Chiral behavior of the heavy meson mixing amplitudes in the standard model and beyond

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    - Bases of $\Delta B = 2$ operators and B-parameters
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Control of theoretical uncertainties in $B_{s,d} - \overline{B}_{s,d}$ mixing

- Δm_{B_d} and Δm_{B_s} are used to constrain the shape of the CKM unitarity triangle and thereby determine the amount of the CP-violation in the SM.
- Theoretical uncertainties in computing the values for the decay constants, $f_{B_{s,d}}$, and "bag" parameters, $B_{B_{s,d}}$.
  - Can, in principle, be computed on the lattice:
  - d-quark cannot be reached directly – extrapolation of results with larger light quark masses is needed.
  - Induces systematic uncertainties.

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- HMχPT allows us to gain some control over these uncertainties:
  - Predicts the chiral behavior of the hadronic quantities.
  - Can be implemented to guide the extrapolation of the lattice results.

Impact of lowest lying heavy meson resonances on HMχPT calculations

- HMχPT combines HQET and spontaneous breaking of chiral symmetry, $SU(3)_L \otimes SU(3)_R \to SU(3)_V$.
- One computes chiral logarithmic corrections which are expected to be relevant at $m_q \ll \Lambda_\chi$.
  - Condition is satisfied for u- and d-quarks.
  - Ambiguous size of the chiral symmetry breaking scale Λ_χ:
    - $4\pi f_\pi \simeq 1$ GeV
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- Heavy-light quark systems are more complicated – first orbital excitations ($j_\ell^P = 1/2^+$) are not far from the ground ($j_\ell^P = 1/2^-$) states.
- Experimental evidence for scalar D_{0s}^* and axial D_{1s} mesons indicates $\Delta_{S_s} \equiv m_{D_{0s}^*} - m_{D_s} = 350$ MeV, $\Delta_{S_q} = 430$ MeV
- Both Δ_{S_s} and Δ_{S_q} are smaller than Λ_χ, m_η, and even m_K.
- Requires revisiting predictions based on HMχPT.

Scope of work

- Investigate the issue on specific examples:
  - Decay constants $f_{B_{d,s}}$, and bag parameters $B_{B_{d,s}}$.
  - SM and supersymmetric (SUSY) effects in the $B_{s,d} - \overline{B}_{s,d}$ mixing amplitudes.
- Focus on the chiral limit of these quantities.

Bases of $\Delta B = 2$ operators and B-parameters

"SUSY basis" of $\Delta B = 2$ operators

$$
\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1-\gamma_5) q^i \, \bar{b}^j \gamma^\mu (1-\gamma_5) q^j \,, \\
O_2 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1-\gamma_5) q^j \,, \\
O_3 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1-\gamma_5) q^i \,, \\
O_4 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1+\gamma_5) q^j \,, \\
O_5 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1+\gamma_5) q^i \,.
\end{aligned}
$$

In SM, only O_1 (left-left) operator is relevant in describing the $B_q - \overline{B}_q$ mixing amplitude.

Introducing bag-parameters, B_{1-5} as measures of the difference with respect to the vacuum saturation approximation (VSA)

$$
\frac{\langle \bar{B}_q^0 | O_{1-5}(\nu) | B_q^0 \rangle}{\langle \bar{B}_q^0 | O_{1-5}(\nu) | B_q^0 \rangle_{\mathrm{VSA}}} = B_{1-5}(\nu) \,.
$$

Reducing the number of independent operators

In HQET, heavy quark spin symmetry ($h^\dagger\gamma_0 = h^\dagger$) imposes relation

$$\langle \bar{B}_q^0 | \widetilde{O}_3 + \widetilde{O}_2 + \frac{1}{2}\widetilde{O}_1 | B_q^0 \rangle = 0.$$

In HMχPT $\left\{ H_q^Q(v) = \frac{1+\not v}{2}\left[P_\mu^{Q*}(v)\gamma^\mu - P^Q(v)\gamma_5\right]_q, H_q^{\bar{Q}}(v) = \left[P_\mu^{\bar{Q}*}(v)\gamma^\mu - P^{\bar{Q}}(v)\gamma_5\right]_q \frac{1-\not v}{2} \right\}$, the bosonized operators are color blind

