Recent developments in radiative B decays

Tobias Huber
Institut für Theoretische Physik E,
RWTH Aachen

EPS Conference on HEP, Manchester, July 19th, 2007

Contents

- The inclusive decay $\bar{B} \to X_s \, \ell^+ \ell^-$
  - Electromagnetic corrections
  - Phenomenology: Branching ratio, forward backward asymmetry
- Transverse asymmetries in $B \to K^* \ell^+ \ell^-$
- The inclusive decay $\bar{B} \to X_s \, \gamma$
  - NNLO results
  - Phenomenology
- Time dependent CP asymmetry in $\bar{B} \to K^* \, \gamma$
- CKM and UT parameters from exclusive $b \to (s, d) \, \gamma$

Apologies for any omissions

The inclusive decay $B \to X_s \ell^+ \ell^-$

- Differential decay width: (q^2 : lepton inv. mass; $\hat{s} \equiv q^2/m_b^2$)

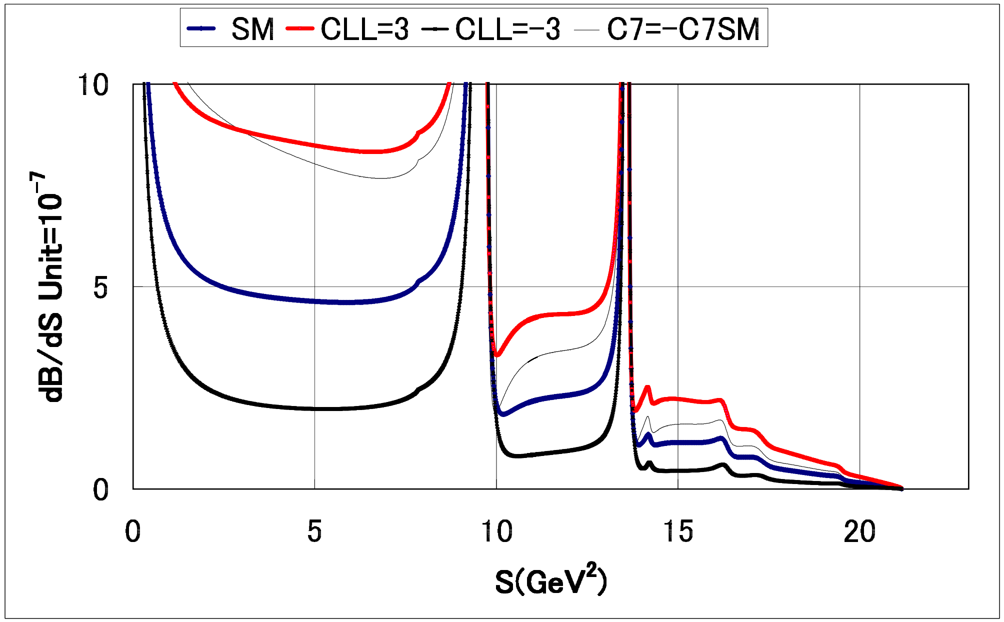
$$\frac{d\Gamma(\bar{B} \to X_s\, l^+ l^-)}{d\hat{s}} = \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 \alpha_{em}^2(\mu)(1-\hat{s})^2}{768\pi^5}$$

$$\times \left\{ \left(4 + \tfrac{8}{\hat{s}}\right) \left|\tilde{C}_7^{eff}\right|^2 + (1+2\hat{s})\left(\left|\tilde{C}_9^{eff}\right|^2 + \left|\tilde{C}_{10}^{eff}\right|^2\right) + 12\,\mathrm{Re}\left(\tilde{C}_7^{eff}\tilde{C}_9^{*\,eff}\right) + \frac{d\Gamma^{brems}}{d\hat{s}} \right\}$$

- Compare to:

$$\Gamma(\bar{B} \to X_s\, \gamma) \propto \left|\tilde{C}_7^{eff}\right|^2$$

- SM size and signs of amplitudes
  - $\tilde{C}_7^{eff} \simeq -0.30$
  - $\tilde{C}_9^{eff} \simeq +4.05$
  - $\tilde{C}_{10}^{eff} \simeq -4.26$

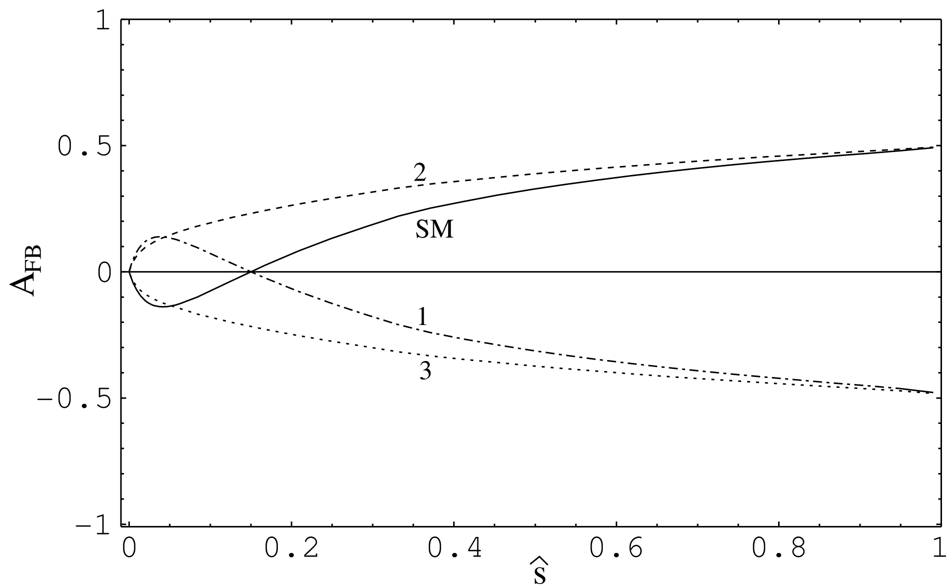


[Akeroyd et. al.]

