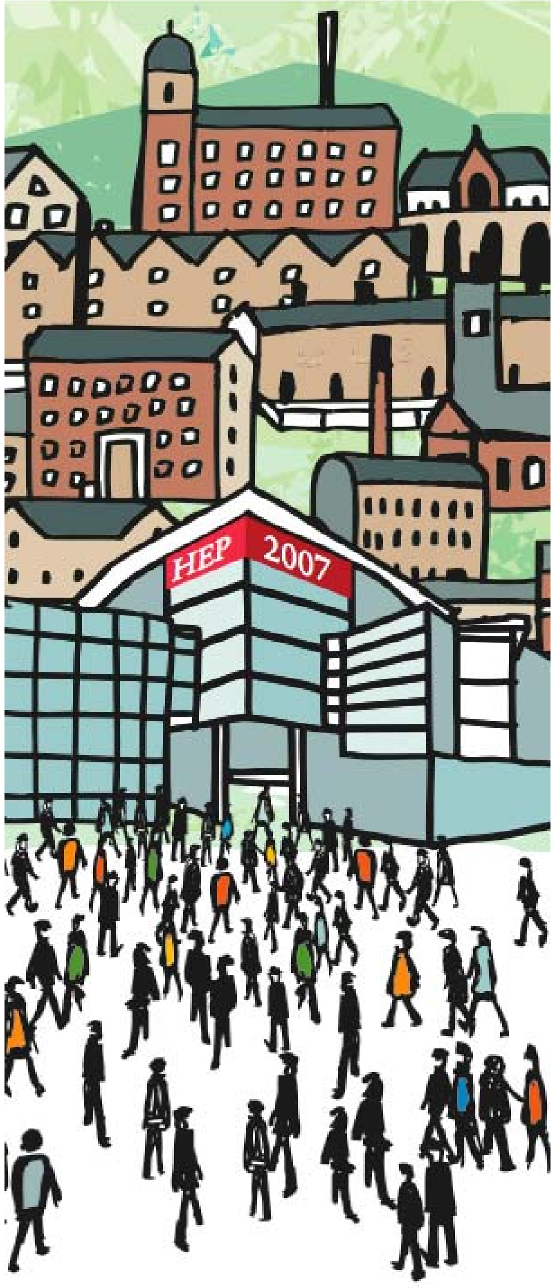
Measurements of $|V_{ub}|$ and $|V_{cb}|$ at Belle

Christoph Schwanda
Austrian Academy of Sciences
Representing the Belle collaboration



European Physical Society
HEP 2007

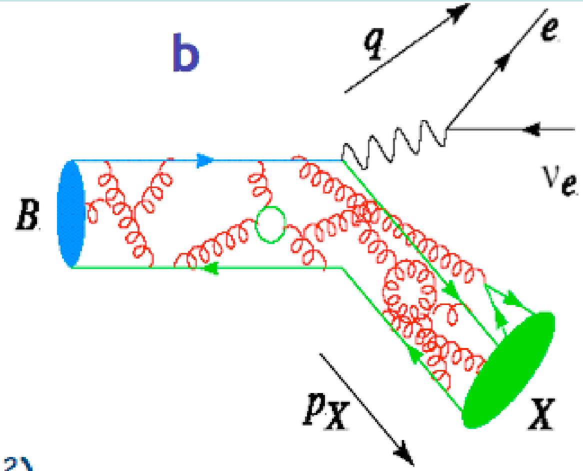
$|V_{cb}|$ from inclusive decays $B \to X_c\, l\, \nu$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

PRD 75, 032001 (2007)
PRD 75, 032005 (2007)
hep-ex/0611047

Theory of the measurement

- Semileptonic width in the framework of Heavy Quark Expansion



$$\Gamma_{sl}(B \to X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 |(1 + A_{ew}) A_{nonpert} A_{pert}|$$

in $1/m_b$ to $O(1/m_b^3)$

in α_s to $O(\alpha_s^2)$

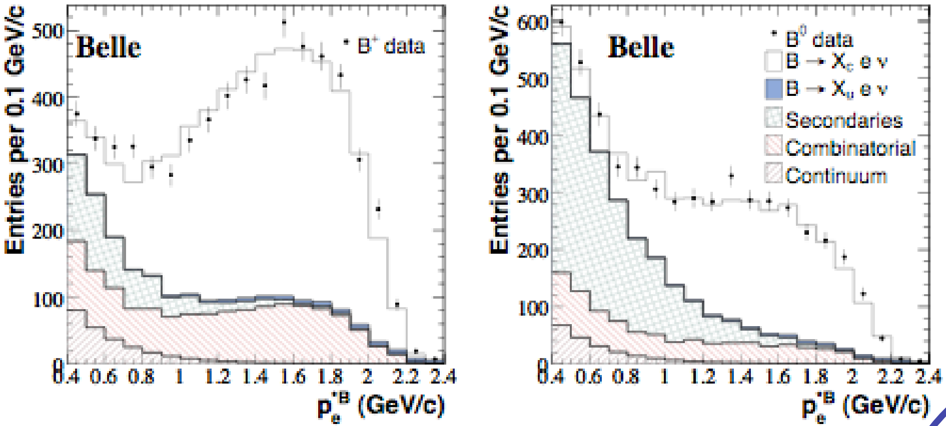
- Two separate calculations available:
  - kinetic running mass [P.Gambino, N.Uraltsev, Eur.Phys.J. C34, 181]
  - 1S mass [C.Bauer, Z.Ligeti, M.Luke, A.Manohar, Phys.Rev. D70, 094017]
- Non-perturbative parameters in the $1/m_b$ expansion

