$D^0 - \bar{D}^0$ Mixing: Theory Introduction

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Outline

- **$D^0 - \bar{D}^0$ mixing in the SM:** box vs. long-distance contributions
- **Observables sensitive to the mixing:** recent experimental progress
- **Combining data:**
  - determination of the mixing parameters
  - Model Independent constraints on New Physics (NP)
- **Application to SUSY**

Basic Formalism

☑ One calculates the time-evolution of the 'flavor-eigenstates' $D^0 = (c\bar{u})$ and $\bar{D}^0$ through

$$i\frac{d}{dt}\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} \quad \text{with} \quad \hat{M} = \begin{pmatrix} M & M_{12} \\ M^*_{12} & M \end{pmatrix}, \; \hat{\Gamma} = \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma^*_{12} & \Gamma \end{pmatrix},$$

because of CPT + Hermiticity

⇨ Mass eigenstates [with masses and widths $m_{1,2}$ and $\Gamma_{1,2}$] are defined through

$$|D_{1,2}\rangle = p\,|D^0\rangle \pm q\,|\bar{D}^0\rangle \quad \text{with} \quad \frac{q}{p} = \sqrt{\frac{M^*_{12} - \frac{i}{2}\Gamma^*_{12}}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

and

Basic mixing observables

$$x = \frac{m_2 - m_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$

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If $|q/p| = 1$ $\Rightarrow$ $|D_1\rangle$ CP even, $|D_2\rangle$ CP odd

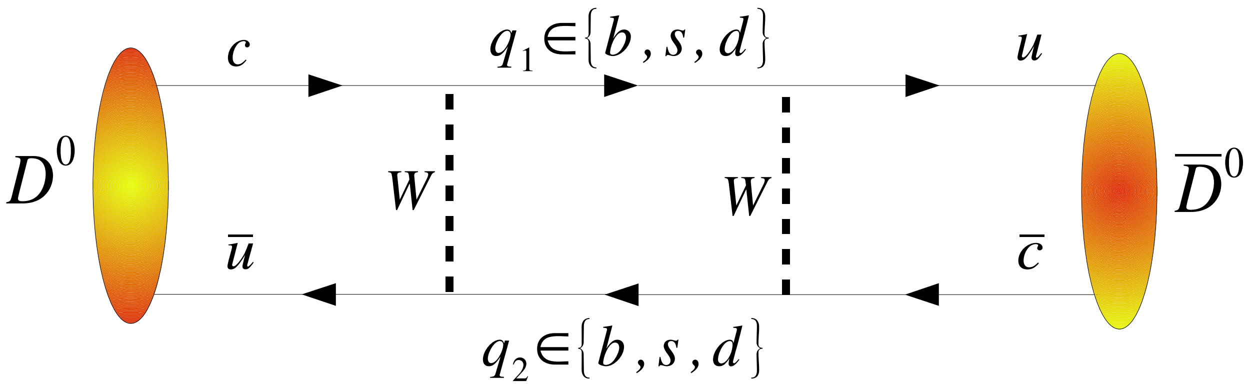
since one can choose phases such that $|D^0\rangle \overset{CP}{\leftrightarrow} |\bar{D}^0\rangle$

If $|q/p| \neq 1$ $\Rightarrow$ Mass eigenstates cannot be chosen as CP eigenstates $\Rightarrow$ CPV in Mixing

$D^0 - \overline{D}^0$ mixing in the SM

- The SM contribution to meson mixings is well-described – in the K, B_d and B_s cases – by box diagrams with W – up-type-quarks

- In the D case one has: $D^0 \sim \begin{pmatrix} c \\ \bar{u} \end{pmatrix}$, so that box contributions for D^0-mixing give



