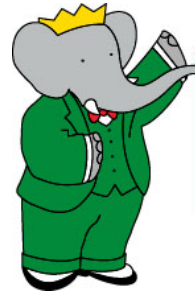


D^0 Mixing from



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Jonathon Coleman

European Physical Society
HEP 2007



19 July 2007

Manchester, England





Charm meson mixing

Why would observation of charm mixing be interesting?

It would *complete the picture* of quark mixing already seen in the K , B , and B_s systems.

K — PR 103, 1901 (1956); PR 103, 1904 (1956).

B — PL B186, 247 (1987); PL B192, 245 (1987).

B_s — PRL 97, 021802 (2006); PRL 97, 242003 (2006).

It would provide new information about processes with *down-type quarks* in the mixing loop diagram.



It would be a significant step toward observation of *CP violation* in the charm sector.

It could possibly indicate *new physics*.



Mixing Phenomenology

Neutral D mesons

are produced as *flavor eigenstates* D^0 and \bar{D}^0 and decay via

$$i\frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

as *mass, lifetime eigenstates* D_1, D_2

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$$

$$|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$

where $|q|^2 + |p|^2 = 1$ and

$$\left(\frac{q}{p} \right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

D_1, D_2 have masses M_1, M_2 and widths Γ_1, Γ_2

Mixing occurs when there is a *non-zero* mass

$$\Delta M = M_1 - M_2$$

or lifetime difference

$$\Delta\Gamma = \Gamma_1 - \Gamma_2$$

For convenience define quantities x and y

$$x = \frac{\Delta M}{\Gamma}, \quad y = \frac{\Delta\Gamma}{2\Gamma}$$

where $\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$



Short- and long-distance effects

Short-distance contributions from mixing box diagrams primarily affect x

b quark is CKM-suppressed
 s and d quarks are GIM suppressed

Expect $O(10^{-5})$ or less

Long-distance contributions primarily affect y

Non-perturbative effects

Expect $O(10^{-2})$ or less

New physics would be indicated if

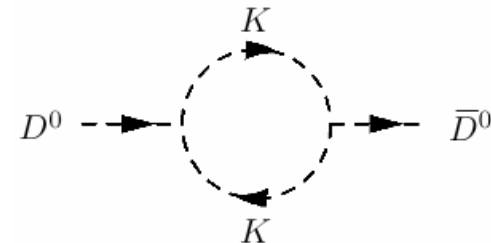
$x \gg y$

CP violation is observed

Short-distance



Long-distance



Patricia Ball, hep-ph/0703245, Moriond 2007:

“The central problem of all these calculations is that the D is too heavy to be treated as light and too light to be treated as heavy.”



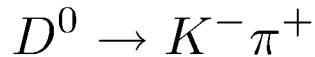
BABAR $D^0 \rightarrow K\pi$ Mixing Analysis

Mixing occurs when a meson produced as a D^0 decays as a \bar{D}^0 or vice versa.

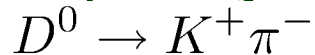
This can be studied by tagging the D^0 flavor at production and at decay.

We use the $D^0 \rightarrow K\pi$ decay mode.

Cabibbo-favored (CF), “right-sign” (RS) decay

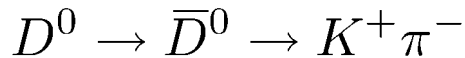


Doubly Cabibbo-suppressed (DCS), “wrong-sign” (WS) decay



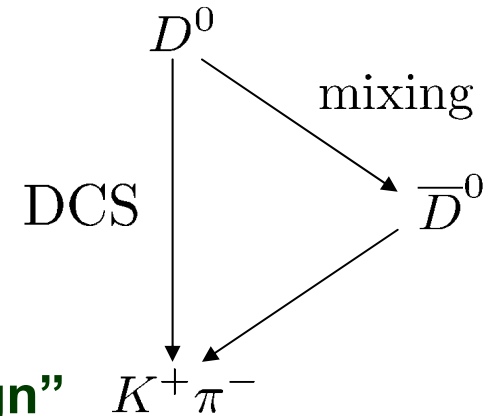
Rate: $\tan^4 \theta_c \approx 0.3\%$

Mixing followed by CF decay (WS)



Rate: 10^{-4} or less

(interference between mixing and DCS can enhance)

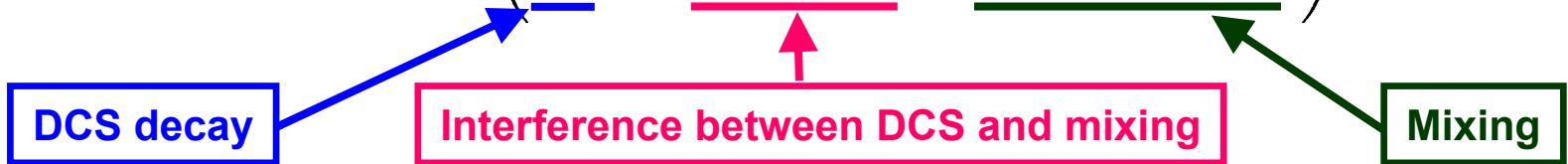




Time-dependent decay rate

Use time dependence to separate DCS and mixing contributions (approximate; for $x, y \ll 1$)

$$\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] \propto e^{-\Gamma t} \left(R_D + \sqrt{R_D} y' \Gamma t + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right)$$



Allows for a strong phase difference $\delta_{K\pi}$ between CF and DCS direct decay

$$x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi}, \quad y' = -x \sin \delta_{K\pi} + y \cos \delta_{K\pi}$$

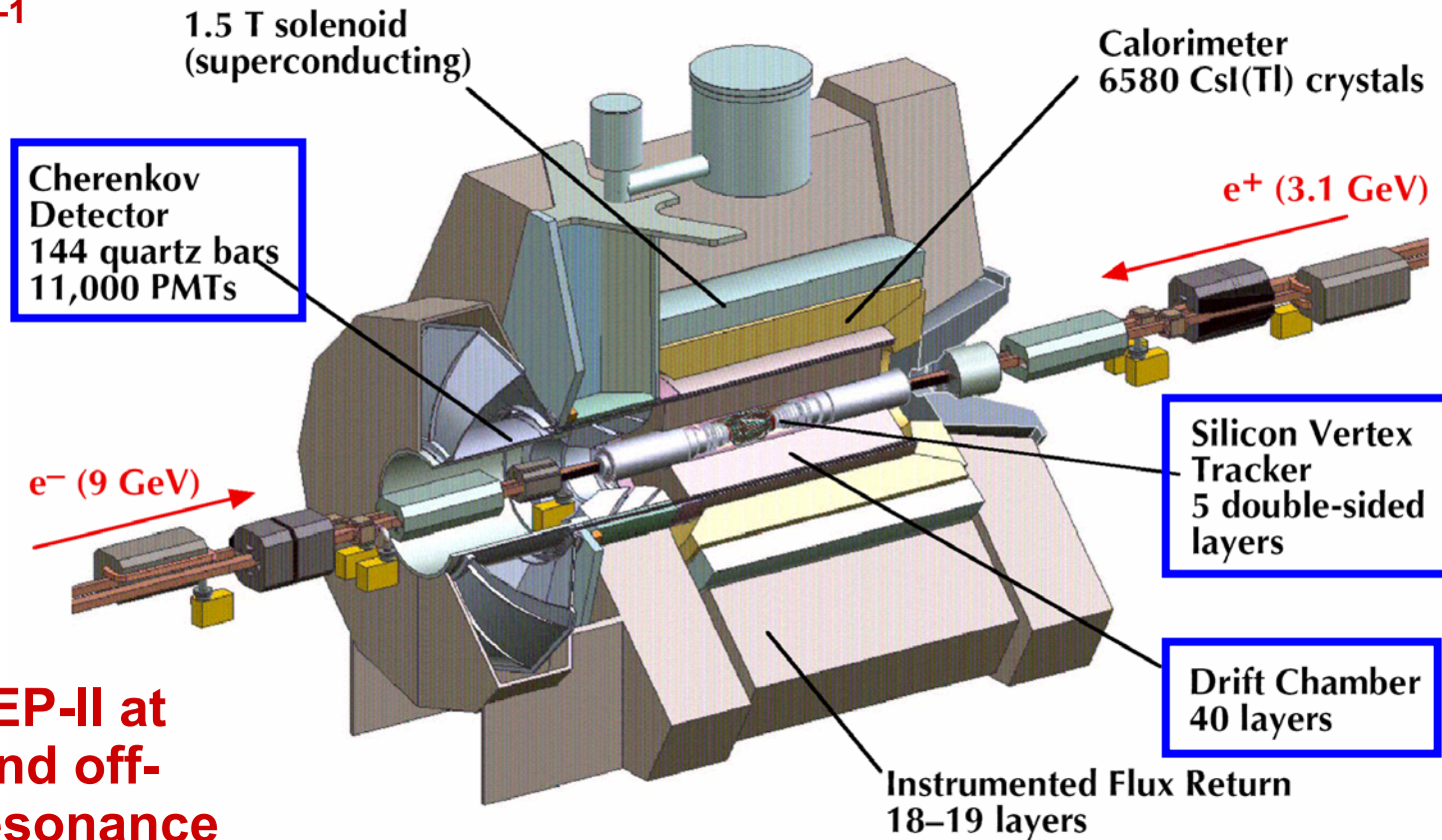
This phase may differ between decay modes.

And may vary over phase space for multi-body decays.



BABAR detector and dataset

Dataset: 384 fb⁻¹



**Collected at PEP-II at
SLAC on- and off-
the $\Upsilon(4S)$ resonance**

NIM A479, 1 (2002)



$D^0 \rightarrow K\pi$ Analysis Method

Identify the D^0 charge conjugation state at prod. & decay using vertices fit to

$$D^{*\pm} \rightarrow \pi_S^\pm D^0, \quad D^0 \rightarrow K^\mp \pi^\pm$$

Determines $m_{K\pi}$, Δm , proper-time t and error δ_t

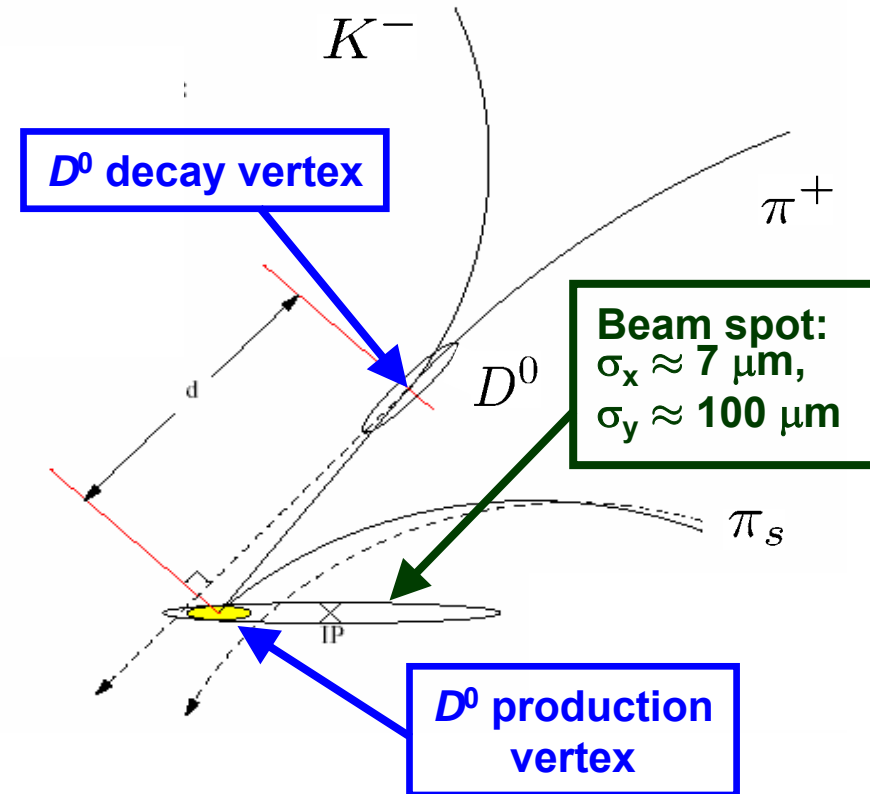
$$\Delta m = m(D_{\text{rec.}}^{*+}) - m(D_{\text{rec.}}^0)$$