	Kinetic scheme	1S scheme
O(1)	m_b, m_c	m_b
$O(1/m^2_b)$	μ^2_π, μ^2_G	λ_1, λ_2
$O(1/m^3_b)$	ρ_D, ρ_{LS}	ρ_1, τ_{1-3}

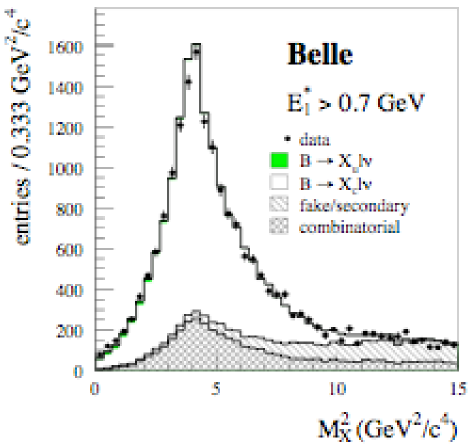
Contain soft QCD physics

- Non-perturbative parameters can be measured from inclusive observables in B decays

Inclusive E_l spectrum
[Phys.Rev. D75, 032001 (2007)]



Inclusive M^2_X spectrum
[Phys.Rev. D75, 032005 (2007)]



rate

shape

$|V_{cb}|$ at 1-2%

Non-perturbative parameters
(m_b, m_c, μ^2_π, …)

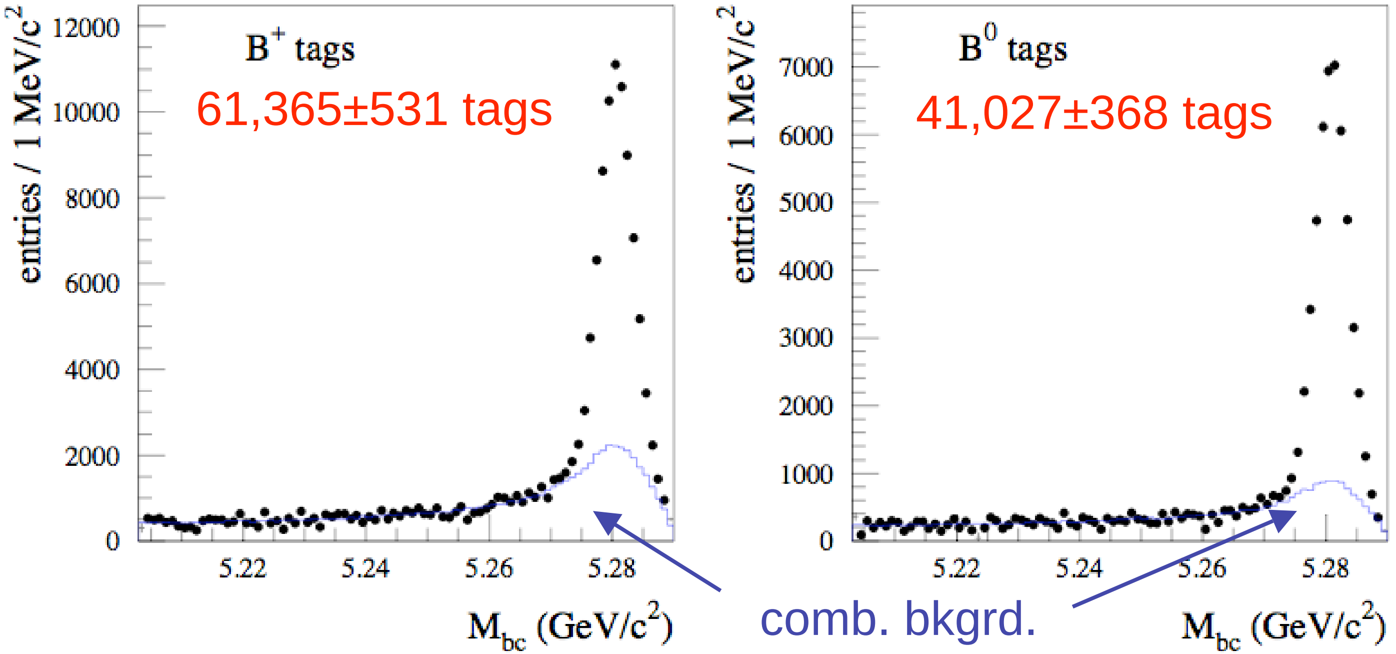
shape

shape

$B \to X_s \gamma$
[hep-ex/0508005]

Full reconstruction tag

- Electron energy and hadronic mass moments are measured using events in which the hadronic decay of one B is fully reconstructed in the modes $B \rightarrow D^* \{\pi,\rho,a_1\}$



