NNLO corrections to event shape variables in electron positron annihilation

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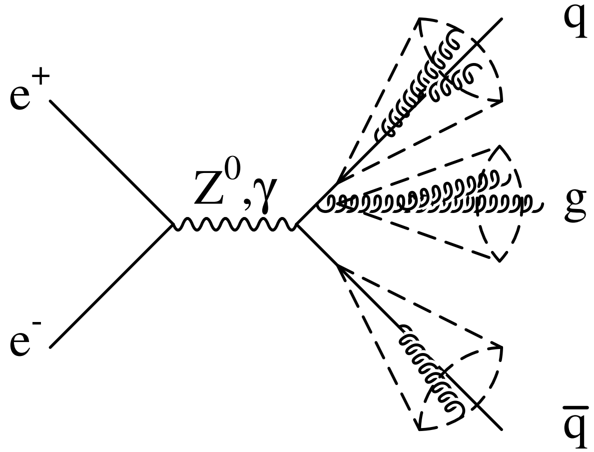
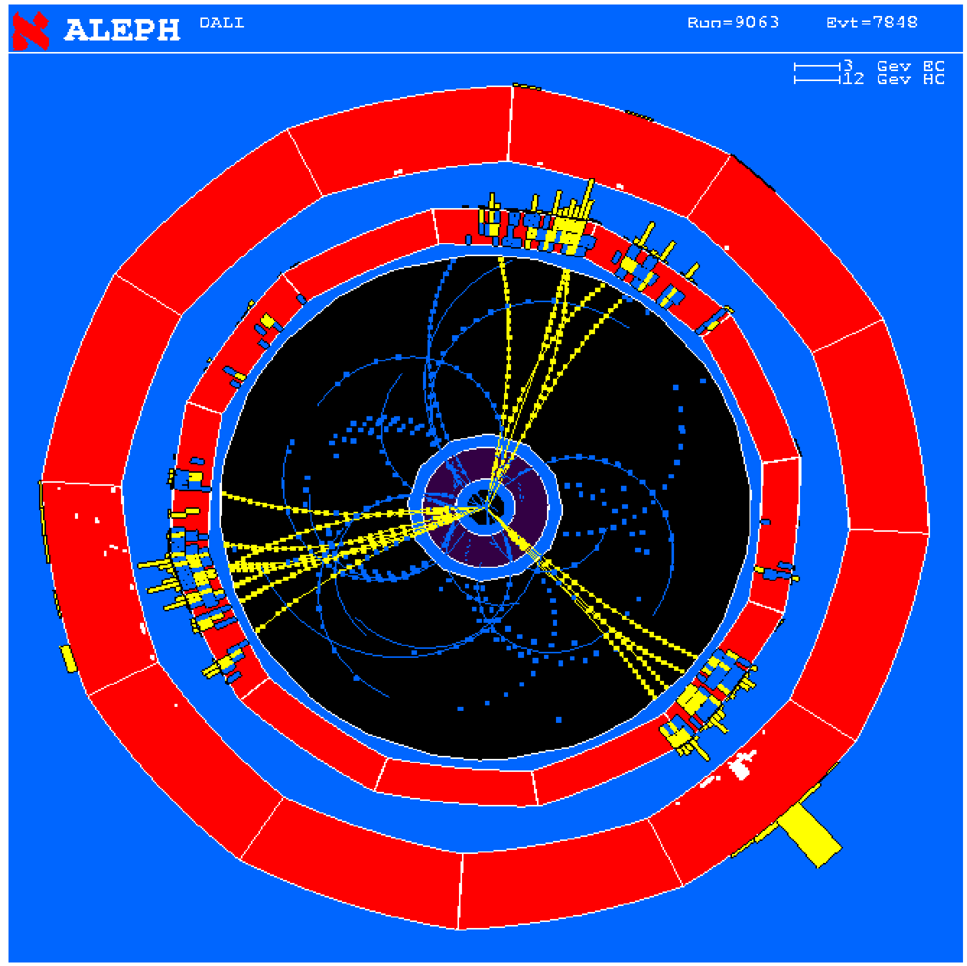
$e^+e^- \to 3$ jets and event shapes

