho_{K\bar{K}})}$$

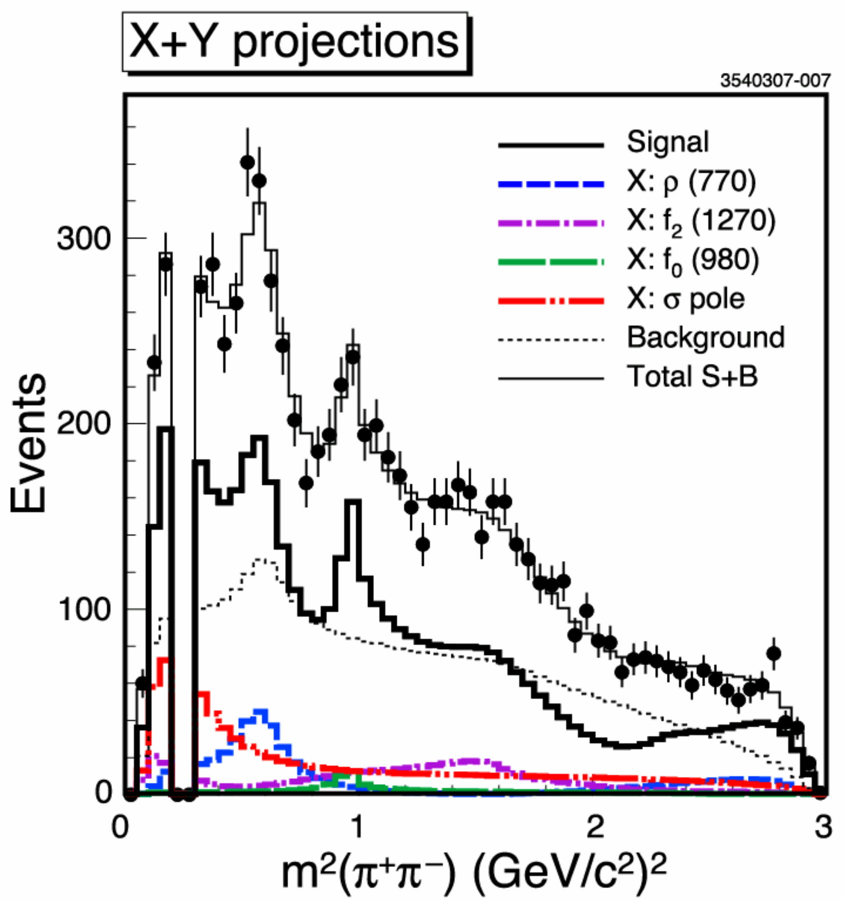
Isobar model results

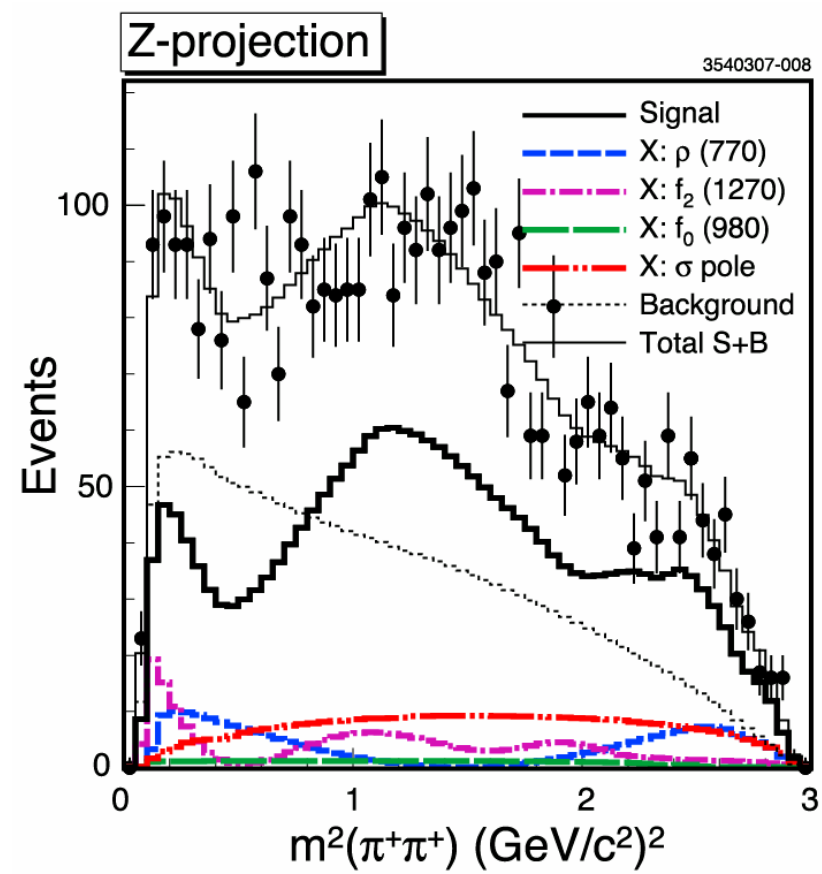
Starting from the contributions clearly seen in the data additional resonances are added or removed one by one to improve the fit. A contribution is kept if the amplitude is significant at more than 3 standard deviations and the phase uncertainty is less than 30^0

Mode	Amplitude, a.u.	Phase, (°)	Fit Fraction, %
$\rho(770)\pi^+$	1(fixed)	0(fixed)	20.0±2.3±0.9
$f_0(980)\pi^+$	1.4±0.2±0.2	12±10±5	4.1±0.9±0.3
$f_2(1270)\pi^+$	2.1±0.2±0.1	–123±6±3	18.2±2.6±0.7
$f_0(1370)\pi^+$	1.3±0.4±0.2	–21±15±14	2.6±1.8±0.6
$f_0(1500)\pi^+$	1.1±0.3±0.2	–44±13±16	3.4±1.0±0.8
σ pole	3.7±0.3±0.2	–3±4±2	41.8±1.4±2.5
$\sum_i FF_i$, %			90.2

Mode	Upper Limit on Fit Fraction, %
$\rho(1450)\pi^+$	<2.4
N.R.	<3.5
I=2 $\pi^+\pi^+$ S-Wave	<3.7
$f_0(1710)\pi^+$	<1.6
$f_0(1790)\pi^+$	<2

Fits to the Mass projections





Other S wave models

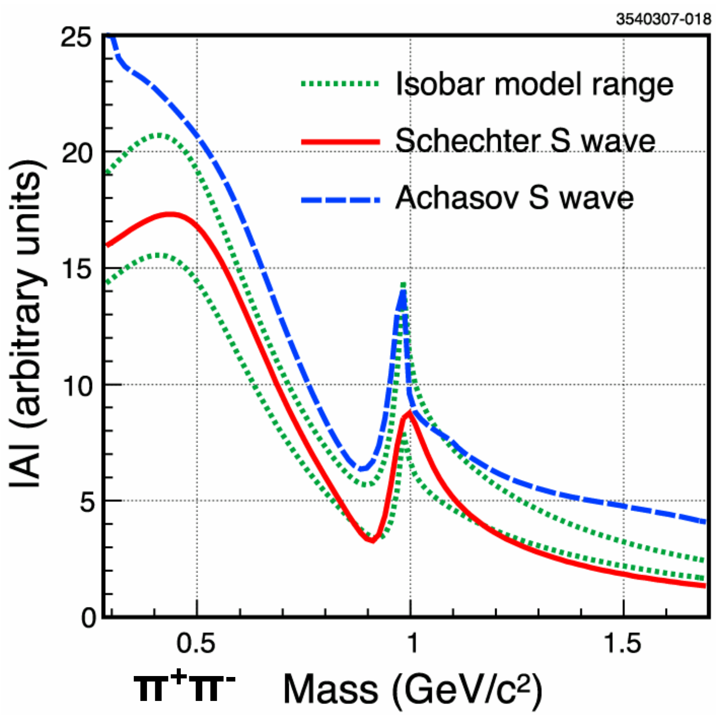
The isobar model does not adequately represent the S wave $\pi^+\pi^-$ where wide resonances overlap so we have used two other models.

