The 2007 Europhysics conference on
High Energy Physics

Semileptonic *B* decays and the inclusive determination of $|V_{ub}|$

Paolo Giordano

University of Turin, Department of Theoretical Physics

Outline

- Status of $|V_{ub}|$ and of its inclusive determination.

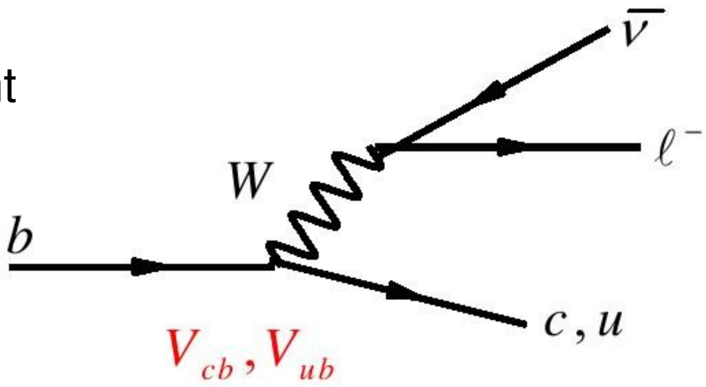
- Description of a new theoretical framework:
  - General features
  - Perturbative and non-perturbative corrections in the kinetic scheme
  - Fermi motion effects
  - Problems in the high q^2 region

- Discussion of results:
  - Extraction of $|V_{ub}|$
  - Theoretical uncertainties

- Conclusions

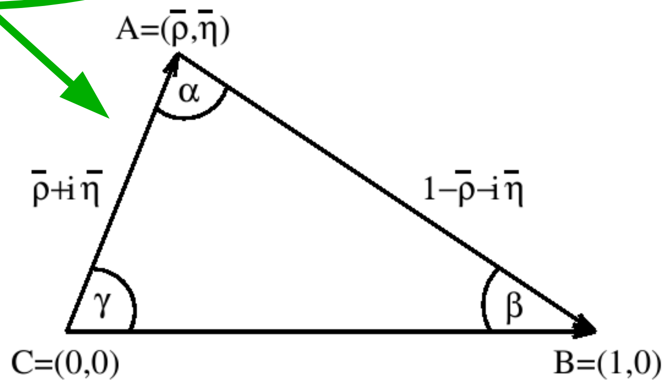
$|V_{ub}|$, $|V_{cb}|$ and the Unitarity Triangle

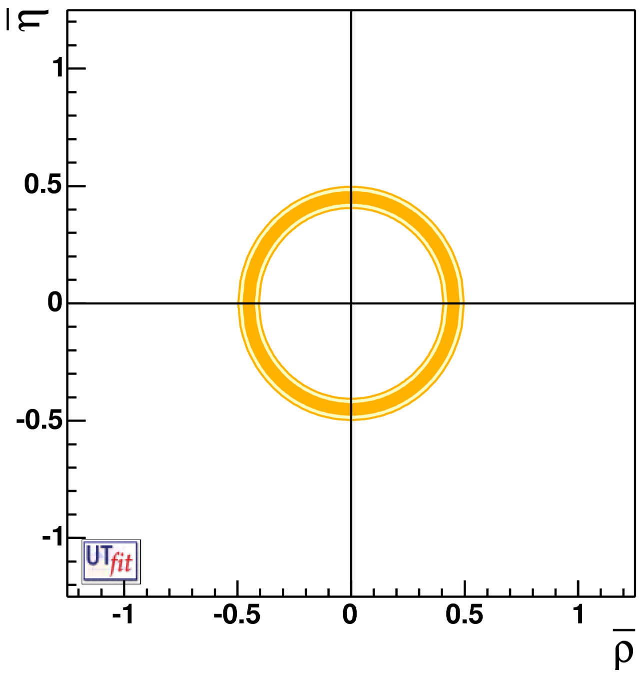
- Semileptonic *B* decays allow for a measurement of $|V_{ub}|$ and $|V_{cb}|$ from tree-level processes.