Vertices fit with beamspot constraint is important

Improves the decay-time error resolution

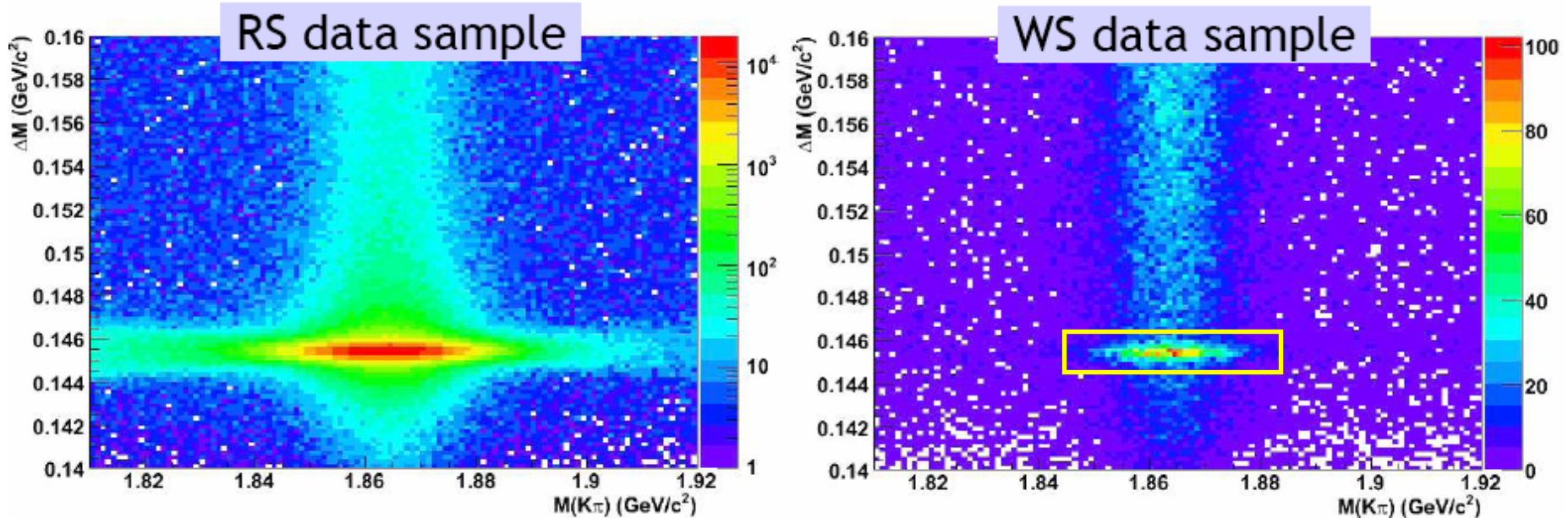
Improves the Δm resolution

Right-sign (RS) decay





RS & WS $m_{K\pi}$, Δm distributions



All fits are over the *full range* shown in the plots

$1.81 \text{ GeV}/c^2 < m_{K\pi} < 1.92 \text{ GeV}/c^2$ and $0.14 \text{ GeV}/c^2 < \Delta m < 0.16 \text{ GeV}/c^2$

Define a *signal region*

$1.843 \text{ GeV}/c^2 < m_{K\pi} < 1.883 \text{ GeV}/c^2$ and $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



Fitting strategy

Fitting is performed in *stages* to reduce demand on computing resources
All stages are *unbinned, extended maximum-likelihood fits*.

1. **RS & WS $m_{K\pi}$ Δm fit.**
Yields PDF shape parameters $m_{K\pi}$, Δm categories.
2. **RS lifetime fit.**
 $m_{K\pi}$, Δm category shape parameters held constant.
Yields D^0 lifetime τ_D and proper-time resolution parameters.
Constrained by the large statistics of the RS sample.
3. **WS lifetime fit.**
Yields parameters describing the WS time dependence.

Small correlation between fitted parameters in the different stages justifies the staged approach.

The WS fit is performed under *three* different assumptions.

Mixing and CP violation (CPV); mixing but no CPV; and no mixing or CPV.

Monte Carlo (MC) simulations *are not used* directly in the data fits.

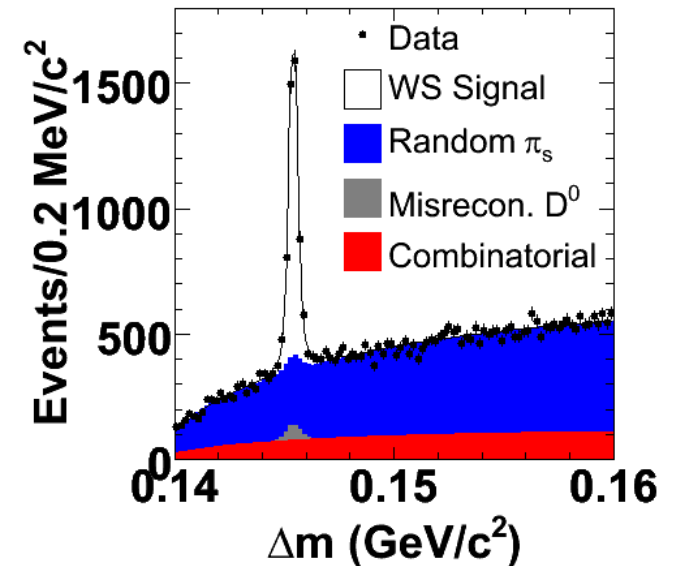
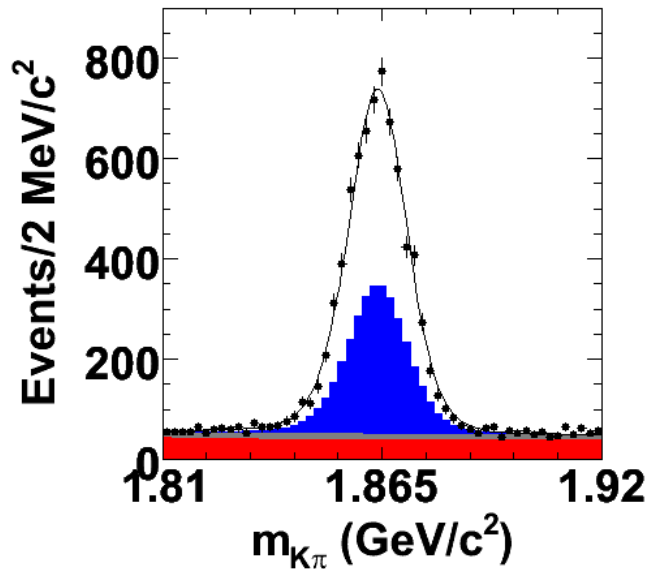
MC simulations used only to motivate the fit PDFs

WS mis-reconstructed D^0 category studied in swapped $K \leftrightarrow \pi$ data.



Wrong-sign $m_{K\pi}$, Δm fit

The $m_{K\pi}$, Δm fit determines the WS b.r. $R_{WS} = N_{WS}/N_{RS}$



BABAR (384 fb⁻¹): $R_{WS} = (0.353 \pm 0.008 \pm 0.004)\%$ (PRL 98,211802 (2007))
BELLE (400 fb⁻¹): $R_{WS} = (0.377 \pm 0.008 \pm 0.005)\%$ (PRL 96, 151801 (2006))

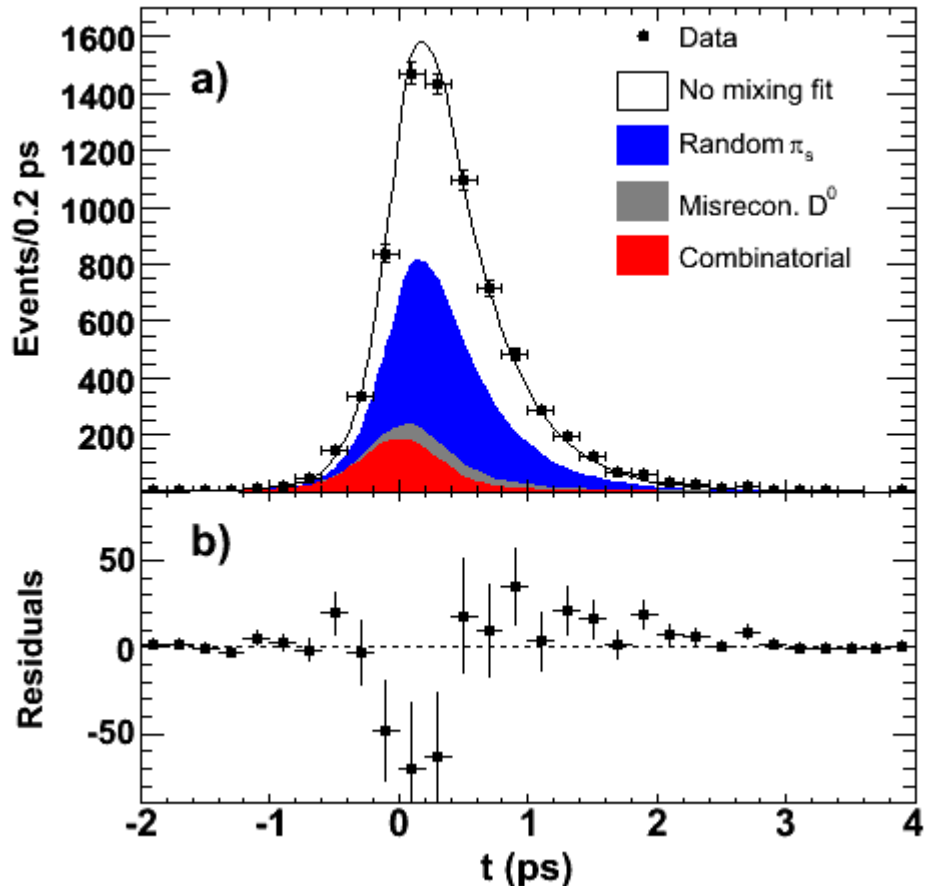


No-mixing WS decay time fit

The parameters fitted are
WS category yields
WS combinatoric shape
parameter

As can be seen in the
residual plot, there are
large residuals.

Residuals = data - fit



WS no-mixing fit projection in signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



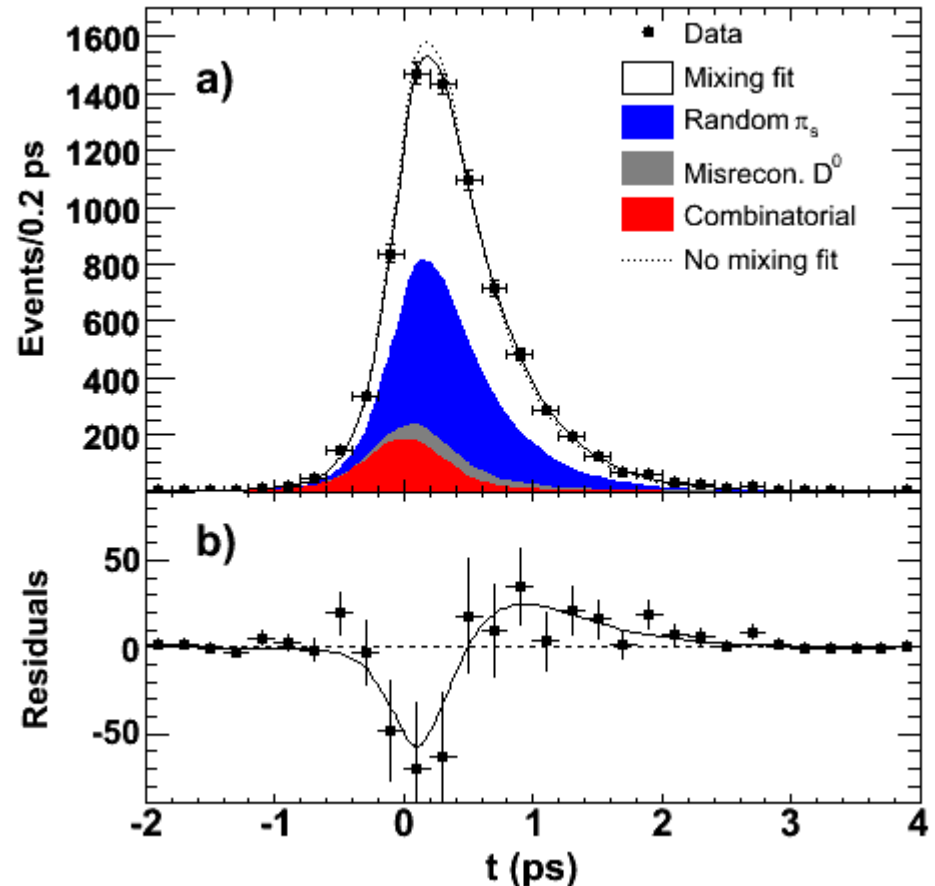
Mixing WS decay time fit

The difference between the no-mixing fit and the fit with mixing is shown in the residuals plot.

The dotted line is the no-mixing fit.

The solid line is the mixing fit.

The fit is significantly improved by allowing for mixing.