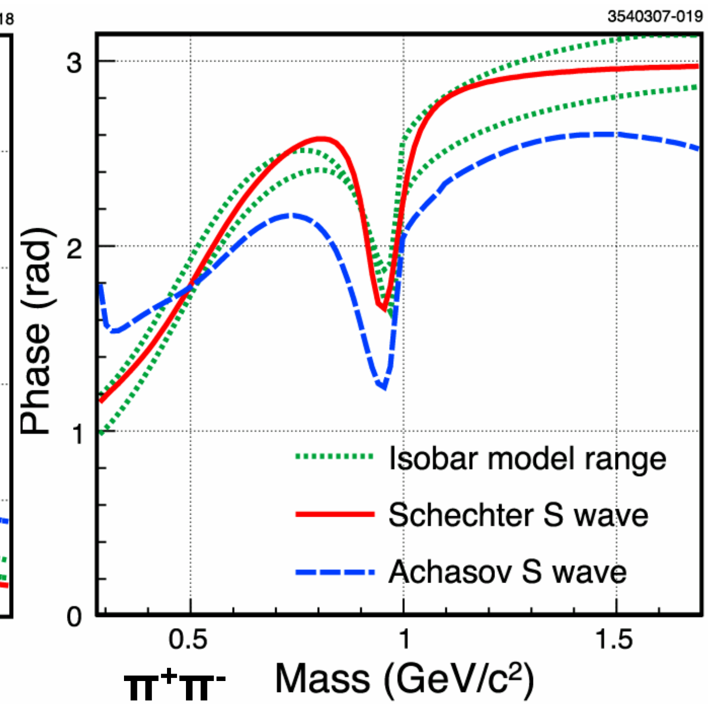
Schechter motivated by chiral invariance parameterizes simultaneously the σ and f_0 (980) with 7 parameters, the bare masses, the strong mixing angle between the σ and f_0 (980), the total S wave amplitude and phase and the relative weak amplitude and phase.
(PRD 64, 014031 (2001) and Int.J.Mod.Phys A20,6149 (2005))

The Achasov model is motivated by field theory and treats the S wave ππ with contributions from non resonant, direct resonance production of σ and f_0(980) and rescattering from several intermediate states such as KK and ππ
(PRD 67, 114018 (2003), Sov.J.Nucl.Phys 32,566(1980), PRD.D55,2663(1997), PRD70,111901 (2004), PRD 67,114018(2003), PRD 73,054029(2006)

Comparison of amplitudes and phases

The two models used to extend the isobar model give amplitudes and phases which are close to the isobar results but are a better physical description of the S wave component





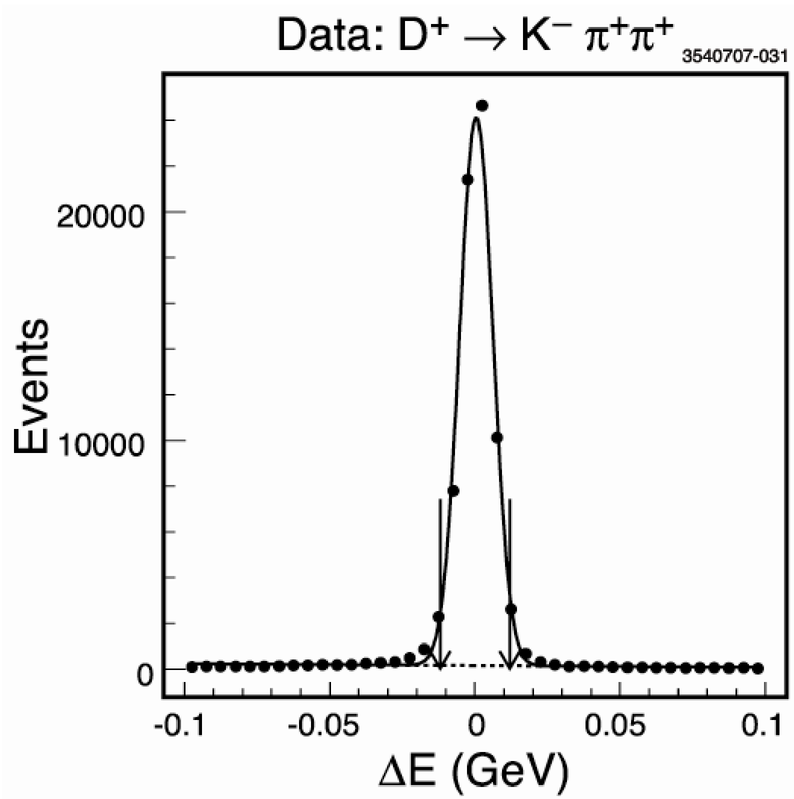
Dalitz analysis of $D^+ \to K^- \pi^+ \pi^+$

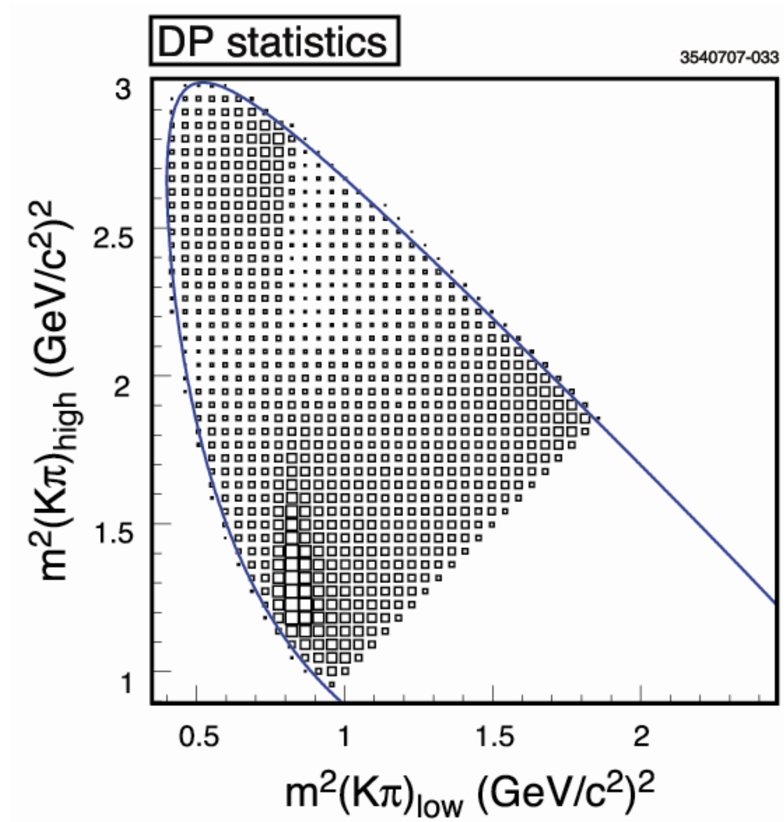
- Experimental publications on $D^+ \to K^- \pi^+ \pi^+$ DPA:
  - Mark III, PL B196, 107 (1987) , ~2000 events
  - NA14, Z. Phys. C50, 11 (1991), 394±33 ev.
  - E691, PR D48, 56 (1993), 4149±79 ev.
  - E687, PL B331, 217 (1994), 8800±98 ev.

  - E791, PRL 89, 121801 (2002), S+6%B=15090, isobar model
  - E791, PR D73, 032004 (2006), the same sample, MIPWA
  - FOCUS, arXiv:0705.2248, S+3%B=53653, IM + K-matrix

- Re-analyses of E791 data:
  - J.A. Oller, FSI in hadronic D decays, PR D71, 054030 (2005).
  - D.V. Bugg, $\sigma, \kappa, f_0(980), a_0(980)$, hep-ex/0510014.

