Decays of Charmed Hadrons at Belle
V.Balagura (ITEP), EPS HEP 2007, July 19, 2007

1. Measurement of $\boldsymbol{B}\left(\boldsymbol{D}_{s}^{+} \rightarrow \boldsymbol{\mu}^{+} \boldsymbol{\nu}_{\mu}\right)$.
2. (a) Observation of $\boldsymbol{D}_{\boldsymbol{s} \mathbf{1}}(\mathbf{2 5 3 6})^{+} \rightarrow \boldsymbol{D}^{+} \boldsymbol{\pi}^{-} \boldsymbol{K}^{+}$,
(b) angular decomposition of S- and D-waves in $\boldsymbol{D}_{s 1}(\mathbf{2 5 3 6})^{+} \rightarrow \boldsymbol{D}^{*+} \boldsymbol{K}_{S}^{0}$.

$$
\underline{D_{s}^{+} \rightarrow \mu^{+} \nu_{\mu}}
$$

Latice $\mathrm{QCD} \Rightarrow f_{B_{d}}$ and $f_{B_{s}} \Rightarrow \underline{B_{d}}$ and $B_{s}$ mixing constraints on CKM unitarity triangle. Latice QCD can be checked and improved in the charm sector using measured $f_{D}$ and $f_{D_{s}^{+}}$.

$$
\Gamma\left(D_{s}^{+} \rightarrow l^{+} \nu_{l}\right)=\frac{G_{F}^{2}}{8 \pi} f_{D_{s}^{+}}^{2} m_{l}^{2} M_{D_{s}^{+}}\left(1-\frac{m_{l}^{2}}{M_{D_{s}^{+}}^{2}}\right)^{2}\left|V_{c s}\right|^{2}
$$



Experimentally $D_{s}^{+} \rightarrow \mu^{+} \nu_{\mu}$ is the best mode

$$
\Gamma\left(D_{s}^{+} \rightarrow e^{+} \nu_{e}\right): \Gamma\left(D_{s}^{+} \rightarrow \mu^{+} \nu_{\mu}\right): \Gamma\left(D_{s}^{+} \rightarrow \tau^{+} \nu_{\tau}\right) \approx 2.5 \cdot 10^{-5}: 1.0: 9.7
$$

$$
\begin{gathered}
\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow \boldsymbol{D}_{s}^{*+}(\boldsymbol{D} \boldsymbol{K} \boldsymbol{X}), \boldsymbol{D}_{\boldsymbol{s}}^{*+} \rightarrow \boldsymbol{D}_{\boldsymbol{s}}^{+} \gamma, \quad \boldsymbol{X}=\boldsymbol{k} \boldsymbol{\pi}^{ \pm}{ }_{\text {or }}\left((\boldsymbol{k}-\mathbf{1}) \pi^{ \pm}\right) \gamma \\
\left.\boldsymbol{k} \leq \mathbf{5} \text {. Everything in red (DKX and } \gamma \text { from } D_{s}^{*+}\right) \text { is fully reconstucted, } \\
D_{s}^{+} \text {peak is searched for in the recoil mass spectrum. } \\
\hline
\end{gathered}
$$

$M_{\text {rec }}((D K X) \gamma)$, right and wrong sign



$$
\begin{gathered}
D^{0,+} \rightarrow K_{S}^{0,-} n \pi \pi^{0, \pm}, n=1,2,3 \\
\left|M_{r e c}(D K X)-M D_{S}^{*+}\right|<150 \mathrm{MeV} / c^{2} \\
p_{D}^{*}>2, p_{D_{S}^{*}}^{*}>3, p_{K}^{*}<2 \mathrm{GeV} / \mathrm{c}^{2}
\end{gathered}
$$

$\mu^{+}$is also reconstructed

$1.94<M_{\text {rec }}(D K X \gamma)<2.00 \mathrm{GeV} / c^{2}$ $p_{\mu}>0.5 \mathrm{GeV} / \mathrm{c}^{2}$
$548 \mathrm{fb}^{-1}$ of data.