$$\propto \sum_{q_1, q_2} \text{CKM}[q_1, q_2] \times S_0[m_1^2, m_2^2]$$

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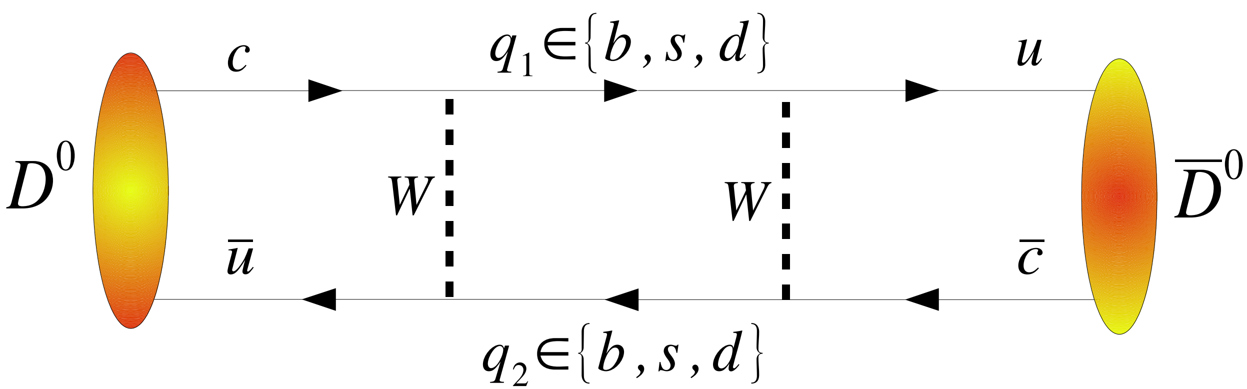

$$\propto \sum_{q_1, q_2} \mathrm{CKM}[q_1, q_2] \times S_0[m_1^2, m_2^2]$$

For $m_1^2 = m_2^2 \ll M_W^2$ one has $S_0[m_1^2, m_1^2] \sim m_1^2 / M_W^2$

With down-type quarks in the loop,
GIM-suppression is extremely effective

Further remarks

☞ In the D-mixing case, one has

$$\frac{[b-b\,\text{contrib.}]}{[s-s\,\text{contrib.}]} = \frac{m_b^2}{m_s^2}\frac{(V_{ub}V_{cb}^*)^2}{(V_{us}V_{cs}^*)^2} \approx (1.8\cdot 10^3)\times(A^4\lambda^8) \approx 10^{-3}$$

Compare with the K, $B_{d,s}$ cases, where the 3rd family (top) contribution is always important [for $B_{d,s}$ it is dominant]

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Consequences:

☺ **[SM box is tiny]:** in principle ideal room for New Physics to show up

☹ **[long distance]:** $m_c \approx$ hadronic scale. K, π intermediate states likely to dominate

Estimates:

$x_{\text{box}} \leq 10^{-5}$

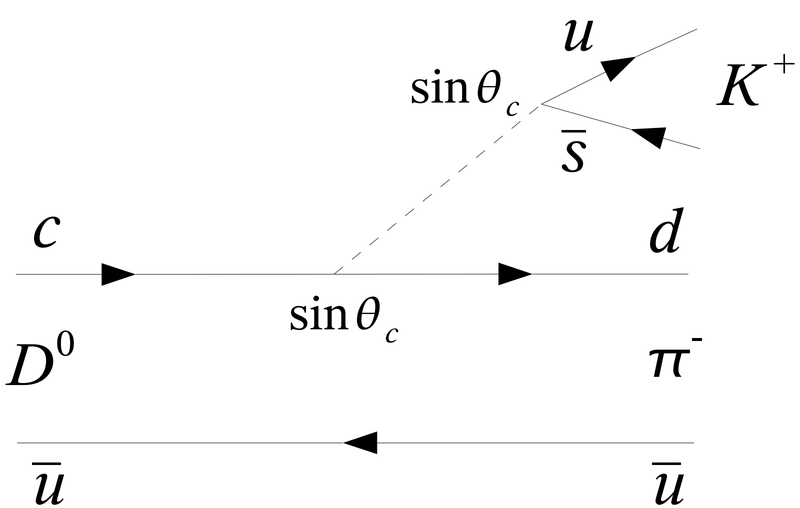
$x_{\text{long dist.}} \leq O(10^{-3})$

See:
Burdman & Shipsey
+ Refs. therein

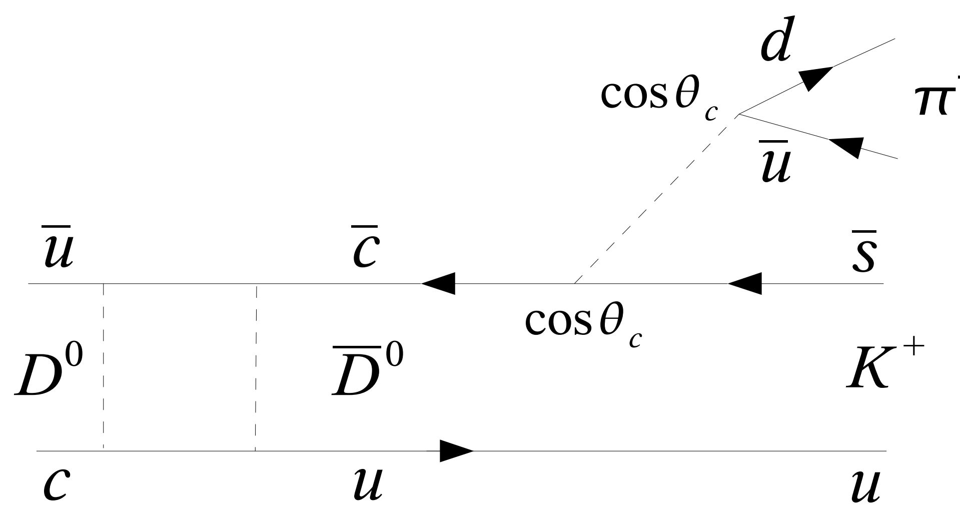
Where to look for $D^0 - \overline{D}^0$ mixing

Example: "wrong sign" $D \to K\,\pi$ decays

Doubly-Cabibbo-Suppressed [DCS]



Mixing + Cabibbo-Favored [CF]



Naïve estimate

$$R_D \equiv \frac{|\langle K^+\pi^-|H_{\rm eff}|D_0\rangle|^2}{|\langle K^-\pi^+|H_{\rm eff}|D_0\rangle|^2} \sim (\tan\theta_c)^4 = 0.3\,\%$$

In the wrong-sign decays, the "DCS" and "mixing + CF" contributions become competitive. Mixing is then measurable.

Basic Formalism: 2

CPV in Interference between decay amplitudes with and without Mixing

Example: $\Gamma(D^0 \to K^+ \pi^-) \sim |$ "DCS" + "mixing+CF" $|^2$

interference

driven by the mixing phase $\phi = Arg\left(\frac{q}{p}\right) \neq 0$

[phases in decay ampl. are negligible: no "direct" CPV]

From the definition of q/p one has

$$\phi = Arg\left(\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}\right)$$

Note

In the D-case, $|M_{12}| \approx |\Gamma_{12}|$ and Arg(M_{12}) small,

$\Rightarrow \quad \phi \approx$ -Arg(M_{12}) × [O(1) factor]

Arg(M_{12}): naïve estimate from SM-boxes

$$\lambda^2 S_0(x_s, x_s) + A^2\lambda^6(\rho - i\eta) S_0(x_s, x_b)$$

Real Part: s-s box — Imaginary Part: s-b box

$$\Rightarrow \quad Arg(M_{12}) \sim A^2\lambda^4\eta \frac{S_0(x_s, x_b)}{S_0(x_s, x_s)} \leq 10^{-2}$$

Recent Experimental Progress

Access to:

BaBar

hep-ex/0703020

- Studies the time dependence of $D^0 \to K^- \pi^+$ [CF] and $D^0 \to K^+ \pi^-$ [DCS] [and C-conj. modes]

$R_D\,,\quad x'^2_\pm = x'^2\,,\quad y'_\pm = y'$ (assuming no CPV)

Primed *x, y* are a rotation of *x, y* by the strong $K\pi$ phase

Belle

0704.1000 [hep-ex]

