V_{ub} determination using $B \to \pi$ form factor from Light-Cone Sum Rules

Alexander Khodjamirian (Uni. Siegen, Germany)

UNIVERSITÄT SIEGEN

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Outline

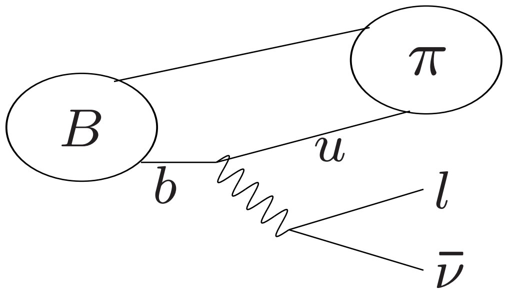
- minireview of exclusive $|V_{ub}|$ determinations, mainly $B \to \pi$
- new (preliminary) results on $f^+_{B\pi}(q^2)$ from LCSR

  [G. Duplancić, A.K., Th. Mannel, B. Melić and N. Offen]

Exclusive $b \to u$ transitions sensitive to $|V_{ub}|$

Channel	BR$\times 10^4$	hadronic input	theory
$B^- \to \tau^- \bar{\nu}_\tau$	$1.79^{+0.56+0.39}_{-0.49-0.46}$ [Belle] $< 1.7 (90\%\ \mathrm{CL})$ [BaBar]	f_B	Lattice $(2\oplus 1)$ QCD SR
$\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$	$1.39 \pm 0.06 \pm 0.06$ [HFAG] q^2-shape [BaBar]	FF $f^+_{B\pi}(q^2)$	Lattice $(2 \oplus 1)$, SCET, LCSR
$\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$	$2.14 \pm 0.21 \pm 0.51 \pm 0.28$ [BaBar] $2.17 \pm 0.54 \pm 0.31 \pm 0.08$[Belle]	three FF's	Lattice (quench.) LCSR("quench.")
$B^- \to l^- \bar{\nu}_l \gamma$	$< 0.05\ (90\%\ \mathrm{CL})$ [BaBar]	two FF's	QCDF,LCSR
$B^- \to \pi^- \pi^0$	0.057 ± 0.004 [HFAG]	hadr. ampl.	input: $B \to \pi$ FF QCDF, SCET

$B \to \pi$ form factor (FF)



$\Rightarrow |V_{ub}|$

- Hadronic matrix element:

$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|\bar{B}^0(p+q)\rangle = f^+_{B\pi}(q^2)(2p_\mu + q_\mu) + \ldots$$

.... - the second FF f^- (f^0) only in $B \to \pi\tau\nu_\tau$

- $B \to \pi\mu\nu_\mu, \pi e\nu_e$ region: $0 < q^2 < (m_B - m_\pi)^2 = q^2_{max} \simeq 26.4 \text{ GeV}^2$

- differential decay distribution (neglecting m_l)

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}(E_\pi^2 - m_\pi^2)^{3/2}|f^+_{B\pi}(q^2)|^2$$

$E_\pi = (m_B^2 + m_\pi^2 - q^2)/(2m_B)$ -pion energy in the B rest frame

Employing the analyticity of $f^+_{B\pi}(q^2)$

- $q^2 \Rightarrow$ complex variable, $f^+_{B\pi}(q^2) \Rightarrow$ analytic function, singularities (poles, branch points) given by unitarity relation (consult your favorite Quantum Field Theory textbook)

- $f^+_{B\pi}(q^2)$ real at $q^2 < m^2_{B^*}$, $m^2_{B^*} > q^2_{max}$, pole at $q^2 = m^2_{B^*}$, branch points (and poles) at $q^2 > (m_B + m_\pi)^2$ (radial excitations of B^*)

- dispersion relation (derived from Cauchy theorem in q^2-plane)

$$f^+_{B\pi}(q^2) = \frac{f_{B^*} g_{B^* B\pi}}{m^2_{B^*} - q^2} + \frac{1}{\pi} \int\limits_{(m_B+m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} f^+_{B\pi}(s)}{s - q^2}$$