$e^+e^- \to 3$ jets
event at LEP

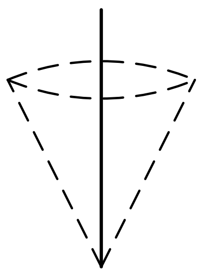




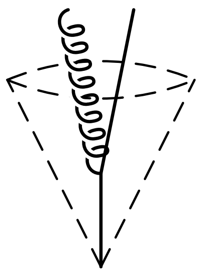
✓ Testing ground for QCD in electron-positron annihilation: fixed order perturbation, infrared resummation, power corrections

Jets in perturbative QCD

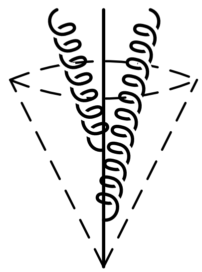
✓ Partons are combined into jets using the same jet algorithm (recombination procedure) in theory as in experiment



LO
each parton forms 1 jet on its own



NLO
2 partons in 1 jet, 1 parton experimentally unresolved



NNLO
3 partons in 1 jet, 2 partons experimentally unresolved

Current state-of-the-art: NLO

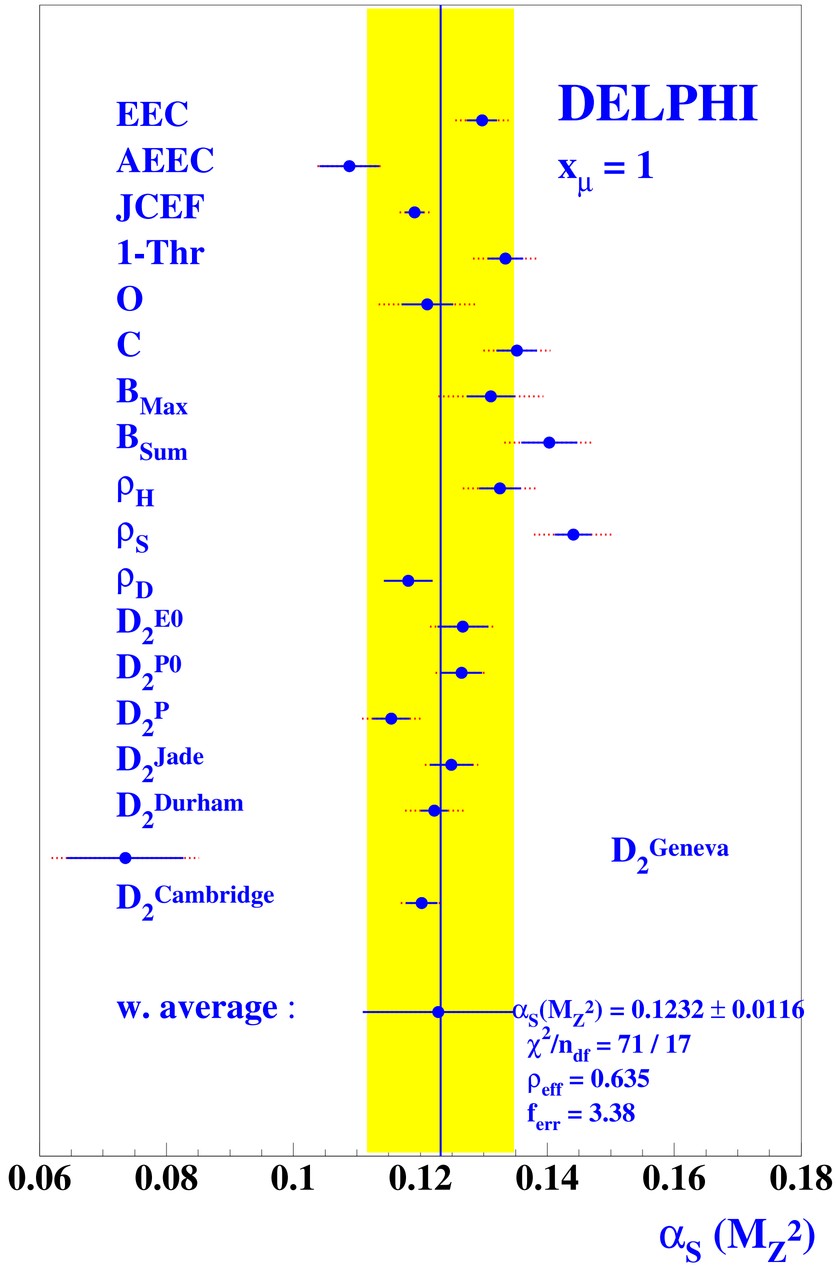
✗ Current error on α_s from jet observables dominated by theoretical error

Need for NNLO:

✓ reduce error on α_s

✓ better matching of parton level and hadron level jets

Indication of importance of higher orders?