WS mixing fit projection in signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



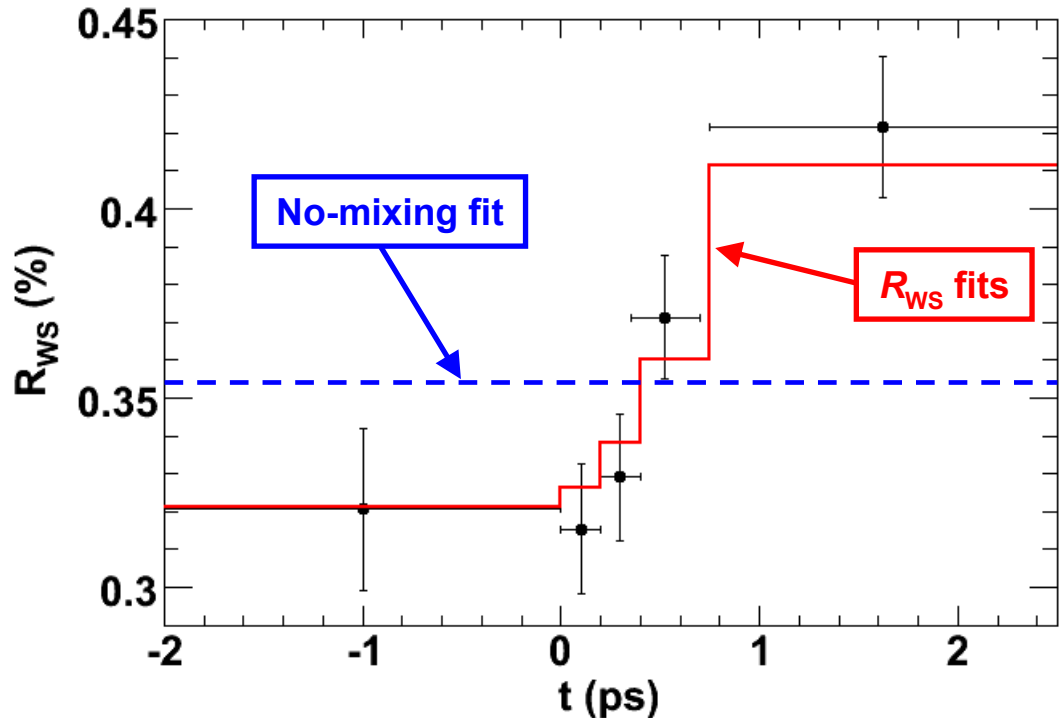
R_{WS} vs. decay-time slices

If mixing is present, it should be evident in an R_{WS} rate that *increases with decay-time*.

Perform the R_{WS} fit in *five time bins* with similar RS statistics.

Cross-over occurs at $t \approx 0.5$ psec

Similar to residuals plot.



Dashed line: standard R_{WS} fit ($\chi^2=24$).
Solid, red line: independent R_{WS} fits to each time bin ($\chi^2 = 1.5$).



Mixing fit likelihood contours

Contours in y' , x'^2 computed from $-2\Delta \ln L$

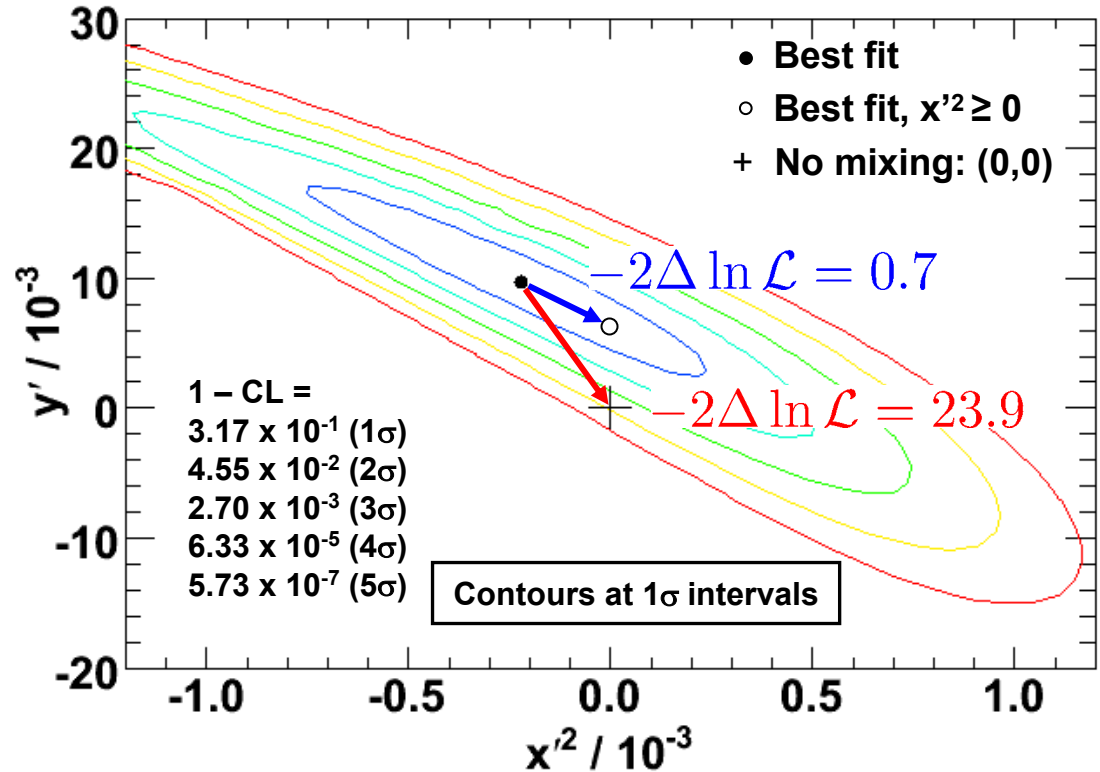
Best-fit point is in the non-physical region $x'^2 < 0$

1σ contour extends into physical region

Correlation: -0.95

Contours include systematic errors

The no-mixing point is at the 3.9σ contour



$$R_D: (3.03 \pm 0.16 \pm 0.10) \times 10^{-3}$$

$$x'^2: (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3}$$

$$y': (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}$$

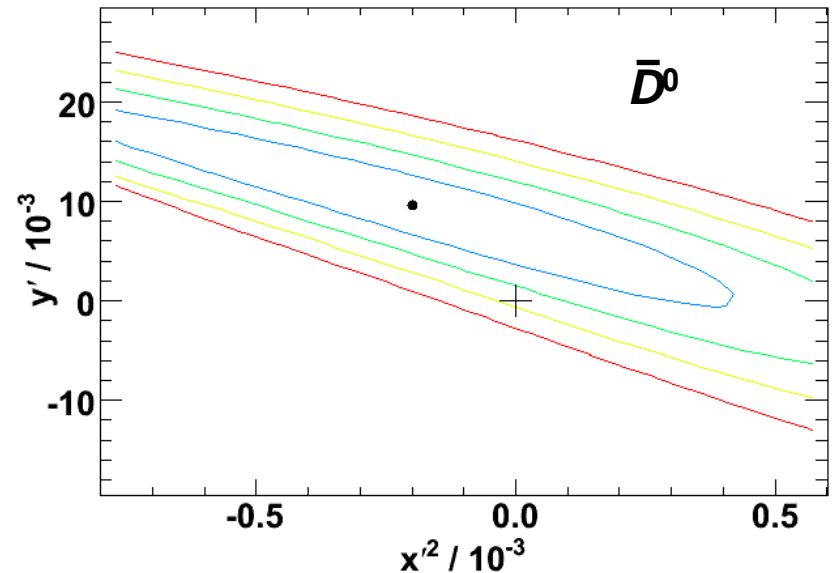
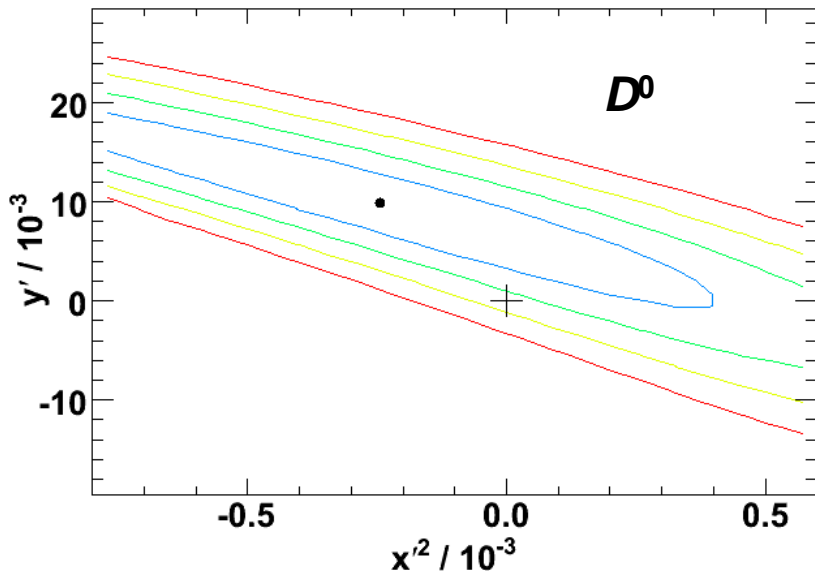


Fits allowing for CP violation

Fit D^0 and \bar{D}^0 decay-time dependence separately.

$$x'^{2+} = (-0.24 \pm 0.43 \pm 0.30) \times 10^{-3}$$
$$y'^+ = (9.8 \pm 6.4 \pm 4.5) \times 10^{-3}$$

$$x'^{2-} = (-0.20 \pm 0.41 \pm 0.29) \times 10^{-3}$$
$$y'^- = (9.6 \pm 6.1 \pm 4.3) \times 10^{-3}$$



No evidence seen for CP violation



List of systematics, validations

Systematics: variations in

Functional forms of PDFs

Fit parameters

Event selection

Computed using full difference with original value

Results are expressed in units of the statistical error

Systematic source	R_D	y'	x'^2
PDF:	0.59σ	0.45σ	0.40σ
Selection criteria:	0.24σ	0.55σ	0.57σ
Quadrature total:	0.63σ	0.71σ	0.70σ

Validations and cross-checks

Alternate fit (R_{WS} in time bins)