Signal shape and efficiency depend on number of prompt particles $n_{X}=k+3=3, \ldots 8$ $\Rightarrow$ number of $D_{s}^{+}$'s $N$ is counted in $n_{X}$ bins, $N=\sum N_{n_{X}^{\text {rec }}}$.
Prompt $\pi^{ \pm}$or $\gamma$ may be not reconstructed $\Rightarrow k_{\text {rec }} \leq k_{\text {true }}, N_{n_{X}^{\text {rec }}}=\sum M_{n_{X}^{\text {rec }}, n_{X}^{\text {true }}} \times N_{n_{X}^{\text {true }}}$ $\Rightarrow$ cross feeds $M$ between $n_{X}$ bins is obtained from MC and is taken into account in the fit.

Fitted background is subtracted

background - from WS
or from generic MC to cross check.

$$
32100 \pm 870 \pm 1210 \mathrm{ev} .
$$


background - by exchanging $\mu^{+} \rightarrow e^{+}$, ratio of $\mu^{+} / e^{+}$efficiencies - from MC.

$$
169 \pm 16 \pm 8 \mathrm{ev} .
$$

$$
\underline{D_{s}^{+} \rightarrow \mu^{+} \nu_{\mu}}
$$

$\frac{B\left(D_{s}^{+} \rightarrow \mu \nu_{\mu}\right), \times 10^{-3} f_{D_{s}^{+}}, \mathrm{MeV}}{\text { Preliminary Belle results }}$

| $6.44 \pm 0.76 \pm 0.52$ | $275 \pm 16 \pm 12$ |
| :--- | :--- |

Previous results
PDG'06

$$
6.1 \pm 1.9
$$

$294 \pm 27$
BaBar

$$
6.74 \pm 0.83 \pm 0.26 \pm 0.66_{\phi \pi} \quad 283 \pm 17 \pm 7 \pm 14_{\phi \pi} \quad \text { PRL 98, } 121801 \text { (2007) }
$$ (last error from $B\left(D_{s}^{+} \rightarrow \phi \pi^{+}\right)$)

CLEO-c $\quad 5.94 \pm 0.66 \pm 0.31 \quad 264 \pm 15 \pm 7 \quad$ hep-ex/0704.0437
$\Rightarrow$ Belle: accuracy is similar to CLEO-c,
direct measurement of absolute $B\left(D_{s}^{+} \rightarrow \mu^{+} \nu_{\mu}\right)$ without norm. channel
C.Davies at EPS-2005: "There has been a revolution in LQCD..."

Unquenched LQCD with $n_{f}=2+1$ [PRL 95, 122002 (2005)]: $f_{D_{s}^{+}}=249 \pm 3 \pm 16 \mathrm{MeV}$.

## Observation of $D_{s 1}(2536)^{+} \rightarrow D^{+} \pi^{-} K^{+}$

$D_{s 1}(2536)^{+}: J^{P}=1^{+}, j_{l}=3 / 2$; known modes $D^{*+} K_{S}^{0}, D^{* 0} K^{+}, D_{s}^{+} \pi^{+} \pi^{-}$. $D^{+} \pi^{-}$cannot come from $D^{* 0}: M_{D^{0}}+M_{\pi^{-}}>M_{D^{* 0}}$.
$x_{P}=p_{D_{s 1}} / \sqrt{s / 4-M_{D_{s 1}}^{2}}>0.8$,
$D^{0} \rightarrow K^{-} \pi^{+}\left|K_{S}^{0} \pi^{+} \pi^{-}\right| K^{-} \pi^{+} \pi^{+} \pi^{-}$, $D^{+} \rightarrow K_{S}^{0} \pi^{+} \mid K^{-} \pi^{+} \pi^{+}$,
$\left|\Delta M_{D}\right|<20 \mathrm{MeV} / \mathrm{c}^{2}$ ( $99 \%$ eff.) ,
$D^{*+} \rightarrow D^{0} \pi+$,
$\left|\Delta M_{D^{*}+}\right|<1.5 \mathrm{MeV} / \mathrm{c}^{2}(98 \%$ eff.)
$462 \mathrm{fb}^{-1}$ of data


data (•) vs. ph. space MC
$\rightarrow$ No visible resonant substructure

$$
\frac{B\left(D_{s 1}^{+} \rightarrow D^{+} \pi^{-} K^{+}\right)}{B\left(D_{s 1}^{+} \rightarrow D^{*+} K^{0}\right)}=3.17 \pm 0.17 \pm 0.36 \%
$$