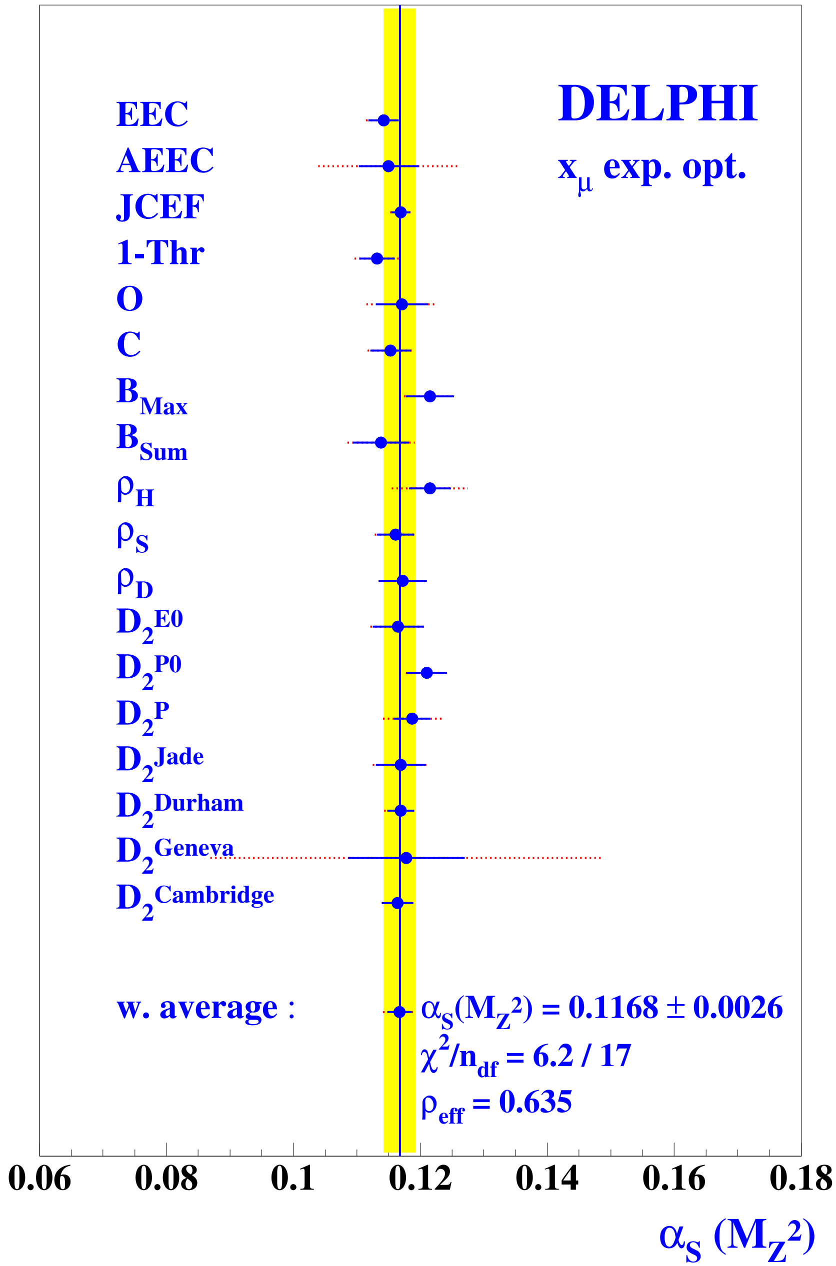


- ✓ Fitting event shapes by varying $\alpha_s(M_Z)$ yield large variations

$$\alpha_s(M_Z) = 0.123 \pm 0.012$$

- ✓ e.g. $1 - T \Rightarrow \ \alpha_s(M_Z) \sim 0.133$
- ✓ A hint that missing higher order terms are large?