Fit RS data for mixing

$$x'^2 = (-0.01 \pm 0.01) \times 10^{-3}$$

$$y' = (0.26 \pm 0.24) \times 10^{-3}$$

Fit generic MC for mixing

$$x'^2 = (-0.02 \pm 0.18) \times 10^{-3}$$

$$y' = (2.2 \pm 3.0) \times 10^{-3}$$

Fit toy MCs generated with various values of mixing

Reproduces generated values

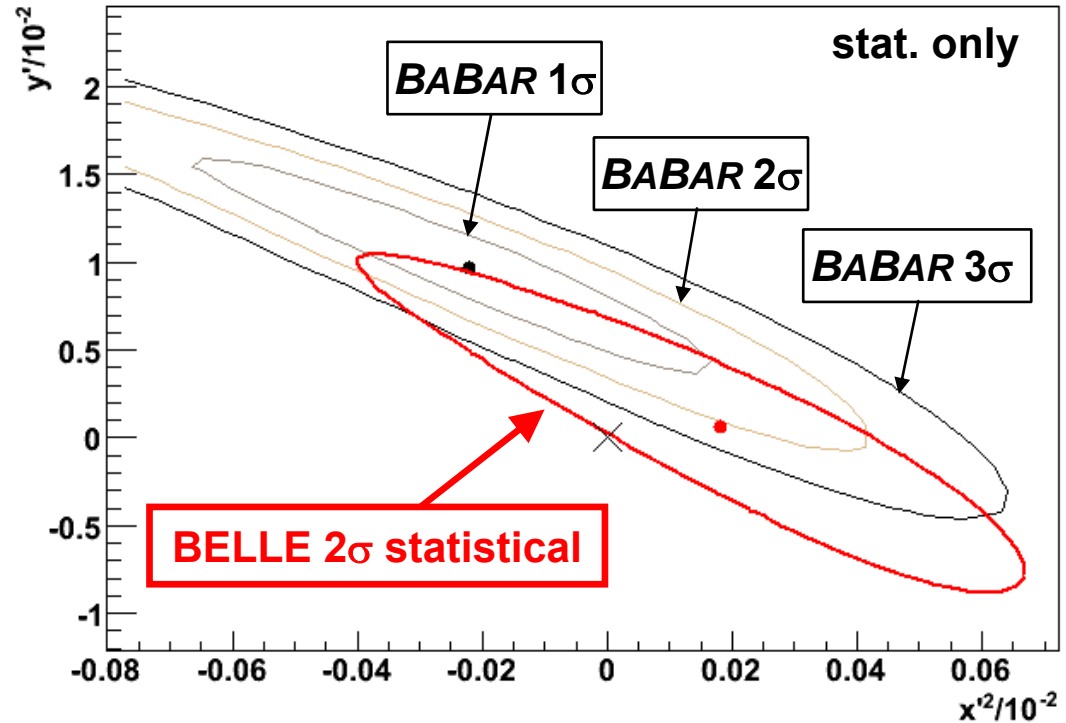
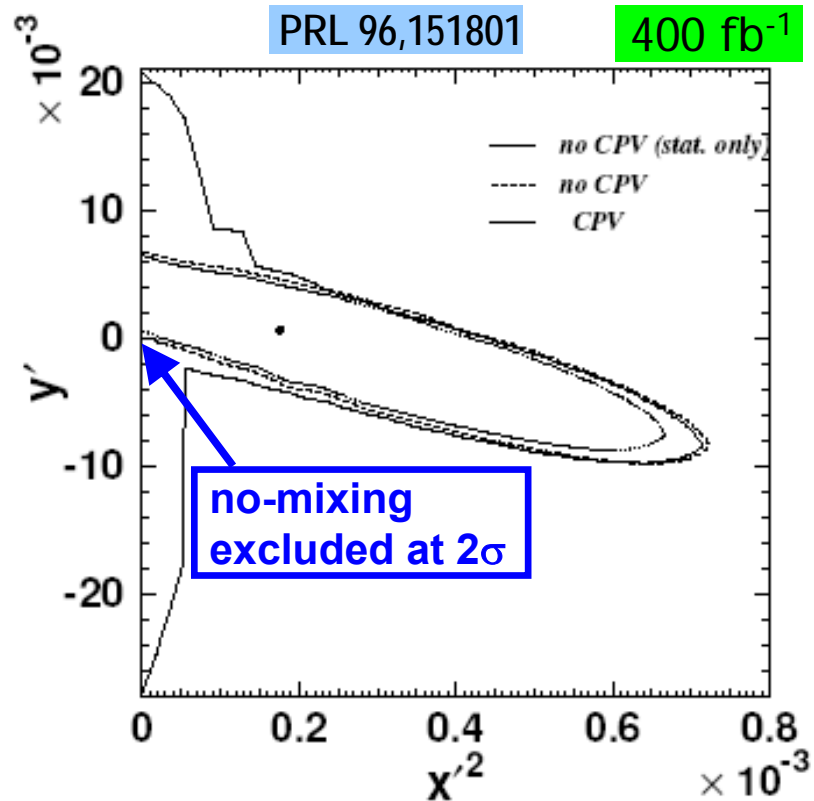
Validation of proper frequentist coverage in contour construction

Uses 100,000 MC toy simulations



BELLE $D^0 \rightarrow K\pi$ result

Results consistent within 2σ





Average $K\pi$ Mixing Results

Heavy flavor averaging group (HFAG)
provides "official" averages

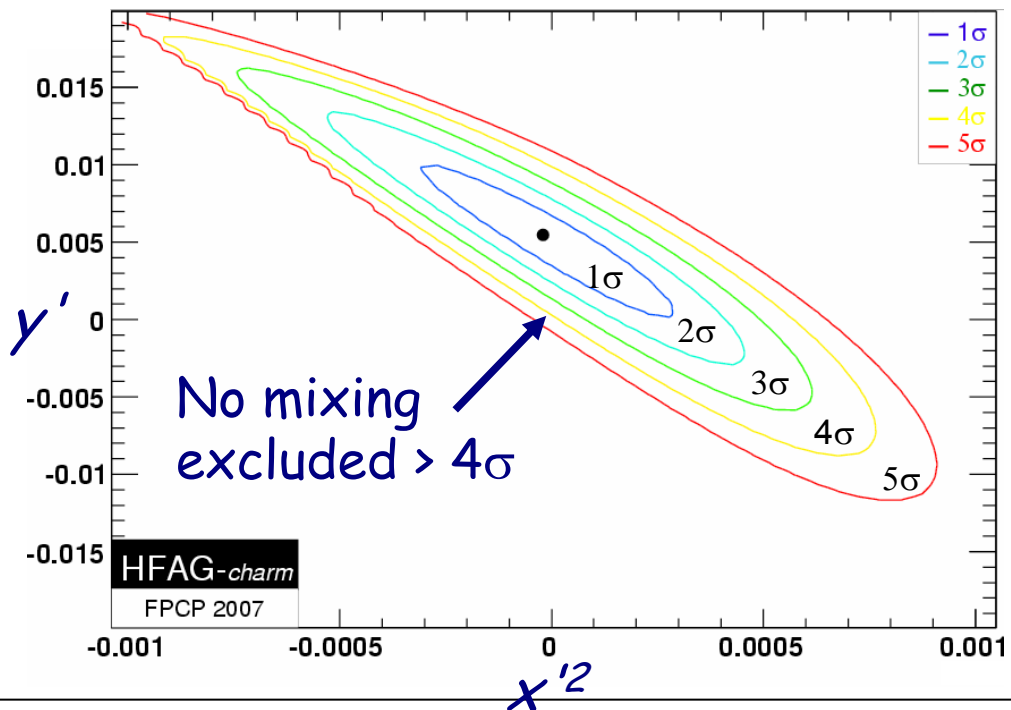
Combine BaBar and Belle likelihoods in 3 dimensions (R_D, x'^2, y')

May 2007 Averages:

$$R_D: (3.30^{+0.14}_{-0.12}) \times 10^{-3}$$

$$x'^2: (-0.01 \pm 0.20) \times 10^{-3}$$

$$y': (5.5^{+2.8}_{-3.7}) \times 10^{-3}$$





Summary

Evidence for mixing at 3.9σ (stat.+syst.)

$$y' = [9.7 \pm 4.4 \text{ (stat.)} \pm 3.1 \text{ (syst.)}] \times 10^{-3}$$

$$x'^2 = [-0.22 \pm 0.30 \text{ (stat.)} \pm 0.219 \text{ (syst.)}] \times 10^{-3}$$

$$R_D = [0.303 \pm 0.016 \text{ (stat.)} \pm 0.010 \text{ (syst.)}] \%$$

(PRL 98,211802 (2007))

