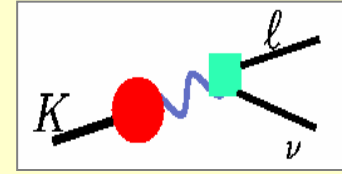
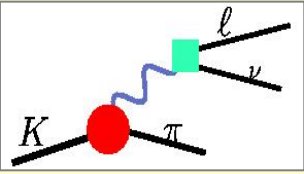


$K_{\ell 3}$ and $K_{\ell 2}$ decays: V_{us}



Federico Mescia

Laboratori Nazionali di Frascati

*on behalf of the
FlaviaNet Kaon Working Group*



V_{us} and the CKM Unitarity:

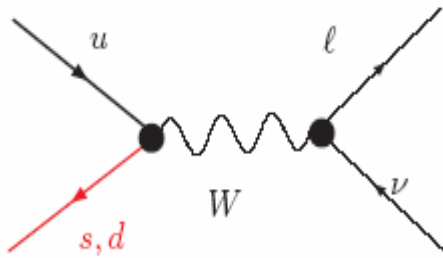
- Quark/Lepton Universality $\Rightarrow |V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 - 1 = 0.9990(8)$
- Lepton Universality $\Rightarrow K_{e2}$

Results from the FlaviaNet KWG

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

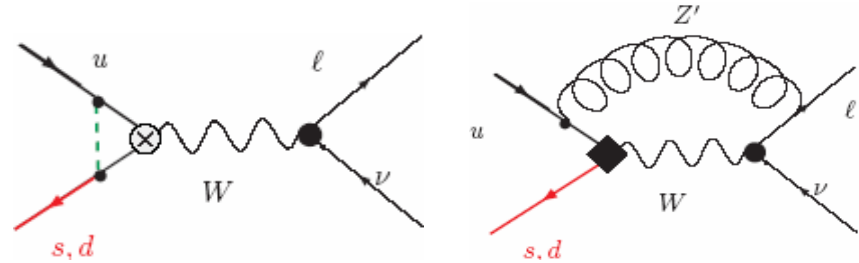
Universality of Weak coupling - $G_F = (g_w/M_w)^2$

Standard Model

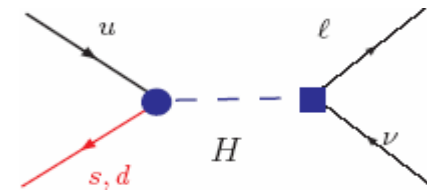


$$\propto G_F^2 |V_{uq}|^2$$

Susy, Little Higgs, Extra Dimension



$$\varepsilon \sim 10^{-3}$$



$$\varepsilon \sim 10^{-2}$$

$$\propto G_F^2 \varepsilon^{CKM}$$

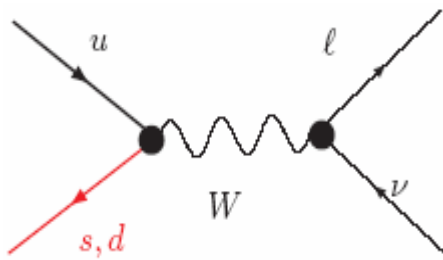
$$|V_{ud}|^2 + |V_{us}|^2 \neq 1$$

CKM Unitarity breaking

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

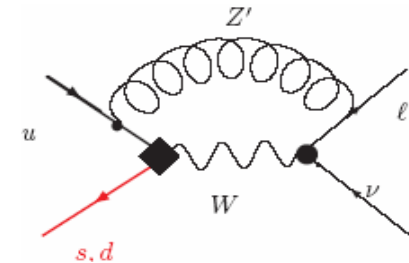
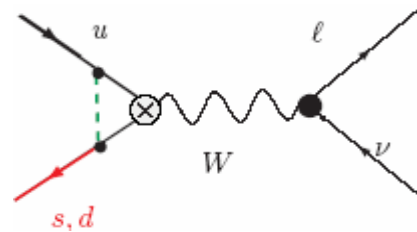
Universality of Weak coupling - $G_F = (g_w/M_w)^2$

Standard Model

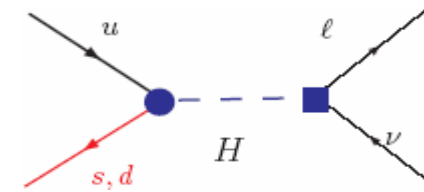


$$\propto G_F^2 |V_{uq}|^2$$

Susy, Little Higgs, Extra Dimension



$$\varepsilon \sim 10^{-3}$$



$$\varepsilon \sim 10^{-2}$$

$$\propto G_F^2 \varepsilon^{CKM}$$

$$G_F^2 \times \left(|V_{ud}|^2 + |V_{us}|^2 \right) \neq 1 \times G_F^2$$

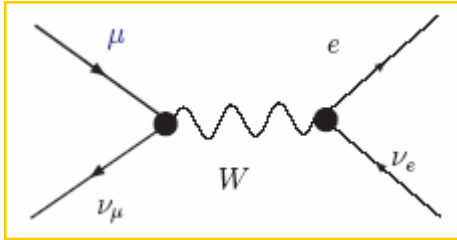
CKM Unitarity breaking

Gauge Universality Breaking

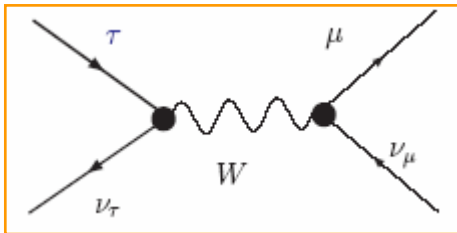
- significantly different from the Unitarity Triangle Test (overall normalisation arbitrary)