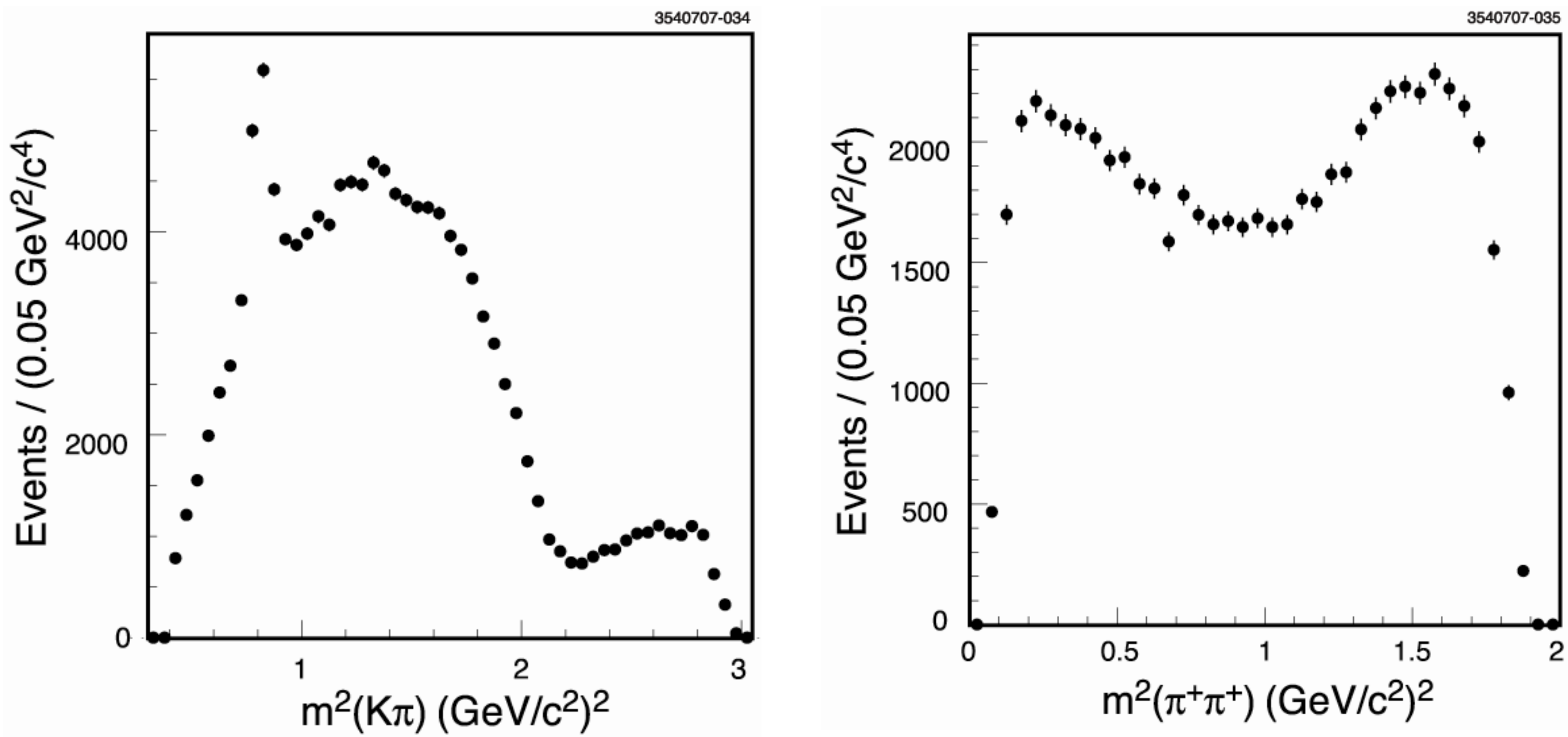
Dalitz analysis of $D^+ \to K^- \pi^+ \pi^+$





281/pb^{-1} statistics ~67K events bkg~1%, is ~×4 of E791 excellent invariant mass resolution.

Mass projections



2 entries/event

Analysis Procedures

- **Compare to E791 using the isobar model**
- **Add an I = 2 ππ S wave contribution**
- **Apply a Quasi Model Independent Partial Wave analysis an extension of E791, PR D73, 032004 (2006):**

Models of E791

PRL 89, 121801 (2002)

We first compare to E791 using the isobar approach. E791 used three models with model C having the best fit quality

- A: float amplitudes and phases
- B: + float m,Γ for $K^*_0(1430)$, float r_D, r_R
- C: + $\kappa\pi^+$ but different $K^*_0(1430)$ pars

Mode	Model A	Model B	Model C
NR	90.9 ± 2.6 1.0 (fixed) $0°$ (fixed)	89.5 ± 16.1 2.72 ± 0.55 $(-49 \pm 3)°$	$13.0 \pm 5.8 \pm 4.4$ $1.03 \pm 0.30 \pm 0.16$ $(-11 \pm 14 \pm 8)°$
$\kappa\pi^+$	... ...	... ...	$47.8 \pm 12.1 \pm 5.3$ $1.97 \pm 0.35 \pm 0.11$ $(187 \pm 8 \pm 18)°$
$\bar{K}^*(892)\pi^+$	13.8 ± 0.5 0.39 ± 0.01 $(54 \pm 2)°$	12.1 ± 3.3 1.0 (fixed) $0°$ (fixed)	$12.3 \pm 1.0 \pm 0.9$ 1.0 (fixed) $0°$ (fixed)
$\bar{K}^*_0(1430)\pi^+$	30.6 ± 1.6 0.58 ± 0.01 $(54 \pm 2)°$	28.7 ± 10.2 1.54 ± 0.75 $(6 \pm 12)°$	$12.5 \pm 1.4 \pm 0.5$ $1.01 \pm 0.10 \pm 0.08$ $(48 \pm 7 \pm 10)°$
$\bar{K}^*_2(1430)\pi^+$	0.4 ± 0.1 0.07 ± 0.01 $(33 \pm 8)°$	0.5 ± 0.3 0.21 ± 0.18 $(-3 \pm 26)°$	$0.5 \pm 0.1 \pm 0.2$ $0.20 \pm 0.05 \pm 0.04$ $(-54 \pm 8 \pm 7)°$
$\bar{K}^*(1680)\pi^+$	3.2 ± 0.3 0.19 ± 0.01 $(66 \pm 3)°$	3.7 ± 1.9 0.56 ± 0.48 $(36 \pm 25)°$	$2.5 \pm 0.7 \pm 0.3$ $0.45 \pm 0.16 \pm 0.02$ $(28 \pm 13 \pm 15)°$
χ^2/ν	167/63	126/63	46/63