No evidence seen for CP violation

Results are consistent with other mixing analyses

Backup slides



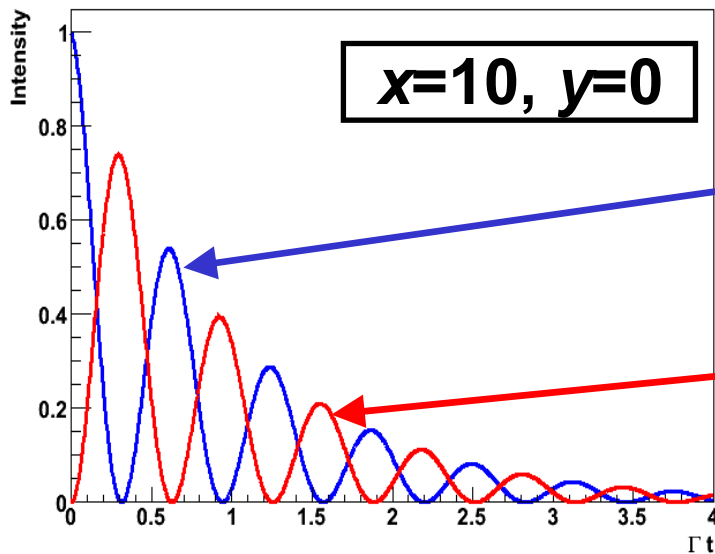


Time-dependent mixing rate

Two illustrations

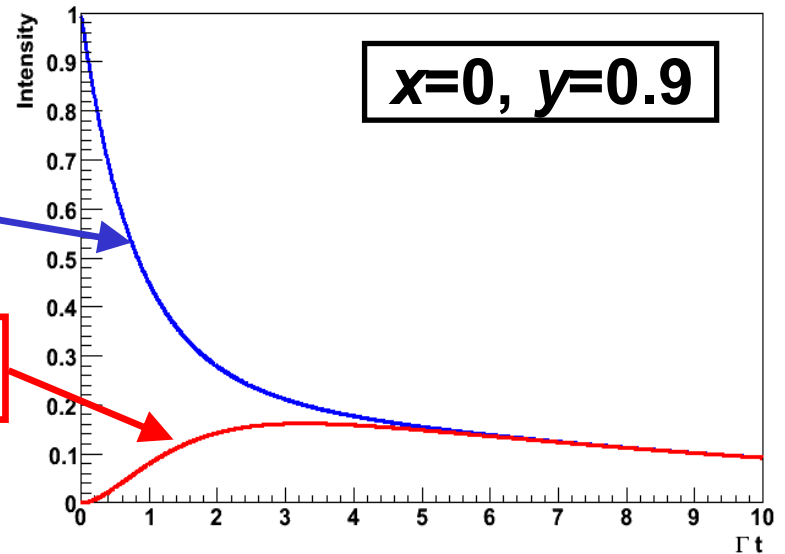
State starts as pure D^0 at $t = 0$

Decays as D^0 or \bar{D}^0



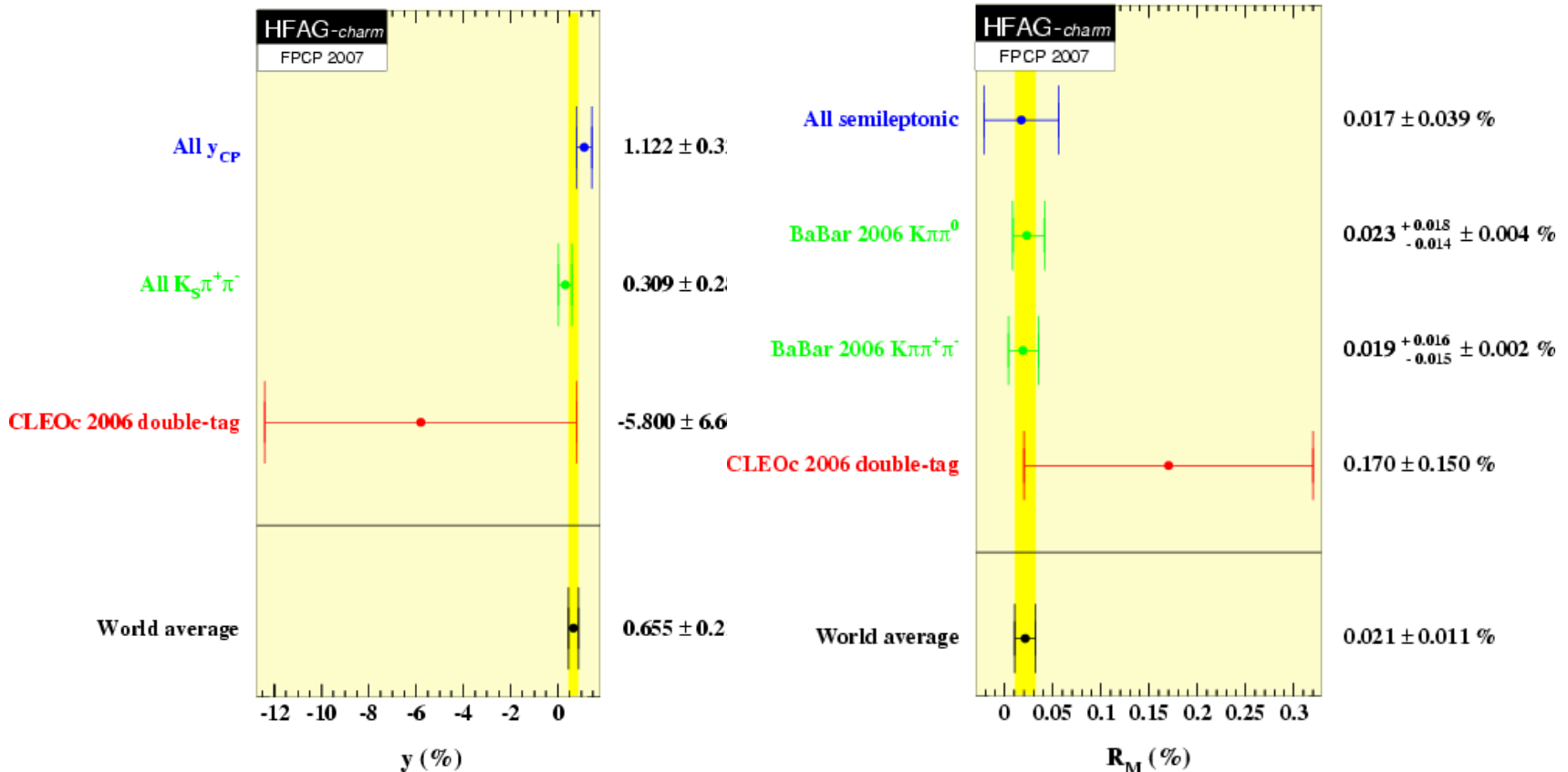
$$|D^0(t)|^2$$

$$|\bar{D}^0(t)|^2$$





HFAG world averages for y_{CP} and R_M

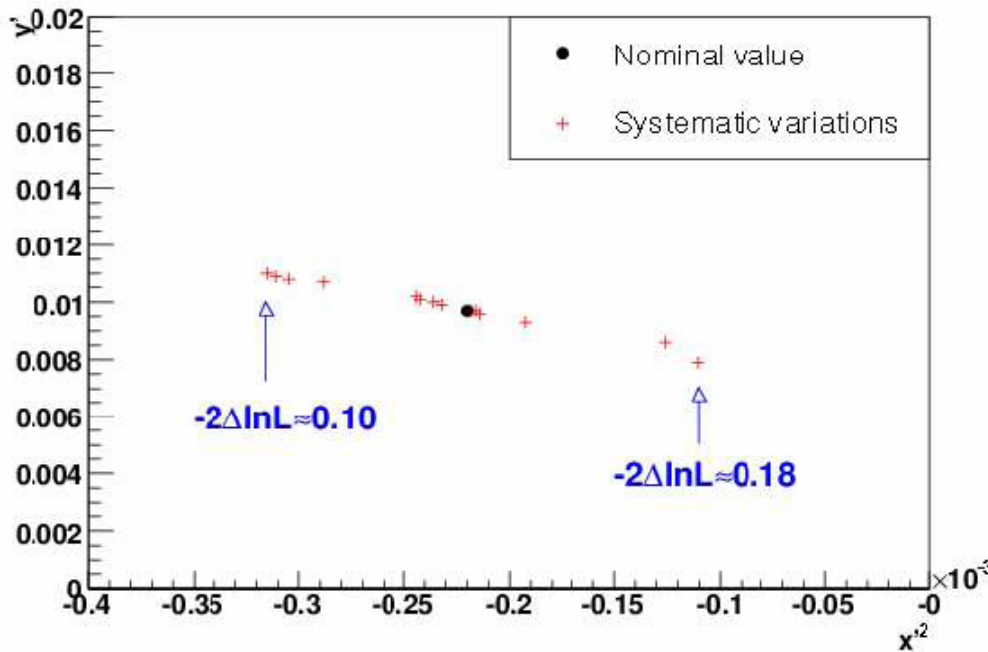


$$y_{CP} = (0.021 \pm 0.011) \%$$

$$R_M = (0.655 \pm 0.211) \%$$

Including Systematics in Contours

Systematic variations produce new mixing parameters sets
 - tend to scatter along correlation axis:



Included in contours as follows:
 - for each variation calculate change in likelihood between new and old point in old likelihood

$$m_i^2 = (-2\Delta\ln L) / 2.3 \quad \leftarrow 1\sigma \text{ in 2D}$$

- Scale likelihood with $\frac{1}{1 + \sum m_i^2}$
 - Should correspond to scaling the statistical uncertainty up

CPV systematics use same scale factor plus tiny correction for charge asymmetry in efficiency

Is this a correct or approximately correct thing to do?



More on systematics

Accounting for systematic errors in contours

■ Sources

- variations in functional form of signal and background terms
- variations in the parameters
- variations in proper time, proper time error and D^* overlap removal criteria

■ (x'^2, y) contours:

- for each variation, compute $s_i^2 = 2 [\ln \mathcal{L}_0 - \ln \mathcal{L}_i] / 2.3$
where \mathcal{L}_0 is the maximum likelihood from the standard fit and \mathcal{L}_i is the likelihood from the standard fit with (x'_i, y'_i) fixed to the values obtained from the fit with the i^{th} variation
 - PDF variations: $\sum s_i^2 = .06$
 - selection criteria: $\sum s_i^2 = .18$
 - total: $\sum s_i^2 = .24$
- divide change in $-2 \log \mathcal{L}$ by the factor $f = 1 + \sum s_i^2 = 1.24$
to account for systematic errors

"Final" Systematics

fit:	y' ($\times 10^{-2}$)	$\delta y'/y'$	R_M ($\times 10^{-4}$)	δR_M^{sys}	m^2
default fit:	0.97 ± 0.44	-	-0.63 ± 1.07	-	-
No offset in core resolution:	1.10 ± 0.44	$+0.30\sigma$	-0.97 ± 1.06	-0.33σ	0.045
offset in all resolution Gaussians:	0.97 ± 0.44	-0.01σ	-0.61 ± 1.07	$+0.02\sigma$	0.000
Proper time error distributions from sidebands, not sPlot:					
widest core Gaussian without per-event errors:	1.01 ± 0.44	$+0.09\sigma$	-0.70 ± 1.07	-0.07σ	0.003
Fix nask factor $s_1 = 1$:	0.93 ± 0.44	-0.09σ	-0.53 ± 1.08	$+0.10\sigma$	0.004
Fix D^0 lifetime to PDG value:	0.97 ± 0.44	-0.00σ	-0.62 ± 1.07	$+0.01\sigma$	0.001
Change Category 3 Model:	0.96 ± 0.44	-0.05σ	-0.61 ± 1.07	$+0.02\sigma$	0.003
Cat.4 ffrom low sideband:	0.85 ± 0.43	-0.28σ	-0.46 ± 1.06	$+0.16\sigma$	0.060
Cat.4 ffrom high sideband:	1.01 ± 0.44	$+0.08\sigma$	-0.65 ± 1.07	-0.02σ	0.011
Vary $(m_{K^*}, \Delta m)$ fit model:	1.00 ± 0.44	$+0.06\sigma$	-0.68 ± 1.07	-0.05σ	0.002
Vary $(m_{K^*}, \Delta m)$ parameters:	1.02 ± 0.44	$+0.10\sigma$	-0.70 ± 1.06	-0.07σ	0.007
$(-1 < t < 3.5)$ ps:	0.86 ± 0.44	-0.26σ	-0.26 ± 1.10	$+0.34\sigma$	0.061
$(-5 < t < 10)$ ps:	1.08 ± 0.44	$+0.24\sigma$	-0.94 ± 1.05	-0.30σ	0.039
$(\delta_1 < 0.4)$ ps:	1.07 ± 0.45	$+0.23\sigma$	-0.87 ± 1.07	-0.22σ	0.023
$(\delta_1 < 0.6)$ ps:	0.79 ± 0.43	-0.41σ	-0.27 ± 1.07	$+0.34\sigma$	0.077
Keep all overlapping candidates	0.99 ± 0.44	$+0.05\sigma$	-0.67 ± 1.06	-0.04σ	0.002
Remove all overlapping candidates	1.09 ± 0.45	$+0.27\sigma$	-0.96 ± 1.07	-0.31σ	0.042
Total variation:		0.71σ		0.70σ	0.306

Systematics summary:

systematic source:	R_D	y'	χ^2
PDF:	0.59σ	0.45σ	0.40σ
selection criteria:	0.24σ	0.55σ	0.57σ
Quadrature total:	0.63σ	0.71σ	0.70σ

Validation: fit to generic Monte Carlo

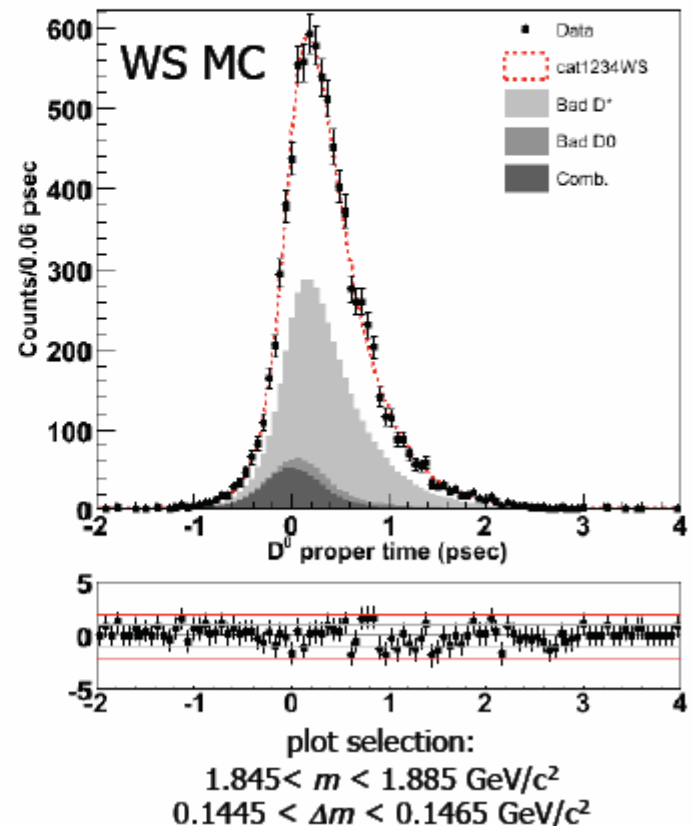
- repeat fitting procedure on R18b generic Monte Carlo sample ($\sim 400 \text{ fb}^{-1}$)
 - WS mixing fit results:

$$y' = (-0.22 \pm 0.30)\%$$

$$\chi^2 = (2 \pm 18) \times 10^{-5}$$

$$R_D = (0.413 \pm 0.014)\%$$

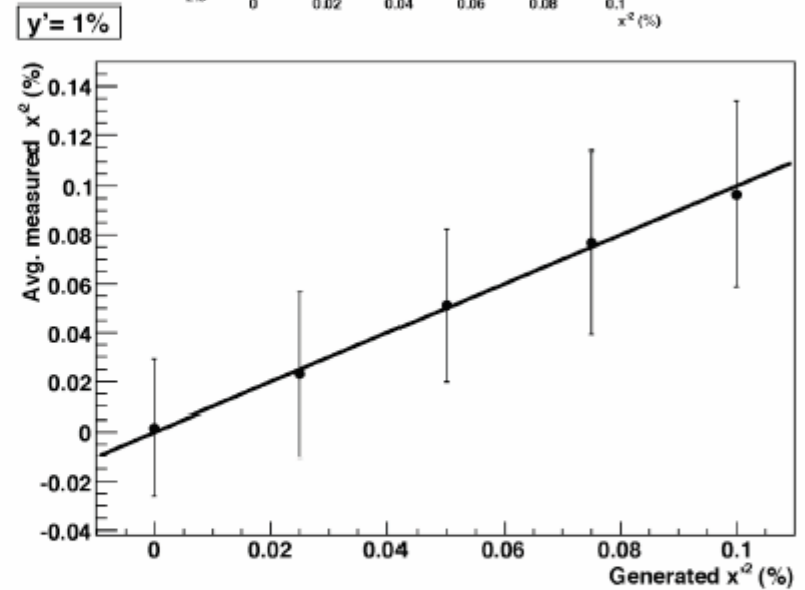
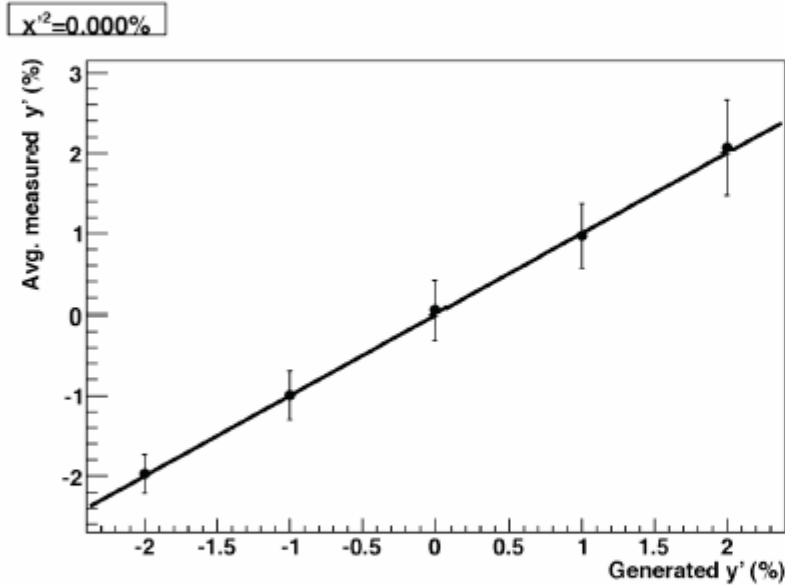
- MC generated without mixing
- No mixing is observed
- R_D consistent with dialed value



Validation: Toy studies

- test for unbiasedness:

100	100	100	100	100
1	48	100	100	100
100	75	51	93	100
100	100	100	100	100

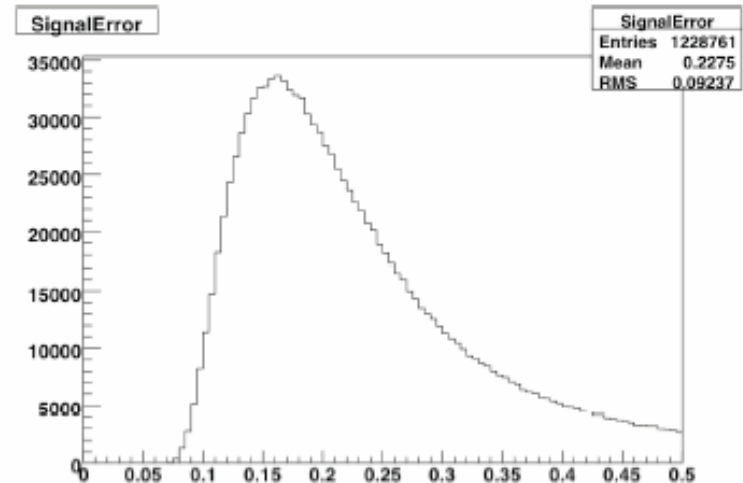


average fitted value of mixing parameter versus generated value.
 Error bars: RMS of fitted values: expected parameter errors
 Straight line has unit slope, 0 intercept.