- The ratio $|V_{ub}/V_{cb}|$ provides an important constraint on the Unitarity Triangle

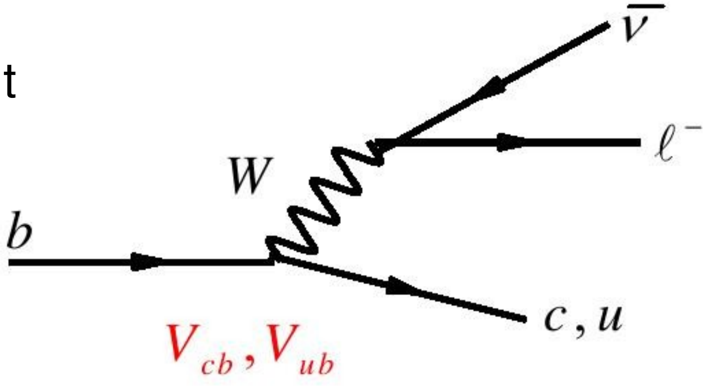
$$R_b = (1 - \frac{\lambda^2}{2})\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$





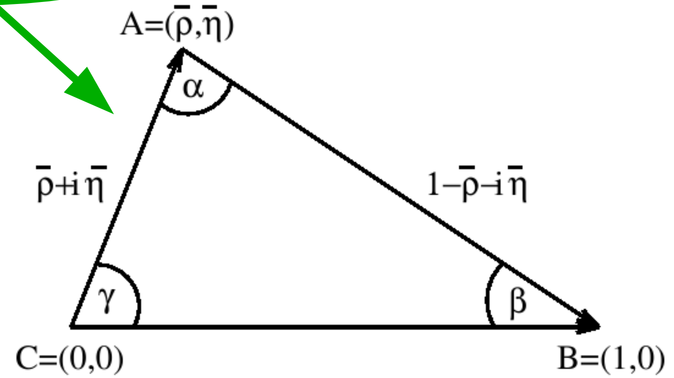
$|V_{ub}|$, $|V_{cb}|$ and the Unitarity Triangle

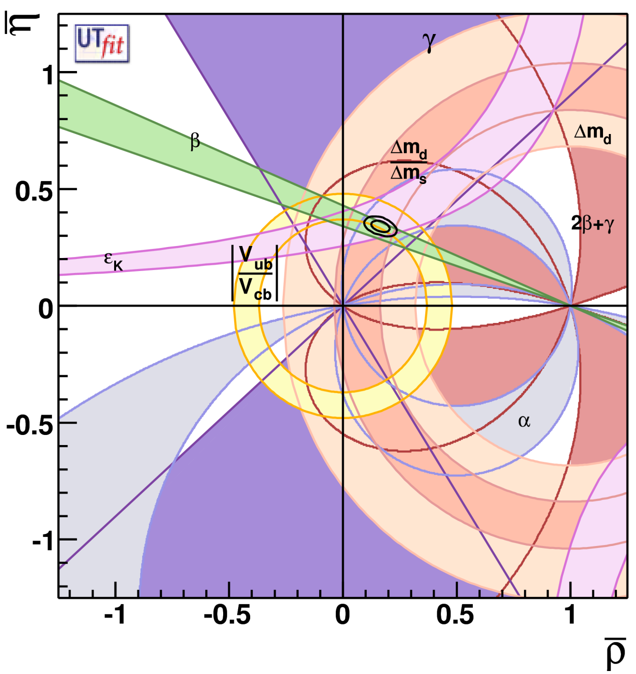
• Semileptonic *B* decays allow for a measurement of $|V_{ub}|$ and $|V_{cb}|$ from tree-level processes.



• The ratio $|V_{ub}/V_{cb}|$ provides an important constraint on the Unitarity Triangle