G_F – Universality

$$G_{CKM}^2 \equiv G_F^2 \times \left(|V_{ud}|^2 + |V_{us}|^2 \right) \neq G_F^2$$



$$G_F = 1.166371(6) \times 10^{-5} \text{ GeV}^{-2}$$

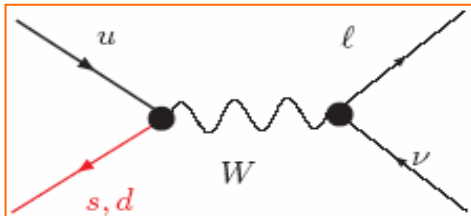


$$G_\tau = 1.1678(26) \times 10^{-5} \text{ GeV}^{-2}$$

$\alpha + M_W + S_W$
[e.w. precision tests]



$$G_{e.w.} = 1.1655(12) \times 10^{-5} \text{ GeV}^{-2}$$




$$G_{CKM} = 1.1658(04) \times 10^{-5} \text{ GeV}^{-2}$$

V_{us} at 0.5%


- V_{us} below 1% makes CKM unitarity competitive to Electro-Weak Precision Test

- V_{us} below 1% makes the CKM unitarity test competitive to Electro-Weak Precision Test

- *In the spirit of the HFAG for B physics, a joint experimental and theory working group has been set up for Kaon physics:*



Working Group on Precise SM Tests in K Decays



Kaon WG home
FlaviaNet home

Master Formulae
Branching Ratios
Lifetimes
Form Factors

Radiative
Corrections
SU(3) Breaking
Form Factors


Contacts

ISTRA+:
[Oleg Yushchenko \(Protvino\)](#)
[Vladimir Obraztsov \(Protvino\)](#)

KLOE:
[Matthew Moulson \(Frascati\) web contact](#)
[Patrizia De Simone \(Frascati\)](#)

KTeV:
[Sasha Glazov \(DESY\)](#)

NA48:
[Rainer Wanke \(Uni. Mainz\)](#)
[Michele Veltri \(Uni. Urbino\)](#)
[Mauro Piccini \(CERN\)](#)



<http://www.lnf.infn.it/wg/vus/>

News
Talks
Acknowledgements

Honorary chair: **Paolo Franzini (LNF)** Coordinators: **Mario Antonelli (LNF)** and **Gino Isidori (LNF)**

- *Measurements* (BR's + Lifetime's + Form Factors) and *Theory inputs* have not trivial correlations, which is crucial to take into account!!!

Averages from the FlaviaNet KWG

- **Vector Weak Universality** \Rightarrow

$$V_{us} f_+(0) = 0.21668(45) \Rightarrow V_{us}^{KI3} = 0.2254(13) \Rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9990(8)$$

- **Vector Lepton Universality** \Rightarrow

$$G_F^\mu / G_F^e = 1.0042(50) \quad [\Rightarrow 0.5\%]$$

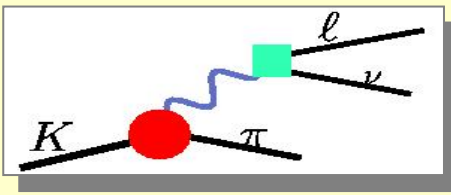


talk by Spadaro

Mind

- **Theory:** QCD uncertainties need much better control!!
- **Experiment:** poor agreement on Form Factors measurements

suggestion for the new generation of *Lattice* and *P326* experiments



V_{us} from K_{l3} decays

$$\Gamma(K_{l3}(\gamma)) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{KI}(\lambda) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{KI}^{EM})$$

with $K = K^+, K^0$; $l = e, \mu$ and $C_K^2 = 1/2$ for K^+ , 1 for K^0

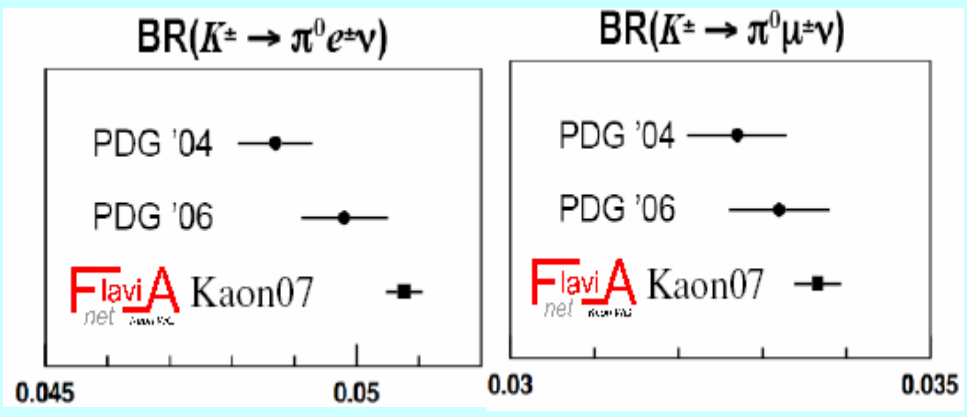
Inputs from experiment:

$\Gamma(K_{l3}(\gamma))$ **Branching ratios** with well determined treatment of radiative decays; **lifetimes**