Employing the analyticity of $f^+_{B\pi}(q^2)$

- analyticity $\oplus$ perturbative QCD bounds
(unitarity for correlation function of heavy-light vector currents)

$\Rightarrow$ knowledge of $f^+_{B\pi}(q^2)$ at a few points tightly bounds ($\simeq$ reproduces) the FF at all $q^2 < q^2_{max}$

mathematical framework: [N. Meiman (1963)];...,
applications to $B \to \pi$ FF: [C.G.Boyd, B.Grinstein, R.Lebed (1995)], L.Lellouch (1996)],..., [T.Becher, R.Hill (2006)],..

- Use of the relation between $f_{B\pi}(q^2)$ and the elastic phase of $\pi B \to \pi B$ strong scattering amplitude:
[Muskhelishvili (1950)], [Omnes (1950's)], ...
applications to $B \to \pi$ FF: [J. Flynn, J. Nieves (2001)]

- Practical use: FF parameterizations

Simplest parameterizations

- dispersion relation as a starting point, replace the integral by an effective pole:

$$[f^+_{B\pi}(q^2)]_{disp.rel} \Rightarrow \frac{r_1}{1-q^2/m^2_{B^*}} + \frac{r_2}{1-q^2/m^2_{B_{fit}}}$$

- BK parameterization [D.Becirevic, A.Kaidalov, 2000]

(3 parameters $\to$ 2 , motivated by HQ limit)

$$[f^+_{B\pi}(q^2)]_{BK} = \frac{f_{B\pi}(0)}{(1-q^2/m^2_{B^*})(1-\alpha q^2/m^2_{B^*})}$$

- a generic 3-parameter two-pole parameterization: [P.Ball, R.Zwicky, 2004]

$$[f^+_{B\pi}(q^2)]_{BZ} = f_{B\pi}(0)\left(\frac{1}{1-q^2/m^2_{B^*}} + \frac{rq^2/m^2_{B^*}}{(1-q^2/m^2_{B^*})(1-\alpha_{BZ}q^2/m^2_{B^*})}\right)$$

reduced to BK at $\alpha_{BZ} = r = \alpha_{BK}$

Advanced parameterizations

- BGL-parameterization [Boyd, Grinstein, Lebed]:
a power expansion with $\sim 4, 5$ parameters

$$[f^+_{B\pi}(q^2)]_{BGL} = \frac{\sum\limits_k a_k [z(q^2, q_0^2)]^k}{P(q^2, m^2_{B^*})\phi(q^2, q_0^2)}$$

$z < 1$, P, ϕ -known functions, $\sum_k a_k^2 \leq 1$ (unitarity), truncated at $k = 2$

- AFHNV parameterization, based on Omnes-representation, with 4 shape parameters: [Albertus, Flynn, Hernandez, Nieves, Verde-Velasco (2005)]

$f^+_{B\pi}(q_i^2)/f^+_{B\pi}(0)$, $0.25 q^2_{max} < q_i^2 < q^2_{max}$ $\oplus$ overall normaliz.

Give me four parameters, and I can fit an elephant.
Give me five, and I can wiggle its trunk"
(J. von Neumann)

Fixing $|V_{ub} f^+_{B\pi}(0)|$ from data

- fit various parameterizations to the exp. differential shape and $BR_{tot}(B \to \pi l \nu_l)$ [HFAG]

- using combined CLEO,BaBar,Belle shape measurement and BGL form [T. Becher, R.Hill , hep-ph/0509090]