Forward backward asymmetry

- Forward backward asymmetry:
  - $$\mathscr{A}_{FB}(q^2) \equiv \frac{d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l > 0) - d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l < 0)}{d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l > 0) + d\mathrm{BR}_{\ell\ell}/dq^2(\cos\theta_l < 0)}$$
  - $$\mathscr{A}_{FB}(\hat{s}) = \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 \alpha_{em}^2(\mu)(1-\hat{s})^2}{768\pi^5}$$
    $$\times\left\{-6\,\mathrm{Re}\left(\tilde{C}_{7,FB}^{eff}\tilde{C}_{10,FB}^{*\,eff}\right) - 3\hat{s}\,\mathrm{Re}\left(\tilde{C}_{9,FB}^{eff}\tilde{C}_{10,FB}^{*\,eff}\right) + A_{FB}^{brems}\right\}$$
- Zero of FBA represents SM precision observable (theor. uncertainty $\sim 5\%$)
- A measurement of $d\mathrm{BR}_{\ell\ell}/d\hat{s}$ and $\mathscr{A}_{FB}(\hat{s})$ can provide information on the sign of $\tilde{C}_7^{eff}$, which again will allow to constrain parameter space of new physics models.

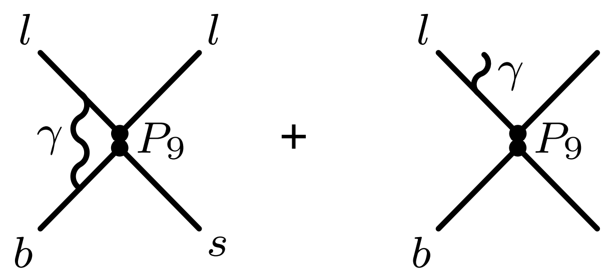
[Gambino,Haisch,Misiak]
[Wyler,Misiak,Cho]



[Ali,Greub,Hiller,Lunghi, $\hat{s} \equiv q^2/m_b^2$]
[1: C10=-C10SM, 2: C7=-C7SM, 3: C(7,10)=-C(7,10)SM]

Perturbative Corrections

- QCD corrections to quark level decay rate are known to NNLO
  [Misiak, Buras, Münz, Bobeth, Urban, Asatrian, Asatryan, Greub, Walker]
  [Bobeth, Gambino, Gorbahn, Haisch, Bieri, Ghinculov, Hurth, Isidori, Yao]
  - reduce NLO diff. BR by about $20-25\%$
  - shift q_0^2 by around $+10-15\%$
  - reduce scale uncertainties from $15-20\%$ to $3-5\%$
- $1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ corrections known *[Falk, Luke, Savage, Ali, Hiller, Handoko, Morozumi]* *[Bauer, Burrell, Buchalla, Isidori, Rey]*
- Motivation for NLO QED corrections
  - They are expected to be larger than N^{3}LO QCD corrections.
  - They reduce $\pm 4\%$ scale uncert. due to $\alpha_e(m_b) \approx 1/133$ vs. $\alpha_e(m_Z) \approx 1/128$.
- IR divergent contributions in QED matrix elements:



  - Contain terms enhanced by $\frac{\alpha_e}{4\pi} \log\left(m_b^2/m_l^2\right)$
  - Contrary to the integrated branching ratio (BR), the differential BR is not an IR safe object with respect to the emission of collinear photons from lepton lines.

NLO QED Matrix Elements

- Include log-enhanced corrections to $|\langle P_7\rangle|^2$, $|\langle P_9\rangle|^2$,$|\langle P_{10}\rangle|^2$, $Re\left[\langle P_7\rangle \langle P_9\rangle^*\right]$, $|\langle P_{1,2}\rangle|^2$, $Re\left[\langle P_{1,2}\rangle \langle P_9\rangle^*\right]$ and $Re\left[\langle P_{1,2}\rangle \langle P_7\rangle^*\right]$
- Presence of $\log\left(\frac{m_b^2}{m_l^2}\right)$ depends on experimental setup due to finite detector resolution for collinear photons
  - not a problem for muons
  - For electrons: cone of opening angle θ_c inside which collinear γ's are included in the reconstructed 4-momentum [Berryhill, Ishikawa]

$$q^2 = (p_+ + p_- + p_\gamma)^2 \qquad m_\ell^2 \leq (p_\ell + p_\gamma)^2 \leq \Lambda^2 \simeq 2E_\ell^2(1 - \cos\theta_c) \qquad \Lambda \sim \mathcal{O}(m_\mu)$$

- We normalize the differential decay width to the semileptonic $\bar{B} \to X_u e\bar{\nu}$ rate
  - removes $m_{b,pole}^5$-factor
  - better than normalization to $\bar{B} \to X_c e\bar{\nu}$ due to absence of phase space factors involving $m_{c,pole}$
- BR expressed in terms of $m_{b,pole}$ and $m_{c,pole}$ contains renormalon ambiguities. They are removed if $1S$ or $\overline{MS}$-masses are used [Hoang,Ligeti,Manohar,Trott]