$I_{KI}(\lambda)$ Phase space integral: λ s parameterize form factor dependence on t :
K_{e3} : only λ_+ (or λ_+, λ_+'')
K_{μ3} : need λ_+ and λ_0

Several new results in the last 2 years
KTeV KLOE NA48 ISTRA+ E865

Evolution of K^\pm BRs



➔ talk by De Lucia & Goudzovski

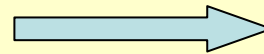
V_{us} from $K_{\ell 3}$ decays

$$\Gamma(K_{l3}(\gamma)) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{Kl}(\lambda) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{Kl}^{EM})$$

with $K = K^+, K^0$; $l = e, \mu$ and $C_K^2 = 1/2$ for K^+ , 1 for K^0

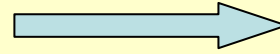
Inputs from theory:

S_{EW} Universal short distance
EW correction (1.0232)



Marciano & Sirlin

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element
at zero momentum transfer ($t=0$)



Lattice (+CHPT)

Delicate Point

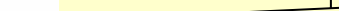
$\Delta_K^{SU(2)}$ Form factor correction for strong
SU(2) breaking

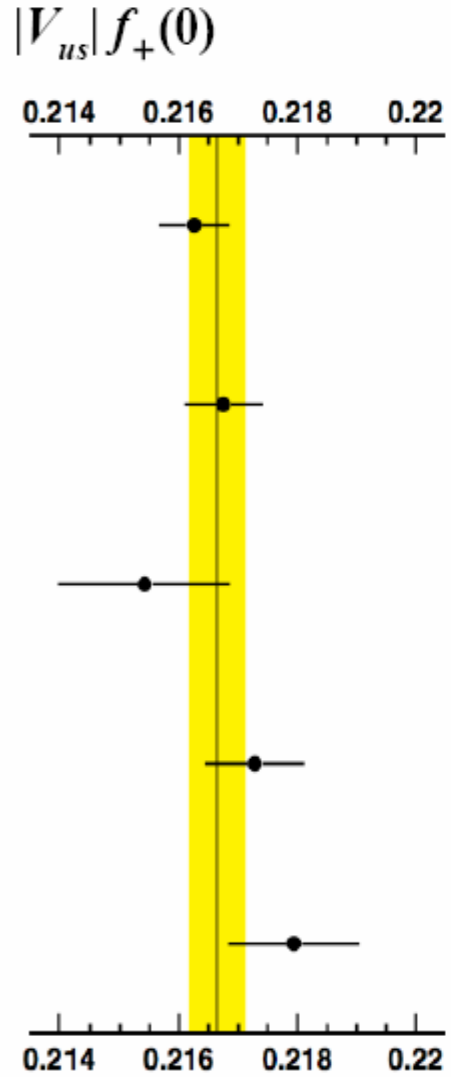


CHPT

Knecht et al. '00
Cirigliano et al. '02-'04
Andre '04; Gatti '05
Moussalam et al. '06
Neufeld prelim. '07

Δ_{Kl}^{EM} Long distance EM effects





		% err	Approx. contrib. to % err from:			
			BR	τ	Δ	Int
$K_L e3$	0.21625(60)	0.28	0.09	0.19	0.15	0.09
$K_L \mu3$	0.21675(66)	0.31	0.10	0.18	0.15	0.15
$K_S e3$	0.21542(144)	0.67	0.65	0.03	0.15	0.09
$K^\pm e3$	0.21728(84)	0.39	0.26	0.09	0.26	0.09
$K^\pm \mu3$	0.21794(111)	0.51	0.40	0.09	0.26	0.15

Average: $|V_{us}|f_+(0) = 0.21664(48)$ $\chi^2/ndf = 2.88/4$ (58%)



$\Delta^{SU(2)}_{exp} = 2.86(39)\%$

→ success of CHPT calculations

$[\Delta^{SU(2)}_{th} = 2.31(22)\%]$

At this point ... using inputs from Br , I_K and δ the quantity $\left(V_{us} \cdot f_+^{K^0\pi^-}(0) \right)$ is measured with small **th.** and **exp.** uncertainties.


$$\left(V_{us} \cdot f_+^{K^0\pi^-} \right)_{\text{FlaviaNet}}^{\text{Exp-Average}} = 0.21668(45) \Rightarrow \Rightarrow \sigma_{rel} \sim 0.21\%$$