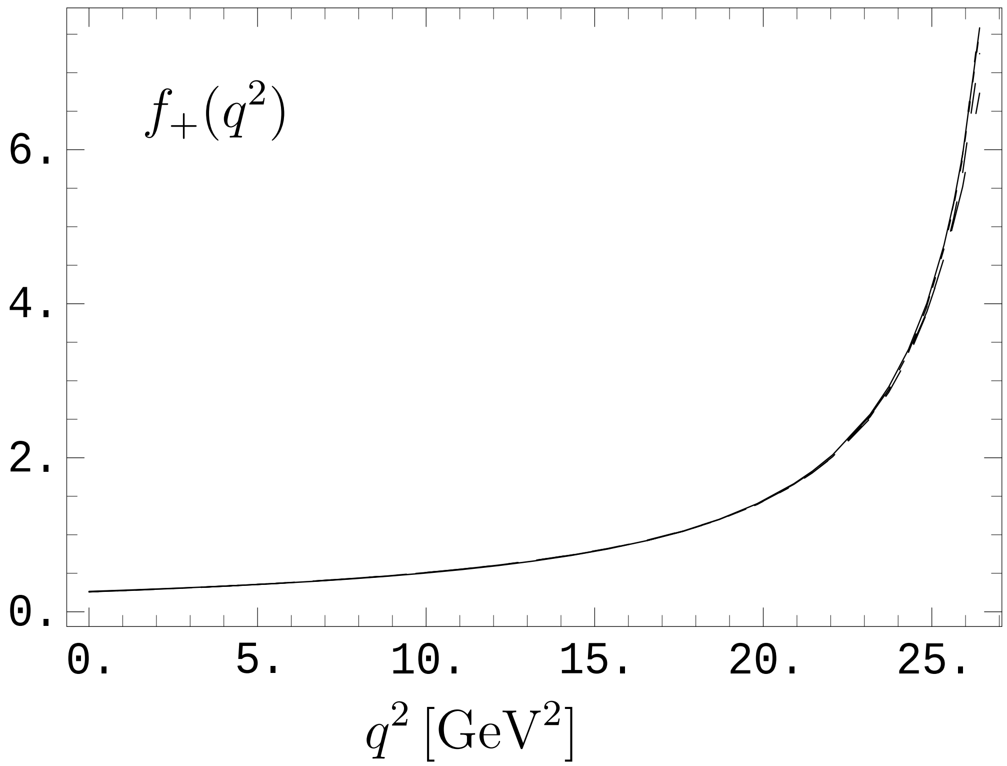
$$|V_{ub} f^+_{B\pi}(0)| = (0.92 \pm [0.11]_{shape} \pm [0.03]_{BR}) \times 10^{-3}$$

- using recent BR q^2-bins [BaBar], fitted five diff. parameterizations (BK, BZ, BGLa, BGLb, AFHNV) [P. Ball , hep-ph/0611108]

$$|V_{ub} f^+_{B\pi}(0)| = (0.91 \pm [0.06]_{shape} \pm [0.03]_{BR}) \times 10^{-3}$$

- it remains to calculate $|f^+_{B\pi}(0)|$ in QCD (or $f^+_{B\pi}(q^2)$ at any single point q^2)

$\Rightarrow$ extract $|V_{ub}|$.



best-fit form factors of 5 different parameterizations, normalized with $|V_{ub}| = (3.50 \pm 0.18) \times 10^{-3}$ (UT fits)
from P. Ball, talk at FPCP 2007, Bled [hep-ph 07052290]

Lattice QCD

- b-quark too heavy, pion too light for the lattice , only small E_π - large $q^2 > 16$ GeV^2 accessible

- recent progress: calculation of heavy-light hadronic matrix elements with $n_f = 2 \oplus 1$ dynamical quark flavours :

- Fermilab-MILC , "Fermilab treatment" of heavy quarks, [M.Okamoto, hep-lat/050113]