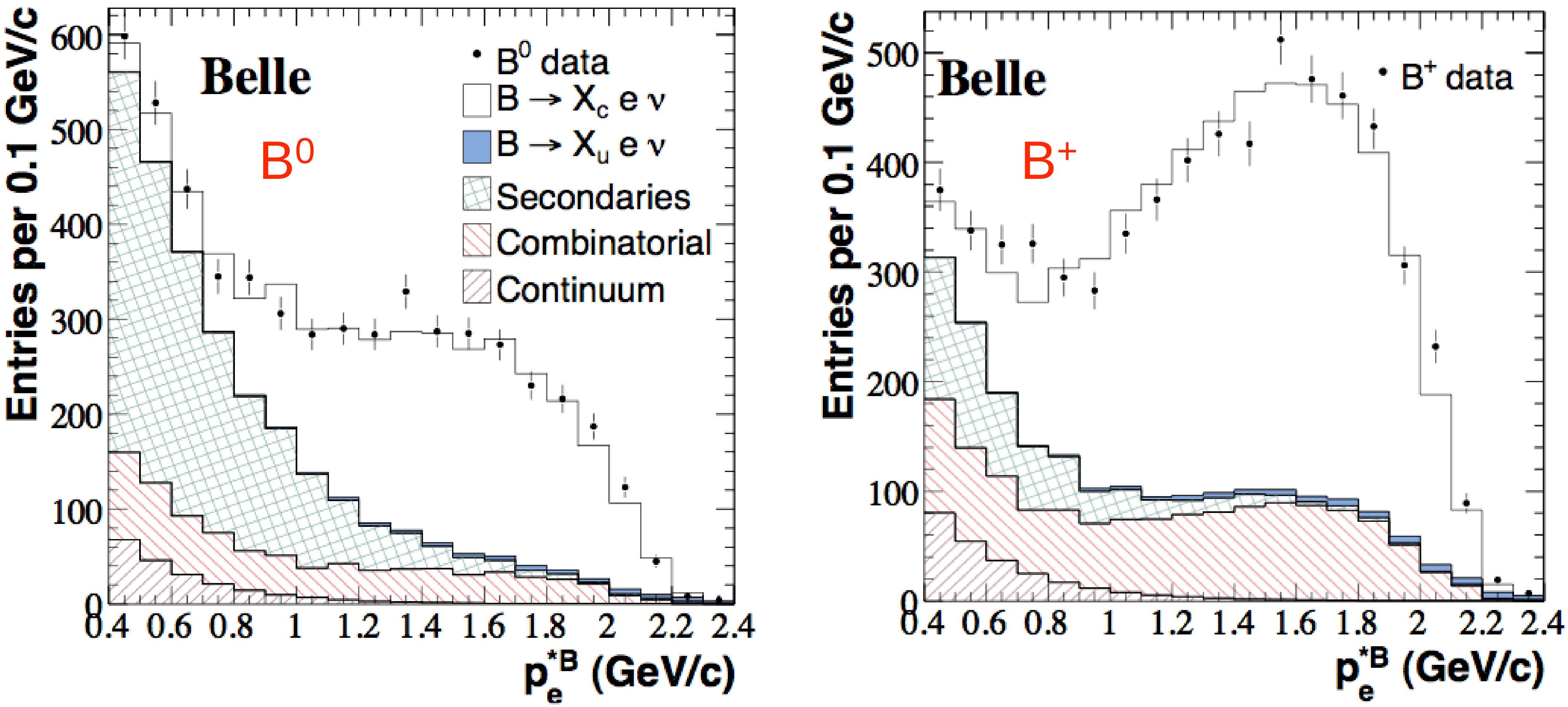
$$M_{bc} = \sqrt{(E_{beam})^2 - (\vec{p}_B)^2}$$

- Advantages
  - Low backgrounds (small background uncertainty)
  - Good M^2_X resolution (0.8 GeV^2/c^4)

Electron energy spectrum in $B \to X_c \, l \, \nu$

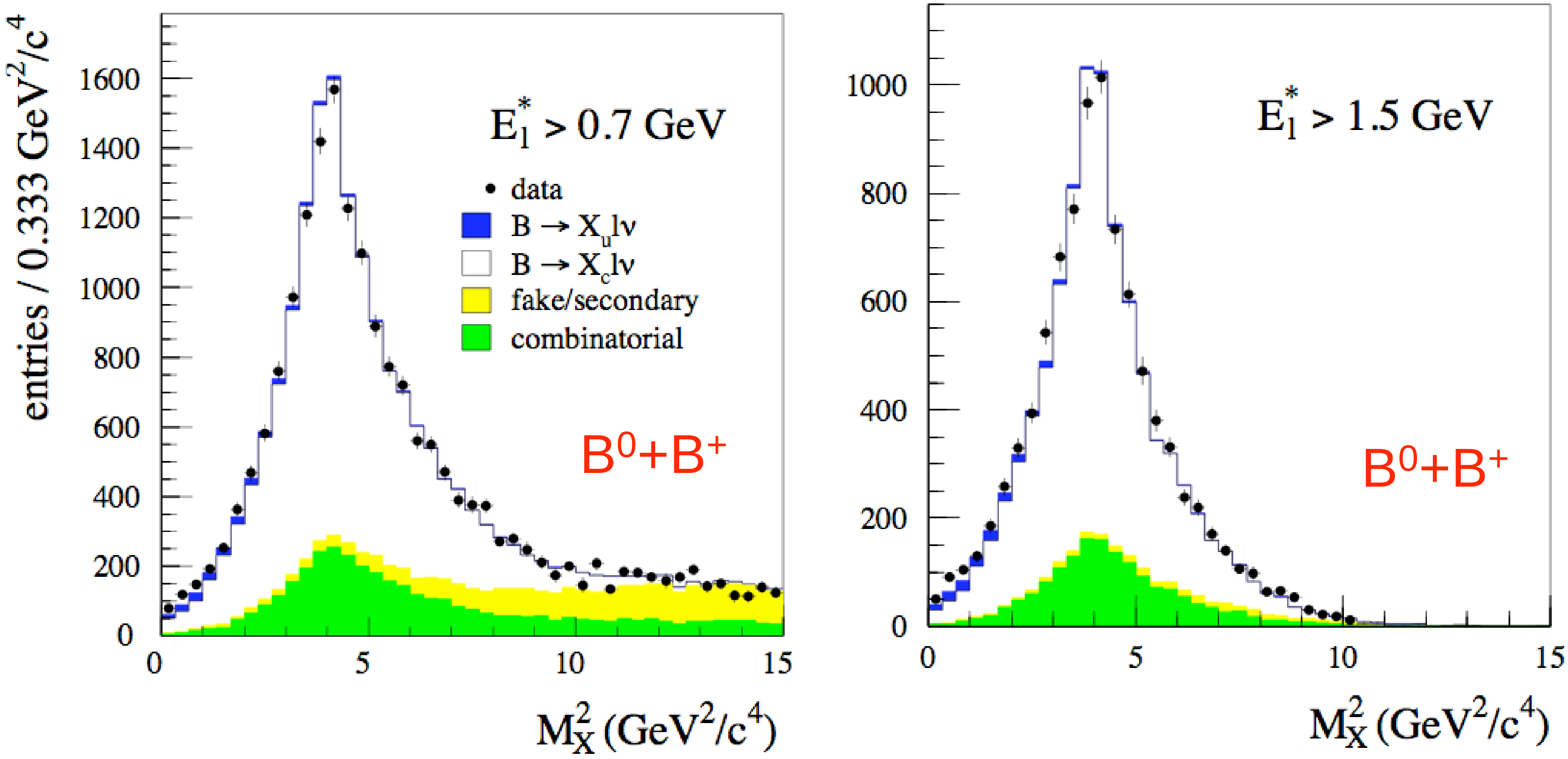
(in the B rest frame)