Now to estimate V_{us} we need $f_+(0) \equiv f_+^{K^0\pi^-}(0)$

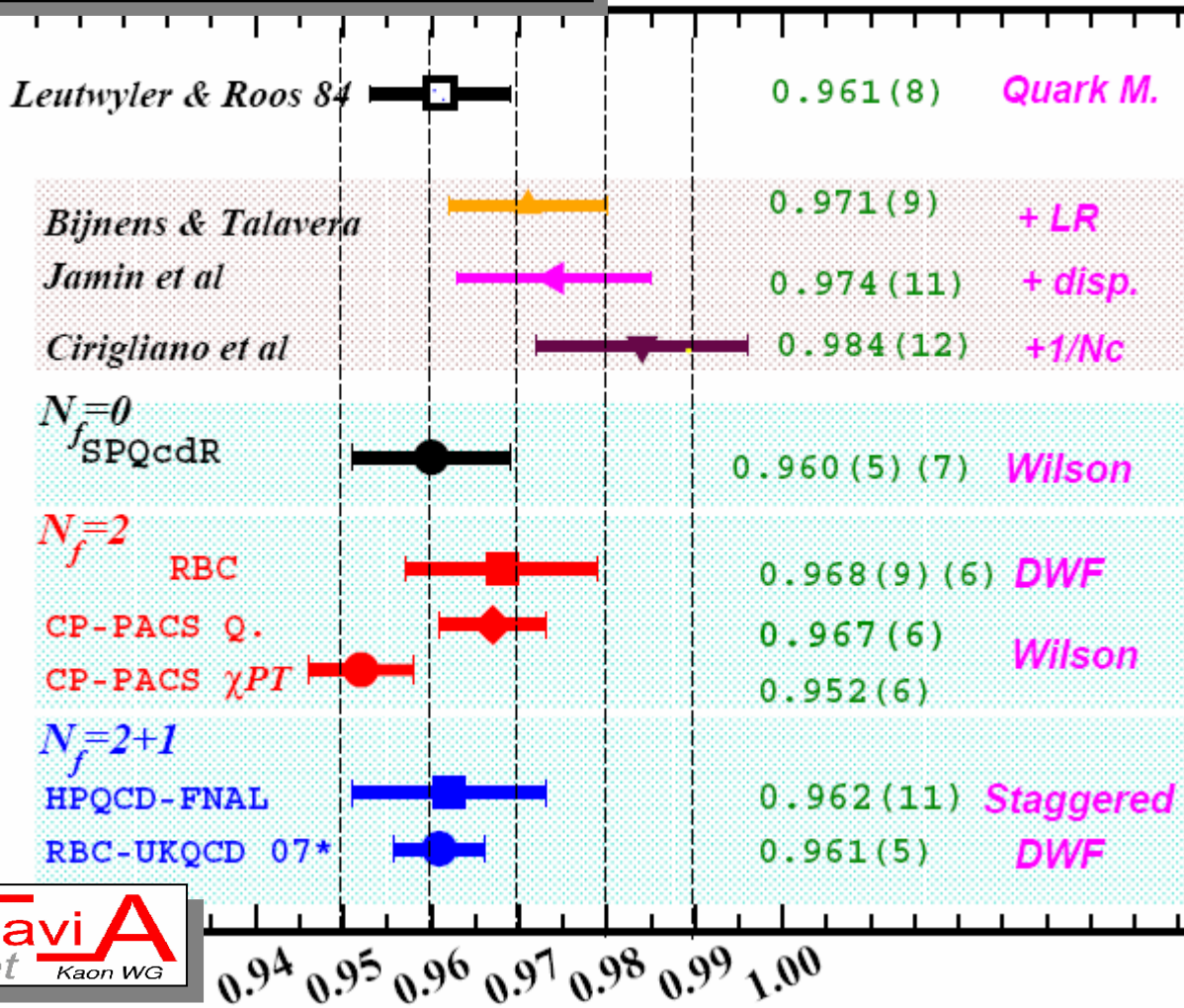
Experimental information diluted by the large theoretical error of $f_+(0)$ $\Rightarrow \sigma_{rel} \sim 1\%$

$f_+^{K^0\pi^-}(\mathbf{0})$ - Evaluations

Q.M.

$-\chi PT+CT-$

- Lattice -



Many Theoretical Approaches

- First estimate by LR in 1984 $\Rightarrow f_+(\mathbf{0})=0.961(8)$

- Present values agree with each other at 1-2% [$f_+(\mathbf{0})\sim 0.96-0.98$]

- Next progress rely on Lattice approach

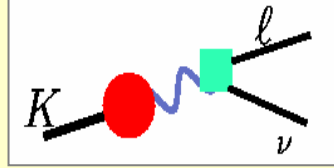
- Many lattice estimates already available, typically at $m_\pi \geq 500$ MeV \Leftrightarrow large chiral uncertainties

- Lattice systematically smaller than χPT -inspired values



- Encouraging result from UKQCD-RBC: $N_f=2+1$, DWF, $m_\pi \geq 300$ MeV
 $f_+(\mathbf{0})=0.961(5) \Rightarrow \sigma \sim 0.5\%$

V_{us} from $K_{\ell 2}$ decays



$$\frac{\Gamma(K^{\pm} \rightarrow \mu^{\pm} \nu(\gamma))}{\Gamma(\pi^{\pm} \rightarrow \mu^{\pm} \nu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_{\mu}^2/m_K^2)^2}{|V_{ud}|^2 f_{\pi}^2 m_{\pi} (1 - m_{\mu}^2/m_{\pi}^2)^2} \times 0.9930(35)$$