- Studies the time dependent Dalitz distribution of $D^0 \to K_S^0 \pi^+ \pi^-$

$x\,,\quad y$

hep-ex/0703036

- Calculates the asymmetry between the decay $D^0 \to K^- \pi^+$ [CF] and decays to CP eigenstates ($K^+ K^-$, $\pi^+\pi^-$)

y_{CP}

Assuming no CPV: $y_{CP} = y$

CLEO

hep-ex/0607078

- Measures the BR of D^0 decays to hadronic flavored states, CP eigenstates, and semilep. states

strong phase $\delta_{K\pi}$

Combining the data

Data can all be expressed in terms of the quantities:

x , y , $\delta_{K\pi}$, ϕ , $|q/p|$

Parameter	Value	Ref.
x'^2_+	$(-0.24 \pm 0.43 \pm 0.30) \cdot 10^{-3}$	[8]
x'^2_-	$(-0.20 \pm 0.41 \pm 0.29) \cdot 10^{-3}$	[8]
y'_+	$(9.8 \pm 6.4 \pm 4.5) \cdot 10^{-3}$	[8]
y'_-	$(9.6 \pm 6.1 \pm 4.3) \cdot 10^{-3}$	[8]
x	$(8.0 \pm 3.4) \cdot 10^{-3}$	[9]
y	$(3.3 \pm 2.8) \cdot 10^{-3}$	[9]
y_{CP}	$(13.1 \pm 4.1) \cdot 10^{-3}$	[10]
A_Γ	$(0.1 \pm 3.4) \cdot 10^{-3}$	[10]
$\cos\delta_{K\pi}$	1.09 ± 0.66	[11]
τ_D	(0.4101 ± 0.0015) ps	[13]

from which one calculates the fundamental mixing parameters:

$$M_{12} = |M_{12}| e^{-i\Phi_{12}}, \qquad \Gamma_{12} = |\Gamma_{12}|,$$

$$|M_{12}|\tau_D = \sqrt{\frac{x^2 + \delta^2 y^2}{4(1-\delta^2)}}, \quad |\Gamma_{12}|\tau_D = \sqrt{\frac{y^2 + \delta^2 x^2}{1-\delta^2}},$$

$$\sin\Phi_{12} = \frac{|\Gamma_{12}|^2 + 4|M_{12}|^2 - (x^2+y^2)|q/p|^2/\tau_D^2}{4|M_{12}\Gamma_{12}|},$$

with $\delta^2 = |p|^2 - |q|^2 \simeq \frac{1}{4}\left(\left|\frac{q}{p}\right|^2 - 1\right)^2$

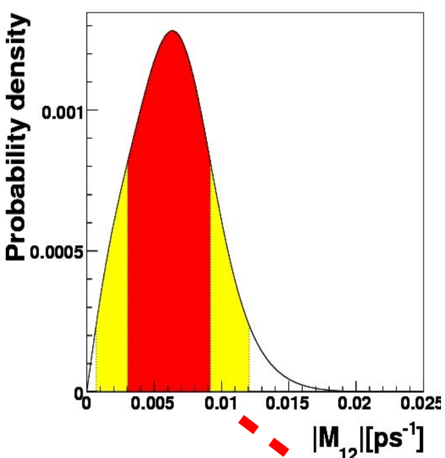
Global Fit

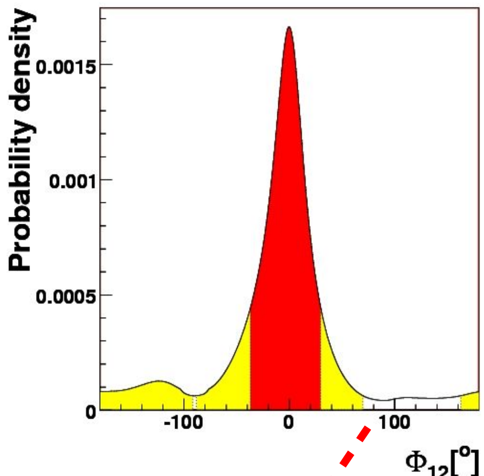
Parameter	68% prob.	95% prob.
x	$(4.8 \pm 2.8) \cdot 10^{-3}$	$[-0.0007, 0.0102]$
y	$(6.1 \pm 1.9) \cdot 10^{-3}$	$[0.0023, 0.0102]$
$\delta_{K\pi}$	$(-21 \pm 43)^\circ$	$[-103^\circ, 45^\circ]$
ϕ	$(-1 \pm 10)^\circ$	$[-46^\circ, 25^\circ]$
$\left|\frac{q}{p}\right| - 1$	0.01 ± 0.20	$[-0.41, 0.64]$
$|M_{12}|$	$(6.1 \pm 3.1) \cdot 10^{-3}$ ps^{-1}	$[0.0007, 0.0121]$ ps^{-1}
Φ_{12}	$(-4 \pm 33)^\circ$	$[-88^\circ, 70^\circ] \cup [163^\circ, 268^\circ]$
$|\Gamma_{12}|$	$(15.5 \pm 4.5) \cdot 10^{-3}$ ps^{-1}	$[0.0067, 0.0247]$ ps^{-1}