140 fb^{-1} Υ(4S) data



Hadronic mass spectrum in $B \rightarrow X_c\, l\, \nu$

140 fb^{-1} Y(4S) data



- The moments are calculated from the unfolded spectra to minimize $X_c\, l\, \nu$ model dependence

electron energy moments as fct. of minimum lepton energy

E_{cut}[GeV]	M_1 [MeV]	M_2 [10^{-3}GeV^2]	M_3 [10^{-3}GeV^3]	M_4 [10^{-3}GeV^4]	$\Delta\mathcal{B}$ [10^{-2}]
0.4	1393.92 ± 6.73 ± 3.02	168.77 ± 3.68 ± 1.53	-21.04 ± 1.93 ± 0.66	64.153 ± 1.813 ± 0.935	10.44 ± 0.19 ± 0.22
0.6	1427.82 ± 5.82 ± 2.55	146.15 ± 2.88 ± 1.08	-11.04 ± 1.35 ± 0.49	45.366 ± 1.108 ± 0.548	10.07 ± 0.18 ± 0.21
0.8	1480.04 ± 4.81 ± 2.13	117.97 ± 2.05 ± 0.55	-3.45 ± 0.83 ± 0.30	28.701 ± 0.585 ± 0.247	9.42 ± 0.16 ± 0.19
1.0	1547.76 ± 3.96 ± 1.45	88.17 ± 1.42 ± 0.36	0.83 ± 0.49 ± 0.20	15.962 ± 0.302 ± 0.142	8.41 ± 0.15 ± 0.17
1.2	1627.79 ± 3.26 ± 1.08	61.36 ± 1.02 ± 0.36	2.40 ± 0.30 ± 0.11	7.876 ± 0.162 ± 0.106	7.11 ± 0.13 ± 0.14
1.4	1719.96 ± 2.58 ± 1.10	38.99 ± 0.71 ± 0.24	2.33 ± 0.16 ± 0.07	3.314 ± 0.080 ± 0.055	5.52 ± 0.11 ± 0.11
1.6	1826.15 ± 1.80 ± 1.03	21.75 ± 0.47 ± 0.22	1.45 ± 0.08 ± 0.05	1.129 ± 0.033 ± 0.032	3.71 ± 0.09 ± 0.07
1.8	1943.18 ± 0.93 ± 1.16	10.14 ± 0.28 ± 0.18	0.68 ± 0.03 ± 0.04	0.283 ± 0.010 ± 0.017	1.93 ± 0.06 ± 0.04
2.0	2077.59 ± 0.21 ± 1.23	3.47 ± 0.13 ± 0.19	0.19 ± 0.01 ± 0.03	0.047 ± 0.002 ± 0.007	0.53 ± 0.02 ± 0.02

hadronic mass moments as fct. of minimum lepton energy

E^*_{min} (GeV)	$\langle M_X^2 \rangle$ (GeV$^2/c^4$)	$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$ (GeV$^4/c^8$)	$\langle M_X^4 \rangle$ (GeV$^4/c^8$)
0.7	$4.403 \pm 0.036 \pm 0.052$	$1.494 \pm 0.173 \pm 0.327$	$20.88 \pm 0.48 \pm 0.77$
0.9	$4.353 \pm 0.032 \pm 0.041$	$1.229 \pm 0.138 \pm 0.244$	$20.18 \pm 0.40 \pm 0.58$
1.1	$4.293 \pm 0.028 \pm 0.029$	$0.940 \pm 0.098 \pm 0.137$	$19.37 \pm 0.33 \pm 0.36$
1.3	$4.213 \pm 0.027 \pm 0.024$	$0.641 \pm 0.071 \pm 0.080$	$18.40 \pm 0.29 \pm 0.26$
1.5	$4.144 \pm 0.028 \pm 0.022$	$0.515 \pm 0.061 \pm 0.064$	$17.69 \pm 0.28 \pm 0.23$
1.7	$4.056 \pm 0.033 \pm 0.022$	$0.322 \pm 0.058 \pm 0.040$	$16.77 \pm 0.32 \pm 0.21$
1.9	$3.996 \pm 0.041 \pm 0.021$	$0.143 \pm 0.056 \pm 0.038$	$16.11 \pm 0.38 \pm 0.20$

(averaged over charged and neutral tags)