$[1 + \delta_{\text{e.m.}}]$

KLOE
PLB 636 (2006)

$$\text{BR}(K^+ \rightarrow \mu^+ \nu(\gamma)) = 0.6366(17)$$

• No competition from non-Lattice approaches

no Ademollo-Gatto protection

$$f_K/f_{\pi} = 1.208(2)^{(+7}_{-14)}$$

MILC '05

reached
rel. err. $\sim 0.7\%$

$$f_K/f_{\pi} = 1.189(7)$$

UKQCD+HPQCD '07

$$N_f = (2+1)_{\text{stag}} \text{ [MILC]}$$

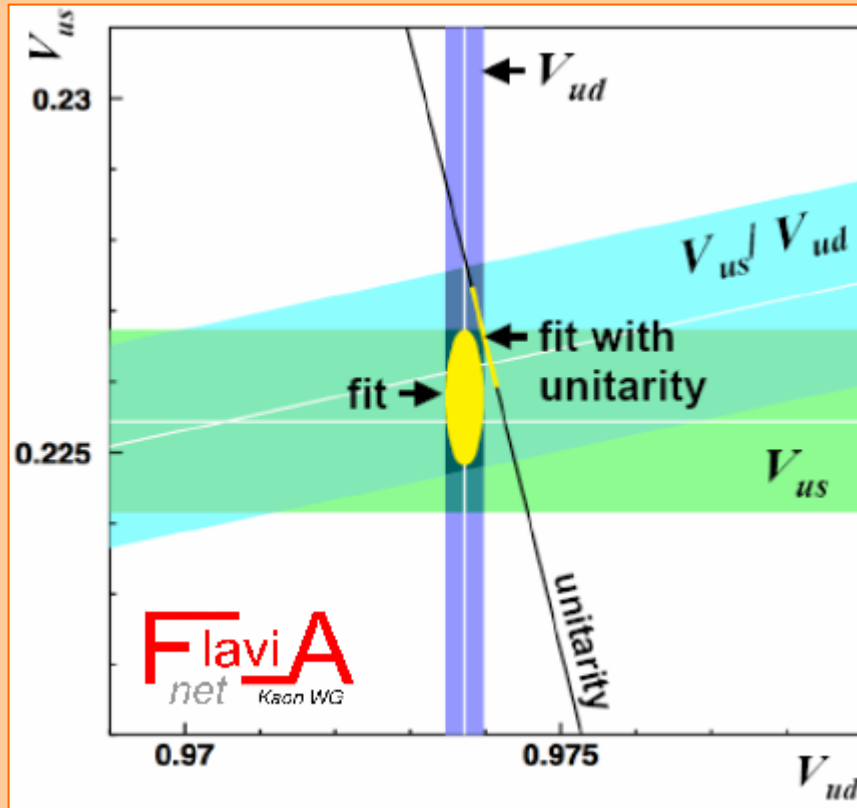
$$V_{us}/V_{ud} = 0.2323(15)$$

Reminder:

Recent unquenched Lattice results are still under scrutiny from other groups

$f_+(0)=0.961(5)$ from UKQCD/RBC'07
 $|V_{us}|=0.2254(13)$ from Kl3

$f_K/f_\pi=1.189(7)$ from HPQCD'07
 $|V_{us}/V_{ud}|=0.2323(15)$ from Kl2



Fit results, no constraint:

$$V_{ud} = 0.97372(26)$$

$$V_{us} = 0.2258(10)$$

$$\chi^2/\text{ndf} = 0.142/1 \text{ (70\%)}$$

Fit results, unitarity constraint:

$$V_{ud} = 0.97398(17)$$

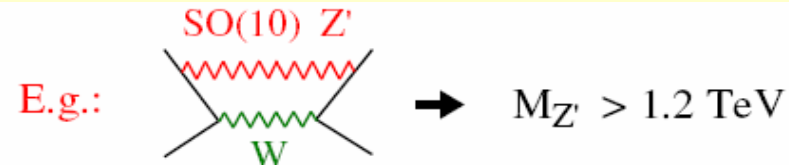
$$V_{us} = 0.2266(7)$$

$$\chi^2/\text{ndf} = 1.92/2 \text{ (38\%)}$$

Agreement with unitarity 0.9σ

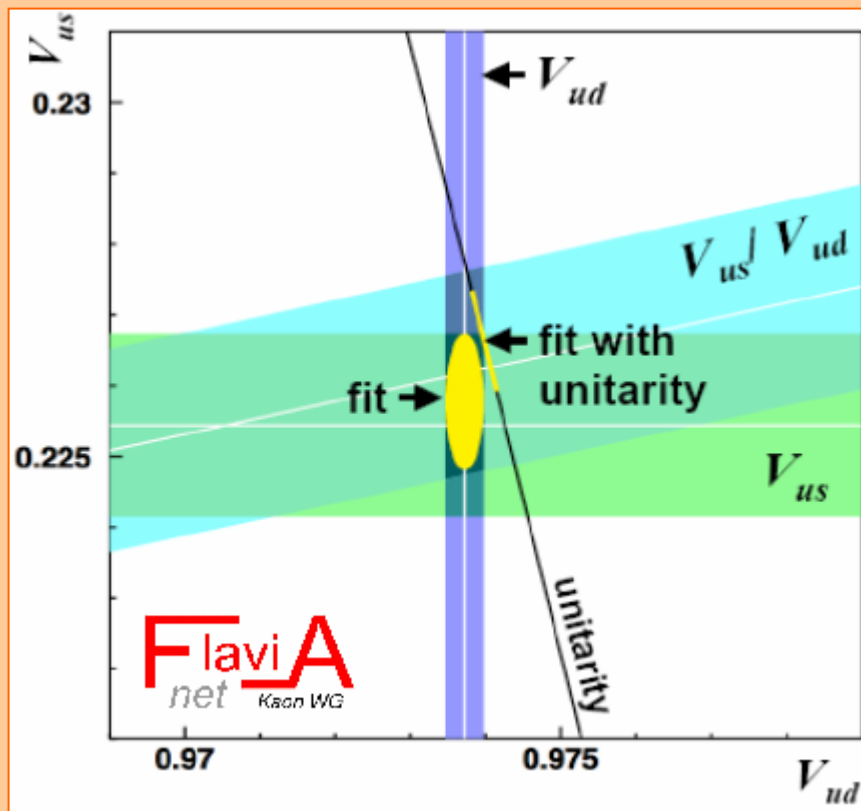
$$|V_{ud}|^2 + |V_{us}|^2 = 0.9990(8)$$

This is a highly non-trivial constraint for NP models...