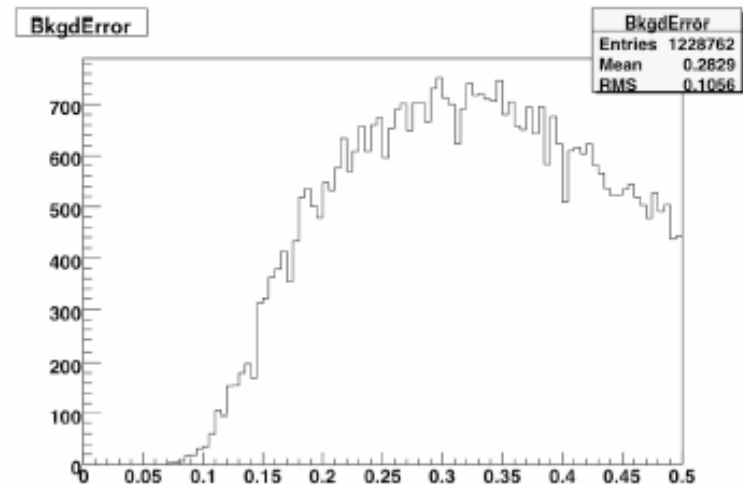
Results indicate no bias in estimating mixing parameters

R18b data decay time error distributions

category 1-3 DecayTimeError sPlot:



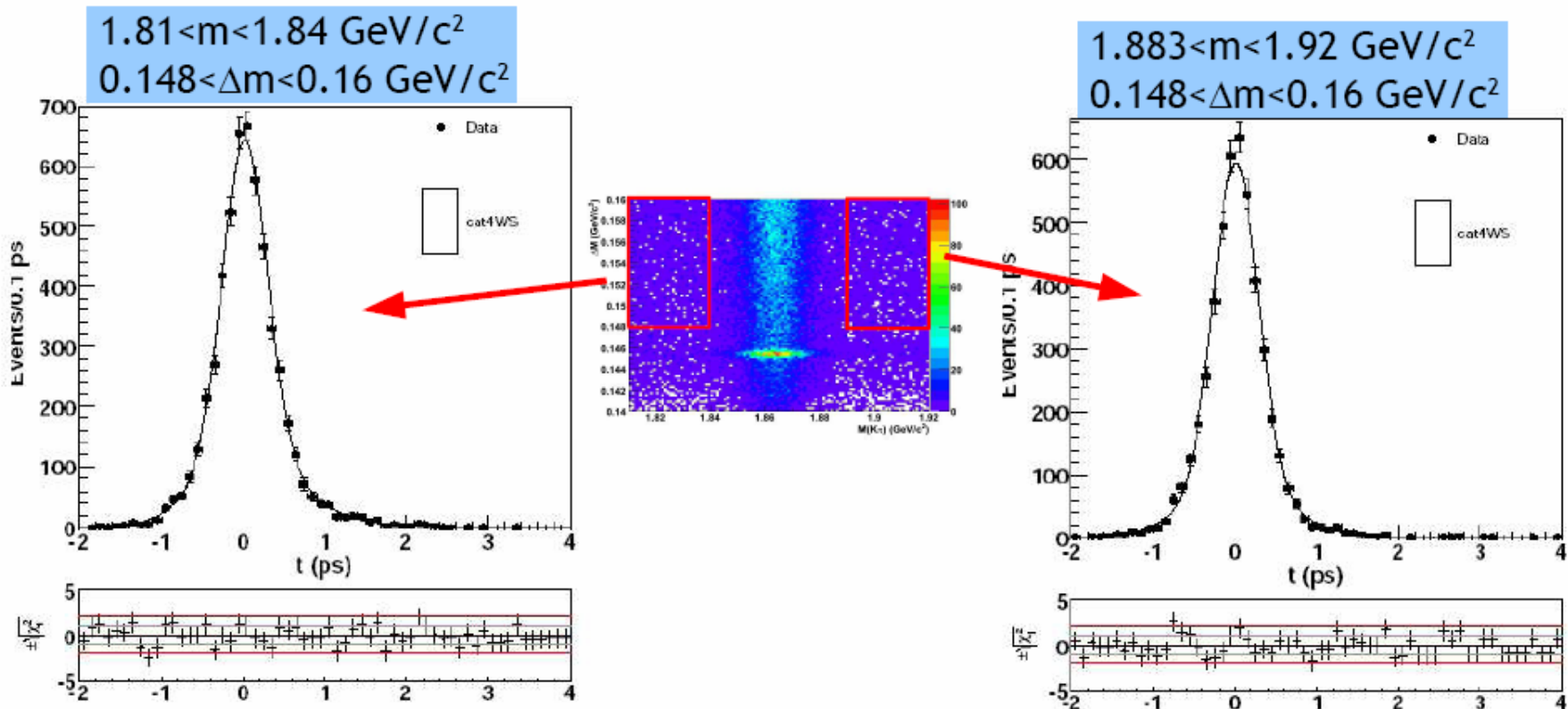
category 4 DecayTimeError sPlot:



Proper Time from Sidebands

Assigning systematic

Instead of fitting proper time for background in full fit, fix it to fits in pure background sidebands:





Time-dependent decay rate

The time-dependent decay rate of an initially-pure D^0 or \bar{D}^0 can be written

$$\begin{aligned}|D^0(t)\rangle &= g_+(t)|D^0\rangle - (q/p)g_-(t)|\bar{D}^0\rangle \\ |\bar{D}^0(t)\rangle &= g_+(t)|\bar{D}^0\rangle - (p/q)g_-(t)|D^0\rangle\end{aligned}$$

where $g_{\pm}(t) = \frac{1}{2}e^{-iMt - \frac{1}{2}\Gamma t} \left(e^{-\frac{i}{2}\Delta Mt - \frac{1}{4}\Delta\Gamma t} \pm e^{+\frac{i}{2}\Delta Mt + \frac{1}{4}\Delta\Gamma t} \right)$

This yields the time-dependent decay rate

$$\begin{aligned}\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] &\propto e^{-\Gamma t} \times \\ &[(|A_f|^2 + |(q/p)\bar{A}_f|^2) \cosh(y\Gamma t) + (|A_f|^2 - |(q/p)\bar{A}_f|^2) \cos(x\Gamma t) \\ &+ 2\text{Re}((q/p)A_f^*\bar{A}_f) \sinh(y\Gamma t) - 2\text{Im}((q/p)A_f^*\bar{A}_f) \sin(x\Gamma t)]\end{aligned}$$



Time-dependent decay rate (1)

Solving the Hamiltonian for the time-dependence of the D_1 , D_2 eigenstates yields

$$|D^0(t)\rangle = g_+(t)|D^0\rangle - (q/p)g_-(t)|\bar{D}^0\rangle$$

$$|\bar{D}^0(t)\rangle = g_+(t)|\bar{D}^0\rangle - (p/q)g_-(t)|D^0\rangle$$

where $g_{\pm}(t) = \frac{1}{2}e^{-iMt - \frac{1}{2}\Gamma t} \left(e^{-\frac{i}{2}\Delta Mt - \frac{1}{4}\Delta\Gamma t} \pm e^{+\frac{i}{2}\Delta Mt + \frac{1}{4}\Delta\Gamma t} \right)$

This yields the approximate time-dependent decay rate (for $x, y \ll 1$)

Mixing

$$\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] \propto e^{-\Gamma t} \left(R_D + \sqrt{R_D} y' \Gamma t + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right)$$

DCS decay

Interference between DCS and mixing



Systematics: decay time resolution

Decay-time resolution

Sum of 3 Gaussians

Narrowest has a non-zero mean of 3.6 fsec

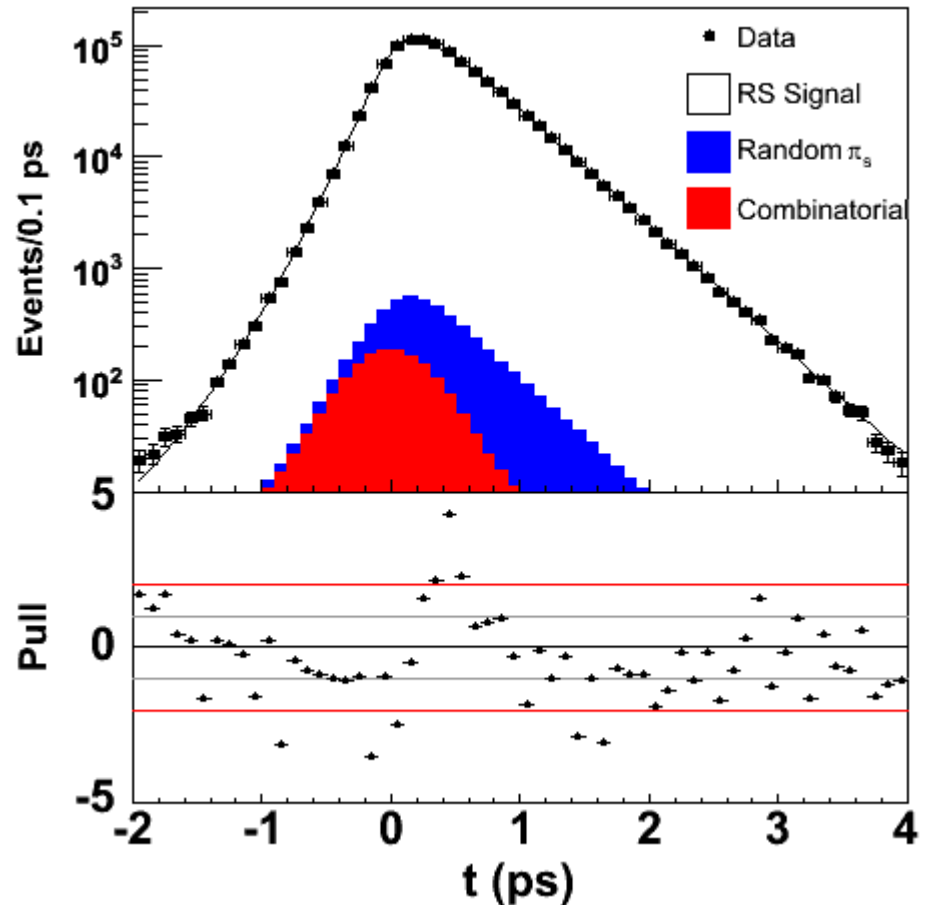
Most likely due to alignment issues.

Also seen in other analyses.

Check by setting offset to zero and refitting for mixing parameters.