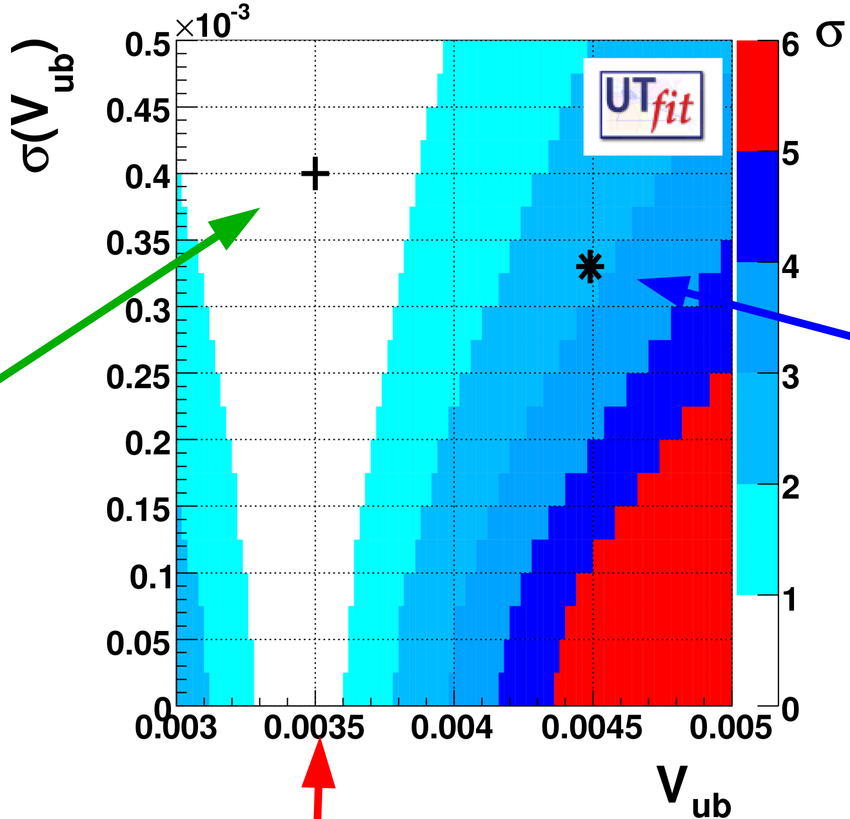
$$R_b = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$





$|V_{ub}|$: where we are

Exclusive determination (lattice QCD, light-cone sum rules)



Inclusive determination

2.5 σ discrepancy (New Physics or something wrong?)

$$|V_{ub}|_{UTfit} = (3.44 \pm 0.16) \times 10^{-3}$$

Inclusive determination: the theoretical approach

• Inclusive decays potentially offer the most accurate way to determine $|V_{ub}|$, through a measurement of the decay rate:

$$|V_{ub}| = \sqrt{\frac{\Gamma^{exp}_{cuts}}{\frac{1}{|V_{ub}|^2}\int_{cuts}\frac{d^3\Gamma_{th}}{dq_0\,dq^2\,dE_\ell}}}$$

Experiment

Theory

• The process involves three different scales:

$M_W >> m_b >> \Lambda_{QCD}$

• The triple differential width is expressed in a double series, using a local Operator Product Expansion (OPE):

• Power corrections known through $O(1/m_b^3)$

• QCD corrections known through $O(\alpha_s^2\beta_0)$

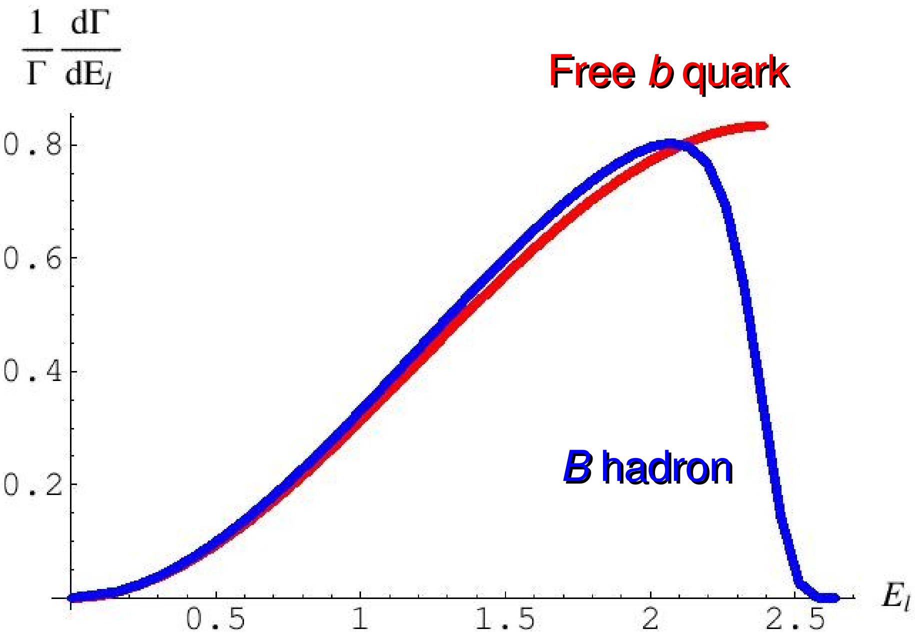
Successfully tested in charmed decays (2 % determination of $|V_{cb}|$)