★ $f_+(0)=0.961(5)$ from UKQCD/RBC'07
 $|V_{us}|=0.2254(13)$ from Kl3

★ $f_K/f_\pi=1.189(7)$ from HPQCD'07
 $|V_{us}/V_{ud}|=0.2323(15)$ from Kl2



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Agreement with unitarity 0.9σ

★ **Reminder:**
Recent unquenched Lattice results are still under scrutiny from other groups

- QCD Test of $\frac{f_K}{f_\pi} / f_+(0)$ by the Callan-Treiman theorem



improved bound on right-handed current and scalar densities

→ Bernard, Oertel, Passemar & Stern

→ Hou - Isidori & Paradisi,

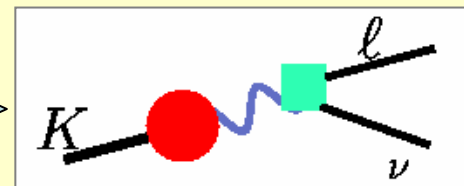
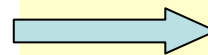
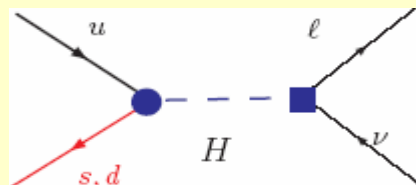
- Lepton Universality Test ⇒

$$B(K \rightarrow e\nu) / B(K \rightarrow \mu\nu)$$

Masiero, Paradisi & Petronzio
large $\tan\beta$ & LFV

Key Point:

scalar operators enhanced by elicity suppression



- Callan-Treiman soft-pion theorem \Rightarrow

$$\left(\frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right) = f_0 (m_K^2 - m_\pi^2) + \delta_{SU(2)}^{CT}$$

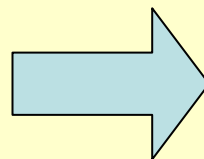
$$1 + \lambda_0^{\text{exp}} \frac{m_K^2 - m_\pi^2}{m_\pi^2} + \Delta$$

*experimental
input*

*disp. relation
+ th. input*

Bernard, Oertel,
Passemar & Stern

*consistency test
between **experiments**
and **Lattice QCD***



- Callan-Treiman soft-pion theorem \Rightarrow

$$\left(\frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right) = 1 + \lambda_0^{\text{exp}} \frac{m_K^2 - m_\pi^2}{m_\pi^2} + \Delta$$

*disp. relation
+ th. input*

Bernard, Oertel, Passemar & Stern

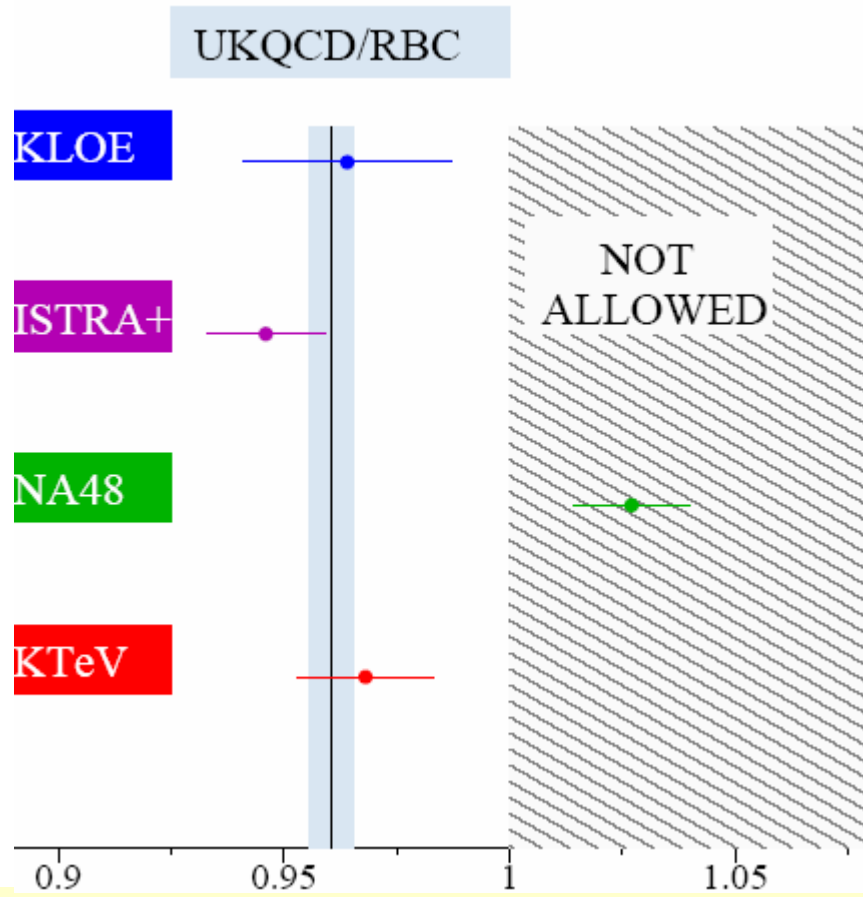
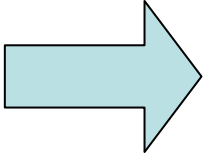
$f_K/f_\pi = 1.189(7)$
from HPQCD'07