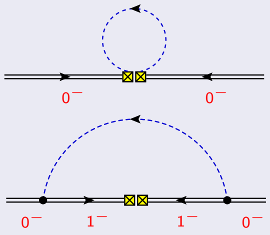
$$\begin{aligned}
\widetilde{O}_1 &= \sum_X \beta_{1X} \mathrm{Tr}\left[(\xi \overline{H}^Q)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi H^{\bar{Q}})_q \gamma^\mu (1-\gamma_5) X\right] + \mathrm{c.t.}\,, \\
\widetilde{O}_2 &= \sum_X \beta_{2X} \mathrm{Tr}\left[(\xi \overline{H}^Q)_q (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi H^{\bar{Q}})_q (1-\gamma_5) X\right] + \mathrm{c.t.}\,, \\
\widetilde{O}_4 &= \sum_X \beta_{4X} \mathrm{Tr}\left[(\xi \overline{H}^Q)_q (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi^\dagger H^{\bar{Q}})_q (1+\gamma_5) X\right] \\
&\quad + \bar{\beta}_{4X} \mathrm{Tr}\left[(\xi H^{\bar{Q}})_q (1-\gamma_5) X\right] \mathrm{Tr}\left[(\xi^\dagger \overline{H}^Q)_q (1+\gamma_5) X\right] + \mathrm{c.t.}\,,
\end{aligned}$$

where $X \in \{1, \gamma_5, \gamma_\nu, \gamma_\nu\gamma_5, \sigma_{\nu\rho}\}$

Chiral logarithmic corrections to SUSY basis bag parameters

All factorisable chiral loop corrections can be absorbed into HMχPT bag-parameter and decay constant definitions ($\beta_x \propto \tilde{B}_x/\hat{f}^2$).

Two nonfactorisable contributions



$$\widetilde{B}_{1d} = \widetilde{B}_1^{\mathrm{Tree}}\left[1 - \frac{1-3g^2}{(4\pi f)^2}\left(\frac{1}{2}m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + \frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right) + \mathrm{c.t.}\right],$$

$$\widetilde{B}_{1s} = \widetilde{B}_1^{\mathrm{Tree}}\left[1 - \frac{1-3g^2}{(4\pi f)^2}\frac{2}{3}m_\eta^2\log\frac{m_\eta^2}{\mu^2} + \mathrm{c.t.}\right],$$

$$\widetilde{B}_{2,4d} = \widetilde{B}_{2,4}^{\mathrm{Tree}}\left[1 + \frac{3g^2 Y \mp 1}{(4\pi f)^2}\left(\frac{1}{2}m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + \frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right) + \mathrm{c.t.}\right]$$

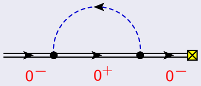
$$\widetilde{B}_{2,4s} = \widetilde{B}_{2,4}^{\mathrm{Tree}}\left[1 + \frac{2}{3}\frac{3g^2 Y \mp 1}{(4\pi f)^2}m_\eta^2\log\frac{m_\eta^2}{\mu^2} + \mathrm{c.t.}\right],$$

Agreement with the full unquenched scenario in the parallel computation of W. Detmold and C.J.D. Lin Phys.Rev.D76 (2007) 014501 [hep-lat/0612028].

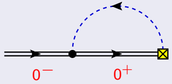
Impact of the $1/2^+$-mesons

- New $J_\ell^P = 1/2^+$ field operators $S_q(v) = \frac{1+\slashed{v}}{2}\left[P^*_{1\,\mu}(v)\gamma_\mu\gamma_5 - P_0(v)\right]_q$.
- New scale parameter $\Delta_S \approx 400$ MeV.

Corrections to the decay constants



Wavefunction renormalization



Weak vertex correction

- In the limit $x = m_\pi/\Delta_S \to 0$ all leading order corrections due to $1/2^+$-mesons are analytic in m_π.
- Kaon and eta logarithms are competitive in size with the terms proportional to $\Delta_S^2 \log(4\Delta_S^2/\mu^2)$
- Relevant chiral logarithmic corrections are those coming from the $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ theory (below the Δ_S scale)

$$\boxed{\hat{f}_q = \alpha\left[1 - \frac{1+3g^2}{2(4\pi f)^2}\frac{3}{2}m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + c_f(\mu)m_\pi^2\right]},$$

$$\boxed{\hat{f}_q^+ = \alpha^+\left[1 - \frac{1+3\tilde{g}^2}{2(4\pi f)^2}\frac{3}{2}m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + c_f^+(\mu)m_\pi^2\right]}.$$