Inclusive determination: limitations of the OPE

• The local OPE describes the partonic decay $b \to u\, \ell\, \nu$. There is a kinematical region of the hadronic decay that cannot be populated by the free quark decay.

• $1/m_b$ corrections to the OPE lead to increasingly singular contributions to the triple differential width (higher derivatives of the Dirac-δ). Predictions need to be "smeared" over sufficiently large regions of the phase space.

• Near the threshold non-local effects become important: leading terms of the OPE must be resummed into a universal distribution function, that describes the motion of the heavy quark inside the meson (Fermi motion).



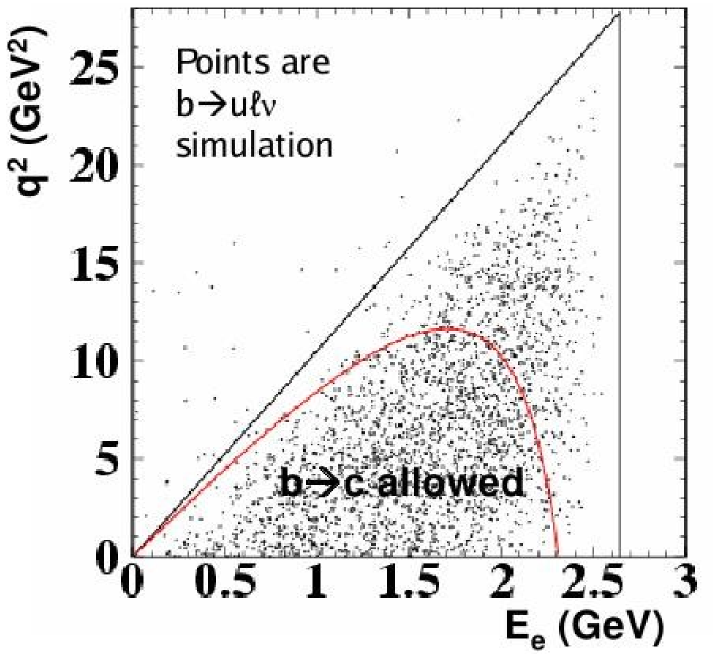
Inclusive determination: the problem with cuts

- The $b \to c$ background is by far dominating over $b \to u$

experiments have to impose cuts that tend to destroy the convergence of the OPE.

$$\frac{\Gamma(b \to u \ell \bar{\nu})}{\Gamma(b \to c \ell \bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$

- Current analyses are performed using:

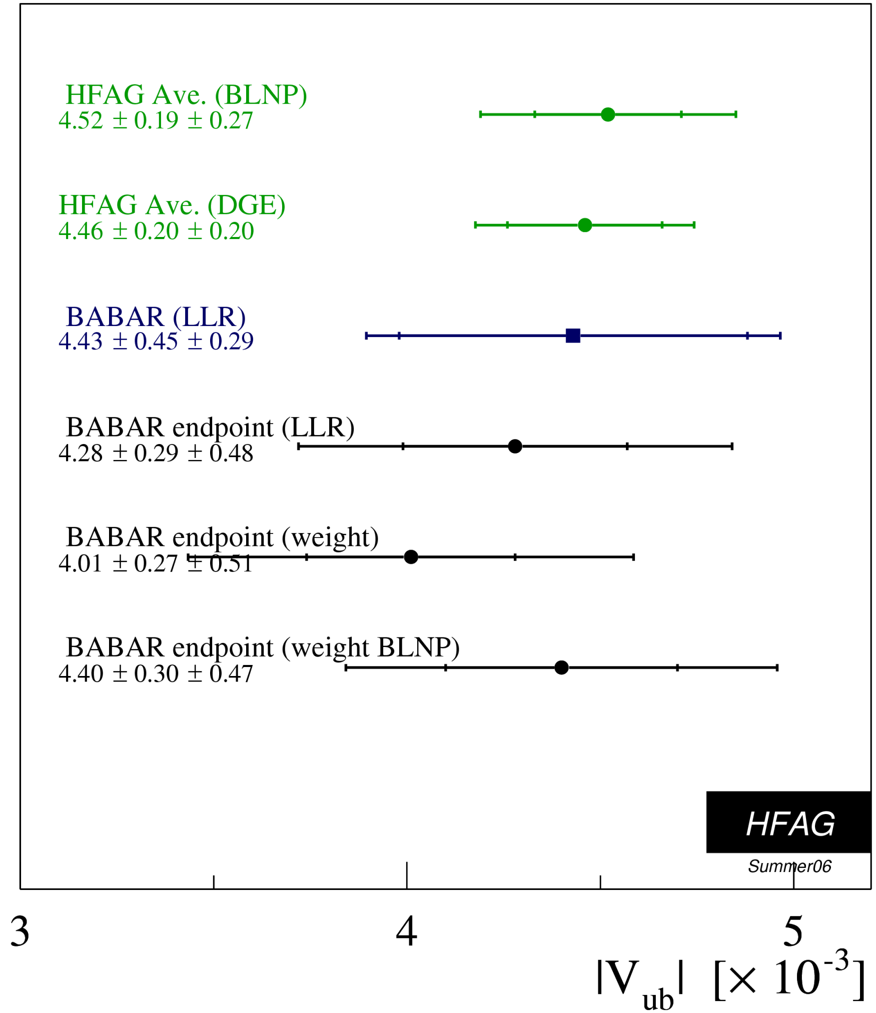


- M_X (upper cut)
- P_+ (upper cut)
- E_ℓ (lower cut)
- M_X / q^2 (lower cut)
- E_ℓ / q^2

Sensitivity to shape function effects (Fermi motion)

Enhanced dependence on Weak Annihilation effects (contributions from the spectator quark at high q^2)

Inclusive determination: different frameworks



S. W. Bosch, B. O. Lange, M. Neubert, G. Paz, hep-ph/0402094

J. Andersen, E. Gardi, hep-ph/0509360

A. K. Leibovich, I. Low, I. Z. Rothstein, hep-ph/9909404

M. Neubert, hep-ph/0104280

B. O. Lange, M. Neubert, G. Paz, hep-ph/0508178

Good agreement between all determinations

A new theoretical framework

P. Gambino, PG, G. Ossola, N. Uraltsev
arXiv:07072493

• Provide the triple differential width of $B \to X_u \ell \nu$ in the whole phase space, including all known perturbative and non-perturbative contributions.

The decay rate and the moments of any spectrum can be computed with any combination of cuts (common to BLNP and DGE methods).

• Work in the kinetic scheme, to separate perturbative and non-perturbative contributions consistently.

• Include all leading and subleading shape function effects, with an OPE based approach.

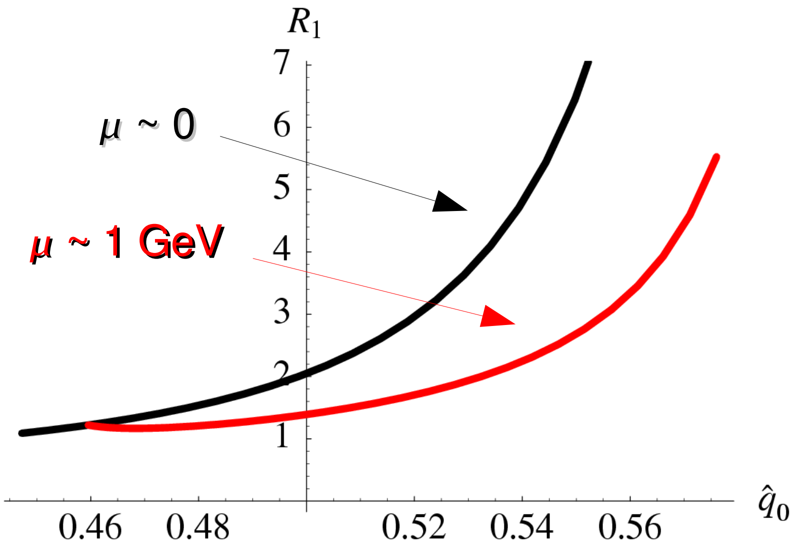
• Model Weak Annihilation effects in the high q^2 region.