+

exp. input

$f_+(0)_{CT}^{\text{exp}}$

*consistency test
between experiments
and Lattice QCD*



- Callan-Treiman soft-pion theorem \Rightarrow

$$\left(\frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right) = 1 + \lambda_0^{\text{exp}} \frac{m_K^2 - m_\pi^2}{m_\pi^2} + \Delta$$

*disp. relation
+ th. input*

Bernard, Oertel, Passemar & Stern

*consistency test
between **experiments**
and **Lattice QCD***

*promising in view of future lattice and exp. data
improved determination of hadronic parameters
useful to test/constraint NP models on*

$$\frac{B(K \rightarrow lv)}{B(\pi \rightarrow lv)} \times \frac{B(n \rightarrow plv)}{B(K \rightarrow \pi lv)} \propto \left(\frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right)^2$$

1. Higgs contributions on scalar operators \Rightarrow

MFV @ large $\tan\beta$
Isidori & Paradisi '06,

$$\frac{B(K \rightarrow lv)}{B(\pi \rightarrow lv)} \times \frac{B(n \rightarrow plv)}{B(K \rightarrow \pi lv)} = \left(\frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right)^2 \times \left(1 - \frac{m_K^2}{m_{H^+}^2} \frac{\tan^2 \beta}{1 + \varepsilon_0 \tan \beta} \right)^2 \frac{(\text{known f.s})}{I_{KI}(\lambda_{+,0})}$$

$\sim 0.3\%$

$\sim 0.2\%$ @ large $\tan\beta$

competitive to $\text{Br}(B \rightarrow \tau\nu)$ at 30% uncertainties

2. Right-Handed Current \Rightarrow

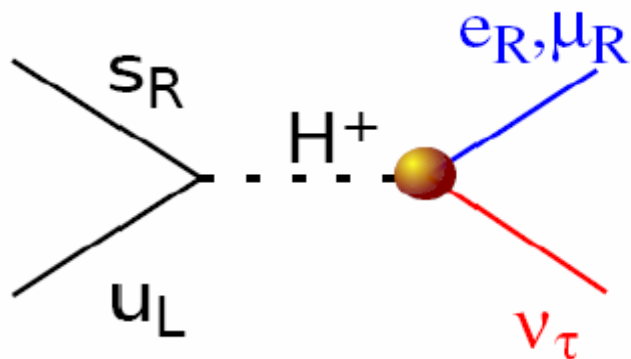
Bernard, Oertel, Passemar & Stern

talk by Oertel on "BSM session"

Lepton Universality Test \Rightarrow $B(K \rightarrow e\nu) / B(K \rightarrow \mu\nu)$

The most popular/exciting NP scenario which could affect c.c. semileptonic decays is the possibility of LFV effects modifying the μ/e ratio in K_{l2}

$$R_K = (1 - \Delta r_K^{e-\mu}) = \frac{\sum_i K \rightarrow e\nu_i}{\sum_i K \rightarrow \mu\nu_i} \simeq \frac{\Gamma_{SM}(K \rightarrow e\nu_e) + \Gamma(K \rightarrow e\nu_\tau)}{\Gamma_{SM}(K \rightarrow \mu\nu_\mu)}$$



$$eH^\pm \nu_\tau \rightarrow \frac{g_2}{\sqrt{2}} \frac{m_\tau}{M_W} \Delta_R^{31} \tan^2 \beta$$

$$\Delta_R^{31} \sim \frac{\alpha_2}{4\pi} \delta_{RR}^{31}$$

$$\Delta_R^{31} \sim 5 \cdot 10^{-4} \quad t_\beta = 40 \quad M_{H^\pm} = 500 \text{ GeV}$$

$$\Delta r_K^{e-\mu} \simeq \left(\frac{m_K^4}{M_{H^\pm}^4} \right) \left(\frac{m_\tau^2}{m_e^2} \right) |\Delta_R^{31}|^2 \tan^6 \beta \approx 10^{-2}$$