## $\underline{\text { Partial Wave Analysis (PWA) of } D_{s 1}(2536)^{+} \rightarrow D^{*+} K_{S}^{0}}$

HQET: $\quad D_{s 1}^{\left(j_{l}=3 / 2\right)} \rightarrow D^{*} K \quad$ D-wave

$$
\left(j_{l}=1 / 2\right) \rightarrow D^{*} K \quad \text { S-wave (but } D_{s 1}(2460)^{+} \rightarrow D^{*} K \text { is energetically forbidden). }
$$

Energy release in $D_{s 1}(2536)^{+} \rightarrow D^{*} K$ is small $\Rightarrow$ D-wave is suppressed $\Rightarrow$ small mixing between $j_{l}=3 / 2$ and $j_{l}=1 / 2$ states

$$
\begin{array}{rlr}
\left|D_{s 1}(2460)^{+}\right\rangle & = & \cos \theta\left|\left(j_{l}=1 / 2\right)\right\rangle+\sin \theta\left|\left(j_{l}=3 / 2\right)\right\rangle \\
\left|D_{s 1}(2536)^{+}\right\rangle & = & -\sin \theta\left|\left(j_{l}=1 / 2\right)\right\rangle+\cos \theta\left|\left(j_{l}=3 / 2\right)\right\rangle
\end{array}
$$

can give a sizeable contribution to $D_{s 1}(2536)^{+}$width.
Belle'03,04: $\frac{B\left(D_{s 1}(2460) \rightarrow D_{s}^{*+} \gamma\right)}{B\left(D_{s 1}(2460) \rightarrow D_{s}^{+} \gamma\right)}=0.31 \pm 0.14$ ( B decays and $e^{+} e^{-}$annihilation)
$\Rightarrow \tan ^{2}\left(\theta+\theta_{0}\right)=0.8 \pm 0.4$, where $\tan ^{2}\left(\theta_{0}\right)=2$
CLEO'94: first attempt to decompose $S$ - and D-waves in $D_{1}(2420)^{0,+} \rightarrow D^{*+} \pi^{-}, D^{0} \pi^{+}$
[PLB 331, 236; PLB 340, 194].
Currently no results on $D_{s 1}(2536)^{+}$exist.
Moreover CLEO method did not allow to perform complete PWA of the decay.
Partial Wave Analysis (PWA) of $D_{s 1}(2536)^{+} \rightarrow D^{*+} K_{S}^{0}$


$$
\begin{gathered}
\frac{d^{3} N}{d(\cos \alpha) d \beta d(\cos \gamma)}=\frac{9}{4 \pi\left(1+2 R_{\Lambda}\right)} \times\left(\operatorname { c o s } ^ { 2 } \gamma \left[\rho_{00} \cos ^{2} \alpha+\frac{1-\rho_{00}}{2} \sin ^{2}\right.\right. \\
+R_{\Lambda} \sin ^{2} \gamma\left[\frac{1-\rho_{00}}{2} \sin ^{2} \beta+\cos ^{2} \beta\left(\rho_{00} \sin ^{2} \alpha+\frac{1-\rho_{00}}{2} \cos ^{2} \alpha\right)\right] \\
+ \\
\frac{\left.\frac{\sqrt{R_{\Lambda}}\left(1-3 \rho_{00}\right)}{4} \sin 2 \alpha \sin 2 \gamma \cos \beta \cos \xi\right)}{4}
\end{gathered}
$$

$\rho_{00}$ - longitudinal polarization, $\sqrt{\mathbf{R}_{\Lambda}} \exp (i \xi)=\frac{A_{1,0}}{A_{0,0}}=z$,
$A_{1,0}$ and $A_{0,0}$ correspond to $D^{*+}$ helicities $\pm 1$ and 0 .
$\frac{d^{3} N}{d(\cos \alpha) d \beta d(\cos \gamma)} \rightarrow \rho_{00}, z=\sqrt{R_{\Lambda}} \exp (i \xi) \rightarrow D / S=\sqrt{2}(z-1) /(1+2 z)=\sqrt{\Gamma_{D} / \Gamma_{S}} \exp (i \eta)$
Last interference term with the phase $\xi$ vanishes after integration over any angle. It does not appear in one- and two-dimensional distributions $\frac{d N}{d(\cos \gamma)}, \frac{d^{2} N}{d(\cos \alpha) d(\cos \gamma)}$ studied by CLEO in 1994 for $D_{1}(2420)$ meson.