Indication of importance of higher orders?



- ✓ Try to estimate higher order effects by varying both $\alpha_s(M_Z)$ and renormalisation scale μ
- ✓ Much more consistent set of $\alpha_s(M_Z)$

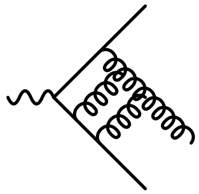
$$\alpha_s(M_Z) = 0.1165 \pm 0.0026$$

- ✓ . . . but extreme variation in fitted values of μ between $0.005M_Z$ and $0.5M_Z$
- ✓ Is this meaningful? Only NNLO can decide.

Ingredients of NNLO calculation

✓ Two-loop matrix elements

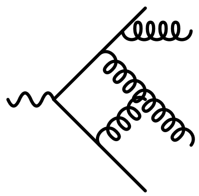
$|\mathcal{M}|^2_{\text{2-loop,3 partons}}$



explicit infrared poles from loop integrals

✓ One-loop matrix elements

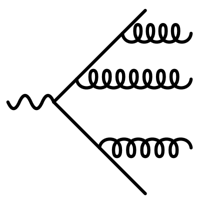
$|\mathcal{M}|^2_{\text{1-loop,3+1 partons}}$



explicit infrared poles from loop integral and implicit infrared poles due to single unresolved radiation

✓ Tree level matrix elements

$|\mathcal{M}|^2_{\text{tree,3+2 partons}}$



implicit infrared poles due to double unresolved radiation

✓ Infrared Poles cancel in the sum

✓ Divergences must be extracted before the jet algorithm can be applied
$\Rightarrow$ Subtraction formalism needed

Infrared Subtraction at NNLO

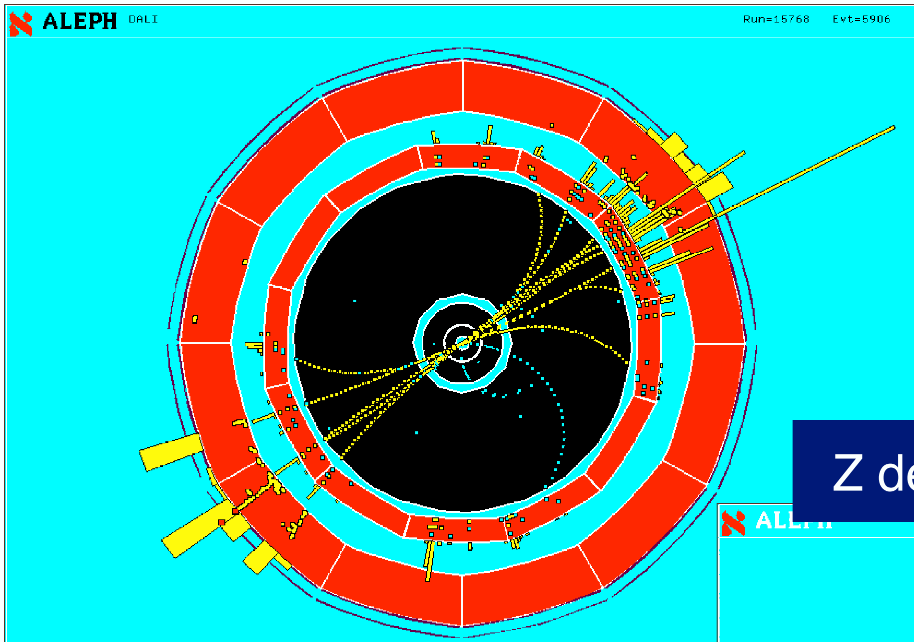
Structure of NNLO 3-jet cross section:

$$\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_5} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_4} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_3} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_5} \mathrm{d}\sigma^S_{NNLO} + \int_{\mathrm{d}\Phi_4} \mathrm{d}\sigma^{VS,1}_{NNLO} \,,
\end{aligned}$$

- ✓ $\mathrm{d}\sigma^S_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^R_{NNLO}$
- ✓ $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- ✓ $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

