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Recent developments on unintegrated parton distributions

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I. Results on small-x final states from $k_\perp$-factorized Monte Carlo event generators

II. Progress toward precise characterizations of u-pdf's: endpoint divergences $x \to 1$

collaboration with H. Jung

INTRODUCTION

▷ Parton distributions unintegrated in transverse momentum are naturally defined for x $\to 0$ via high-energy factorization

$\hookrightarrow$ • basic QCD tool for small-x resummations

• implemented in Monte Carlo generators for HERA physics ($+$ LHC)

▷ But their relevance goes beyond small-x physics:

- Sudakov effects; infrared-sensitive processes
- polarized scattering; exclusive observables
- can be utilized for general-purpose Monte-Carlo's?

$\Rightarrow$ Q: How to define $k_\perp$ distributions gauge-invariantly over the whole phase space?

OUTLINE

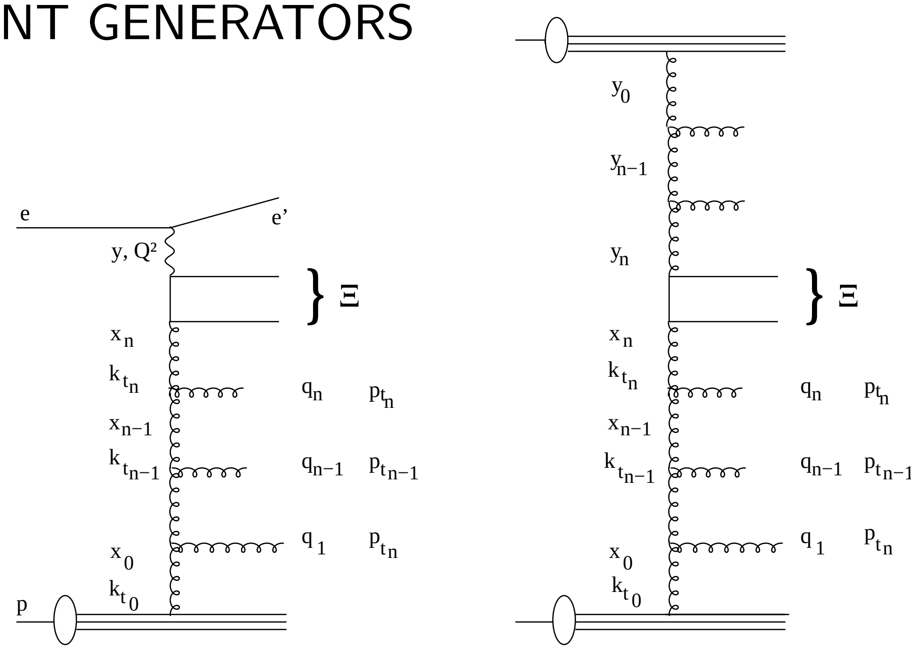
Part I: Unintegrated parton distributions and Monte Carlo generators

- hadronic final states in DIS at $x \ll 1$
- multi-jet distributions; angular correlations

Part II: Open issues on precise characterizations of updf's

- incomplete KLN cancellations near $x = 1$
- subtractive regularization method

UPDF's AND MONTE CARLO EVENT GENERATORS



(a) (b)

◇ $k_\perp$-dependent matrix elements

◇ backward evolution for initial-state cascade

- parton emission in initial state only allowed in angular-ordered region of phase space

⇒ correct treatment of $x \ll 1$ region (logarithmic accuracy)

- need corrections for collinear and $x \sim 1$ region (included partially in present MC)

Existing implementations:

CASCADE	www.quark.lu.se/~hannes/cascade
SMALLX	Marchesini & Webber, 90's
LDCMC	www.thep.lu.se/~leif/ariadne