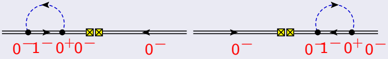
Bag-parameters

Operator bosonization receives new contributions

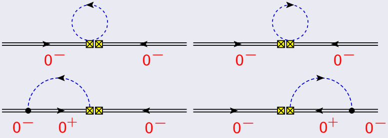
$$\begin{aligned}\widetilde{O}_1 &= \sum_X \beta_{1X} \mathrm{Tr}\left[\left(\xi \overline{H}^Q\right)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[\left(\xi H^{\bar{Q}}\right)_q \gamma^\mu (1-\gamma_5) X\right] \\ &\quad + \beta'_{1X}\left\{ \mathrm{Tr}\left[\left(\xi \overline{H}^Q\right)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q \gamma^\mu (1-\gamma_5) X\right] + \mathrm{h.c.}\right\} \\ &\quad + \beta''_{1X} \mathrm{Tr}\left[\left(\xi \overline{S}^Q\right)_q \gamma_\mu (1-\gamma_5) X\right] \mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q \gamma^\mu (1-\gamma_5) X\right].\end{aligned}$$

Similarly for $\widetilde{O}_2$ and $\widetilde{O}_4$.

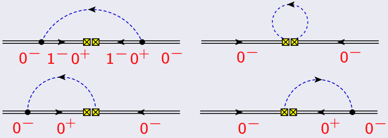
Chiral loop corrections



Wavefunction renormalization



Factorisable contributions cancel against weak vertex corrections



Nonfactorisable contributions

Bag-parameters

Typical loop integrals involving the new Δ_S scale probe large pion momenta in the chiral limit. The two scales (m_π and Δ_S) do not decouple as in the case of $\hat{f}$. We attempt an expansion:

$$
\begin{aligned}
& -2(4\pi)^2 v_\mu v_\nu \times i\mu^\epsilon \int \frac{d^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{(p^2 - m_\pi^2)(\Delta_S - v \cdot p)^2} \\
= \quad & -\frac{2(4\pi^2)}{\Delta_S^2} v_\mu v_\nu \left[i\mu^\epsilon \int \frac{d^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{p^2 - m_\pi^2} + \mathcal{O}(1/\Delta_S^2) \right] \\
\to \quad & -\frac{m_\pi^4}{2\Delta_S^2} \log \frac{m_\pi^2}{\mu^2} + \ldots ,
\end{aligned}
$$

- Expansion around the decoupling limit of the positive parity states.
- Series of local operators with Δ_{SH} dependent prefactors.
- Effective counter terms of a theory with no positive parity mesons.
- Like for the decay constants, the relevant chiral expansion of the bag-parameters is the one derived in the $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ theory.

$$
\widetilde{B}_{1q} = \widetilde{B}_1^{\mathrm{Tree}} \left[1 - \frac{1-3g^2}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{B_1}(\mu) m_\pi^2 \right]
$$

$$
\widetilde{B}_{2,4q} = \widetilde{B}_{2,4}^{\mathrm{Tree}} \left[1 + \frac{3g^2 Y \mp 1}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{B_{2,4}}(\mu) m_\pi^2 \right]
$$

Conclusions

- We revisited the computation of the $B_{s,d} - \overline{B}_{s,d}$ mixing amplitudes in the framework of HMχPT.
- We considered chiral logarithmic corrections to the SM as well as SUSY bag parameters.
- We studied the impact of the near scalar mesons to the predictions derived in HMχPT in which these contributions were previously ignored.
  - Their contributions are **competitive in size** and thus they cannot be ignored nor separated from the discussion of the **kaon and/or η-meson loops**.
  - They **do not spoil the pion logarithmic corrections to the decay constants and bag-parameters**. The formulae derived in HMχPT can still (and should) be used to guide the chiral extrapolations of the lattice results, albeit for the pion masses lighter than Δ_S.
- **Side-result:** chiral logarithmic corrections to the scalar meson decay constants are the same as for the pseudoscalar ones, modulo replacement $g \to \widetilde{g}$.