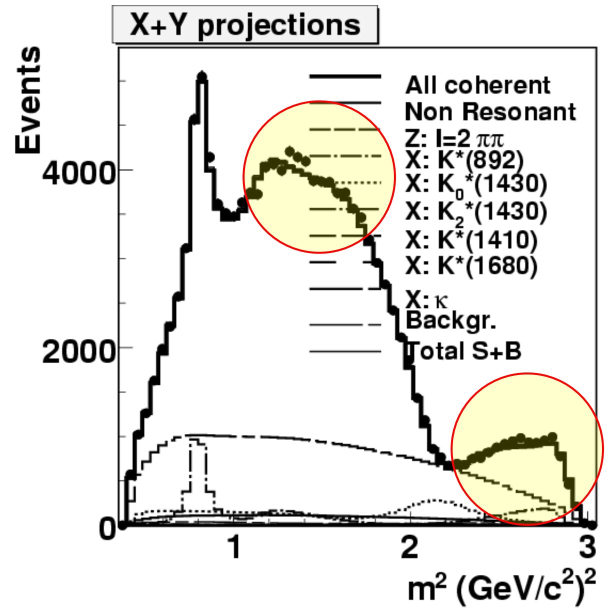
Model C uses

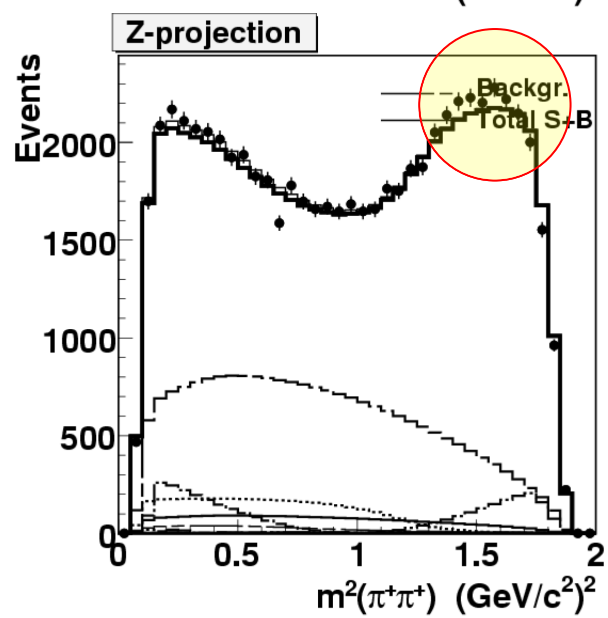
- $K^*(892)$, $K^*_0(1430)$, $K^*(1680)$, $K^*_2(1430)$, $\kappa\pi^+$ and NR
- Gaussian form factors are used for the $K^*_0(1430)$ and the κ and for all Kπ with non zero spin the radii in the Blatt-Weisskopf form factors are fixed with $r_D = 5\text{GeV}^{-1}$ and $r_R = 1.5\text{GeV}^{-1}$

Comparison with E791, Model C

CLEO-c preliminary

Mode	Parameter	E791	CLEO-c
NR	a (a.u.)	1.03±0.30±0.16	8.0±1.0
	ϕ (°)	−11±14±8	−19±9
	FF (%)	13.0±5.8±4.4	10.4±1.3
$\overline{K}^*(892)\pi^+$	a (a.u.)	1 (fixed)	1 (fixed)
	ϕ (°)	0 (fixed)	0 (fixed)
	FF (%)	12.3±1.0±0.9	11.2±1.4
$\overline{K}^*_0(1430)\pi^+$	a (a.u.)	1.01±0.10±0.08	3.1±0.1
	ϕ (°)	48±7±10	48±3
	FF (%)	12.5±1.4±0.5	10.5±1.3
	m (MeV/c²)	1459±7±12	1461±3
	Γ (MeV/c²)	175±12±12	169±5
$\overline{K}^*_2(1430)\pi^+$	a (a.u.)	0.20±0.05±0.04	0.98±0.04
	ϕ (°)	−54±8±7	−29±4
	FF (%)	0.5±0.1±0.2	0.40±0.04
$\overline{K}^*(1680)\pi^+$	a (a.u.)	0.45±0.16±0.02	6.7±0.5
	ϕ (°)	28±13±15	29±4
	FF (%)	2.5±0.7±0.3	1.36±0.16
$\kappa\pi^+$	a (a.u.)	1.97±0.35±0.11	4.8±0.3
	ϕ (°)	−173±8±18	−165±5
	FF (%)	47.8±12.1±5.3	31.2±3.6
Breit-Wigner	m (MeV/c²)	797±19±43	805±11
	Γ (MeV/c²)	410±43±87	453±21
Formfactor	r_κ(GeV^{-1})	1.6±1.3	1.5(fixed)
	r_D(GeV^{-1})	5.0±0.5	5 (fixed)
Other $R \to K\pi$	r_R(GeV^{-1})	1.5(fixed)	1.5(fixed)
Goodness	χ^2/ν, Prob.	46/63(???)	448/388, P=2%





The fit probability for C is ~2% and the fit significantly under estimates the data in the range 1.3 < $m^2(\pi^+\pi^+)$ < 1.6 (GeV/c²)²

QMIPWA analysis

- binned complex amplitudes for Kπ amplitude and phase with 26 uniform bins of 0.1GeV and 26 amplitudes and 26 phases

- ππ amplitude and phase is defined by 18 bins in ππ from 0.1 – 1.9GeV
- Breit-Wigner for the narrow resonances.

- S wave: Breit-Wigner $K^*_0(1430)$, binned S wave amplitude which replaces κ and non resonant contributions in the isobar model.

- P wave: Breit-Wigner $K^*(892)$, binned P wave amplitude which replaces $K^*(1680)$ in the isobar model

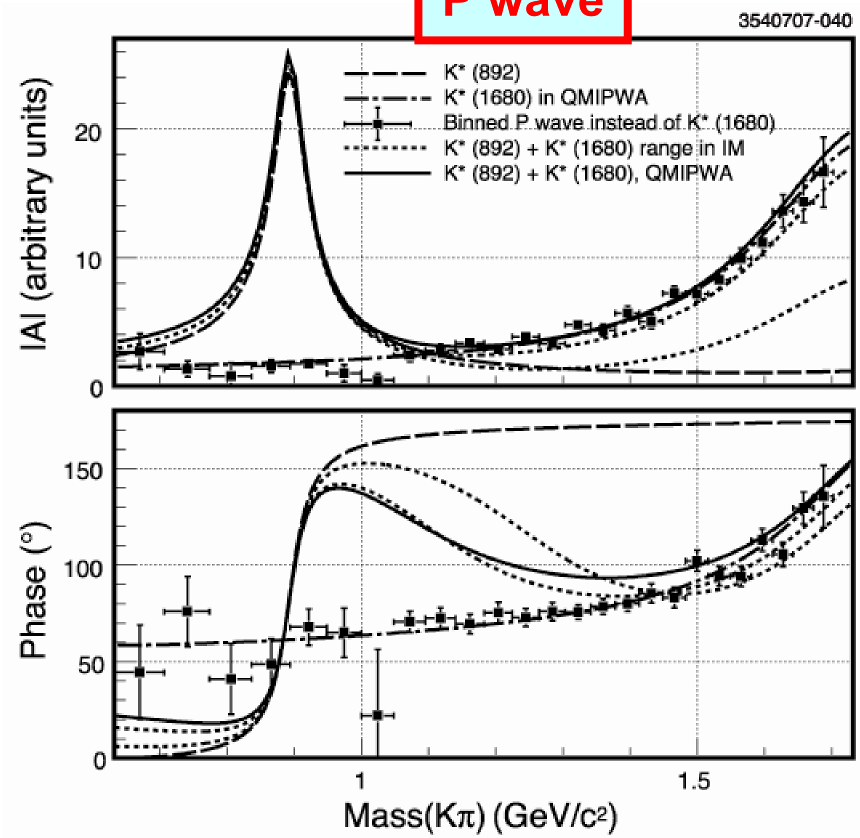
- D wave: binned amplitude which replaces $K^*_2(1430)$ in the isobar model

- The parameters of the binned functions describing the S, P, and D waves are allowed to float one wave at a time