Golec-Biernat et al., hep-ph/0703317

Höche et al., arXiv:0705.4577

See Proceedings Workshop "HERA and the LHC" for full references

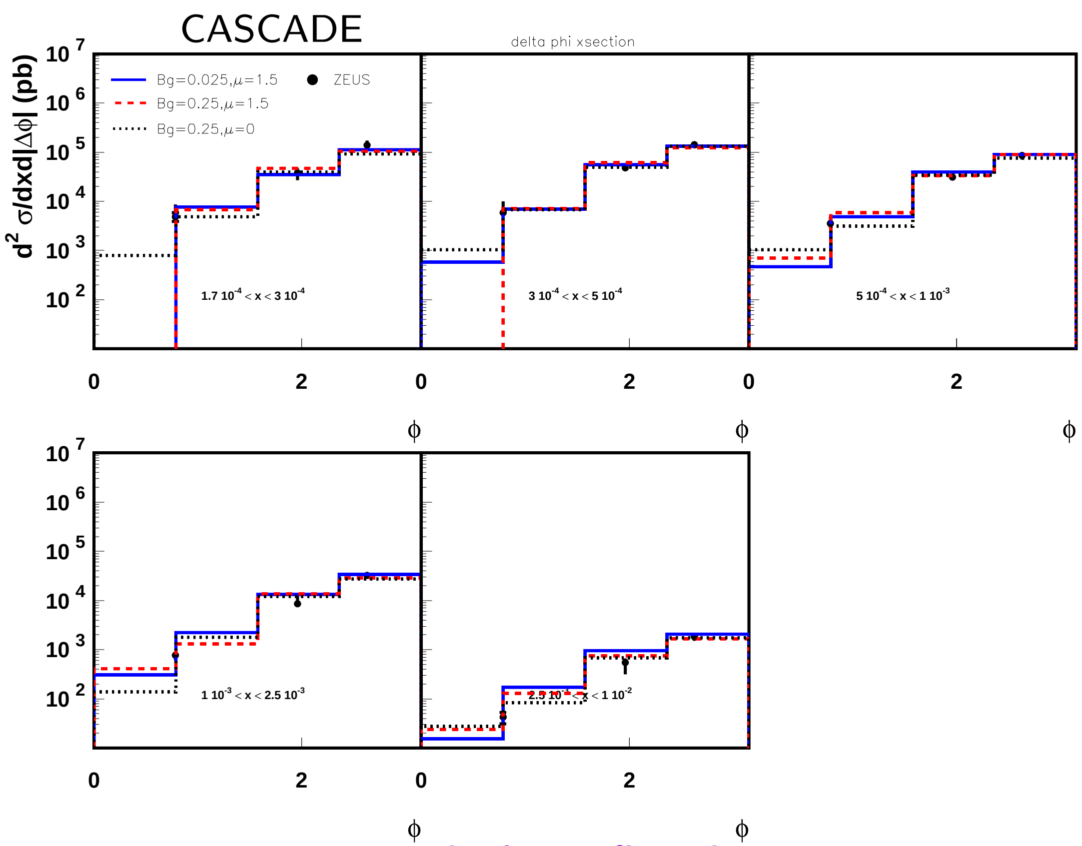
Example:

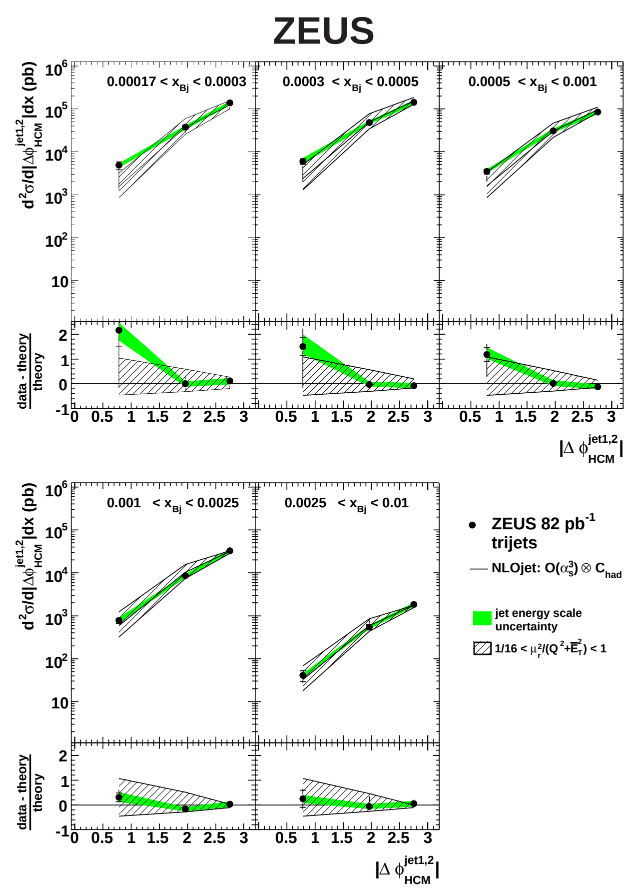
- multijet production in DIS at $x \ll 1$