χ^2 changes by -0.3σ

y' changes by $+0.3\sigma$

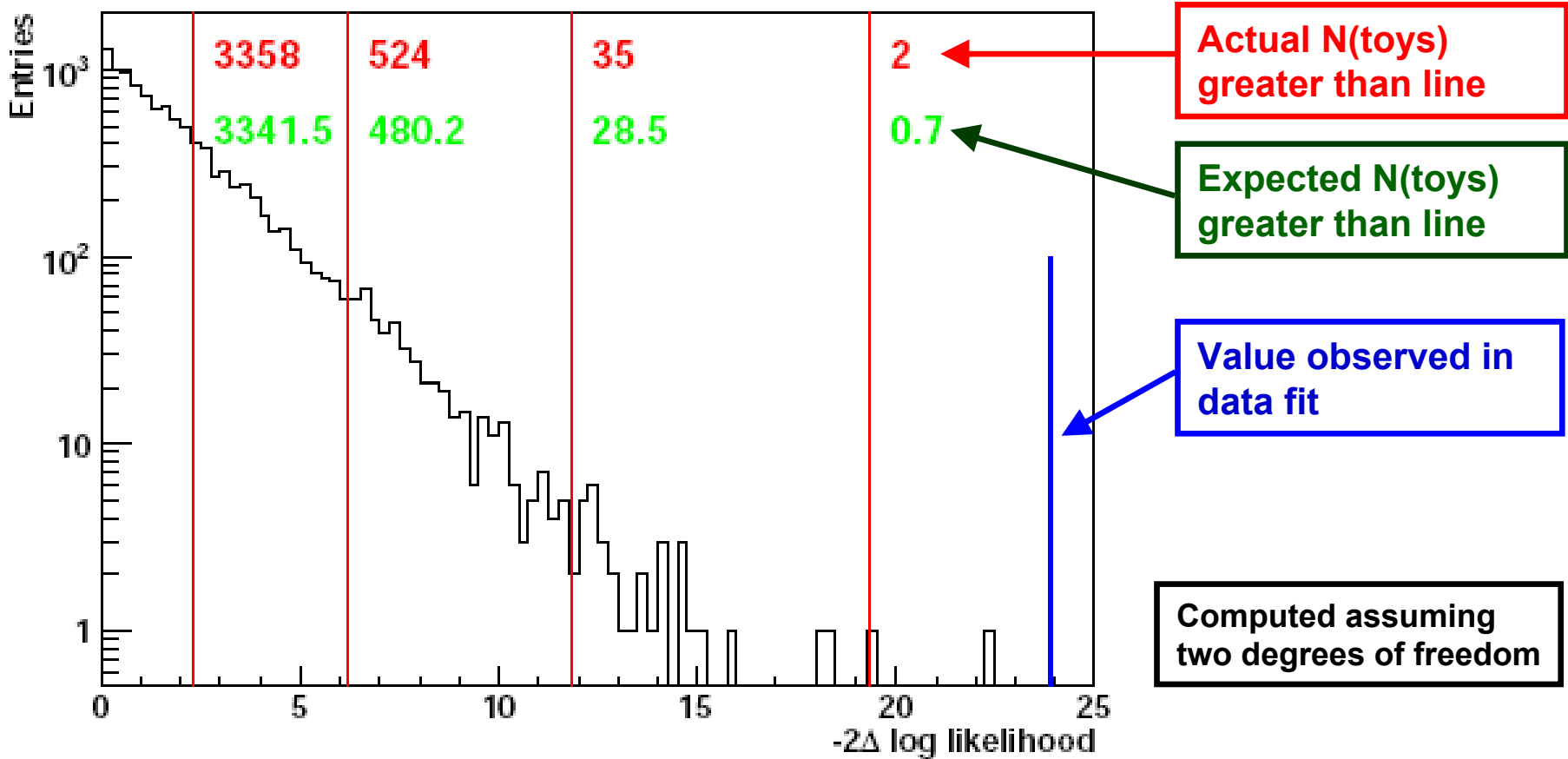


RS decay time fit with zero offset.



Validation: $-2\Delta\ln L$ frequentist coverage

Generated >10000 toys without mixing to test frequentist coverage





Mixing Hamiltonian

Neutral D^0 and \bar{D}^0 mesons are produced as flavor eigenstates of the strong interaction.

Their time development is governed by a 2×2 effective Hamiltonian

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

which has physical eigenstates D_1, D_2 that are linear combinations of the flavor eigenstates

$$\begin{aligned} |D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle \\ |D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle \end{aligned} \quad \text{where} \quad \left(\frac{q}{p} \right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

and $|q|^2 + |p|^2 = 1$.

The states D_1, D_2 possess masses M_1, M_2 and lifetimes Γ_1, Γ_2 .



Masses, lifetimes, and amplitudes

We define mass and lifetime *differences* and *averages* of the physical eigenstates D_1, D_2

$$\Delta M = M_1 - M_2, \quad \Delta\Gamma = \Gamma_1 - \Gamma_2, \quad M = \frac{M_1 + M_2}{2}, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

and the parameters $x = \frac{\Delta M}{\Gamma}$ and $y = \frac{\Delta\Gamma}{2\Gamma}$.

We also define weak (H_w) *decay amplitudes* to *CP-conjugate* final states $f = K^+\pi^-, \bar{f} = K^-\pi^+$ as

$$A_f = \langle f | H_w | D^0 \rangle, \quad \bar{A}_f = \langle f | H_w | \bar{D}^0 \rangle, \quad A_{\bar{f}} = \langle \bar{f} | H_w | D^0 \rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f} | H_w | \bar{D}^0 \rangle$$

WS

RS

RS

WS



Event selection details

Perform a beam-constrained fit to the full decay chain

$$D^{*\pm} \rightarrow \pi_s^\pm D^0, D^0 \rightarrow K^\mp \pi^\pm$$

Require fit probability > 0.001

$$\delta t < 0.5 \text{ ps}$$

$$-2 < t < 4 \text{ ps}$$

Select the D^0

$$\text{CM } p_D > 2.5 \text{ GeV}/c$$

K, π particle identification

$$1.81 < m_{K\pi} < 1.92 \text{ GeV}/c^2$$

Select the D^{*+}

$$\text{CM } p_\pi < 0.45 \text{ GeV}/c$$

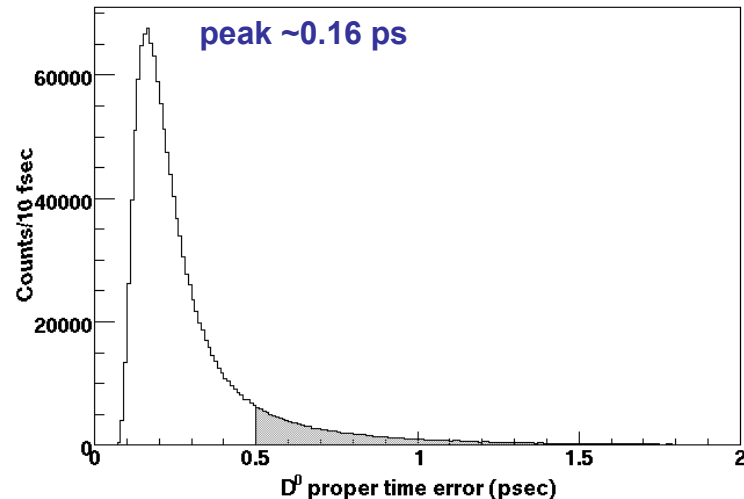
$$p_\pi > 0.1 \text{ GeV}/c \text{ in lab frame}$$

$$0.14 < \Delta m < 0.16 \text{ GeV}/c^2$$

If multiple D^{*+} candidates share tracks in the event:

Select candidate with greatest fit probability

Event selection, fitting procedures are *finalized before examining* the mixing results





Separating signal and backgrounds

Signal and backgrounds have differing behavior in $m_{K\pi}$ and Δm .

We define four categories:

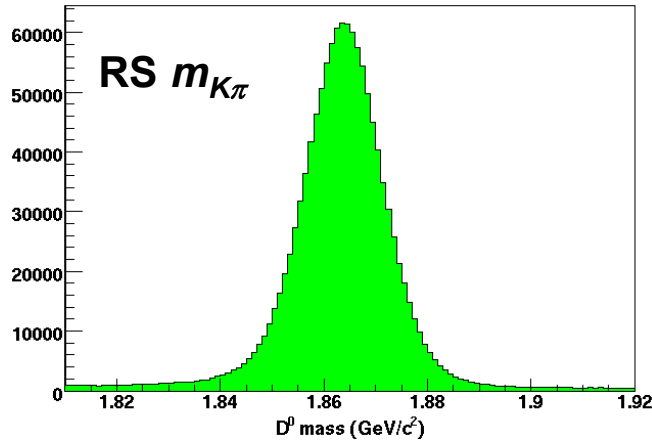
Signal, random π_s , mis-reconstructed D^0 , and combinatoric.

Category	Description	Peaking Behavior
RS signal	$D^0 \rightarrow K^- \pi^+$ signal	$m_{K\pi}$ and Δm
RS random π_s	Correctly-reconstructed D^0 combined with an incorrect slow pion	$m_{K\pi}$
RS mis-recon. D^0	Mis-reconstructed D^0 from $D^0 \rightarrow Kl^+\nu$, $D^0 \rightarrow \pi l^+\nu$, $D^0 \rightarrow \pi^+\pi^-$, $D^0 \rightarrow K^+K^-$	Δm
RS combinatoric	Combinatoric background	non-peaking
WS signal	$D^0 \rightarrow K^+\pi^-$ signal	$m_{K\pi}$ and Δm
WS random π_s	Correctly-reconstructed D^0 combined with an incorrect slow pion	$m_{K\pi}$
WS mis-recon. D^0	Doubly mis-identified $D^0 \rightarrow K^- \pi^+$ decays and $D^0 \rightarrow \pi^+\pi^-$, $D^0 \rightarrow K^+K^-$ reflections	Δm
WS combinatoric	Combinatoric background	non-peaking

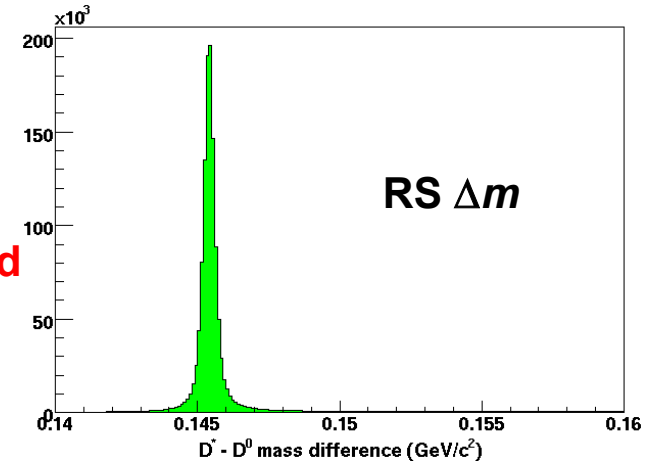


RS & WS $m_{K\pi}$, Δm projections

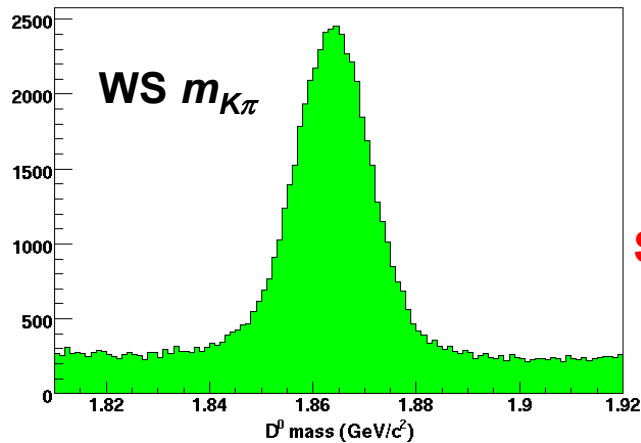
counts/1 MeV/c²



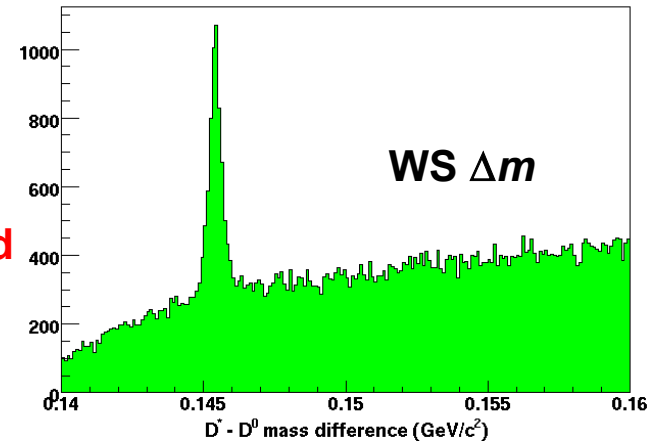
1,229,000
RS candidates
Signal:background
 $\approx 100:1$



counts/0.1 MeV/c²



64,000
WS candidates
Signal:background
 $\approx 1:1$





Validation: fit for mixing in RS sample

Fit the RS data using the WS mixing PDF

$$x' = (-0.01 \pm 0.01) \times 10^{-3}$$

$$y' = (0.26 \pm 0.24) \times 10^{-3}$$

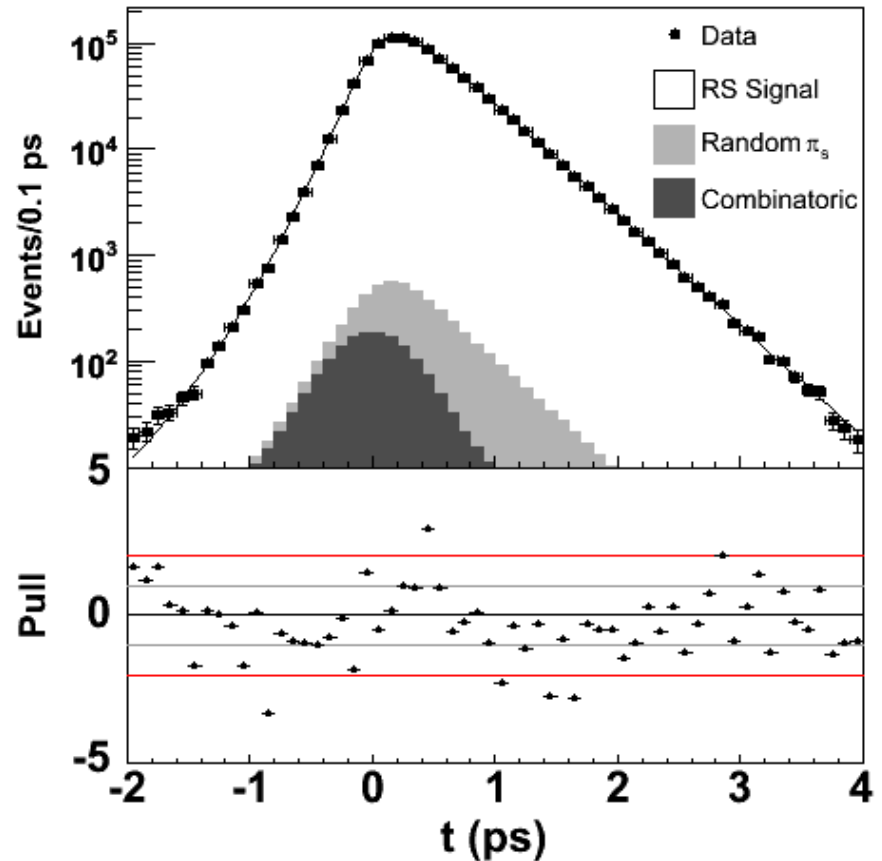
The change in $-2\Delta\ln L$ is 1.4

A very stringent test

RS sample $270\times$ larger than WS sample

Conclusion:

D^0 decay-time distribution is properly described.