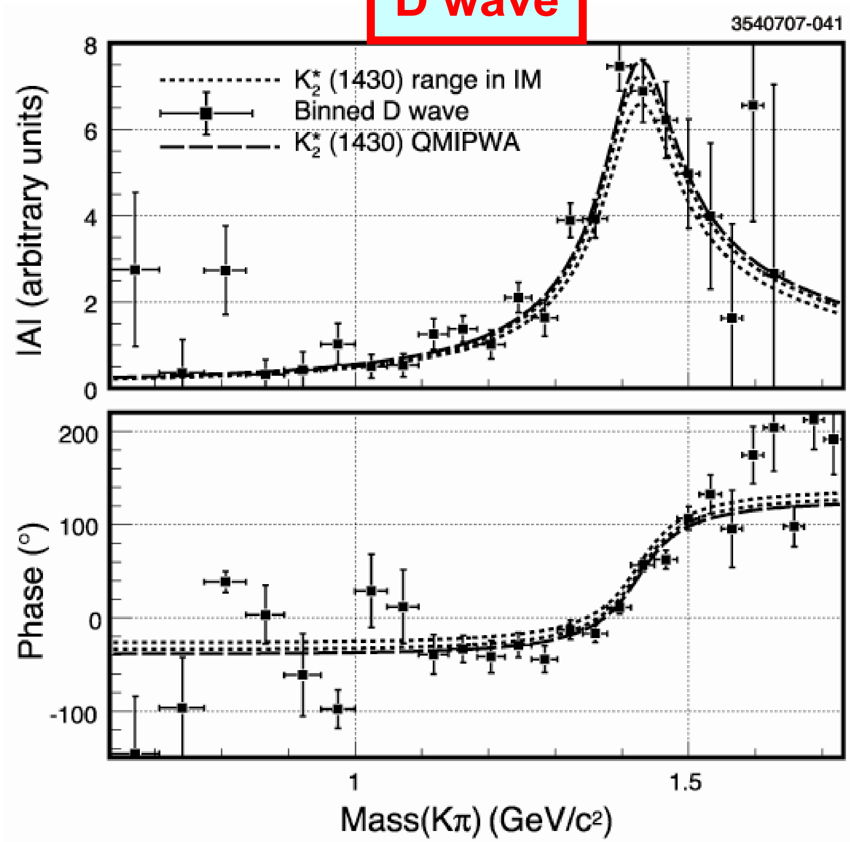
Measured Kπ P and D waves
Isobar model and QMIPWA