## PDF for 3-D Unbinned ML Fit

$$
\mathcal{P}(\alpha, \beta, \gamma)=\left(1-f_{b}\right) \cdot \frac{d^{3} N}{d(\cos \alpha) d \beta d(\cos \gamma)} \cdot \frac{\epsilon(\alpha, \beta, \gamma)}{\langle\epsilon\rangle_{\mathrm{avr}}}+f_{b} \cdot \mathcal{P}_{b c k}(\alpha, \beta, \gamma)
$$

Background subtracted and efficiency corrected projections: data (•) vs. 3-D fit results


## 3-D Fit Quality and Results

PDF is made of $\frac{d^{3} N}{d(\cos \alpha) d \beta d(\cos \gamma)} \bigoplus$ efficiency $\bigoplus$ background.

1) $\frac{d^{3} N}{d(\cos \alpha) d \beta d(\cos \gamma)}$ depends on fit parameters only quadratically or linearly;
2) efficiency is almost flat in all projections: $\epsilon=$ const $\Rightarrow$ results change by $\leq 1 / 3 \sigma_{\text {stat }}$;
3) background fraction $f_{b}=9 \%$ is small;
$\Rightarrow$ fit is simple.
Fit of 1000 toy MC samples: no bias found. FCN is worse than in data in $33 \%$ of cases.

$$
\begin{gathered}
A_{1,0} / A_{0,0}=\sqrt{3.6 \pm 0.3} \exp ( \pm i \cdot(1.27 \pm 0.16)) \\
D / S=(0.63 \pm 0.07) \cdot \exp ( \pm i \cdot(0.77 \pm 0.03))
\end{gathered}
$$

Contrary to HQET, S-wave dominates:

$$
\Gamma_{S} / \Gamma_{\text {total }}=0.72 \pm 0.05
$$

Longitudinal polarization in the region $x_{P}>0.8$

$$
\rho_{00}=0.490 \pm 0.013
$$

## Fit in Bins of $D_{s 1}^{+}$Recoil Mass, $M_{\mathrm{rec}}=\sqrt{\left(2 E_{\mathrm{beam}}^{*}-E_{D_{s 1}^{+}}^{*}\right)^{2}-\left(p_{D_{s 1}^{+}}^{*}\right)^{2}}$

Parameters $R_{\Lambda}$ and $\xi$ are independent of $M_{\text {rec }}$ within statistical errors $\Rightarrow$ fixed to values from overall fit


Resolution: $\sim 70 \mathrm{MeV} / c^{2}$ at $2 \mathrm{GeV} / c^{2}$, $1 / M_{\text {rec }}$ dependence. Indication of two-body $e^{+} e^{-} \rightarrow D_{s 1}^{+} X$, $X=D_{s}^{+}, D_{s}^{*+}, D_{s}^{* *+}$.
$\rho_{00}$ : also some structure at low $M_{\text {rec }}$; for $e^{+} e^{-} \rightarrow D_{s 1}^{+} D_{s}^{+}$: low $\rho_{00}$, rises to $\approx 0.5$ at higher $M_{\text {rec }}$.

## Conclusions

1. First preliminary results on $D_{s}^{+} \rightarrow \mu \nu_{\mu}$; systematics is to be thoroughly evaluated.

|  | $B\left(D_{s}^{+} \rightarrow \mu \nu_{\mu}\right), \times 10^{-3}$ | $f_{D_{s}^{+}}, \mathrm{MeV}$ |
| :---: | :---: | :---: |
| Belle | $6.44 \pm 0.76 \pm 0.52$ | $275 \pm 16 \pm 12$ |
| PDG'06 | $6.1 \pm \mathbf{1 . 9}$ | $294 \pm \mathbf{2 7}$ |
| BaBar | $6.74 \pm \mathbf{0 . 8 3} \pm \mathbf{0 . 2 6} \pm \mathbf{0 . 6 6 _ { \phi \pi }}$ | $283 \pm \mathbf{1 7} \pm \mathbf{7} \pm \mathbf{1 4} 4_{\phi \pi}$ |
| CLEO-c | $5.94 \pm \mathbf{0 . 6 6} \pm \mathbf{0 . 3 1}$ | $264 \pm \mathbf{1 5} \pm \mathbf{7}$ |
| Unquen. LQCD | $[$ PRL 95, 122002 (2005)] | $249 \pm 3 \pm 16$ |