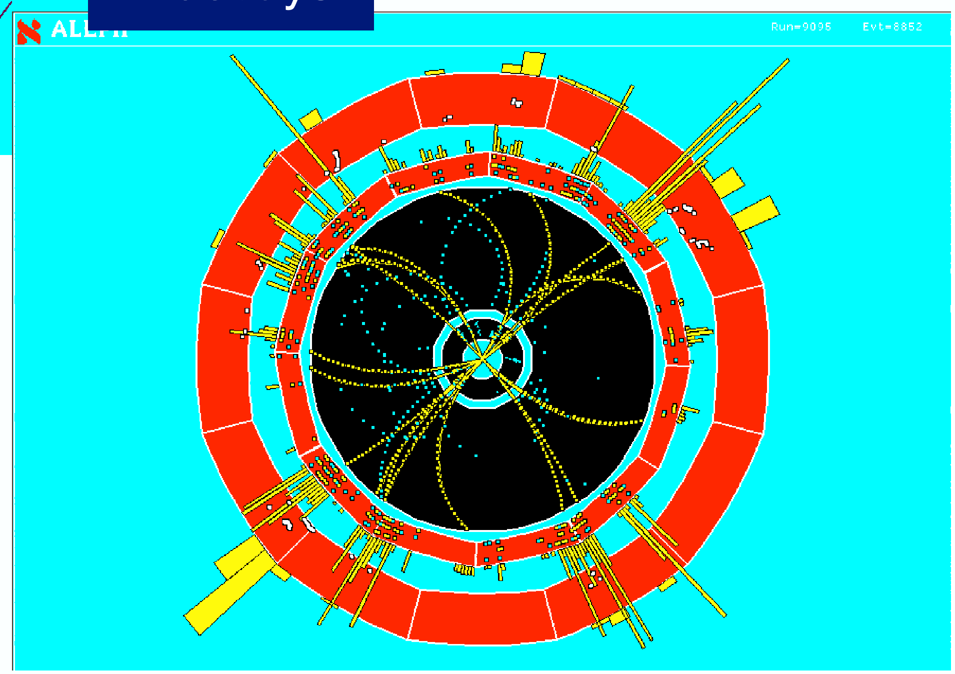
Each line above is finite numerically and free of infrared ϵ-poles