CLEO-c fit results

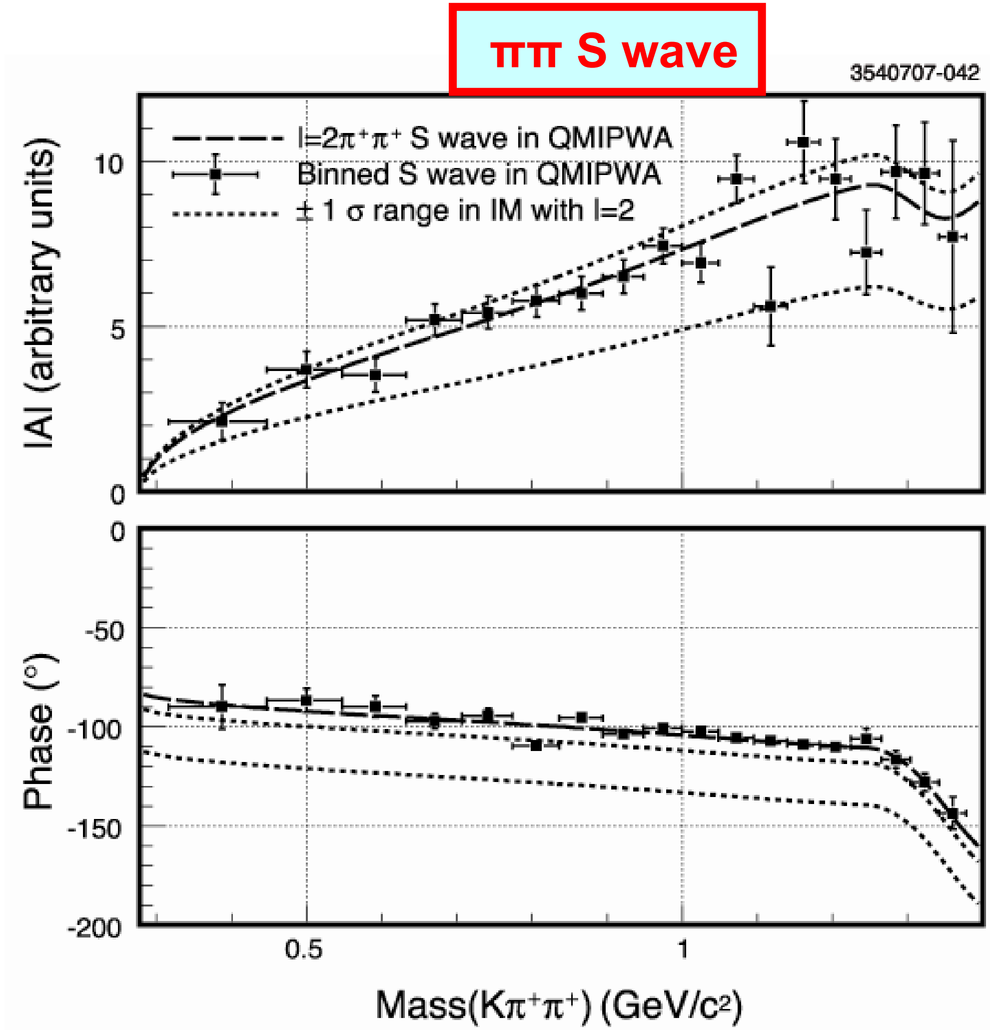
P wave



D wave

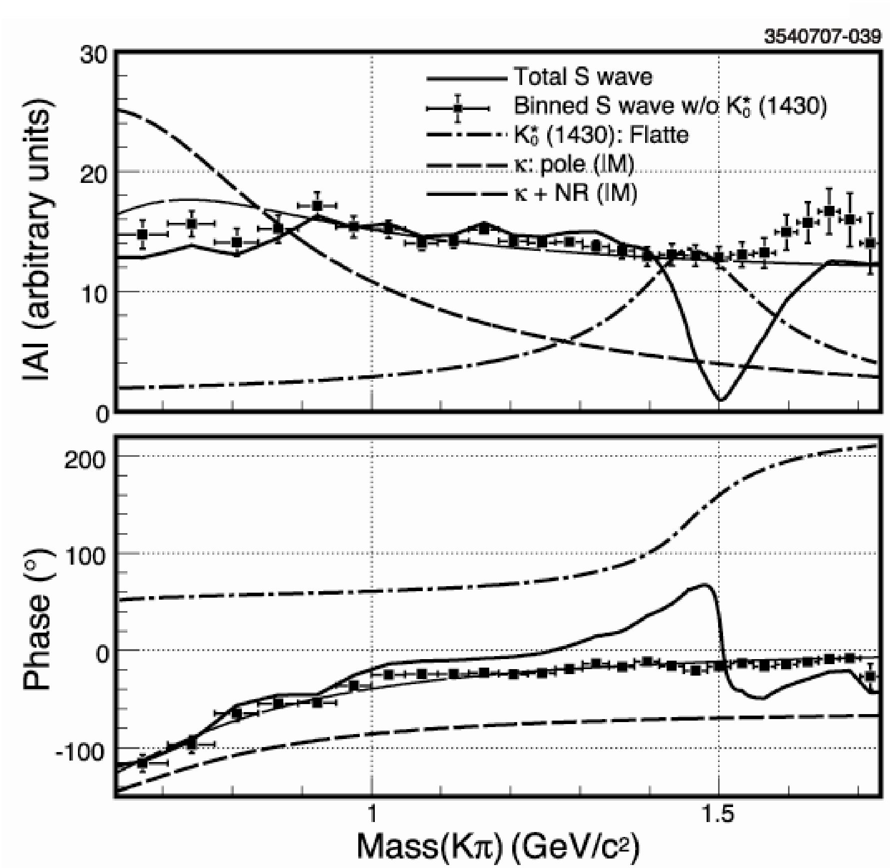


Kπ and ππ S waves
Isobar model and QMIPWA

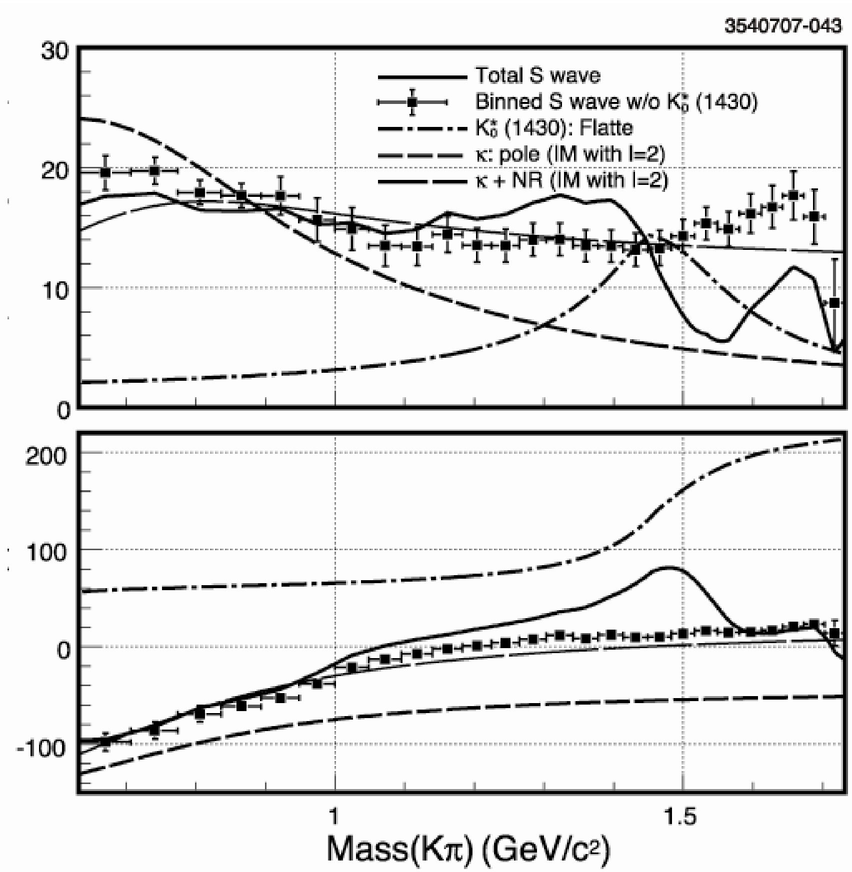


Kπ S wave in Isobar and QMIPWA

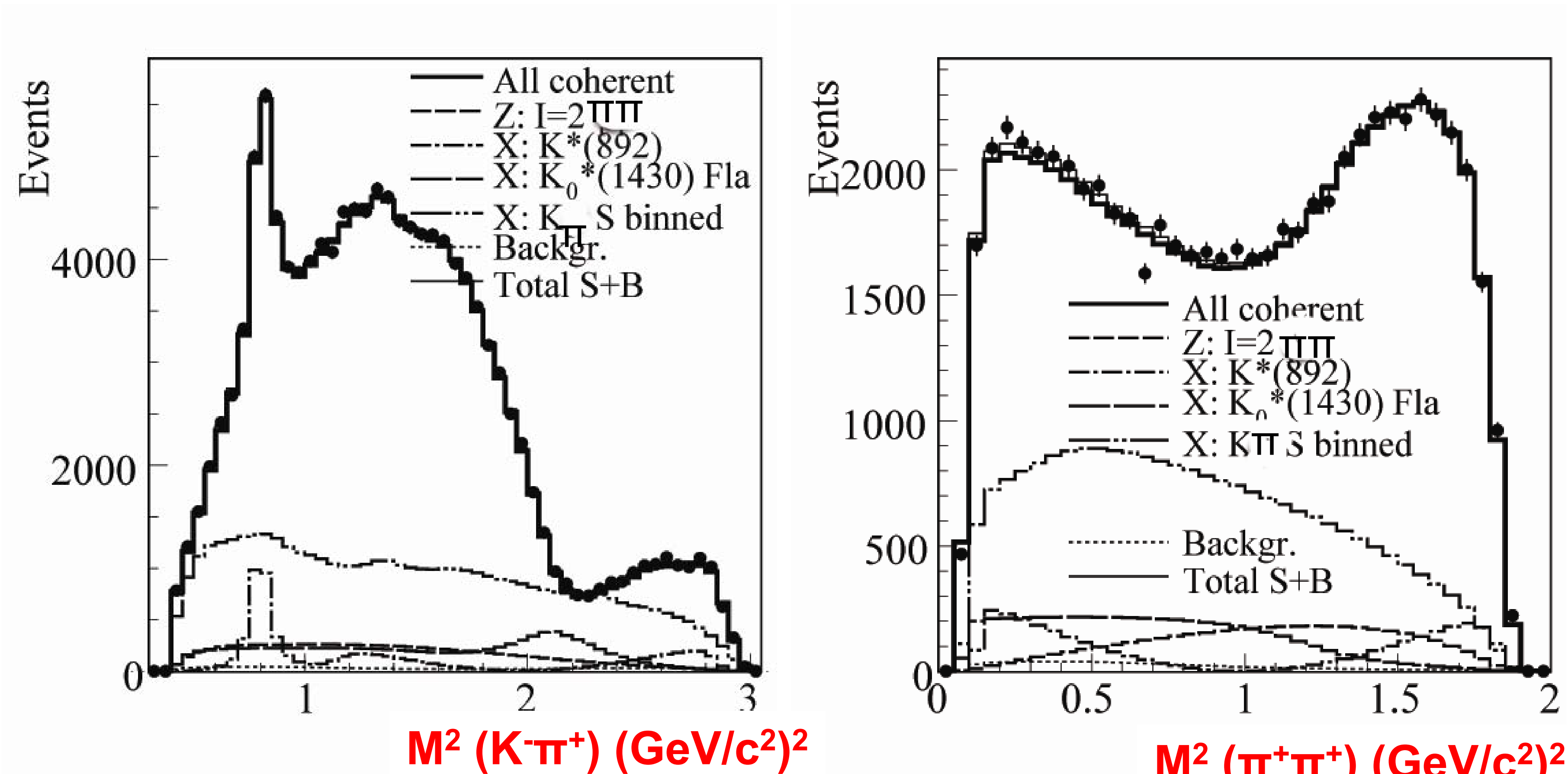
Kπ S wave



Kπ S wave with I = 2 ππ



Mass projections



Comparison Summary

Comparison of CLEO-c and E791. Only statistical errors are shown for CLEO-c. Systematic errors will have similar magnitudes

	Model C		QMIPWA	
Mode	E791 [5]	CLEO-c	E791 [6]	CLEO-c
NR	13.0±5.8±4.4	10.4±1.3	see S wave	see S wave
$\overline{K}^{*}(892)\pi^{+}$	12.3±1.0±0.9	11.2±1.4	11.9±0.2±2.0	10.0±0.3
$\overline{K}_{0}^{*}(1430)\pi^{+}$	12.5±1.4±0.5	10.5±1.3	see S wave	11.4±3.6
$\overline{K}_{2}^{*}(1430)\pi^{+}$	0.5±0.1±0.2	0.40±0.04	0.2±0.1±0.1	0.476±0.014
$\overline{K}^{*}(1680)\pi^{+}$	2.5±0.7±0.3	1.36±0.16	1.2±0.6±1.2	2.52±0.08
$\kappa\pi^{+}$	47.8±12.1±5.3	31.2±3.6	see S wave	see S wave
Total S wave	73±15	52±4	78.6±1.4±1.8	67.4±1.3
χ^2/ν, Prob.(%)	46/63, 94%	448/388, 2%	277/277, 47.8%	368/346, 19.5%

Discussion: S, P & D waves

- **Kπ S wave (we do not distinguish I=1/2, 3/2)**
  - **Amplitude is almost constant below $K_0^*(1430)$**
  - **Binned wave shows a minor deviation comparing to the isobar model**
  - **Phase shows slow variation from -100° to ~0, well described by the complex pole + $K_0^*(1430)$**
- **I=2 $\pi^+\pi^+$ S wave: required, gives slight change for Kπ S wave**
- **Kπ P and D waves: consistent with isobar model**