Model-Independent constraints on New Physics

Ciuchini *et al.*
hep-ph/0704204

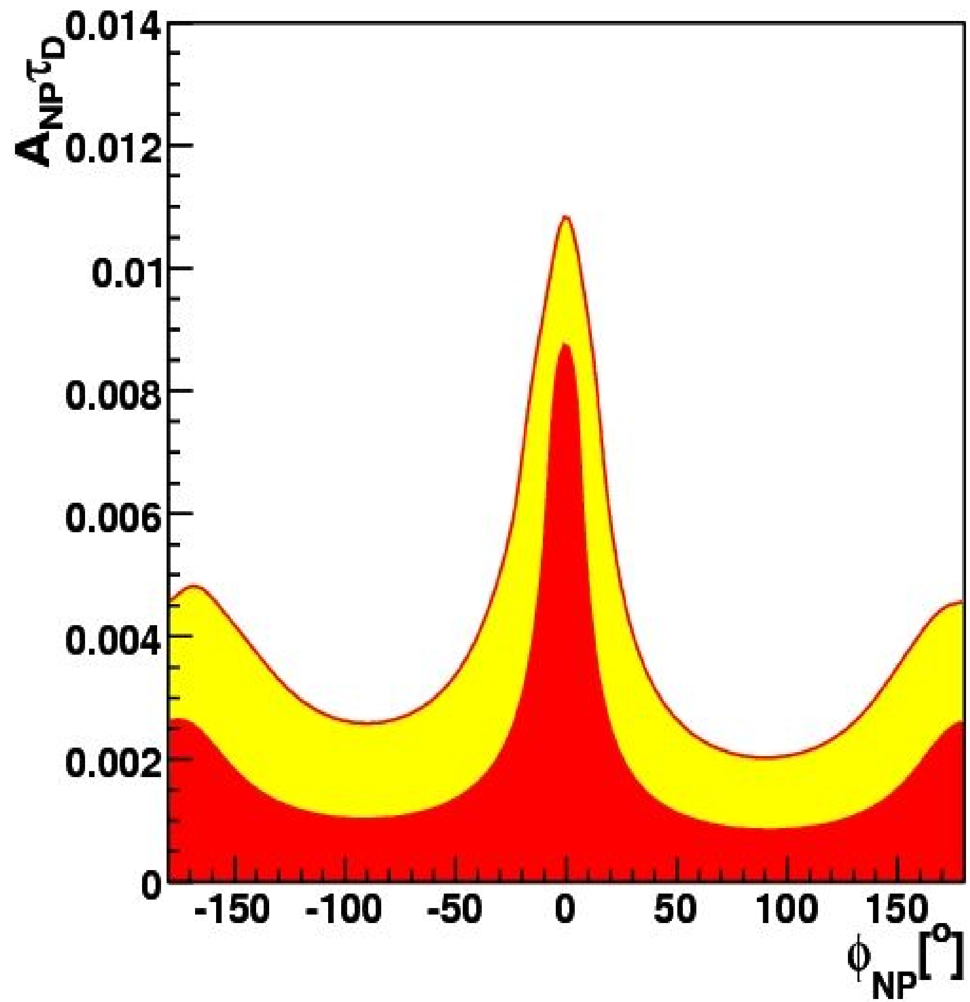
☑ The determination of M_{12} can be used to place constraints on any extension of the SM