Thrust as an example



Thrust → 1

Z decays



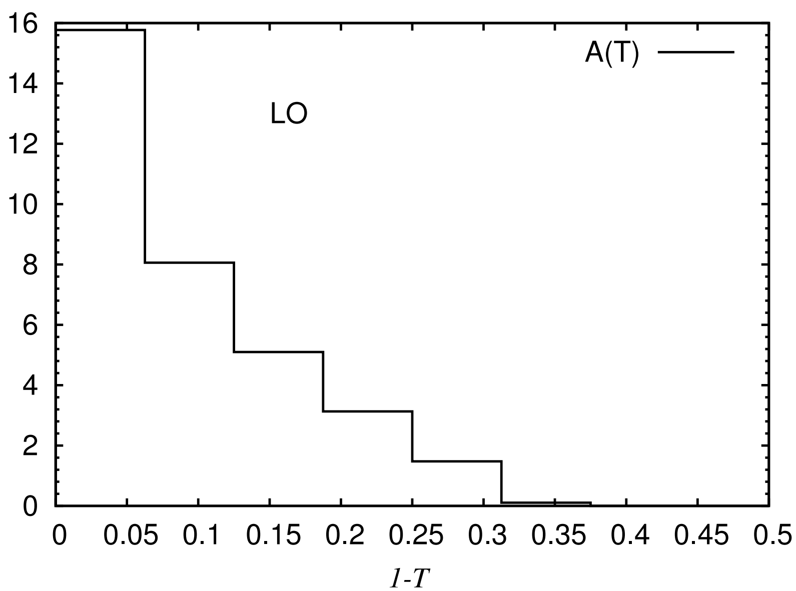
Thrust → 1/2

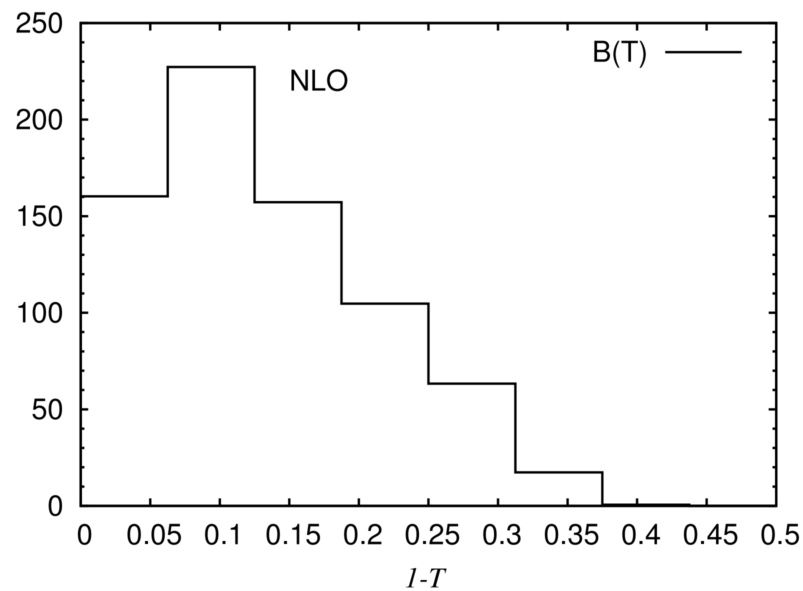
Perturbative expansion for Thrust

$$(1-T)\,\frac{1}{\sigma_{\mathrm{had}}}\,\frac{\mathrm{d}\sigma}{\mathrm{d}T} \;=\; \left(\frac{\alpha_s}{2\pi}\right) A(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(B(T) - 2\,A(T)\right) + \left(\frac{\alpha_s}{2\pi}\right)^3 \left(C(T) - 2\,B(T) - 1.64\,A(T)\right)$$

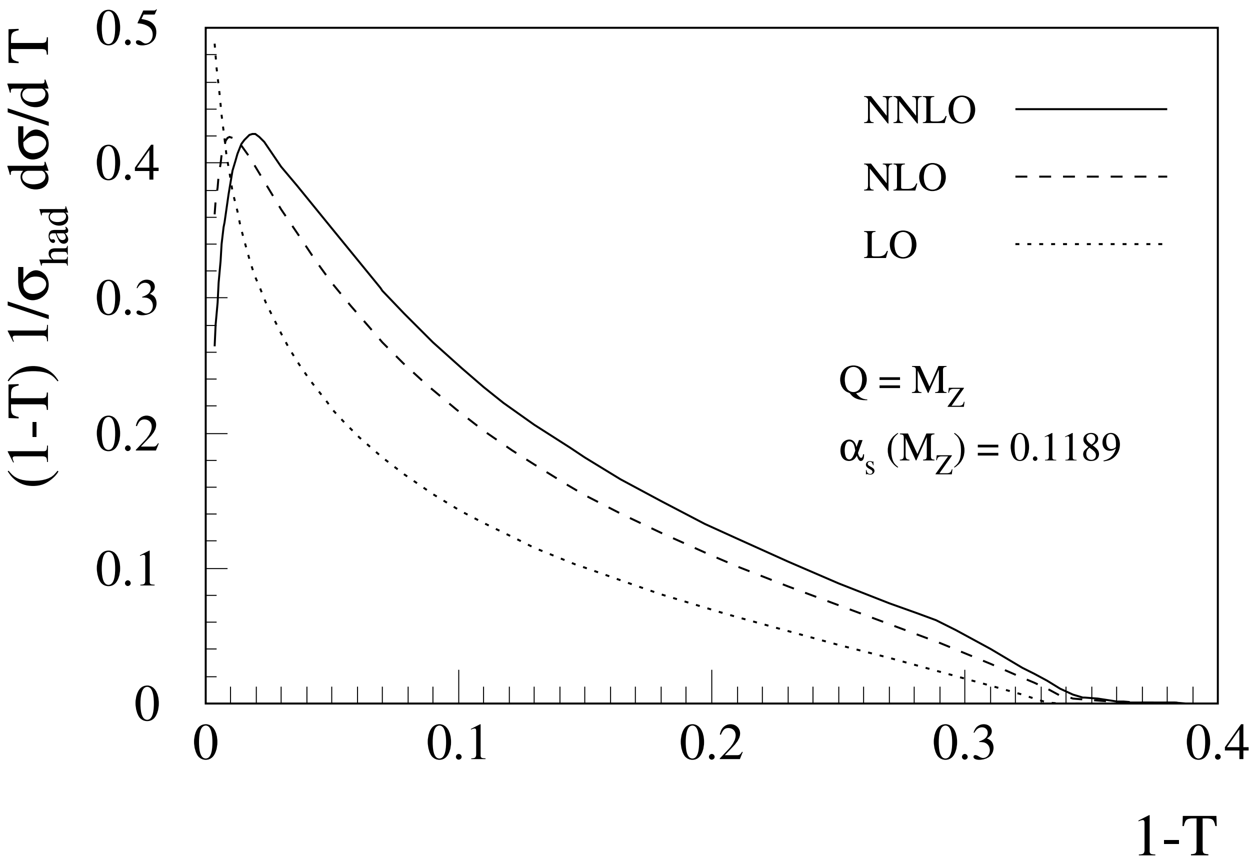
with LO contribution $A(T)$, NLO contribution $B(T)$