• Combine all elements in a numerical C++ code, available for experiments.

The kinetic scheme

- The perturbative and non-perturbative regimes are separated by a hard Wilsonian cutoff μ ~ 1 GeV.

- Soft contributions are absorbed in the definitions of the heavy quark parameters and of the distribution function:
$m_b(\mu)$, $\mu_\pi^2(\mu)$, $\rho_D^3(\mu)$...



- Structure functions diverge less severely (left divergences due to collinear effects).

- The kinetic scheme was applied successfully to $B \to X_c \ell \nu$ and $B \to X_s \gamma$

- A whole set of new calculations has been performed to account for the cutoff.
$O(\alpha_s^2 \beta_0)$ corrections included for the first time.

Fermi motion[1]

• The moments of the distribution function are determined by the q_0 moments of the local OPE.

• The leading order shape function is independent of the process and shared by radiative decays. Subleading effects break universality ⇒ there is a different shape function for each of the 3 structure functions $W_{1\text{-}3}$ and for each value of q^2:

Leading shape function ($m_b \to \infty$) Finite-m_b shape function

$$F(k_+) \longrightarrow F_i(k_+, q^2, \mu)$$

Structure function (i = 1,2,3)

q^2 dependence

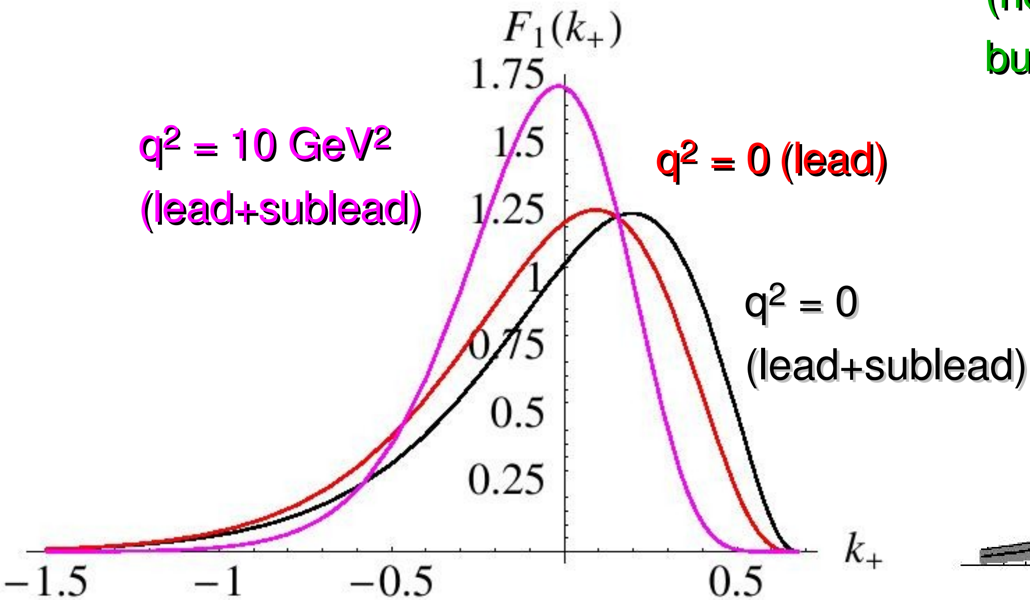
cutoff dependence (gluons with $E_g < \mu$)

• We don't split dominant and subdominant contributions ⇒ more efficient approach than introducing many subleading shape functions.