Input to the $|V_{cb}|$ fit

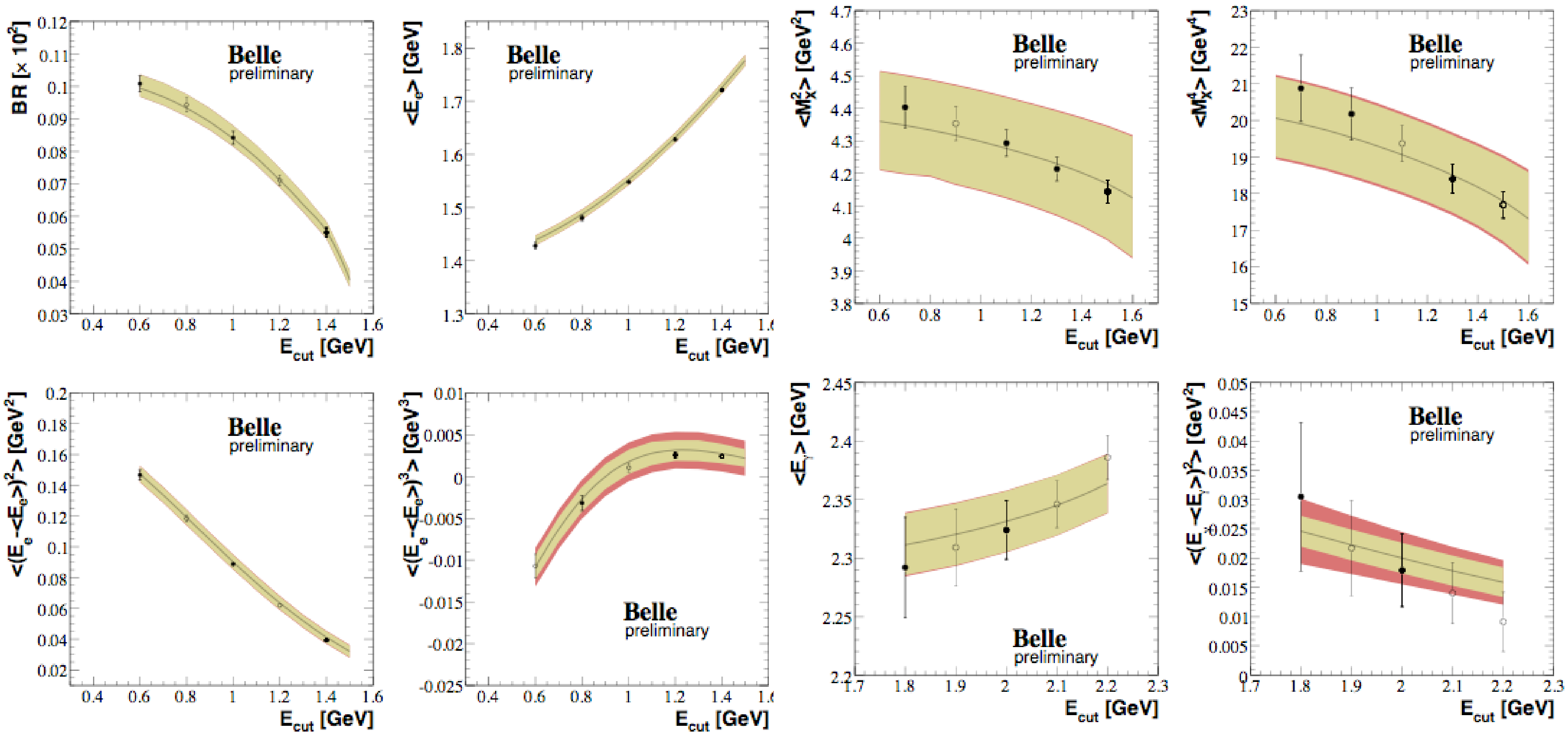
- Interpretation of the data using two sets of theoretical calculations

	1S scheme	kinetic scheme
Lepton moments $\langle E_\ell^n \rangle_{E_{\rm min}}$	$n=0$ $E_{\rm min} = 0.6,\ 1.0,\ 1.4$	$n=0$ $E_{\rm min} = 0.4,\ 0.8$
	$n=1$ $E_{\rm min} = 0.6,\ 0.8,\ 1.0,\ 1.2,\ 1.4$	$n=1$ $E_{\rm min} = 0.4,\ 0.8,\ 1.0,\ 1.2\ 1.4$
	$n=2$ $E_{\rm min} = 0.6,\ 1.0,\ 1.4$	$n=2$ $E_{\rm min} = 0.4,\ 0.8,\ 1.0,\ 1.2\ 1.4$
	$n=3$ $E_{\rm min} = 0.8,\ 1.2$	$n=3$ $E_{\rm min} = 0.4,\ 0.8,\ 1.0,\ 1.2\ 1.4$
Hadron moments $\langle M_X^{2n} \rangle_{E_{\rm min}}$	$n=1$ $E_{\rm min} = 0.7,\ 1.1,\ 1.3,\ 1.5$	$n=1$ $E_{\rm min} = 0.7,\ 0.9,\ 1.1,\ 1.3$
	$n=2$ $E_{\rm min} = 0.7,\ 0.9,\ 1.3$	$n=2$ $E_{\rm min} = 0.7,\ 0.9,\ 1.1,\ 1.3$
Photon moments $\langle E_\gamma^n \rangle_{E_{\rm min}}$	$n=1$ $E_{\rm min} = 1.8,\ 2.0$	$n=1$ $E_{\rm min} = 1.8,\ 1.9,\ 2.0$
	$n=2$ $E_{\rm min} = 1.8,\ 2.0$	$n=2$ $E_{\rm min} = 1.8,\ 1.9,\ 2.0$