Results, BR, 1 GeV$^2 < q^2 < 6$ GeV^2

- Including all NNLO QCD, non-pert., and NLO-QED corrections: *[Lunghi, Misiak, Wyler, TH]*
  - $$BR(\bar{B} \to X_s ee) = (1.64 \pm 0.08_{scale} \pm 0.06_{m_t} \pm 0.015_{m_b} \pm 0.025_{C,m_c} \pm 0.02_{\alpha_s(M_Z)} \pm 0.015_{CKM} \pm 0.026_{\mathrm{BR}_{sl}}) \cdot 10^{-6}$$
  - $$BR(\bar{B} \to X_s \mu\mu) = (1.59 \pm 0.08_{scale} \pm 0.06_{m_t} \pm 0.015_{m_b} \pm 0.024_{C,m_c} \pm 0.02_{\alpha_s(M_Z)} \pm 0.015_{CKM} \pm 0.026_{\mathrm{BR}_{sl}}) \cdot 10^{-6}$$
- Experimental values:
  - $BR(\bar{B} \to X_s ll) = (1.493 \pm 0.504_{stat.} {}^{+0.411}_{-0.321}{}_{sys.}) \cdot 10^{-6}$ *[Belle, 152 M evts.]*
  - $BR(\bar{B} \to X_s ll) = (1.8 \pm 0.7_{stat.} \pm 0.5_{sys.}) \cdot 10^{-6}$ *[BaBar, 89 M events]*
  - weighted average: $(1.60 \pm 0.51) \cdot 10^{-6}$
- With reversed sign of $\widetilde{C}_7^{\mathrm{eff}}$
  - $BR(\bar{B} \to X_s ee) = 3.19 \cdot 10^{-6}$
  - $BR(\bar{B} \to X_s \mu\mu) = 3.11 \cdot 10^{-6}$ $\Rightarrow$ SM-sign of $\widetilde{C}_7^{\mathrm{eff}}$ is favored *[Gambino,Misiak,Haisch]*
- Subdivided results for two bins $[1, 3.5]$ GeV^2 and $[3.5, 6]$ GeV^2:
  - $BR(ee, [1, 3.5]) = (0.92 \pm 0.06) \cdot 10^{-6}$ $\quad BR(\mu\mu, [1, 3.5]) = (0.88 \pm 0.05) \cdot 10^{-6}$
  - $BR(ee, [3.5, 6]) = (0.72 \pm 0.05) \cdot 10^{-6}$ $\quad BR(\mu\mu, [3.5, 6]) = (0.71 \pm 0.05) \cdot 10^{-6}$

Forward backward asymmetry **PRELIM.**

- Forward backward asymmetry: $[\frac{\mathrm{d}\mathcal{A}(\bar{B}\to X_s\ell^+\ell^-)}{\mathrm{d}\hat{s}}]\,/\,[\frac{\mathrm{d}\mathcal{B}(\bar{B}\to X_s\ell^+\ell^-)}{\mathrm{d}\hat{s}}]$
  - Each of the brackets is normalized to Γ_u and gets fully expanded in the couplings, but no overall expansion is done.
- Analysis of zero q_0^2 of forward backward asymmetry *[Hurth,Lunghi,TH]*

$$
\begin{aligned}
q^2_{0,\mu\mu} &= \Big[3.543 \pm 0.075_{\rm scale} \pm 0.003_{m_t} \pm 0.03_{m_c,C} \pm 0.05_{m_b} \pm 0.074_{\alpha_s(M_Z)}\Big]\,\mathrm{GeV}^2 \ , \\
q^2_{0,ee} &= \Big[3.421 \pm 0.07_{\rm scale} \pm 0.003_{m_t} \pm 0.03_{m_c} \pm 0.046_{m_b} \pm 0.07_{\alpha_s(M_Z)}\Big]\,\mathrm{GeV}^2 \ .
\end{aligned}
$$

  - NNLO zero lies within error bars of NLO analysis
- Integrated FBA for different bins: (num. and denom. integrated separately)

[Hurth,Lunghi,TH]

  - $\bar{\mathcal{A}}_{ee[1,3.5]} = (\,-8.20 \pm 0.90\,)\,\%$, $\quad \bar{\mathcal{A}}_{\mu\mu[1,3.5]} = (\,-9.17 \pm 0.90\,)\,\%$
  - $\bar{\mathcal{A}}_{ee[3.5,6]} = (\,7.61 \pm 0.61\,)\,\%$, $\quad \bar{\mathcal{A}}_{\mu\mu[3.5,6]} = (\,7.12 \pm 0.64\,)\,\%$
  - $\bar{\mathcal{A}}_{ee[1,6]} = (\,-1.27 \pm 0.78\,)\,\%$, $\quad \bar{\mathcal{A}}_{\mu\mu[1,6]} = (\,-1.93 \pm 0.81\,)\,\%$

BR, high-q^2 region **PRELIM.**

- Branching ratio integrated over $q^2 > 14.4$ GeV^2 *[Hurth,Lunghi,TH]*
  - $BR(\bar{B} \to X_s ee) = (2.15 \pm 0.56) \cdot 10^{-7}$
  - $BR(\bar{B} \to X_s \mu\mu) = (2.47 \pm 0.58) \cdot 10^{-7}$
  - Dominant error from uncertainties in non-perturbative corrections
- Experimental values:
  - $BR(\bar{B} \to X_s ll) = (4.18 \pm 1.17_{stat.} {}^{+0.61}_{-0.68}{}_{sys.}) \cdot 10^{-7}$ *[Belle, 152 M evts.]*
  - $BR(\bar{B} \to X_s ll) = (5 \pm 2.5_{stat.} {}^{+0.8}_{-0.7}{}_{sys.}) \cdot 10^{-7}$ *[BaBar, 89 M events]*
- Recent analysis: Normalization to semilept. $B \to X_u \ell\nu$ rate *with the same cut* reduces significantly the theoretical error from non-perturbative uncertainties. *[Ligeti,Tackmann]*
- Recent observation: 3rd independent combination of Wilson Coefficients: ($z = \cos\theta$)

$$\frac{d^2\Gamma}{dq^2\, dz} = 3/8\, \left[(1+z^2)\, H_T(q^2) \,+\, 2\, z\, H_A(q^2) + 2\,(1-z^2)\, H_L(q^2)\right]$$