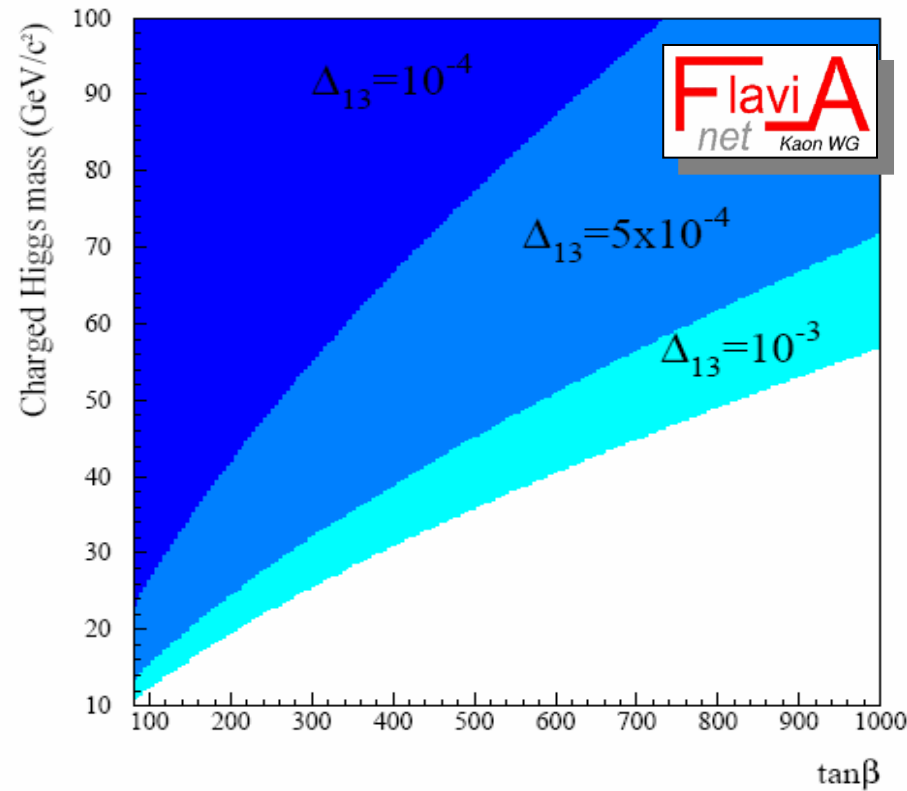
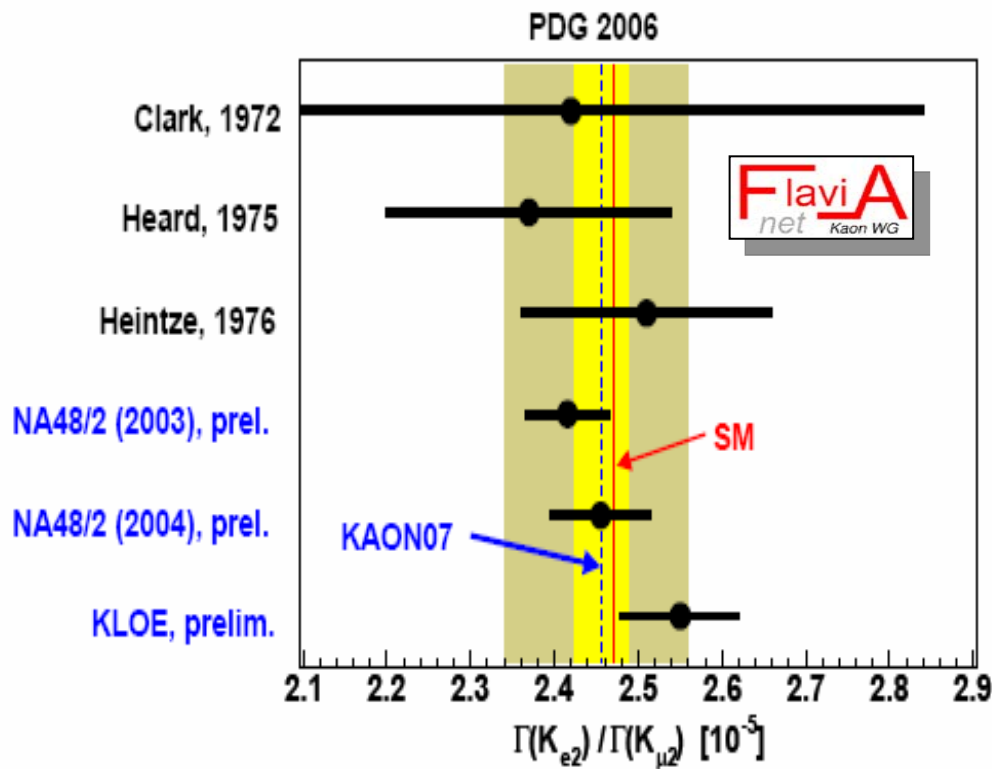
Masiero Paradisi Petronzio '06

key ingredients
for visible effects
in SUSY:

- Large $\tan \beta$, $M_H < 1 \text{ TeV}$
- Large LFV slepton mixings, $\delta_{3j} \sim \mathcal{O}(1)$, ($m_{SUSY} \geq 1 \text{ TeV}$)

Limit on LFV in H⁺ coupling ⇒

$$B(K \rightarrow e\nu) / B(K \rightarrow \mu\nu)$$



	$\Gamma(K_{e2}) / \Gamma(K_{\mu2}) [10^{-5}]$
PDG 2006 [1]	2.45 ± 0.11
NA48/2 prel. ('03) [22]	$2.416 \pm 0.043 \pm 0.024$
★ NA48/2 prel. ('04) [23]	$2.455 \pm 0.045 \pm 0.041$
★ KLOE prel. [24]	$2.55 \pm 0.05 \pm 0.05$
SM prediction	2.472 ± 0.001

Recent Results at $O(1-2\%)$

⇒ 0.3% from P326 in the fall of 2007

talk by Fantecchi on "BSM session"

★ @Kaon07



$$\Gamma(K_{e2}) / \Gamma(K_{\mu2}) = (2.457 \pm 0.032) \times 10^{-5}$$

Lessons from Kaon Physics

1. V_{us} at 0.1% precision not impossible

⇒ significant SM test competitive with EWPT

2. To reach this goal a collaboration between theorist and experimentalist is essential

⇒ *e.m corrections*

⇒ strong *SU(2) corrections*

⇒ chiral extrapolation of lattice results

⇒ correlated data analysis