- Points with E_{min} > 1.5 GeV ($X_c\, l\, \nu$) and E_{min} > 2 GeV ($X_s\, \gamma$) are excluded for theoretical reasons
- Correlations of experimental and theoretical uncertainties are taken into account

Fit result in the 1S mass scheme

preliminary

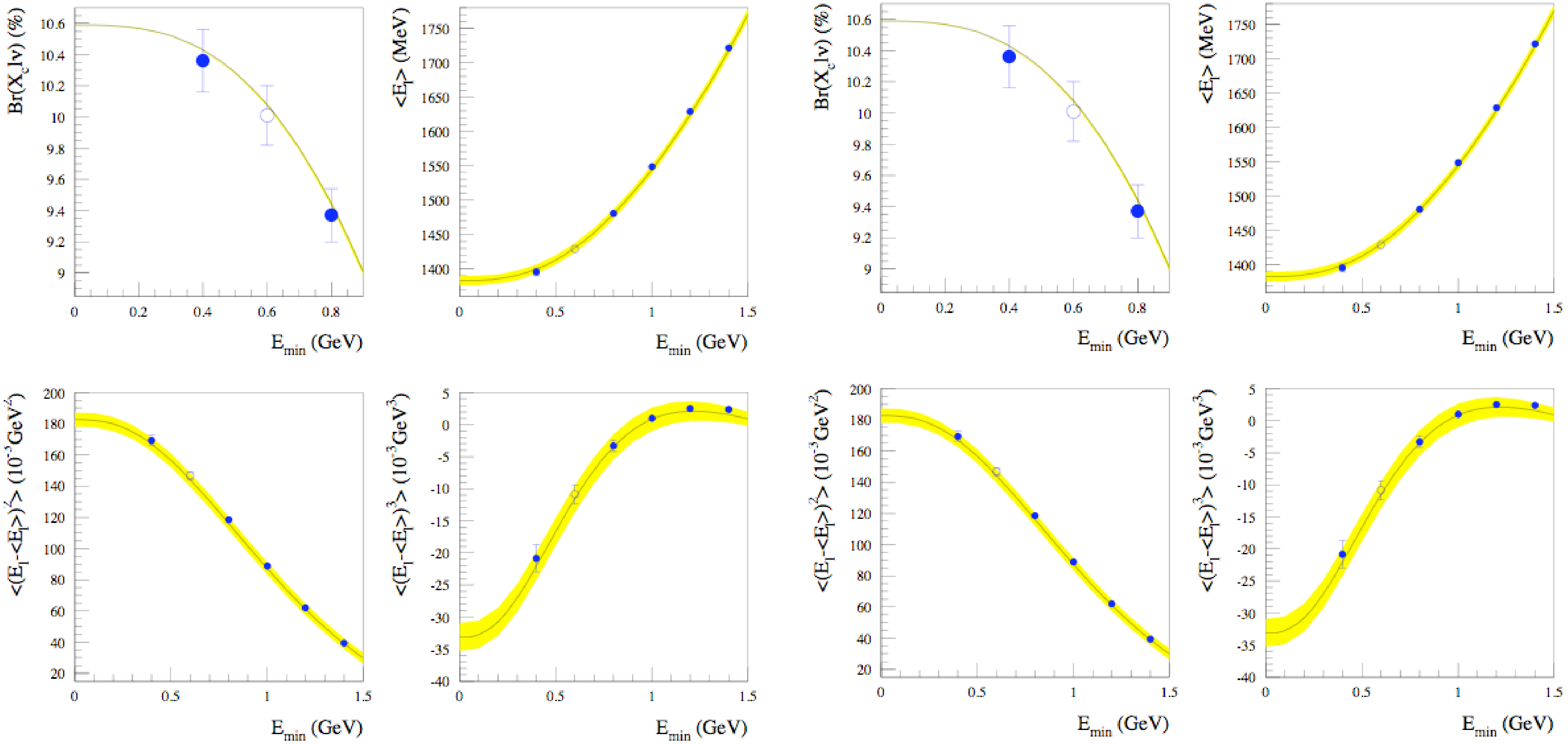


Green: fit error, red: theory error

χ^2/dof. = 5.7/17

Fit result in the kinetic running mass scheme

preliminary



yellow: theory error

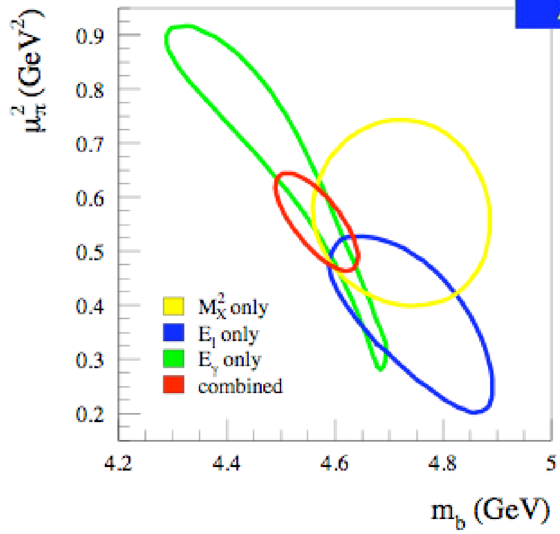
χ^2/dof. = 17.8/24

Global fit to Belle data

Kinetic scheme ($X_c l\nu + X_s\gamma$ data)