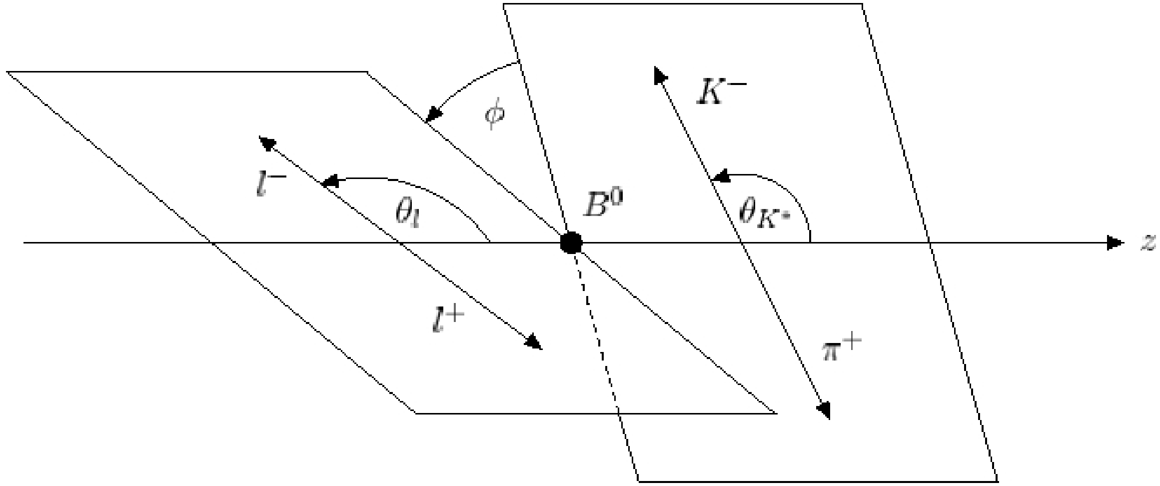
- Note: $$\frac{d\Gamma}{dq^2} = H_T(q^2) \,+\, H_L(q^2)\,, \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\, H_A(q^2)$$

[Lee,Ligeti,Stewart,Tackmann]

Transversity amplit. in $B \to K^*(K\pi)\ell^+\ell^-$

- For an on-shell K^*, the decay $B^0 \to K^{*0}(\to K^-\pi^+)\ell^+\ell^-$ is described by s (lepton inv. mass), and three angles θ_l, θ_ℓ, ϕ.

$$\frac{d^4\Gamma}{ds\, d\cos\theta_l\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi}\sum_{i=1}^{9} I_i(s,\theta_{K^*})\, f_i(\theta_l,\phi)$$



  - I_i depend on the four K^* spin amplitudes $A_\parallel$, $A_\perp$, A_0, A_t. The f_i are the corresponding angular distribution functions

- In the limit of a heavy quark and a large E_{K^*} the seven $B \to K^*$ form factors reduce to two universal ones.

- Those form factors cancel out in specific transverse asymmetries, which then depend on short-distance information only:

$$A_T^{(1)}(s) = \frac{-2\mathrm{Re}(A_\parallel\, A_\perp^*)}{|A_\perp|^2 + |A_\parallel|^2}\,, \qquad A_T^{(2)}(s) = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}$$

Transversity amplit. in $B \to K^*(K\pi)\ell^+\ell^-$

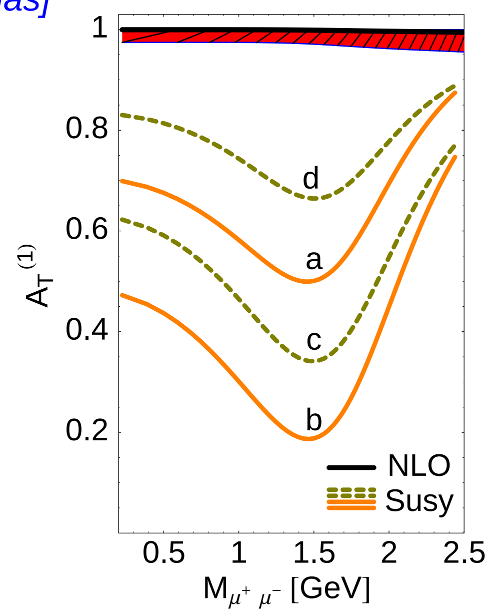
- Including next-to-leading corrections and integrating over the low di-muon mass region $2\,m_\mu \leq M_{\mu\mu} \leq 2.5$ GeV: (without $\Lambda_{\rm QCD}/m_b$ corrections) *[Krüger,Matias]*

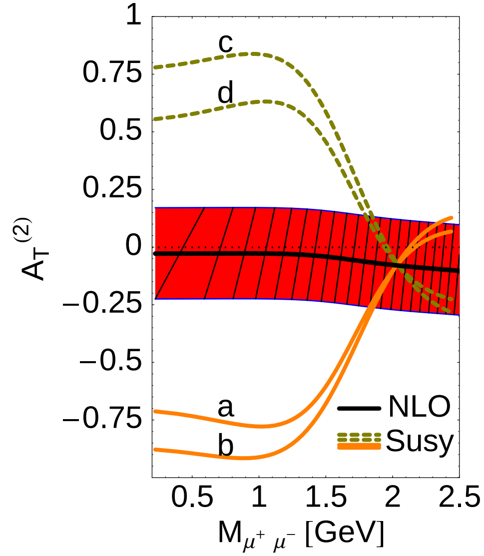
$$A_T^{(1)} = 0.9986 \pm 0.0002\,, \qquad A_T^{(2)} = -0.0043 \pm 0.003$$

- Transverse asymmetries provide theoretically clean way to analyse the chiral structure of the $b \to s$ current.