$\hookrightarrow$

Azimuthal correlation in three-jet cross sections

ZEUS, arXiv:0705.1931

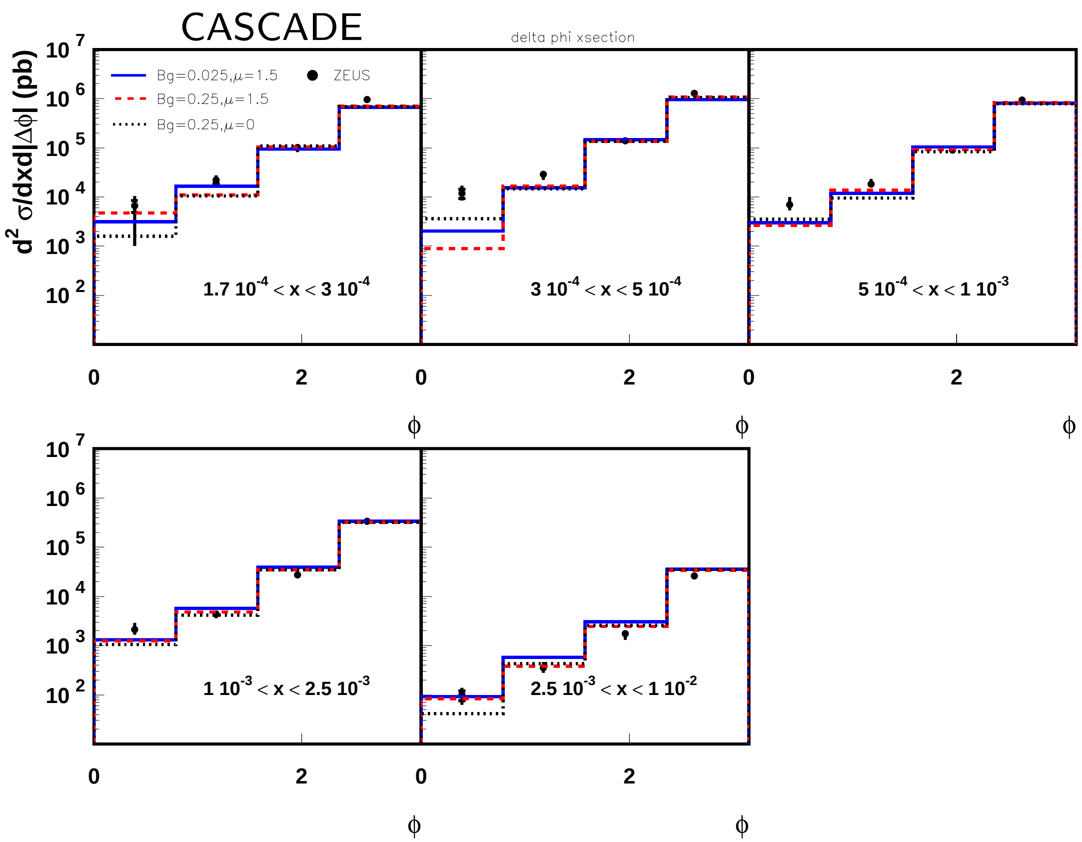


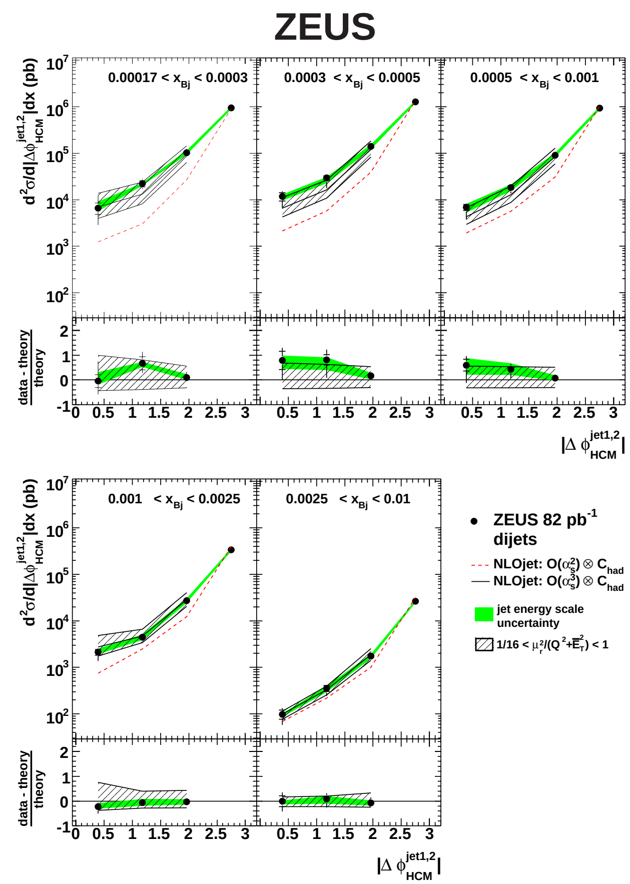