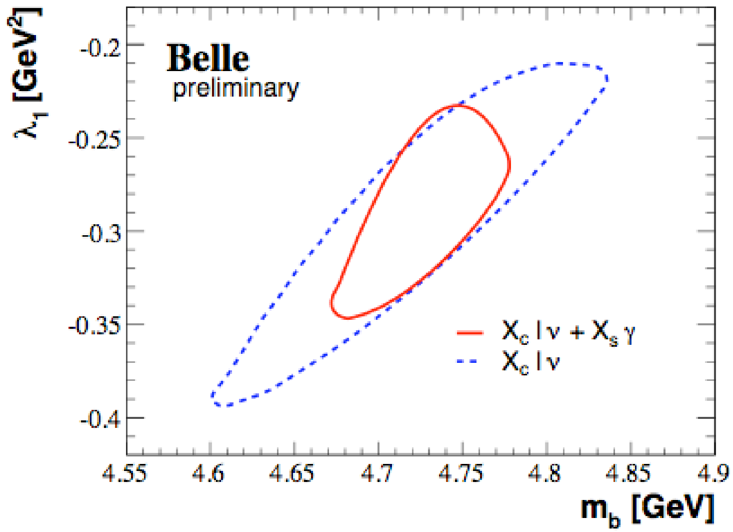
$|V_{cb}| = (41.93 \pm 0.65_{fit} \pm 0.07_{\alpha s} \pm 0.63_{th}) \times 10^{-3}$
$m_b = 4.564 \pm 0.076$ GeV
$m_c = 1.105 \pm 0.116$ GeV $\chi^2/\text{dof.} = 17.8/24$

[hep-ex/0611047] preliminary



1S scheme ($X_c l\nu + X_s\gamma$ data)

$|V_{cb}| = (41.49 \pm 0.52_{fit} \pm 0.20_{\tau}) \times 10^{-3}$
$m_b = 4.729 \pm 0.048$ GeV
$\lambda_1 = -0.30 \pm 0.04$ GeV^2 $\chi^2/\text{dof.} = 5.7/17$



The result for m_b compatible after scheme translation

$|V_{ub}|$ from exclusive semileptonic decays $b \rightarrow u$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \boxed{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

hep-ex/0610054

Theory of the measurement

- $B \to \pi\, l\, \nu$ decay rate

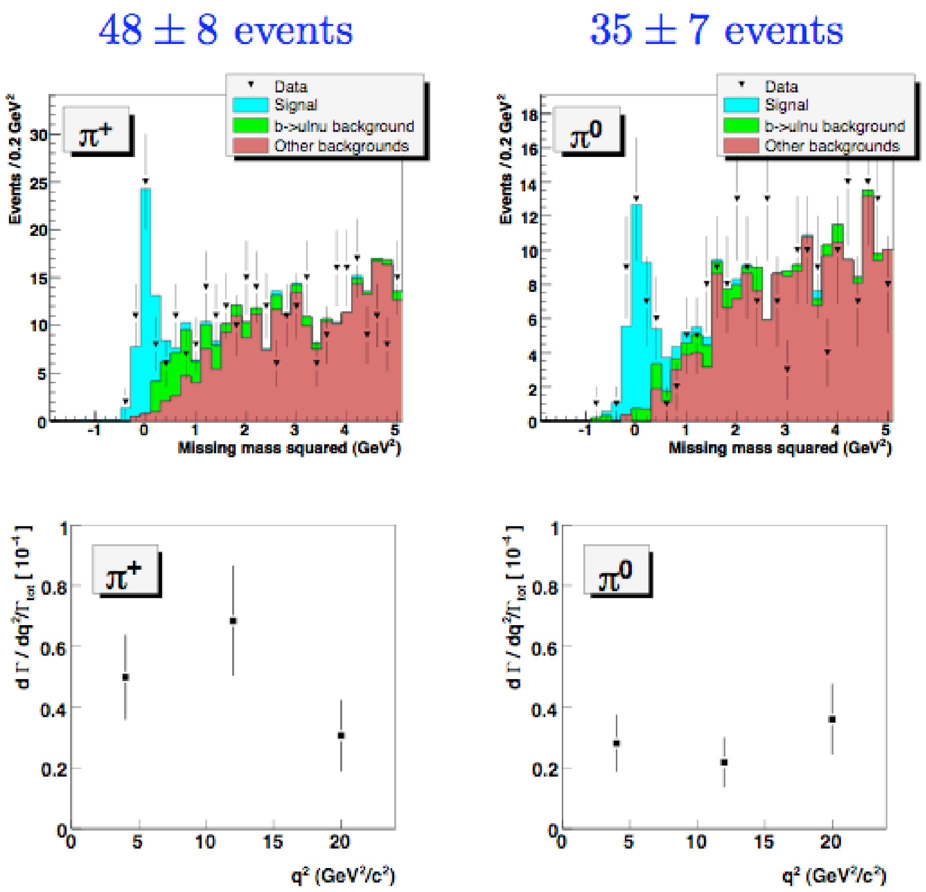
$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_b^3} \lambda(q^2)^{3/2} |f_+(q^2)|^2 \,, \quad q^2 = (p_\ell + p_\nu)^2$$

- Need form factor shape and normalization for $|V_{ub}|$
- Available form factor calculations
  - Relativistic quark models
    - ISGW2 [Phys. Rev. D52, 2783 (1995)]
  - Light cone sum rules (LCSR) in the region $q^2 < 14\ \mathrm{GeV}^2$
    - Ball-Zwicky [Phys. Rev. D71, 014015 (2005)]
  - Lattice QCD in the region $q^2 > 16\ \mathrm{GeV}^2$
    - HPQCD [Phys. Rev. D73, 074502 (2006); erratum ibid.D75, 119906 (2007)]
    - FNAL [Nucl. Phys. Proc. Suppl. 140, 464 (2005)]