  Example: MSSM with R-parity and non-MFV in down-squarks soft-breaking terms

[Lunghi,Matias]





The inclusive decay $\bar{B} \to X_s\gamma$

- $\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{th.,NLO}} = (3.60 \pm 0.30) \times 10^{-4}$ *[Gambino,Misiak'01]*
  - for $E_\gamma > 1.6$ GeV in the restframe of the $\bar{B}$.
  - main errors from m_c, m_b, scales, $\alpha_s(M_Z)$
  - m_c dependence pronounced since it first enters at NLO
- $\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp.}}_{E_\gamma > 1.6\mathrm{GeV}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$ *[HFAG'06]*
  - Errors are "combined stat. and sys." "shape function", "$b \to d\gamma$ fraction"
- At future colliders: 5% uncertainty can be reached experimentally (more statistics, lower E_γ)
- Motivation for NNLO precision calculation. NNLO SM prediction:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{E_\gamma > 1.6\mathrm{GeV}}_{\mathrm{SM}} = (3.15 \pm 0.23) \times 10^{-4}$$

[Misiak,Steinhauser,Gorbahn,Haisch,Bobeth,Urban,Hurth,Bieri,Greub,Melnikov,Mitov,Czakon]

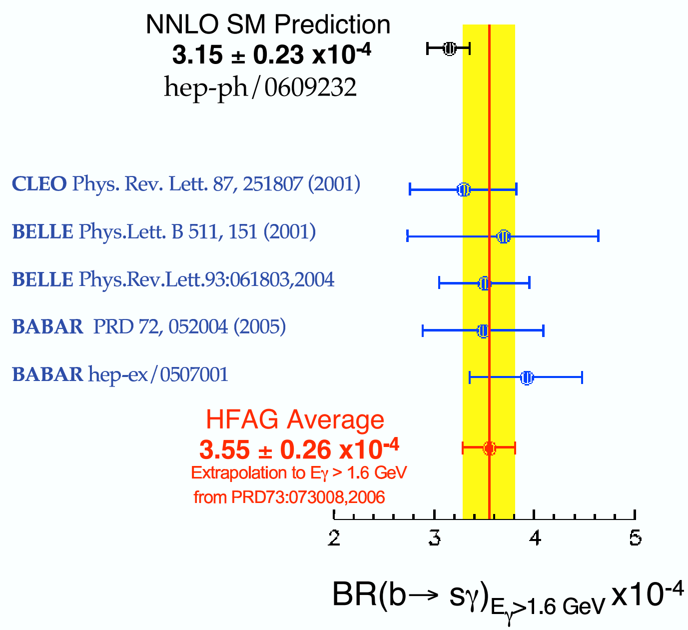
[Blokland,Czarnecki,Ślusarczyk,Tkachov,Asatrian,Hovhannisyan,Poghosyan,Ewerth,Ferroglia,Gambino]

NNLO SM prediction

- Decomposition of total error
  - unknown non-perturbative

$\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ contribution: 5% *[Lee,Neubert,Paz]*

  - parametric uncertainties
    $(m_b, \alpha_s(M_Z), \mathcal{B}_{\rm SL}^{\rm exp.}, \ldots)$: 3%
  - m_c interpolation in
    matrix elements of $P_{1,2}$: 3 %
  - scale dependence on μ_c, μ_b, μ_0
    (estim. of higher order effects): 3%

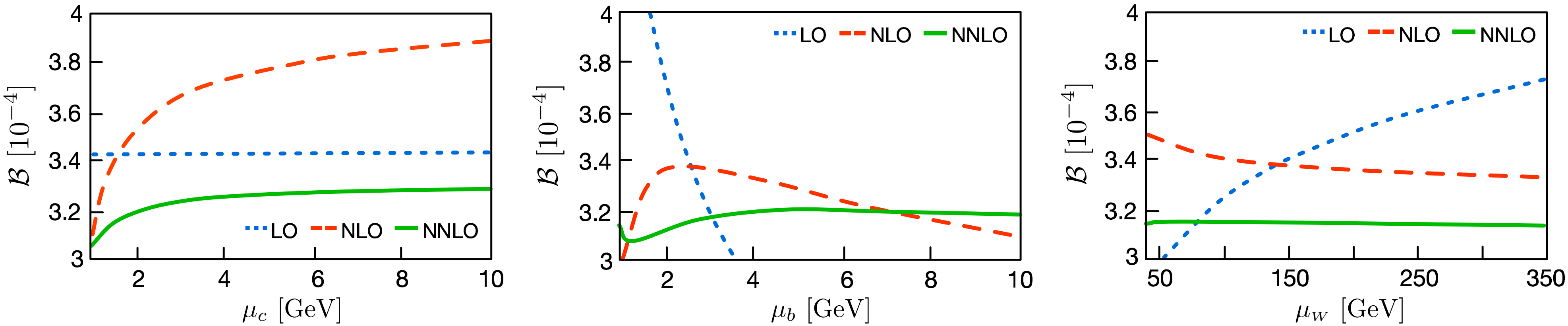


[O. Buchmüller]

- But: A low cut ~ 1.8 GeV might not guarantee that a description in terms of a local OPE is sufficient (due to sensitivity to scale $\Delta = m_b - 2E_\gamma$) *[Becher,Neubert]*
  - Multiscale OPE with 3 short distance scales (m_b, $\sqrt{m_b\,\Delta}$, Δ) needed to connect shape function and local OPE region.
  - Using SCET, effects at the 5 % level found not by power corr. $\Lambda_{\rm QCD}/\Delta$ but by perturbative ones: $\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm SM}^{E_\gamma > 1.6{\rm GeV}} = (2.98 \pm 0.26) \times 10^{-4}$

- Renormalization scale dependence.