- unintegrated gluon fitted from DIS data

- Note small-x shower (away from back-to-back ϕ)

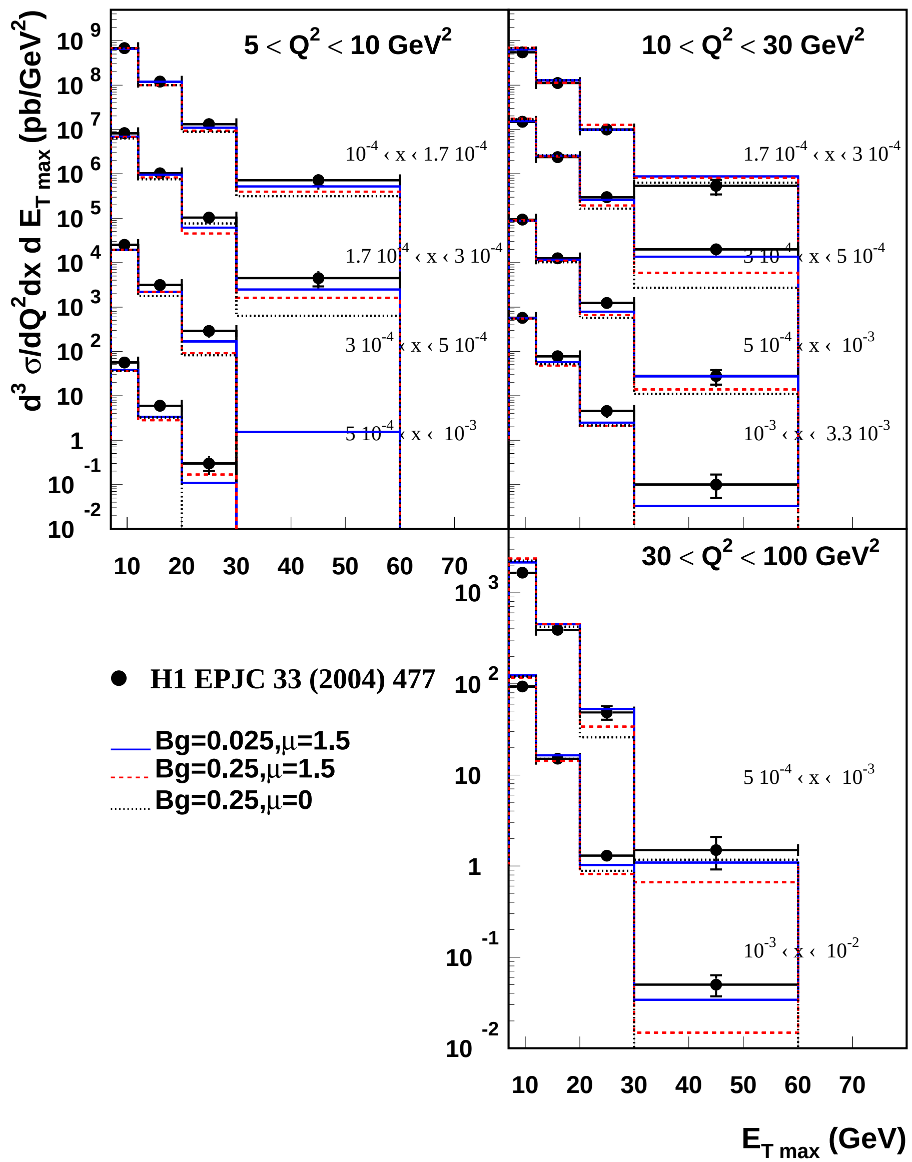
Azimuthal correlation in di-jet cross sections