Accuracy is similar to LQCD, statistically limited.
2. (a) $\frac{\mathcal{B}\left(D_{s 1}^{+} \rightarrow D^{+} \pi^{-} K^{+}\right)}{\mathcal{B}\left(D_{s 1}^{+} \rightarrow D^{*+} K^{0}\right)}=(3.17 \pm 0.17 \pm 0.36) \%$, no clear resonant substructure.
(b) $D_{s 1}(2536)^{+} \rightarrow D^{*+} K_{S}^{0}$ PWA:
amplitude ratio $D / S=(0.63 \pm 0.07) \cdot \exp ( \pm i \cdot(0.77 \pm 0.03))$
Contrary to $H Q E T, \underline{S \text {-wave dominates: }} \Gamma_{S} / \Gamma_{\text {total }}=0.72 \pm 0.05$
$\Rightarrow$ mixing with $j_{l}=1 / 2$ state $\left(D_{s 1}(2460)^{+}\right)$.
(c) Spin of $D_{s 1}(2536)^{+}$with $x_{P}>0.8$ prefers to align
transverse to momentum: $\rho_{00}=0.490 \pm 0.013$.
First measurement for $P$-wave $\boldsymbol{c}$-meson in fragmentation.
HQET: $\rho_{00}=\frac{2}{3}\left(1-w_{3 / 2}\right)$ [Phys. Rev. D 49, 3320 (1994)]
$\Rightarrow$ Falk-Peskin parameter $w_{3 / 2}=0.266 \pm 0.019$ for $x_{P}>0.8$
$\Rightarrow$ predictions for angular distributions of other $\boldsymbol{j}_{l}=\mathbf{3} / \mathbf{2}$ meson decays.

## Backup Slides

## $D_{s}^{+} \rightarrow \mu^{+} \nu_{\mu}$ Signal and Background in MC




$$
\begin{gathered}
\frac{D_{s 1}(2536)^{+} \rightarrow D^{+} \pi^{-} K^{+} \text {cross check: }}{D^{+} \text {from } D_{s 1}^{+} \text {signal and side bands, }} \\
\left|\Delta M_{D \pi K}\right|<5, \quad 10<\left|\Delta M_{D \pi K}\right|<20
\end{gathered}
$$



$1249 \pm 66 \mathrm{ev}$. (expected $1262 \pm 65$ )
background level ( $-0.9 \pm 0.8$ )

## Mixing of $j_{l}=1 / 2$ and $j_{l}=3 / 2$ States

Some information on $\theta$ is obtained from the ratio of electromagnetic decay rates $D_{s 1}(\mathbf{2 4 6 0}) \rightarrow D_{s}^{+} \gamma, D_{s}^{*+} \gamma$, since only the ${ }^{1} P_{1}$ state in $D_{s 1}(2460)$ undergoes E1 transition to $D_{s}^{+}$and only the ${ }^{3} P_{1}$ state to $D_{s}^{*+}$. The bases $\left|j_{l}\right\rangle$ and $\left|{ }^{2 S+1} P_{1}\right\rangle$ are related by the rotation by the angle $\theta_{0}, \tan \theta_{0}=-\sqrt{2}$. Therefore the angle between the bases $\left|D_{s}^{+}\right\rangle$and $\left.\left.\right|^{2 S+1} P_{1}\right\rangle$ is $\theta+\theta_{0}$.

$$
\left|D_{s}^{+}\right\rangle \xrightarrow{\theta}\left|j_{l}\right\rangle \xrightarrow{\theta_{0}}\left|\begin{array}{c}
1_{P_{1}} \\
3_{P_{1}}
\end{array}\right\rangle \xrightarrow{E 1}\left|\begin{array}{c}
D_{s}^{+} \gamma \\
D_{s}^{*+} \gamma
\end{array}\right\rangle
$$