  Central values: $\mu_c = 1.224$ GeV, $\mu_b = m_b^{1S}/2 = 2.35$ GeV, $\mu_W = 2M_W$



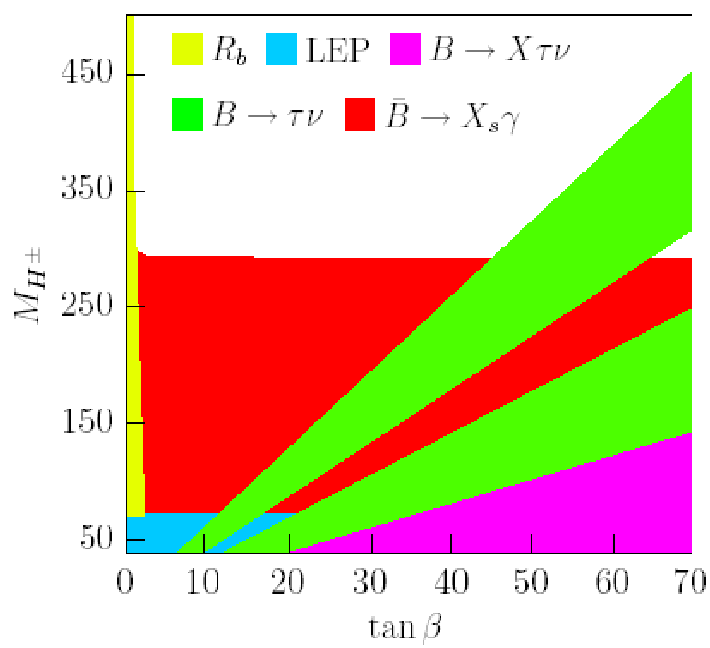
[Misiak et.al.'06; plots courtesy of U. Haisch]

- Precision on theoretical and exptl. side allow to constrain new physics parameter space

  Expample: $M_{H^\pm}$ in type II 2HDM

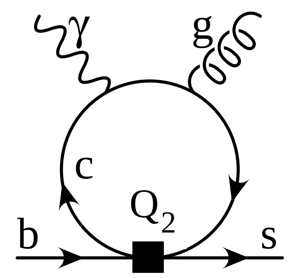
  - $M_{H^\pm}$ > 295 GeV at 95% C.L.
  - independent of $\tan\beta$

  [Misiak et.al., Haisch]



Br. ratio and asymmetries in $B \to K^*\gamma$

- Photon is predominantly left-handed (l.h.) in b and right-handed (r.h.) in $\bar{b}$ decays due to $$Q_7 = \bar{s}\,\sigma^{\mu\nu}\,F_{\mu\nu}\,(m_b\,P_R + m_s\,P_L)\,b$$
  - Naive suppression of r.h. photon emission by a factor of $\mathcal{O}(m_s/m_b)$
  - Suppression partially removed by emission of an additional gluon in diagrams involving $$Q_2 = (\bar{c}\,\gamma^\mu\,P_L\,b)(\bar{s}\,\gamma_\mu\,P_L\,c)$$
  - Resulting suppression factor is $\mathcal{O}(\alpha_s)$ in inclusive and $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ in exclusive decays, both for $b \to s\,\gamma$ and for $b \to d\,\gamma$.



[Grinstein,Grossman,Ligeti,Pirjol]
[Grinstein,Pirjol]

- Possible enhancement of $\mathcal{O}(m_i/m_b)$ from helicity flip on heavy internal lines in NP models such as l.-r. sym. models, SUSY, Warped ED, anomalous r.h. top couplings
- Helicity amplitudes add incoherently in braching ratio, but interfere in time dep. CP asymmetry.

[Atwood,Gronau,Soni]

Time dep. CP asymmetry in $B \to K^*\gamma$

$$A_{\rm CP} = \frac{\Gamma(\bar{B}^0(t) \to \bar{K}^{*0}\gamma) - \Gamma(B^0(t) \to K^{*0}\gamma)}{\Gamma(\bar{B}^0(t) \to \bar{K}^{*0}\gamma) + \Gamma(B^0(t) \to K^{*0}\gamma)} = S\,\sin(\Delta m_B\,t) - C\,\cos(\Delta m_B\,t)$$

- S involves interference of photons with different polarisation
- Time dep. CP asymmetry small in SM, irrespective of hadronic uncertainties
- Prime candidate for "null test" of SM *[Gershon,Soni]*
- Analysis combining QCD-factorisation with QCD sum rules on the light-cone to estimate long-distance photon emission and soft-gluon emission from quark loops yields *[Ball,Zwicky;Ball,Jones,Zwicky]*

$$S = -0.022 \pm 0.015^{+0}_{-0.1} \qquad \text{and} \qquad S\,|_{\text{soft gluons}} = 0.005 \pm 0.01$$

- Conservative dimensional estimate (from a SCET based analysis) gives

$$\left| S|_{\text{soft gluons}} \right| \approx 0.06$$

[Grinstein,Pirjol]
[Grinstein,Grossman,Ligeti,Pirjol]

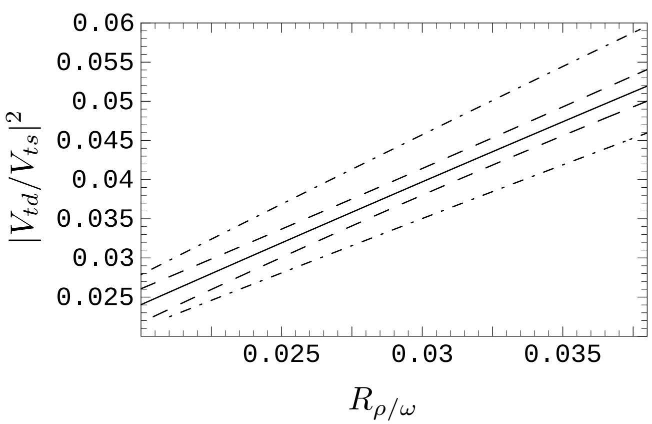
- Calculation in pQCD yields $S_{\rm pQCD} = -0.035 \pm 0.017$ *[Matsumori,Sanda]*
  - effects mainly from hard gluons, soft ones treated in model dependent way
- Experiment: $S = -0.28 \pm 0.26$ *[HFAG'06]*