- Large correction from order-α_s^2 to order-α_s^3 for small x and small ϕ

Inclusive jet E_T distribution



Remarks

▷ Physical picture from $k_\perp$-factorized MC is being probed quantitatively with

- inclusive cross sections
- detailed multi-jet correlations

▷ Main limitations still come from

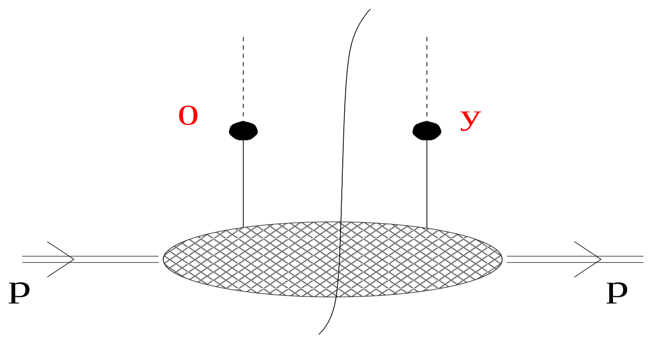
- limited knowledge of updf's
- treatment of evolution

  (how to combine Regge/Sudakov form factors?
  subleading logs? how do multiple interactions
  affect the picture? ...)

▷ Further: status of factorization proofs?

- established only in simplest cases
- e.g.: factorization-breaking from soft gluon exchanges
  revisited in Collins & Qiu, arXiv:0705.2141
  (potential N^{3}LO effect in conventional calculations)

HOW TO CHARACTERIZE UPDF'S WITH PRECISION?



Collins & Zu, 2005

Boer & Mulders, 2003, 1998

Belitsky et al., 2004; Brodsky et al., 2001

. . .

$p = (p^{+}, m^{2}/2p^{+}, 0_{\perp})$

$$\widetilde{f}(y) = \langle\ P \mid \overline{\psi}(y)\ V_y^{\dagger}(n)\ \gamma^{+}\ V_0(n)\ \psi(0) \mid P\ \rangle \quad , \qquad y = (0, y^{-}, y_{\perp})$$

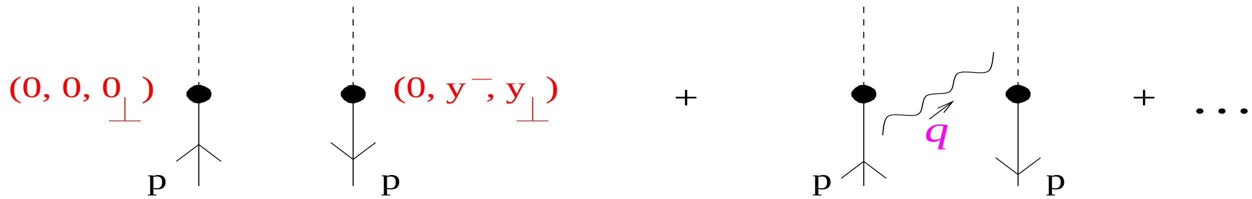
$$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^{\infty} d\tau\ n \cdot A(y + \tau\ n) \right)$$

- Fine at tree level
- Difficulties arise beyond this level

$\hookrightarrow$

Suppose a gluon is absorbed or emitted by eikonal line:

$n = (0, 1, 0_\perp)$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx'\, dk'_\perp\, P_R(x', k'_\perp)$$

$$\text{where} \qquad P_R = \frac{\alpha_s\, C_F}{\pi^2} \left[\frac{1}{1-x}\, \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \to 1\} \right] \qquad \rho = \text{IR regulator}$$

↑ *endpoint singularity* $(q^+ \longrightarrow 0, \ \forall \ k_\perp)$

Physical observables:

$$\mathcal{O} = \int dx\, dk_\perp\, f_{(1)}(x, k_\perp)\, \varphi(x, k_\perp)$$

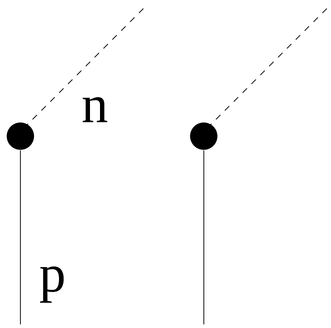
$$= \int dx\, dk_\perp\, [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\, P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences from incomplete KLN cancellation aditionally, put cut-off on the endpoint region:

▷ e.g.: Monte-Carlo generators using u-pdf's

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Chen, Idilbi & Ji, hep-ph/0607003

Ji, Ma & Yuan, hep-ph/0503015

earlier work by Collins; Korchemsky

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim k_\perp / \sqrt{4\eta}$

Drawbacks:

- good for leading accuracy, but makes it difficult to go beyond
- $\int dk_\perp \; f(x, k_\perp, \mu, \eta) = F(x, \mu, \eta) \neq$ ordinary pdf

UPDF'S WITH SUBTRACTIVE REGULARIZATION

H, hep-ph/0702196

Collins, hep-ph/0304122

- Endpoint divergences $x \to 1$ from incomplete KLN cancellation

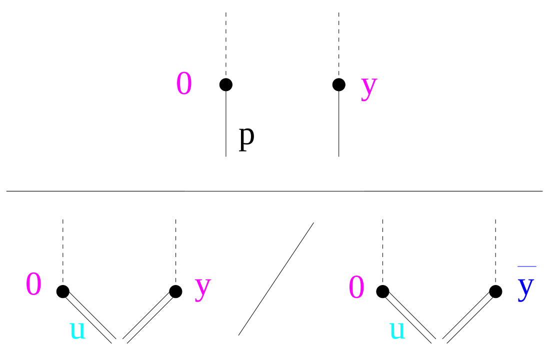
btractive method: more systematic than cut-off. Widely used in NLO calculations.

rmulation suitable for operator matrix elements: Collins & H, 2001.

- gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) =$$

$$\frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \ / \ \langle 0|V_{\bar{y}}(u)V_{\bar{y}}{}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

◊ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$

One loop: $[\zeta = (p^{+2}/2)u^-/u^+]$

$$f^{(\mathrm{subtr})}_{(1)}(x,k_\perp) = P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp)$$
$$- \quad W_R(x,k_\perp,\zeta) + \delta(k_\perp)\int dk'_\perp\,W_R(x,k'_\perp,\zeta)$$

with $P_R = \alpha_s\left\{1/[(1-x)\ (k_\perp^2 + m^2(1-x)^2)] + \ldots\right\}$ = real emission prob.

$W_R = \alpha_s\left\{1/[(1-x)\ (k_\perp^2 + 4\zeta(1-x)^2)] + \ldots\right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$

$$\Rightarrow \mathcal{O} = \int dx\ dk_\perp\ f^{(\mathrm{subtr})}_{(1)}(x,k_\perp)\ \varphi(x,k_\perp)$$
$$= \int dx\ dk_\perp\ \{P_R\ [\varphi(x,0_\perp) - \varphi(1,0_\perp)] + (P_R - W_R)\ [\varphi(x,k_\perp) - \varphi(x,0_\perp)]\}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

CONCLUSIONS

◇ $k_\perp$-MC with updf's and initial-state shower

being applied to description of multi-jet final states at small x

- Example: angular jet correlations in DIS

- Open issues on factorization, lack of complete KLN cancellation

$\Rightarrow$ need to address new problems compared to ordinary pdf's

▷ endpoint divergences ($x \to 1$):

- more transparent representation in coordinate space
- subtractive method as an alternative to cut-off method