Belle studied $D_{s 1}(\mathbf{2 4 6 0}) \rightarrow D_{s}^{+} \gamma, D_{s}^{*+} \gamma$ decays using $D_{s 1}(\mathbf{2 4 6 0})$ from both $B$ decays and from $e^{+} e^{-}$annihilation. The ratio of decay rates is found to be $0.4 \pm 0.3$ and $0.28 \pm 0.17$, respectively. The average value is
$\frac{B\left(D_{s 1}(2460) \rightarrow D_{s}^{*+} \gamma\right)}{B\left(D_{s 1}(2460) \rightarrow D_{s}^{+} \gamma\right)}=0.31 \pm 0.14$. Following Y. Yamada et.al, PR C72, 065202 (2005) and taking into account the phase space difference between $D_{s}^{+} \gamma$ and $D_{s}^{*+} \gamma$, this gives the constraint

$$
\tan ^{2}\left(\theta+\theta_{0}\right)=0.8 \pm 0.4
$$

It should be compared with $\tan ^{2}\left(\theta_{0}\right)=2$ corresponding to the absence of the mixing.

## $\underline{\text { Spin Alignment of } D_{s 1}^{+} \text {from } e^{+} e^{-} \text {Annihilation }}$

Production of heavy mesons $(Q \bar{q})$ in HQET: A. Falk and M. Peskin, PR D49, 3320 (1994). The fragmentation process is so fast that the color magnetic forces do not have time to act and the spins of $Q$ and $\bar{q}$ are uncorrelated.
Predictions for ( $D, D^{*}$ ):

1) $D^{*}$ mesons are produced unpolarized.

Confirmed by CLEO in 1991, 1998 and checked also by HRS'87, TPC'91, OPAL'97, SLD'97.
2) $D^{*}$ and $D$ mesons are produced in fragmentation according to the number of helicity states in a $3: 1$ ratio.
However, average over $e^{+} e^{-}$, hadro-production, photo-production etc. experiments: $P_{V}=V /(V+P)=0.594 \pm 0.010 \ll 0.75$ [A. David, PL B644, 224 (2007)].

There are no similar measurements for the P-wave states. Contrary to the ( $D, D^{*}$ ) case, HQET predicts that $j_{l}=3 / 2$ doublet can be produced aligned.
The probabilities for the light degree of freedom to have helicity $-3 / 2,-1 / 2,1 / 2,3 / 2$ are expressed via one parameter $w_{3 / 2}$ as

$$
\frac{1}{2} w_{3 / 2}, \frac{1}{2}\left(1-w_{3 / 2}\right), \frac{1}{2}\left(1-w_{3 / 2}\right), \frac{1}{2} w_{3 / 2}
$$

respectively. By adding uncorrelated $c$-quark spin and resolving the $c \bar{s}$ system into $1^{+}$and $2^{+}$states, one can calculate their alignment.

For $j_{l}=1 / 2$ doublets two helicity states should have equal probabilities due to $P$ conservation. As a result all three helicity states of $D^{*}$ and the fourth state of $D$ should be equally populated.

For $D_{s 1}(2536)^{+}$the probability of zero helicity is $\rho_{00}=\frac{2}{3}\left(1-w_{3 / 2}\right)$. Perturbative QCD calculations give $w_{3 / 2}=29 / 114 \approx 0.254$ [Y.-Q. Chen and M. Wise, PR D50, 4706 (1994)] and $\rho_{00} \approx 0.497$. ARGUS'89 analysis of angular distributions in $D_{2}^{*}(2460) \rightarrow D \pi$ decay gives an upper limit $w_{3 / 2}<0.24$ at $90 \%$ CL. This is the only available experimental number in $j_{l}=3 / 2$ charm sector. Once $w_{3 / 2}$ is measured, one can make definite predictions for the angular distributions of the remaining $j_{l}=3 / 2$ meson decays (e.g. on the decays of $D_{s 2}(2573)^{+}$state). $j_{l}=3 / 2$ mesons are the lowest states which can be used to perform first nontrivial tests of HQET in fragmentaion process.