Summary/Future

By April 1st, 2008 CLEO-c will complete data taking and continue and extend the analyses of three body D and D_s decays

3770	$D\overline{D}$	$\sim 800 pb^{-1}$
4170	$D_s\overline{D}^{(*)}_s$	$\sim 630 pb^{-1}$

The analysis of $K^0_s\pi\pi$ and $K^0_L\pi\pi$ is continuing and we expect ~1600 CP tags in $800 pb^{-1}$ with an error in φ_3 of $< 4^0$

http://agenda.hepl.phys.nagoya-u.ac.jp/getFile.py/access?contribId=8&sessionId=2&resId=0&materialId=slides&confId=0

Backup slides

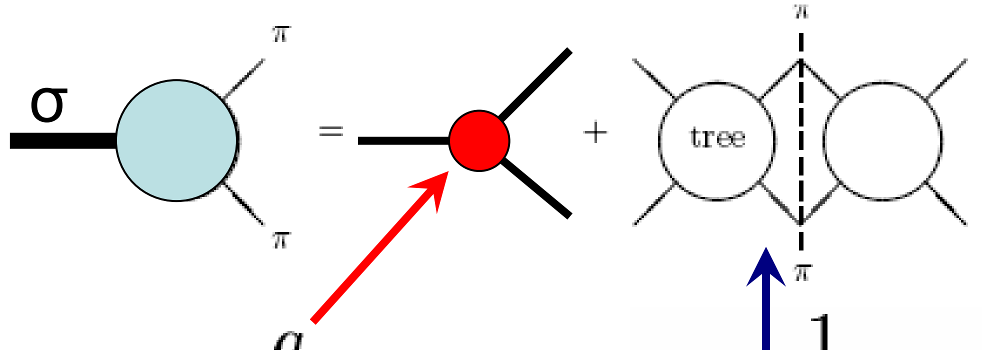
Intermediate $\pi^+\pi^-$ states from PDG

Resonance	Mass (MeV/c^2)	Width (MeV/c^2)	$\mathcal{B}(R \to \pi\pi)$ (%)
$\rho^0(770)$	775.8±0.5	150.3±1.6	~100
$f_2(1270)$	1275.4±1.2	185.1±3.5	84.8±2.5
$f_0(1370)$	1200 to 1500	200 to 500	seen
	1350±50	265±40	with $f_0(1500)$
	1410±50	-.-	w/o $f_0(1500)$
	1434±18±9	173±32±6	
$\rho_0(1450)$	1465±25	400±60	seen
$f_0(1500)$	1507±5	109±7	34.9±2.3
$f_0(1710)$	1714±5	140±10	seen, $K\bar{K}$-domin.
$f_0(1790)$	1790^{+40}_{-30}	270^{+60}_{-30}	seen
σ	$478^{+24}_{-23}\pm17$	$324^{+42}_{-40}\pm21$	seen?
$f_2'(1525)$	1525±5	73±6	0.82±0.15
$\rho_3(1690)$	1688.8±2.1	161±10	23.6±1.3
$\rho(1700)$	1720±20	250±100	seen
$f_2(1950)$	1945±13	475±19	seen
$f_4(2050)$	2034±11	222±19	17.0±1.5

J.Schechter: Production amplitude

- ππ⟶ππ
  scattering amplitude:

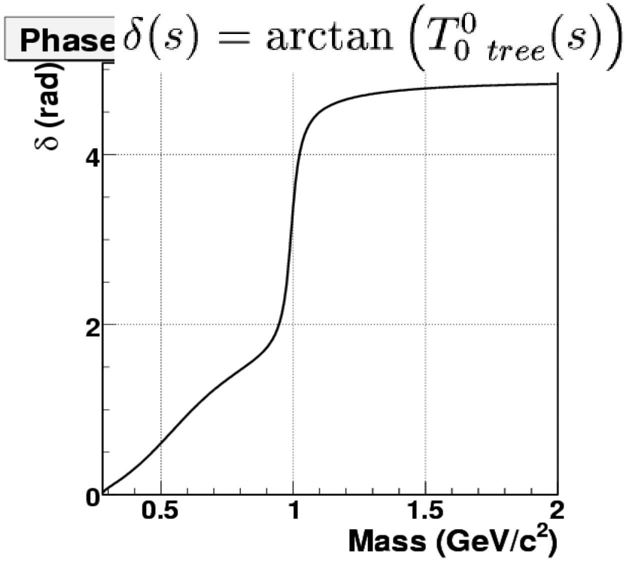
$$T_0^0(s) = \frac{T_0^0{}_{tree}(s)}{1 - iT_0^0{}_{tree}(s)}$$



$$\frac{g_{\sigma\pi\pi}}{m_\sigma^2 - s} \qquad \frac{1}{1 - iT_0^0{}_{tree}}$$

- σ⟶ππ
  production amplitude:

$$\delta(s) = \arctan\left(T_0^0{}_{tree}(s)\right)$$



$$\mathcal{A}_\sigma = \frac{g_{\sigma\pi\pi}}{m_\sigma^2 - s} \cdot \frac{1}{1 - iT_0^0{}_{tree}} = \frac{g_{\sigma\pi\pi}}{m_\sigma^2 - s} \cdot \cos\delta \cdot e^{i\delta}$$

- Add one more resonance in prod. amp.:

$$\mathcal{A} = \mathcal{A}_\sigma + \mathcal{A}_{\tilde{\sigma}} = \cos\delta \cdot e^{i\delta}\left[\frac{1}{m_\sigma^2 - s} + \frac{ae^{i\phi}}{m_{\tilde{\sigma}}^2 - s}\right]$$

N.Achasov: model parameters

$$g_{f_0 K^0 \bar{K}^0} = g_{f_0 K^+ K^-}, \quad g_{R\pi^0\pi^0} = g_{R\pi^+\pi^-}/\sqrt{2}, \quad g_{R\pi\pi} = \sqrt{3/2}\, g_{R\pi^+\pi^-}$$

$$g_{f_0\eta\eta} = -g_{f_0\eta'\eta'} = \frac{2\sqrt{2}}{3} g_{f_0 K^+ K^-}, \quad g_{f_0\eta'\eta} = -\frac{\sqrt{2}}{3} g_{f_0 K^+ K^-}$$

$$g_{\sigma\eta\eta} = g_{\sigma\eta'\eta'} = \frac{\sqrt{2}}{3} g_{\sigma\pi^+\pi^-}, \quad g_{\sigma\eta'\eta} = \frac{1}{3\sqrt{2}} g_{\sigma\pi^+\pi^-}$$

Parameter	Value in Fit#1
m_{f_0}, MeV	984.1
m_σ, MeV	461.9
$g_{f_0 K^+ K^-}$, GeV	4.3
$g_{f_0 \pi^+\pi^-}$, GeV	-1.8
$g_{\sigma K^+ K^-}$, GeV	0.55
$g_{\sigma \pi^+\pi^-}$, GeV	2.4
$C_{f_0\sigma}$	0.047
b_0	4.9
b_1	1.1
b_2	1.36
Λ, MeV	172.2
m_1, MeV	765.4
m_2, MeV	368.9
Λ_K, GeV	1.24

- **Coupling constant ratios from 4-quark model.**
- **Model parameters were obtained in fit to data from scattering experiments**
- **Background phase pars.**