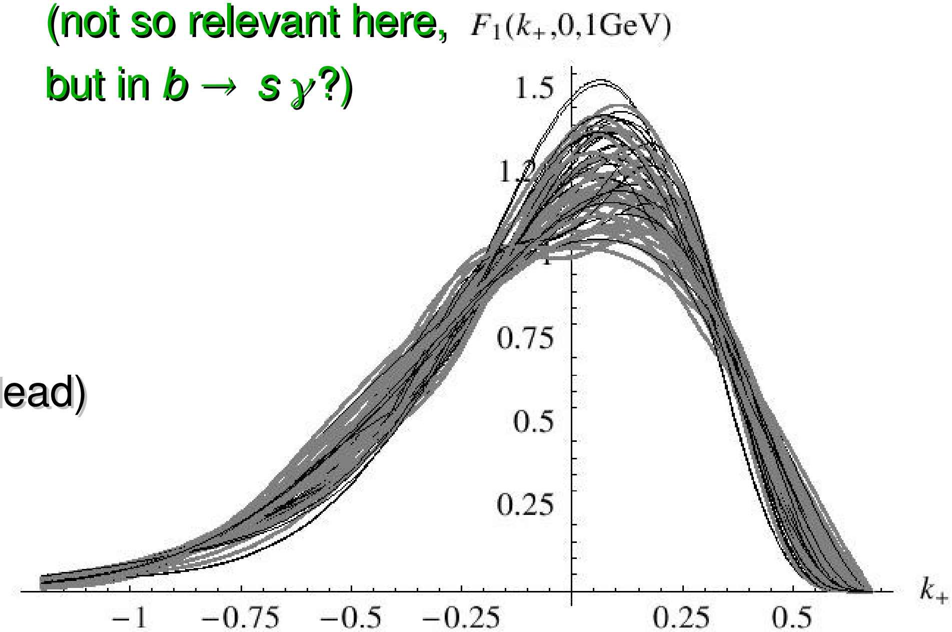
Fermi motion[2]

• Hadronic structure functions are defined *via* the convolution of the distribution functions with the perturbative structure functions:

$$W_i(q_0, q^2) = m_b^{n_i}(\mu) \int dk_+ \; F_i(k_+, q^2, \mu) \; W_i^{pert}\left[q_0 - \frac{k_+}{2}\left(1 - \frac{q^2}{m_b M_B}\right), q^2, \mu\right]$$



Functional form dependence
(not so relevant here,
but in $b \to s\gamma$?)



The high q^2 tail: the symptoms

• The poor convergence of the OPE at high q^2 emerges in a number of pathological features (non-analytic terms, weaker suppression of subleading contributions, negative variance of the distribution functions...).

The formalism developed at low q^2 is no more applicable.

• The q^2 spectrum becomes negative and diverges at the endpoint, due to the contribution of the Darwin term:

$$\frac{d\Gamma}{d\hat{q}^2} \sim \frac{\rho_D^3}{6m_b^3}\left[20\,\hat{q}^6 + 66\,\hat{q}^4 + 48\,\hat{q}^2 + 74 - \frac{96}{1-\hat{q}^2}\right] + \ldots$$

• At the level of integrated rate the singularity is removed by a new operator:

$$\delta\Gamma \sim \left[C_{\mathrm{WA}}\, B_{\mathrm{WA}}(\mu_{\mathrm{WA}}) - \left(8\ln\frac{m_b^2}{\mu_{\mathrm{WA}}^2} - \frac{77}{6}\right)\frac{\rho_D^3}{m_b^3} + \mathcal{O}(\alpha_s)\right]$$