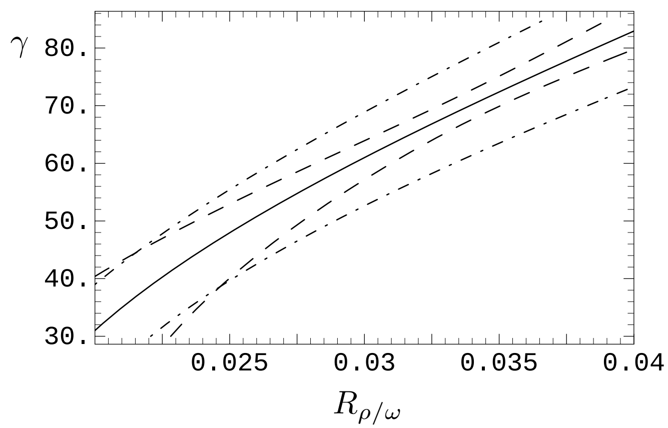
CKM and UT param. from excl. $b \to (s,d)\gamma$

- Consider ratios of BR's: $R_{\rho/\omega} \equiv \frac{\bar{\mathcal{B}}(B \to (\rho,\omega)\gamma)}{\bar{\mathcal{B}}(B \to K^*\gamma)}$, $R_\rho \equiv \frac{\bar{\mathcal{B}}(B \to \rho\gamma)}{\bar{\mathcal{B}}(B \to K^*\gamma)}$
  - Branching ratios are CP- and isospin averaged
- Knowledge of $R_{\rho/\omega}$ and R_ρ (and few other parameters) allows to extract $|V_{td}/V_{ts}|$ and UT angle γ. [Ball,Jones,Zwicky]
  - Extraction of γ involves a degeneracy $\gamma \leftrightarrow 2\pi - \gamma$ due to dependence on $\cos\gamma$.
  - Combining it with tree-level CP asymmetries in $B \to D^{(*)} K^{(*)}$, where $\gamma \leftrightarrow \pi + \gamma$, allows for *unambiguous* determination of γ (if unitarity of V_{CKM} is assumed).

$$|V_{td}/V_{ts}| = 0.199^{+0.022}_{-0.025}(\mathrm{exp}) \pm 0.014(\mathrm{th}) \quad \leftrightarrow \quad \gamma = (61.0^{+13.5}_{-16.0}(\mathrm{exp})^{+8.9}_{-9.3}(\mathrm{th}))^\circ \quad \text{BaBar}$$

$$|V_{td}/V_{ts}| = 0.207^{+0.028}_{-0.033}(\mathrm{exp})^{+0.0014}_{-0.0015}(\mathrm{th}) \quad \leftrightarrow \quad \gamma = (65.7^{+17.3}_{-20.7}(\mathrm{exp})^{+8.9}_{-9.2}(\mathrm{th}))^\circ \quad \text{Belle}$$





[Ball,Jones,Zwicky]

Credits

- Tobias Hurth
- Enrico Lunghi
- Mikołaj Misiak
- Daniel Wyler
- Ulrich Haisch

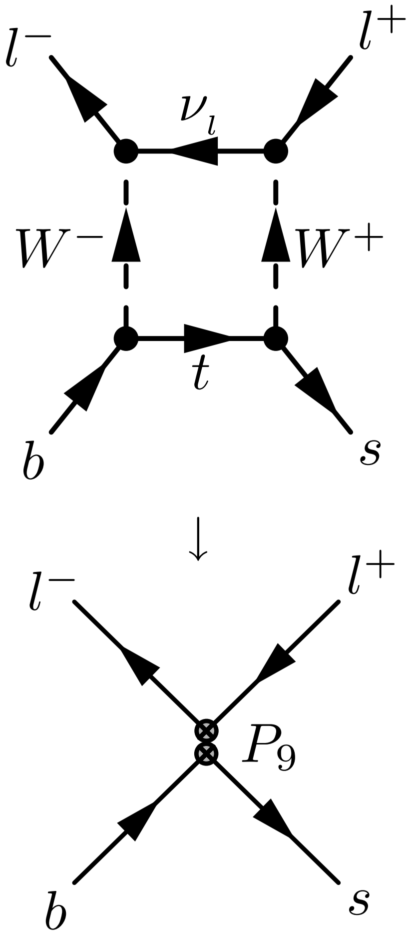
Backup slides

Numerical Inputs

$\alpha_s(M_z) = 0.1182 \pm 0.0027$	$m_e = 0.51099892$ MeV
$\alpha_e(M_z) = 1/127.918$	$m_\mu = 105.658369$ MeV
$s_W^2 \equiv \sin^2\theta_W = 0.2312$	$m_\tau = 1.77699$ GeV
$\lvert V_{ts}^* V_{tb}/V_{cb}\rvert^2 = 0.967 \pm 0.009$	$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)$ GeV
$BR(B \to X_c e\bar{\nu})_{\text{exp}} = 0.1061 \pm 0.0017$	$m_b^{1S} = (4.68 \pm 0.03)$ GeV
$M_Z = 91.1876$ GeV	$m_{t,\text{pole}} = (172.7 \pm 2.9)$ GeV
$M_W = 80.426$ GeV	$m_B = 5.2794$ GeV
$\lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12\ \text{GeV}^2$	$C = 0.58 \pm 0.01$
$\lambda_1 = -0.27 \pm 0.04\ \text{GeV}^2$	$\rho_1 = 0.06 \pm 0.06\text{GeV}^3, \quad f_1 = 0$