S wave amplitude for Dalitz plot

$$A_{SW}(m_x, m_y, m_z) = A_{\pi^+\pi^-}(m_x) + A_{\pi^+\pi^-}(m_y) + A_{\pi^+\pi^+}(m_z)$$

$$\begin{aligned}
A_{\pi^+\pi^-}(m) &= 16\pi c_{\pi\pi} \\
&+ L_{\pi^+\pi^-}(m|c_{\pi\pi}, d_{\pi\pi}) \cdot \left(\frac{2}{3}T_0^0(m) + \frac{1}{3}T_0^2(m)\right) \\
&+ L_{\pi^0\pi^0}(m|c_{\pi^0\pi^0}, d_{\pi^0\pi^0}) \cdot \left(\frac{2}{3}T_0^0(m) + \frac{2}{3}T_0^2(m)\right) \\
&+ L_{K^+K^-}(m|c_{K^+K^-}, d_{K^+K^-}) \cdot T_0^0(K^+K^- \to \pi^+\pi^-, m) \\
&+ L_{K^0\overline{K}^0}(m|c_{K^0\overline{K}^0}, d_{K^0\overline{K}^0}) \cdot T_0^0(K^0\overline{K}^0 \to \pi^+\pi^-, m) \\
&+ c_{D^+R\pi^+} \cdot e^{i\delta_B^{\pi\pi}(m)} \cdot T_{0\ D R\pi}^{0\ res}(m).
\end{aligned}$$

- K<u>K</u> terms have similar shape and are joined in the fit

$$A_{\pi^+\pi^+}(m) = L_{\pi^+\pi^+}(m|c_{\pi\pi}, d_{\pi\pi}) \cdot T_0^2(m)$$

Dalitz plot fit formalism

- **Log likelihood**

$$\mathcal{L} = -2\sum_{n=1}^{N} \log PDF(x_n, y_n)$$

- **PDF**

$$PDF(x,y) = \begin{cases} \varepsilon(x,y) \\ B(x,y) \\ fN_S|\mathcal{M}(x,y)|^2\varepsilon(x,y) + (1-f)N_B B(x,y) \end{cases}$$

- **Efficiency and background x-y symmetric 2D 3d poly**

$$\varepsilon(x,y) = 1 + E_1(\hat{x}+\hat{y}) + E_2(\hat{x}^2+\hat{y}^2) + E_3(\hat{x}^3+\hat{y}^3) + E_{xy}\hat{x}\hat{y} + E_{xyn}(\hat{x}^2\hat{y}+\hat{x}\hat{y}^2)$$

- **Matrix element**

$$\mathcal{M} = \sum_R c_R A_R \Omega_R F_R$$

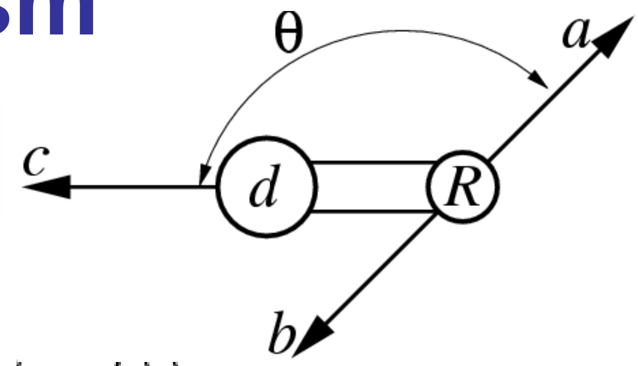
- **Fit fraction**

$$FF_R = \frac{\int |\mathcal{M}_R|^2 dm_x^2 dm_y^2}{\int |\sum_r \mathcal{M}_r|^2 dm_x^2 dm_y^2}$$

- **Fit goodness**

$$\chi^2 = \sum_{bin=1}^{N_{bins}} \frac{(n_{bin} - \mu_{bin})^2}{\mu_{bin}}$$

PWA Formalism



$$s = m^2(K^-\pi_1^+),\ t = m^2(K^-\pi_2^+),\ \text{and}\ u = m^2(\pi_1^+\pi_2^+)$$

$$s + t + u = m_D^2 + m_K^2 + 2m_\pi^2$$

$$\mathcal{M}(s,t) = A(s,t) + A(t,s) + A_{L=0}^{I=2}(u(s,t))$$

$$A(s,t) = \sum_{L=0}^{L_{max}} (-2P_aP_c)^L \mathcal{P}_L(\cos\theta)\mathcal{F}_D^L(P_c^*, r_D)\mathcal{A}_L(s)$$

$$\begin{aligned}
\mathcal{A}_0(s) &= c_{NR} + W_\kappa + W_{K_0^*(1430)} + W_{S\ binned}, \\
\mathcal{A}_1(s) &= W_{K^*(892)} + W_{K^*(1410)} + W_{K^*(1680)} + W_{P\ binned}, \\
\mathcal{A}_2(s) &= W_{K_2^*(1430)} + W_{D\ binned}, \\
\mathcal{A}_0^2(s) &= W_S^{I=2} + W_{S\ binned}^{I=2},
\end{aligned}$$

$$\mathcal{W}_R(m) = \frac{1}{m_R^2 - m^2 - i m_R \Gamma_{R,total}(m)}$$

$$W_{L\ binned}(s) = a_{Lk(s)} \cdot e^{i\phi_{Lk(s)}}$$

$$\Gamma_{R,total}(m) = \Gamma_R \frac{m_R}{m}\left(\frac{p}{p_R}\right)^{2L+1}\left[\frac{\mathcal{F}_R^L(p, r_R)}{\mathcal{F}_R^L(p_R, r_R)}\right]^2$$

$$W_R = c_R \cdot \mathcal{W}_R(s) \cdot \mathcal{F}_R^L(P_a, r_R)$$

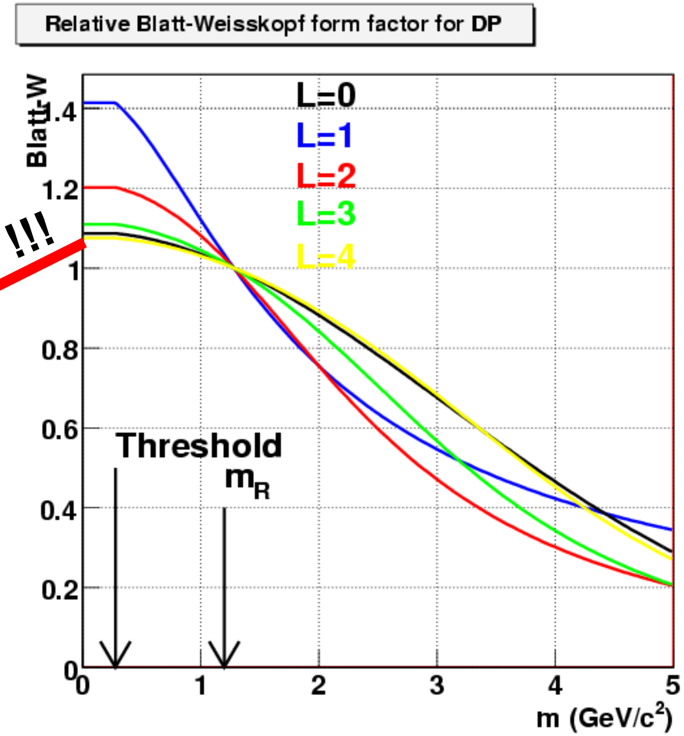
$$c_R = a_R e^{i\phi_R}$$

$$\mathcal{W}_R(m) = \frac{1}{s_R - m^2}$$

Blatt-Weisskopf form factors

- B.-W. barrier and form factors:

$$B_R^L(p,r) = p^L \cdot \mathcal{F}_R^L(p,r)$$



- E791 uses form factors:

L=0 behaves as L=4 !!!

$$\begin{aligned}
\mathcal{F}_R^0(q,r) &= e^{-(rq)^2/12} && \text{scalar,} \\
\mathcal{F}_R^1(q,r) &= [1+(rq)^2]^{-\frac{1}{2}} && \text{vector,} \\
\mathcal{F}_R^2(q,r) &= [9+3(rq)^2+(rq)^2]^{-\frac{1}{2}} && \text{tensor.}
\end{aligned}$$

- CLEO use similar form factors, but
  - $\mathcal{F}^0{}_R(q,r)$ = 1 for scalar
  - re-normalized @ m_R : $\mathcal{F}^L{}_R(q,r)$ / $\mathcal{F}^L{}_R(q_R,r)$ = 1