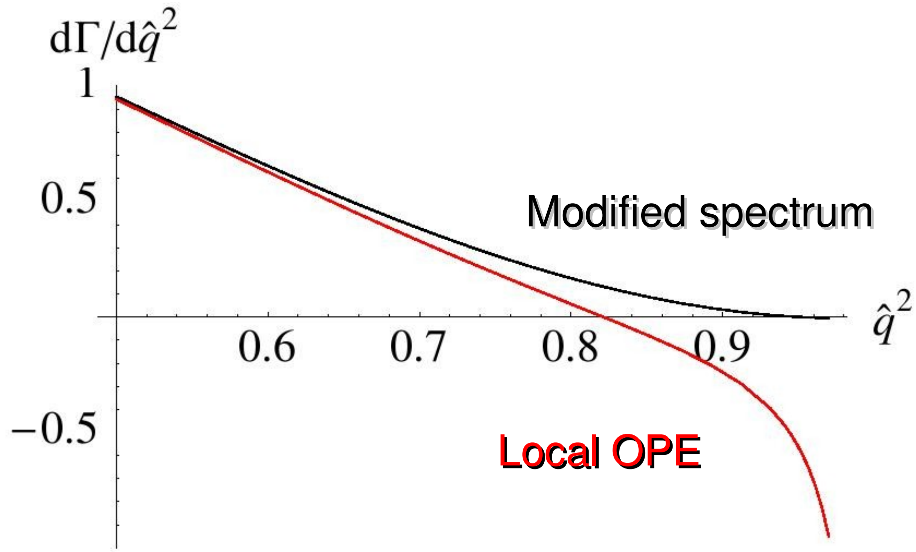
but we are interested in differential distributions as well...

The high q^2 tail: the cure

- First method (default):
  - Model the tail, requiring well-behaved positive spectra.
  - Introduce a WA contribution:

    $$X\,\delta(1-\hat{q}^2)$$ (good)

  - Does not provide a triple differential distribution at high q^2 (bad).
  - Still matches local OPE moments to a great accuracy.



- Second method:
  - "Freeze" the distribution functions at $q^2 \sim 11$ GeV^2 and use them for the convolution at higher q^2.
  - Provides a triple differential width in the whole phase space (good).
  - Does not match local OPE moments at high q^2.

We use the difference of the two methods to estimate the theoretical uncertainty

Extraction of $|V_{ub}|$ and theoretical uncertainties

A Belle analysis with $M_X \leq 1.7\,\mathrm{GeV}$ and $E_\ell > 1.0\,\mathrm{GeV}$ hep-ex/0505088

B Belle and Babar analyses with $M_X \leq 1.7\,\mathrm{GeV}$, $q^2 > 8\,\mathrm{GeV}^2$, and $E_\ell > 1.0\,\mathrm{GeV}$ hep-ex/0505088, hep-ex/0509040

C Babar with $E_\ell > 2.0\,\mathrm{GeV}$ hep-ex/0507017

cuts	$\lvert V_{ub}\rvert \times 10^3$	f	exp	par	pert	tail model	q_*^2	X	ff	tot th
A	3.99	0.69	6.7	3.9	1.9	1.7	2.2	$^{+0.0}_{-2.8}$	$^{+2.4}_{-1.1}$	$\pm 5.1^{+2.4}_{-3.9}$
B	4.60	0.36	7.3	3.8	2.6	3.0	4.2	$^{+0.0}_{-5.1}$	$^{+1.4}_{-0.5}$	$\pm 6.9^{+1.4}_{-5.6}$
C	4.22	0.28	5.7	4.7	3.2	0.8	0.8	$^{+0.0}_{-6.4}$	$^{+1.2}_{-0.7}$	$\pm 5.8^{+1.2}_{-7.1}$

High q2 tail

- All measurements are sensitive to the high q^2 region.

An instructive exercise: combine Belle results (A) and (B) to get a prediction with an *upper* cut on q2: $M_X \leq 1.7\,\mathrm{GeV}$, $q^2 < 8\,\mathrm{GeV}^2$, and $E_\ell > 1.0\,\mathrm{GeV}$

$$|V_{ub}| = 3.28 \times 10^{-3} \pm 4.4^{+1.7}_{-2.6}\%\,(\mathrm{th})$$

A dedicated analysis should be performed

Summary

- The present inclusive determination of $|V_{ub}|$ falls 2.5 σ away from the value preferred by the global UT fit.

- A new theoretical framework has been developed, which has several advantages:
  - Triple differential width with most precise pert and non-pert corrections
  - Separation of hard and soft regimes with a Wilsonian cutoff
  - Leading and subleading Fermi motion effects

- The estimates obtained for $|V_{ub}|$ are in agreement with other determinations.

- The critical role played by the high q^2 region has been pointed out. Even a small contribution from Weak Annihilation decreases $|V_{ub}|$ significantly.

- The analysis suggests to perform measurements with an *upper* cut on q^2.