- HPQCD [E.Gulez et. al, hep-lat/0601021 (+errata)], use NRQCD,

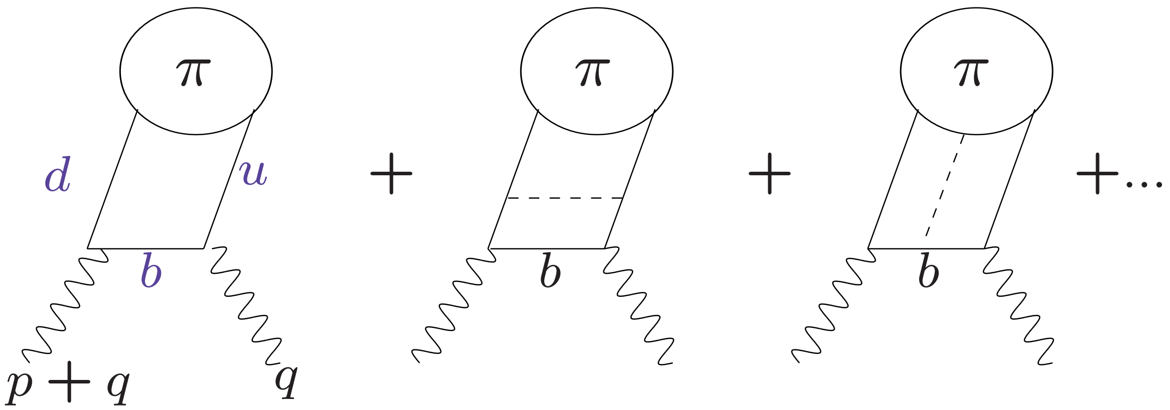
$q^2[GeV^2]$	$f^+_{B\pi}(q^2)$
17.34	1.101 ± 0.053
18.39	1.273 ± 0.099
19.45	1.458 ± 0.142
20.51	1.627 ± 0.185
21.56	1.816 ± 0.126

fit to BZ param.

$$\frac{1}{|V_{ub}|^2} \int_{16GeV^2}^{q^2_{max}} dq^2 d\Gamma/dq^2 = 2.07 \pm 0.57\text{ps}^{-1}$$

(dominant error from chiral extrapolation)

Light-Cone Sum Rules



- The correlation function: $q^2, (p+q)^2 \ll m_b^2,$ b-quark highly virtual

$$F_\lambda(q,p) = i\int d^4x e^{iqx}\langle \pi(p) \mid T\{\bar{u}(x)\gamma_\lambda b(x), \bar{b}(0)i\gamma_5 d(0)\} \mid 0\rangle$$

- operator-product-expansion (OPE) near the light-cone, $x^2 \sim 0$

- universal input: $\langle \pi(p)|\bar{u}(x)....d(0)|0\rangle \sim \varphi_\pi^{(t)}(u)$

-pion distribution amplitudes (DA)of twist $t = 2, 3, 4..$

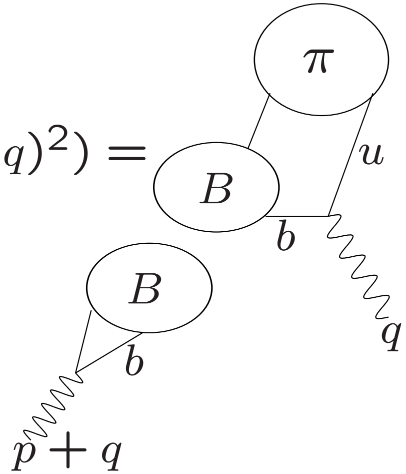
[P. Ball, R. Zwicky (2001),2004].

[G. Duplancić, A.K., Th. Mannel, B. Melić and N. Offen, paper in preparation]

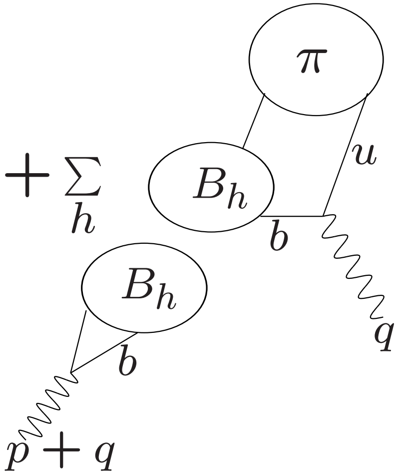
Derivation of LCSR

- Hadronic dispersion relation in $(p+q)^2$: $(q^2 \ll m_b^2$ fixed$)$

$F(q^2,(p+q)^2) =$



$+ \sum_h$



$f_B f^+_{B\pi}(q^2)$ $\qquad$ $\sum_{B_h} \rightarrow \textit{duality}\ (s_0^B)$