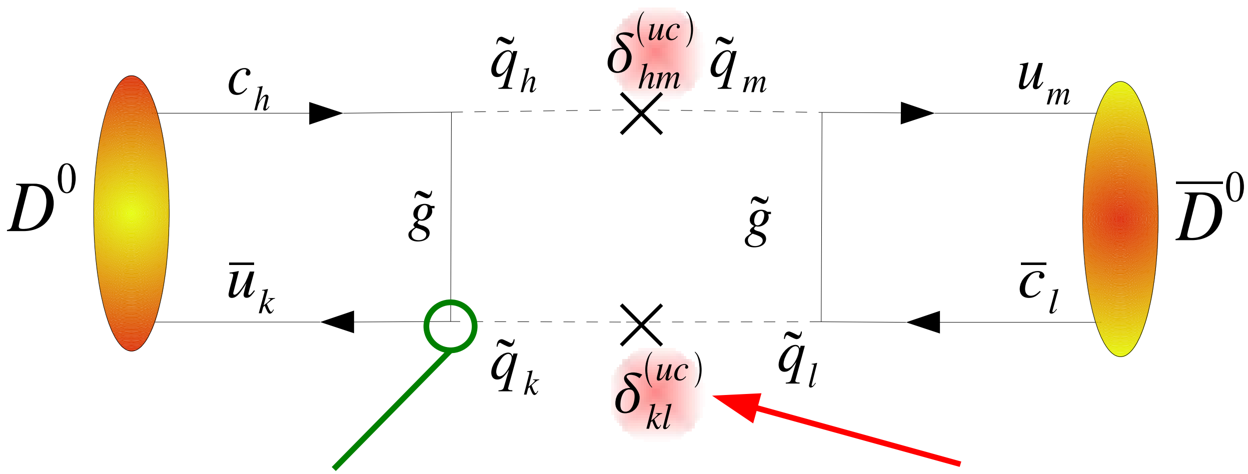
$$M_{12} = |M_{12}| \; e^{-i\Phi_{12}}$$
$$= A_{SM} + A_{NP} \; e^{i\Phi_{NP}}$$

we assume it flatly distributed in [-0.015, 0.015]/ps



Application: Constraints on the 'general' MSSM

- ☑ In the MSSM, squarks get (most of their) masses via 'soft-terms'. The latter are off-diagonal in flavor-chirality and generate FCNCs.
- ☑ When soft-terms are completely general, one can suppose SUSY contributions to $\Delta F = 2$ be dominated by quark-squark-gluino boxes.



basis where q-s-g vertices are flavor-diagonal

⇨ flavor-violation will be manifest in the (off-diagonal) squark propagators: flavor-violating Mass Insertions

General Structure

$$\text{SUSY corrections} \sim \left(\frac{\delta}{M_{\text{SUSY}}} \right)^2 \times f\,(\text{SUSY mass ratios})$$

Constraints on *Re[δ]* and *Im[δ]*

General procedure: Our matrix element is written as follows

$$\langle \overline{D}^0 | H_{eff}^{\Delta C=2} | D^0 \rangle = A_{SM} + A_{SUSY} \left(Re(\delta_{AB}^{(uc)})^2 + i\, Im(\delta_{AB}^{(uc)})^2 \right)$$

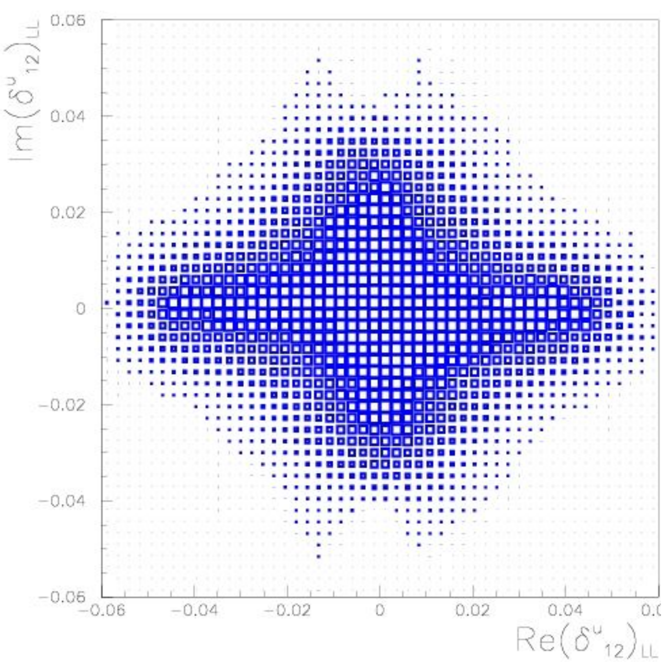
SM part

SUSY part proportional "to a given δ^2_{AB}"