Perspective

- Similar conclusions regarding the impact of the $1/2^+$-mesons on leading chiral logarithms have been reached in other processes
  - Effective HMχPT couplings between heavy and light mesons (g,h,$\tilde{g}$)
    S. Fajfer and J.K., Phys.Rev.D74 (2006) 074023 [hep-ph/0606278]
  - Isgur-Wise functions in semileptonic B to D meson decays (ξ,$\tau_{1/2}$,$\tilde{\xi}$)
    J.O.Eeg, S. Fajfer and J.K., Accepted for publication in JHEP [arXiv:0705.4567]

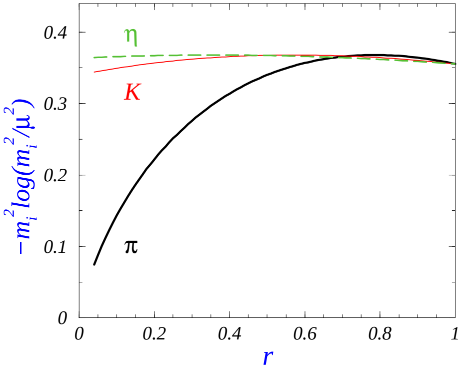
Appendix

Chiral extrapolation

$$m_\pi^2 = \frac{8\lambda_0 m_s}{f^2} r,$$

$$m_K^2 = \frac{8\lambda_0 m_s}{f^2} \frac{r+1}{2},$$

$$m_\eta^2 = \frac{8\lambda_0 m_s}{f^2} \frac{r+2}{3},$$



Chiral extrapolation

Extracting the effective coupling dependence on the pseudo-Goldstone masses.

$$\frac{1}{m_j^2}\frac{\mathrm{d}g^{\mathrm{eff.}}_{P_a^* P_b \pi^i}}{\mathrm{d}\log m_j^2} = \frac{\mathbf{g}}{(4\pi f)^2} \times \left\{ \frac{\lambda^j_{ac}\lambda^j_{ca} + \lambda^j_{bc}\lambda^j_{cb}}{2}\left[-3\mathbf{g}^2 - \mathbf{h}^2\left(1 - \frac{6\Delta^2_{SH}}{m_j^2}\right)\right] + \frac{\lambda^j_{ac}\lambda^i_{cd}\lambda^j_{db}}{\lambda^i_{ab}}\left[\mathbf{g}^2 - \mathbf{h}^2\frac{\tilde{\mathbf{g}}}{\mathbf{g}}\left(1 - \frac{6\Delta^2_{SH}}{m_j^2}\right)\right]\right\}$$

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$$\times \left\{ \frac{\lambda^j_{ac}\lambda^j_{ca} + \lambda^j_{bc}\lambda^j_{cb}}{2}\left[-3\mathbf{g}^2 - \mathbf{h}^2\left(1 - \frac{6\Delta^2_{SH}}{m_j^2}\right)\right]\right.$$
$$\left. + \frac{\lambda^j_{ac}\lambda^i_{cd}\lambda^j_{db}}{\lambda^i_{ab}}\left[\mathbf{g}^2 - \mathbf{h}^2\frac{\tilde{\mathbf{g}}}{\mathbf{g}}\left(1 - \frac{6\Delta^2_{SH}}{m_j^2}\right)\right]\right\}$$

Large Δ_{SH} dependence.

Chiral extrapolation

$$\mu^{(4-D)} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^\mu q^\nu}{(q^2 - m^2)(v \cdot q - \Delta)}$$

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Loop integral expansion in $1/\Delta_{SH}$

$$\Rightarrow \quad \mu^{(4-D)} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^\mu q^\nu}{(q^2 - m^2)} \frac{-1}{\Delta} (1 + \frac{q \cdot v}{\Delta} + \ldots)$$

(All even orders vanish.)

$$\left(1 - \frac{6\Delta_{SH}^2}{m_j^2}\right) \Rightarrow \frac{m_j^2}{4\Delta_{SH}^2}$$

- Expansion around the decoupling limit of the positive parity states.
- Series of local operators with Δ_{SH} dependent prefactors.
- Effective counter terms of a theory with no positive parity mesons.

Restrictions on bosonized operators matching to HQET

Contraction of Lorentz indices and HQET parity conservation requires the same X to appear in both traces of a summation term. Any insertions of $\not{v}$ can be absorbed via $\not{v}H = H$, while any nonfactorisable contribution with a single trace over Dirac matrices can be reduced to this form by using the 4×4 matrix identity

$$\begin{aligned} 4\mathrm{Tr}(AB) &= \mathrm{Tr}(A)\mathrm{Tr}(B) + \mathrm{Tr}(\gamma_5 A)\mathrm{Tr}(\gamma_5 B) + \mathrm{Tr}(A\gamma_\mu)\mathrm{Tr}(\gamma^\mu B) \\ &+ \mathrm{Tr}(A\gamma_\mu\gamma_5)\mathrm{Tr}(\gamma_5\gamma^\mu B) + 1/2\mathrm{Tr}(A\sigma_{\mu\nu})\mathrm{Tr}(\sigma^{\mu\nu} B). \end{aligned}$$