- matching at $\langle -(p+q)^2 \rangle \sim \mu^2 \sim m_b \Lambda$ and using duality

$$[F((p+q)^2, q^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2 - (p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s,q^2)]_{OPE}}{s-(p+q)^2}$$

- inputs:

$*$ m_b, α_s, (full QCD), we use $\overline{MS}$ mass : $m_b(m_b) = 4.164 \pm 0.025$ GeV

[J. Kühn,M. Steinhauser, C. Sturm]

$*$ $\varphi_\pi^{(t)}(u)$, $t = 2, 3, 4$,
main sensitivity to a_2, a_4 -parameters of twist 2 DA (Gegenbauer moments)

$*$ s_0^B, M^2 fixed by calculating m_B,
$*$ we fix a_2, a_4, s_0^B by fitting q^2 dep. to the measured slope of FF

$*$ f_B - Determined from two-point sum rule;

$f_B = 210 \pm 19$ MeV [M. Jamin, B. Lange (2001)]

- uncertainties:

$*$ induced by variation of (universal) input parameters, errors of exp.inputs

$*$ parton-hadron duality (suppressed with Borel transformation,constrained by q^2 shape)

- recent LCSR results: (with one-loop pole mass m_b)

$$f^+_{B\pi}(0) = 0.258 \pm 0.031 \quad \text{[P.Ball, R.Zwicky(2004)]}$$

$$f^+_{B\pi}(0) = 0.26 \pm 0.02_{[a^\pi_{2,4}]} \pm 0.03_{[param]} \text{ (without twist-3 NLO)}$$

[A. K., T. Mannel, M. Melcher and B. Melic, PRD (2005), hep-ph/0509049]

- preliminary result of our new LCSR calculation with $\overline{MS}$ mass and tw.-2,3 NLO:

$$f^+_{B\pi}(0) = 0.25 \pm 0.04$$

(combining all individual uncertainties in quadrature)

Recent $|V_{ub}|$ determinations using $f^+_{B\pi}$

Method	$\|V_{ub}\| \times 10^3$	use of	ref.
Lattice $n_f = 3$	$3.78 \pm 0.25 \pm 0.52$	BK	Fermilab/MILC '05
Lattice $n_f = 3$	$3.35 \pm 0.25 \pm 0.50$	BZ-par.	HPQCD '07
comb.	$3.54 \pm 0.17 \pm 0.44$	BGL$\oplus$ Latt. $\oplus$ SCET $B \to \pi\pi$	Arnesen et al. '05
comb.	$3.7 \pm 0.2 \pm 0.1$	BGL$\oplus$ Latt.	Becher,Hill '06
comb.	$3.47 \pm 0.29 \pm 0.03$	Omnes repr. $\oplus$ Latt $\oplus$ LCSR (BZ)	Flynn, Nieves '07
LCSR	$3.5 \pm 0.4 \pm 0.1$	BGL	Ball '06

- using $|V_{ub} f^+(0)|$ from P. Ball analysis, our *preliminary* result:
$|V_{ub}| = (3.64 \pm [0.3]_{exp} \pm [0.58]_{th}) \times 10^{-3}$

- UT fits (only CP- observables): $|V_{ub}| = (3.50 \pm 0.18) \times 10^{-3}$ [Utfit, CKMfitter]

- $B \to X_u l \nu_l$ ("selected" analyses): $|V_{ub}| = 4.10 \pm 0.30(exp) \pm 0.29(th) \times 10^{-3}$
[M.Neubert, talk at FPCP-2007, Bled]