Ellis, Ross, Terrano





Thrust distribution

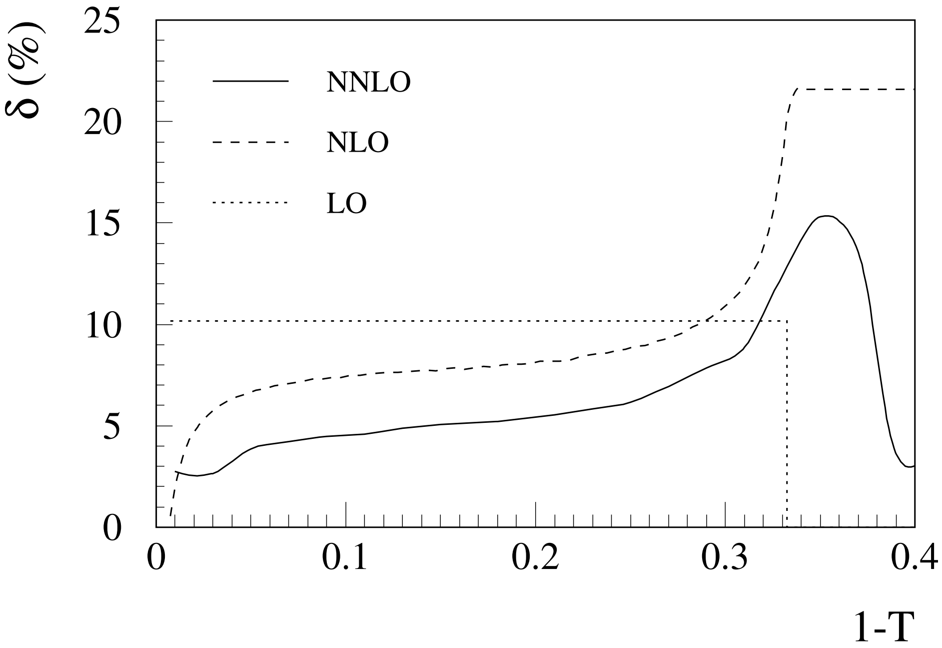


- ✓ enhancement by around (15-20)% over the range $0.03 < (1 - T) < 0.33$
- ✓ $T \to 0$ region influenced by large $\ln(1 - T)$ effects
- ✓ $(1 - T) > 0.33$ kinematically forbidden at LO

Gehrmann, Gehrmann-De Ridder, EWNG, Heinrich, arXiv:0707.1285

Theoretical uncertainty

✓ Estimate theoretical uncertainty by varying the renormalisation scale



✓ vary $\mu = [M_Z/2, 2M_Z]$

✓ determine minimal and maximal values

$$\delta = \frac{\max_\mu(\sigma(\mu)) - \min_\mu(\sigma(\mu))}{2\sigma(\mu = M_Z)}$$