RS mixing fit projection in signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



Validation: fit for mixing in MC

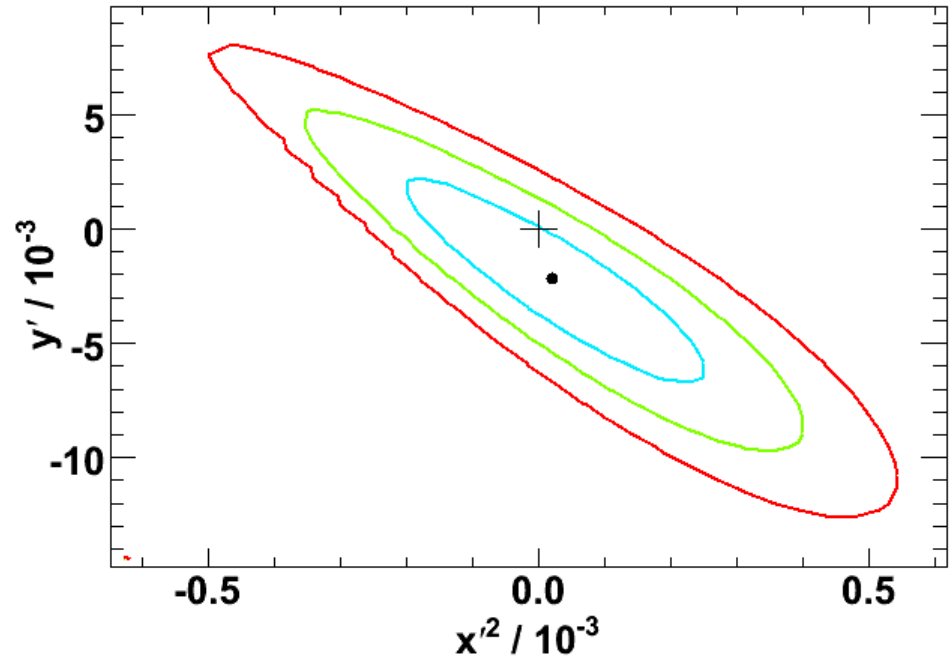
Fit MC for mixing

MC generated with no mixing

Fit finds no mixing signal:

$$x'^2 = (-0.02 \pm 0.18) \times 10^{-3}$$

$$y' = (2.2 \pm 3.0) \times 10^{-3}$$



Result of mixing fit to MC
(which has no mixing).
Contours are at 1σ , 2σ , and 3σ



CP violation

CP violation (CPV) can be classified as occurring

- ◆ **In direct decay:** $|\bar{A}_{\bar{f}}/A_f| \neq 1$
where $A_f = \langle f|H_w|D^0\rangle$, $\bar{A}_{\bar{f}} = \langle \bar{f}|H_w|\bar{D}^0\rangle$
- ◆ **In mixing:** $|q/p| \neq 1$
- ◆ **In the interference between them:** $\text{Im} \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right) \neq 0$

CPV introduces an asymmetry

in the time-dependence between D^0 and \bar{D}^0 decays

$$\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] \propto e^{-\Gamma t} \times \left[R_D + \sqrt{R_D} \left| \frac{q}{p} \right| (y' \cos \varphi - x' \sin \varphi) \Gamma t + \left| \frac{q}{p} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]$$
$$\frac{d\Gamma}{dt} [|\bar{D}^0(t)\rangle \rightarrow \bar{f}] \propto e^{-\Gamma t} \times \left[R_D + \sqrt{R_D} \left| \frac{p}{q} \right| (y' \cos \varphi + x' \sin \varphi) \Gamma t + \left| \frac{p}{q} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]$$

where φ is the phase angle of $\lambda_f = \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right)$.



Mixing and CPV fit results

Fit results for all three cases:

(1) No mixing or CPV; (2) mixing but no CPV; and (3) CPV and mixing.
 R_D changes between no-mixing and mixing fits.

Fit type	Parameter	Fit Results ($/10^{-3}$)
No CP viol. or mixing	R_D	$3.53 \pm 0.08 \pm 0.04$
No CP violation	R_D	$3.03 \pm 0.16 \pm 0.10$
	x'^2	$-0.22 \pm 0.30 \pm 0.21$
	y'	$9.7 \pm 4.4 \pm 3.1$
CP violation allowed	R_D	$3.03 \pm 0.16 \pm 0.10$
	A_D	$-21 \pm 52 \pm 15$
	x'^{2+}	$-0.24 \pm 0.43 \pm 0.30$
	y'^+	$9.8 \pm 6.4 \pm 4.5$
	x'^{2-}	$-0.20 \pm 0.41 \pm 0.29$
	y'^-	$9.6 \pm 6.1 \pm 4.3$



Systematics

Investigate

Variations in functional forms of PDFs

Variations in the fit parameters

Variations in the event selection

Computed using full difference with original value

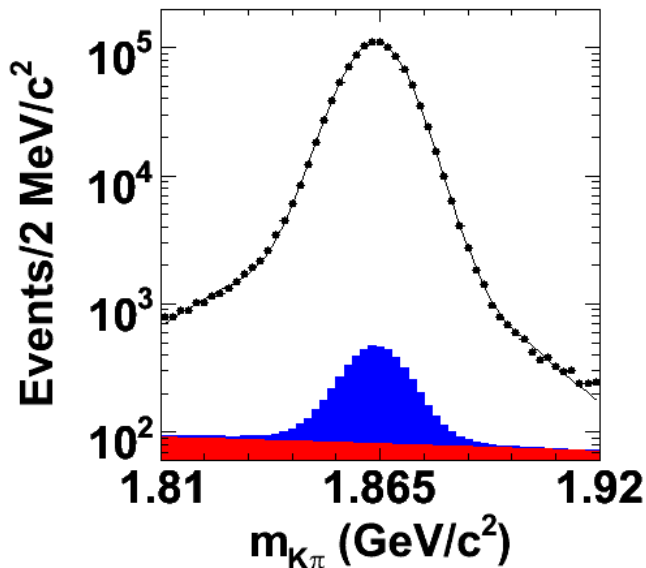
Results are expressed in units of the statistical error

Systematic source	R_D	y'	χ^2
PDF:	0.59σ	0.45σ	0.40σ
Selection criteria:	0.24σ	0.55σ	0.57σ
Quadrature total:	0.63σ	0.71σ	0.70σ



Right-sign $m_{K\pi}$, Δm fit

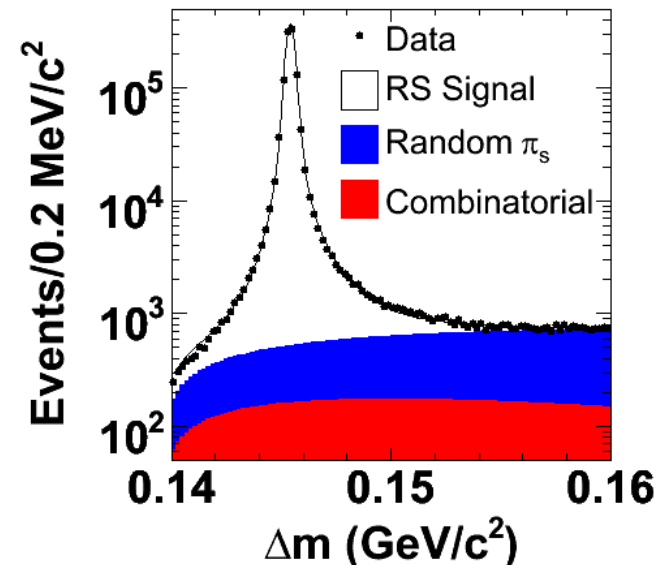
Shown are the fits to right-sign data for $m_{K\pi}$ (left) and Δm (right).



$1,141,500 \pm 1,200$
RS signal events

The mis-reconstructed D^0 category is not included in the RS fit.

This background is too small to be reliably determined.





RS proper decay-time fit

The parameters fitted are

D^0 lifetime τ_D

Resolution parameters

Including a 3.6 fsec offset

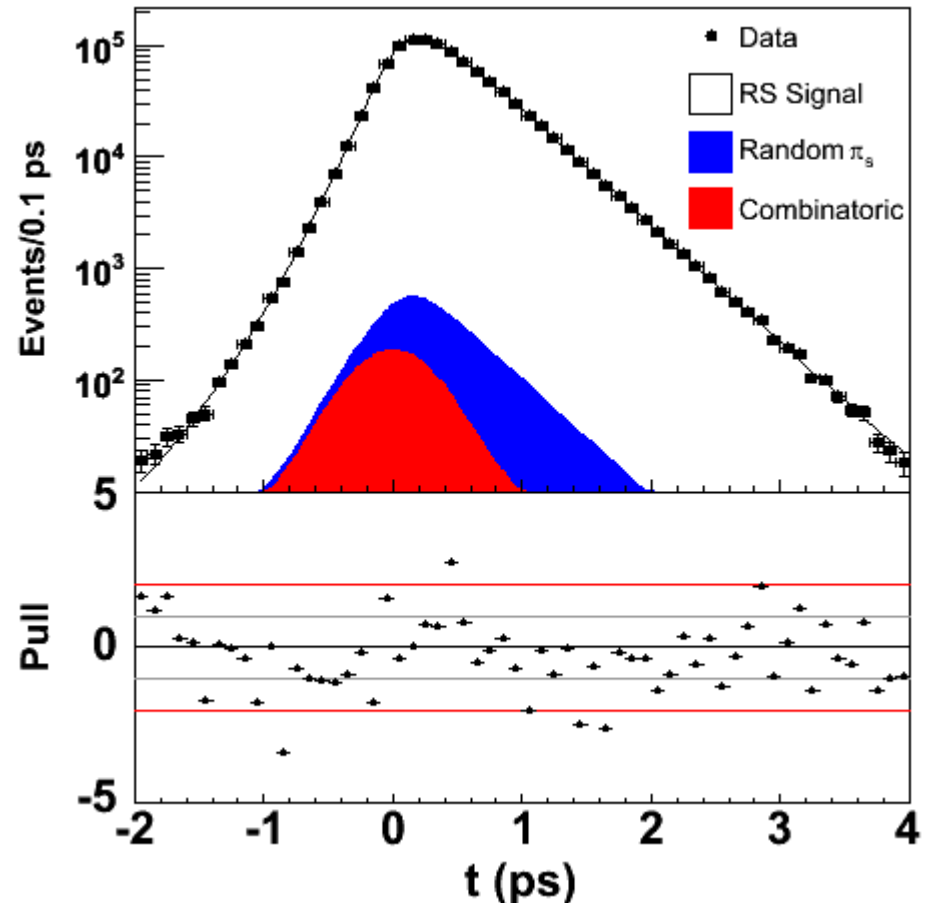
Signal, background category yields

Consistency check

Fitted $\tau_D = (410.3 \pm 0.6)$ fsec

(statistical error only)

(PDG 2006: 410.1 ± 1.5 fsec)



RS fit projection in the signal region

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Validation: $-2\Delta\ln L$ frequentist coverage

Generated $>100,000$ toys without mixing to test frequentist coverage