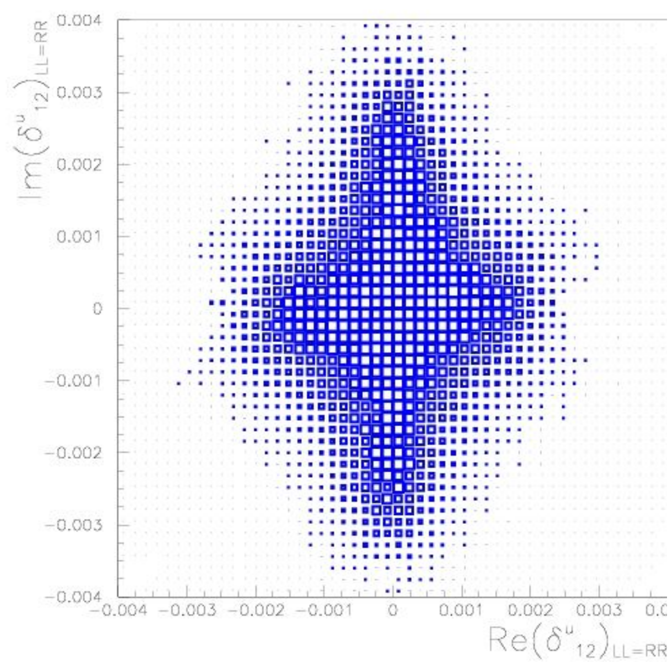
Always pairs of δ's appear in the ampliude

We switch on one δ^2_{AB} at a time: *LL* only (= *RR* only), *LL=RR*, *LR* only (= *RL* only)

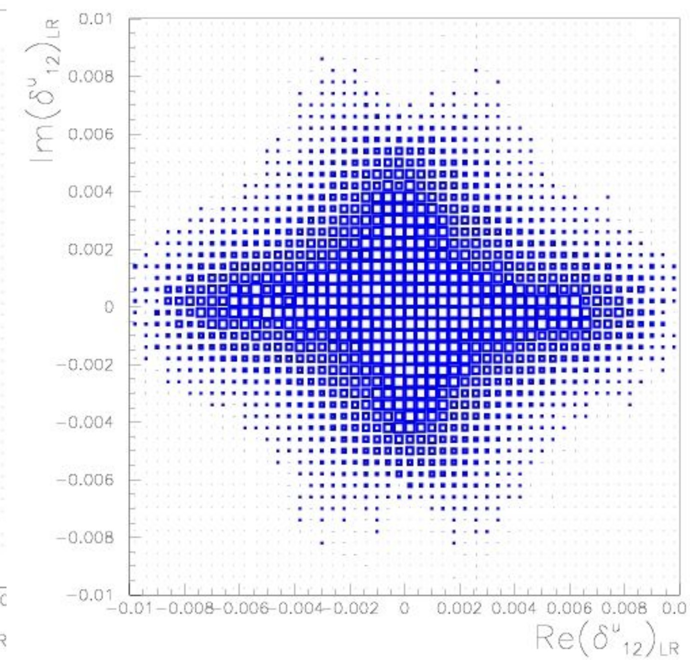
$$\left| (\delta_{12}^{(uc)})_{LL} \right| < 0.037$$



$$\left| (\delta_{12}^{(uc)})_{LL=RR} \right| < 0.006$$



$$\left| (\delta_{12}^{(uc)})_{LR} \right| < 0.002$$



$$m_{\tilde{q}} = m_{\tilde{g}} = 350\, GeV$$

Conclusions

- The measurement of $D^0 - \overline{D}^0$ mixing represents an outstanding experimental achievement

- It is a major step forward for Flavor Physics in general: unique channel to explore down-quark-mediated $\Delta F=2$

- A clear-cut assessment of NP effects in *e.g.* the oscillation frequency is spoiled by the still poor control of the long-distance, SM, contributions

- In this situation, sufficiently ‘strong’ statements on NP can only be done in cases of **very invasive** new FCNC sources, like the general MSSM

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- ☑ However, we do have a **Golden Channel for NP**: observation of (large) CPV

  ☞ Immune to hadronic uncertainties

  Clear NP signature if, *e.g.*, $\Phi_{12} > 10^{-2}$