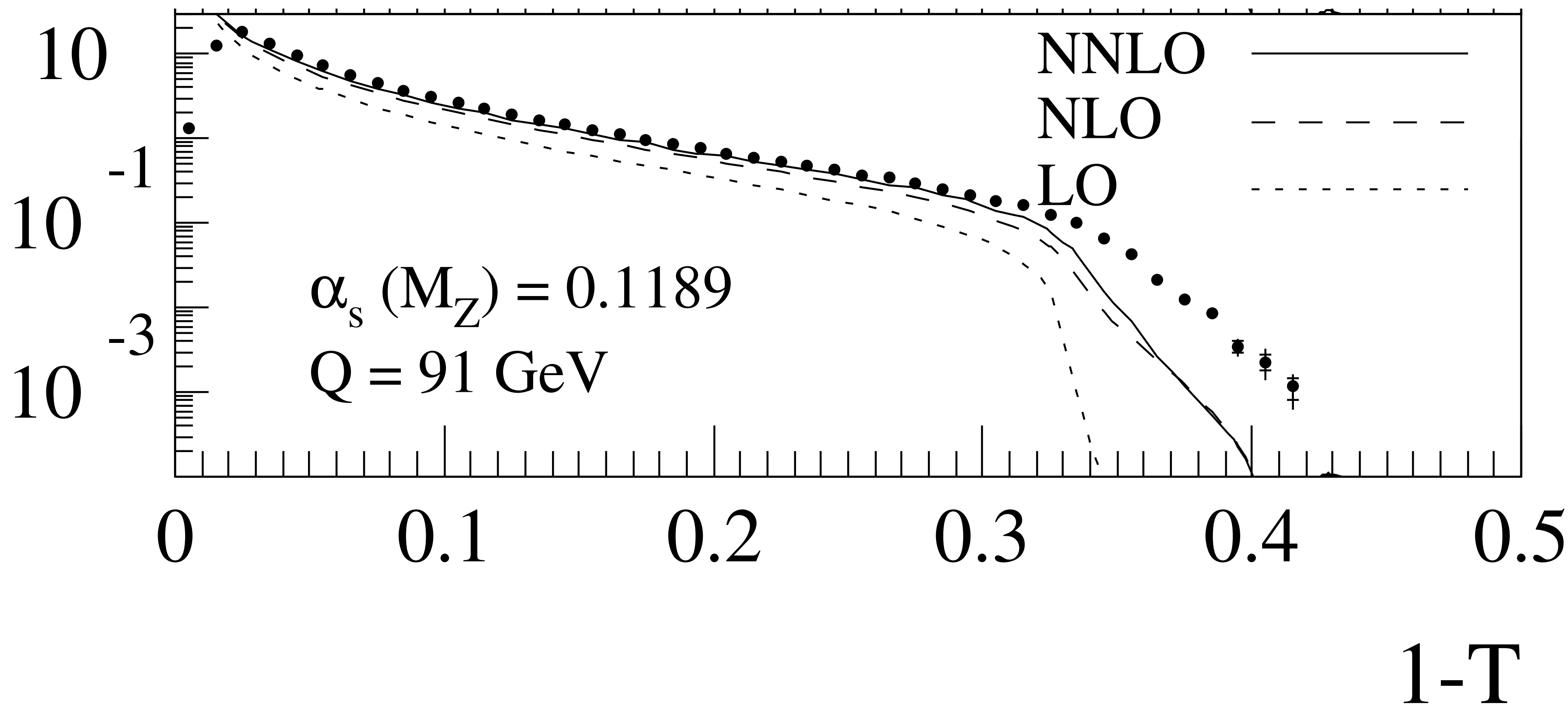
✓ inclusion of NNLO corrections stabilizes the prediction

✓ δ is reduced by almost a factor two between NLO and NNLO

Gehrmann, Gehrmann-De Ridder, EWNG, Heinrich, arXiv:0707.1285

Comparison with data

ALEPH



✓ fixed order perturbative prediction now closer to data with $\mu = M_Z$

⇒ refit of α_s from data required (and is in progress)

Gehrmann, Gehrmann-De Ridder, EWNG, Heinrich, arXiv:0707.1285

Outlook

- ✓ Numerical implementation of parton level NNLO code for three-jet event shape variables now completed and checked
  - ✓ Based on EERAD2 code for $e^+e^- \to 4$ jets

    Campbell, Cullen, EWNG
  - ✓ Two-loop matrix elements computed in terms of 2-d harmonic polylogarithms
  - ✓ Infrared cancellation scheme based on 3 and 4 parton antenna functions, derived from physical matrix elements
- ✓ first results obtained for thrust distribution
- ✓ application to other event shapes in progress
- ✓ NNLO determination of $\alpha_s(M_Z)$ from LEP event shape data ongoing