$B \to \pi l \nu$

- Fully reconstruct the hadronic decay of the other B
- Reconstruct π and l on the signal side
- Extract signal from missing mass squared distribution

531 fb^{-1} Y(4S) data



$$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu) = (1.49 \pm 0.26(stat) \pm 0.06(syst)) \times 10^{-4}$$

$$\mathcal{B}(B^+ \to \pi^0 \ell^+ \nu) = (0.86 \pm 0.17(stat) \pm 0.06(syst)) \times 10^{-4}$$

[hep-ex/0610054] preliminary

Extraction of $|V_{ub}|$

- Partial BF in bins of q^2

preliminary

Mode	$\Delta\mathcal{B}\,[10^{-4}]$ $0 < q^2 < 8$ (GeV^2/c^2)	$8 < q^2 < 16$ (GeV^2/c^2)	$q^2 > 16$ (GeV^2/c^2)	$\mathcal{B}\,[10^{-4}]$ Sum (GeV^2/c^2)
$B \to \pi^+\ell\nu$	$0.50 \pm 0.14 \ \pm 0.02$	$0.68 \pm 0.18 \ \pm 0.03$	$0.31 \pm 0.12 \ \pm 0.01$	$1.49 \pm 0.26 \ \pm 0.06$
$B \to \pi^0\ell\nu$	$0.28 \pm 0.09 \ \pm 0.02$	$0.22 \pm 0.08 \ \pm 0.04$	$0.36 \pm 0.12 \ \pm 0.02$	$0.86 \pm 0.17 \ \pm 0.06$

- Extraction of $|V_{ub}|$ from respective q^2 regions

	q^2 range	$\lvert V_{ub}\rvert$ (10^{-3})
Ball-Zwicky	< 16 GeV^2	$3.56 \pm 0.29_{exp} \pm 0.47_{th} \pm 0.01_{\tau B0}$
HPQCD	> 16 GeV^2	$3.52 \pm 0.49_{exp} \pm 0.48_{th} \pm 0.01_{\tau B0}$
FNAL	> 16 GeV^2	$3.74 \pm 0.52_{exp} \pm 0.51_{th} \pm 0.01_{\tau B0}$

(assuming isospin symmetry)

Summary

- $|V_{cb}|$ inclusive [hep-ex/0611047] preliminary

	$\lvert V_{cb}\rvert$ (10^{-3})
1S scheme	$41.49 \pm 0.52_{fit} \pm 0.20_{\tau B}$
kinetic scheme	$41.93 \pm 0.65_{fit} \pm 0.07_{\alpha s} \pm 0.63_{th}$

  – $|V_{cb}|$ has been measured to 1-2% precision using inclusive B decays
  – Good agreement between different theoretical frameworks

Summary (cont.)

- $|V_{ub}|$ exclusive [hep-ex/0610054] preliminary

	q^2 range	$\|V_{ub}\|$ (10^{-3})
Ball-Zwicky	< 16 GeV^2	$3.56 \pm 0.29_{exp} \pm 0.47_{th} \pm 0.01_{\tau B0}$
HPQCD	> 16 GeV^2	$3.52 \pm 0.49_{exp} \pm 0.48_{th} \pm 0.01_{\tau B0}$
FNAL	> 16 GeV^2	$3.74 \pm 0.52_{exp} \pm 0.51_{th} \pm 0.01_{\tau B0}$

  - Measurements of exclusive $B \to X_u\, l\, \nu$ decays with full reconstruction tag are becoming available
  - Statistically limited at the moment but background systematics are small compared to other tagging methods

Backup slides

The CKM mechanism

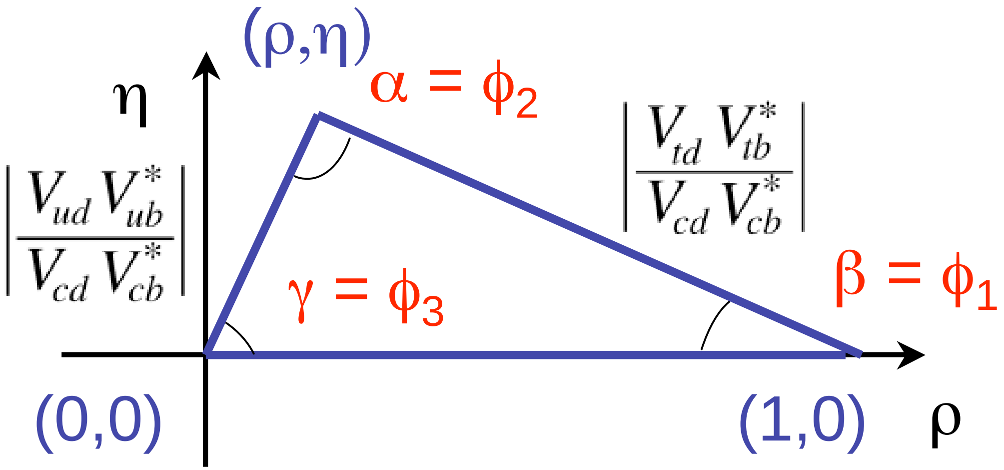
- The charged current interaction in the SM

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}}\,\overline{u_{Li}}\,\gamma^\mu\,(V_{\rm CKM})_{ij}\,d_{Lj}\,W_\mu^+ + \text{h.c.}$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

[Kobayashi, Maskawa, Prog. Theor. Phys. 49, 652 (1973)]

- $V_{\rm CKM}$ is a unitary 3x3 matrix;
  it contains three real parameters and one complex phase
- Its unitarity is commonly represented by the unitarity triangle



$$\alpha \equiv \varphi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \varphi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

Belle Detector