Effective Lagrangian

$$\mathscr{L}_{eff} = \mathscr{L}_{QCD\times QED}(u,..,b,e,\mu,\tau) + \frac{4G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \cdot \left[\sum_{i=1}^{10} C_i\, P_i + \underbrace{\sum_{i=3}^{6} C_{iQ}\, P_{iQ} + C_b\, P_b}_{for\ QED\ corrections} \right]$$

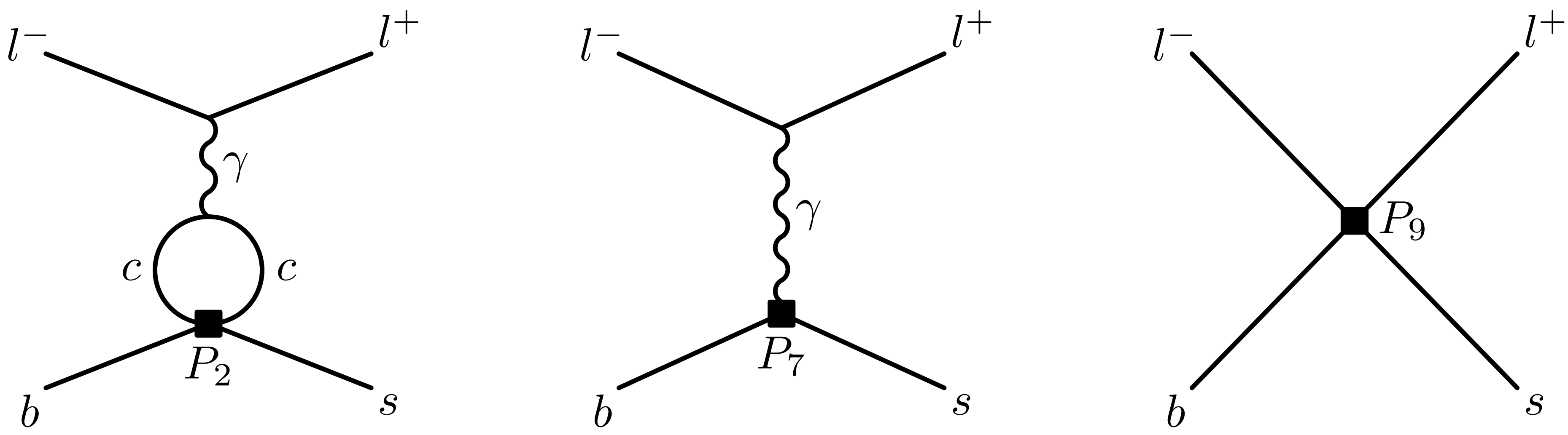


- C_i: Wilson Coefficients
  - scale dependent effective couplings, process independent
  - $C_i(\mu_W)$ obtained by matching on full theory
  - $C_i(\mu_b)$ obtained by solving perturbatively the RGE $\quad \mu \dfrac{d}{d\mu}\vec{C}(\mu) = \hat{\gamma}^T(\mu)\,\vec{C}(\mu)$
  - $\vec{C}(\mu_b) = \hat{R}\,\vec{C}(\mu_W)$

Operators in the EFT

$$P_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L), \quad P_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q),$$
$$P_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L), \quad P_5 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q),$$
$$P_3 = (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q), \quad P_6 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma T^a b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma T^a q),$$

$$P_7 = \frac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu}, \quad P_9 = (\bar{s}_L\gamma_\mu b_L)\sum_l(\bar{l}\gamma^\mu l),$$
$$P_8 = \frac{g}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}, \quad P_{10} = (\bar{s}_L\gamma_\mu b_L)\sum_l(\bar{l}\gamma^\mu\gamma_5 l),$$



Operators in the EFT

$$P_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L), \quad P_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q),$$

$$P_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L), \quad P_5 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q),$$

$$P_3 = (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q), \quad P_6 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma T^a b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma T^a q),$$

$$P_7 = \frac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu}, \quad P_9 = (\bar{s}_L\gamma_\mu b_L)\sum_l(\bar{l}\gamma^\mu l),$$

$$P_8 = \frac{g}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}, \quad P_{10} = (\bar{s}_L\gamma_\mu b_L)\sum_l(\bar{l}\gamma^\mu\gamma_5 l),$$

$$P_{3Q} = (\bar{s}_L\gamma_\mu b_L)\sum_q Q_q(\bar{q}\gamma^\mu q),$$

$$P_{4Q} = (\bar{s}_L\gamma_\mu T^a b_L)\sum_q Q_q(\bar{q}\gamma^\mu T^a q),$$

$$P_{5Q} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)\sum_q Q_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q),$$

$$P_{6Q} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma T^a b_L)\sum_q Q_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma T^a q),$$

$$P_b = \frac{1}{12}\left[(\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma b_L)(\bar{b}\gamma^\mu\gamma^\nu\gamma^\sigma b) - 4(\bar{s}_L\gamma_\mu b_L)(\bar{b}\gamma^\mu b)\right].$$

Numerical values of couplings at $\mu = \mu_b$

- Numerical values for $\widetilde{\alpha}_{\rm s}(\mu_b)$ and $\kappa(\mu_b)$ with $\mu_b = 5$ GeV
  - $\widetilde{\alpha}_{\rm s}(\mu_b) \;=\; 0.0170$
  - $\kappa(\mu_b) \;=\; 0.0354$