Calculation of Charged Current DIS at three loops

Mikhail Rogal

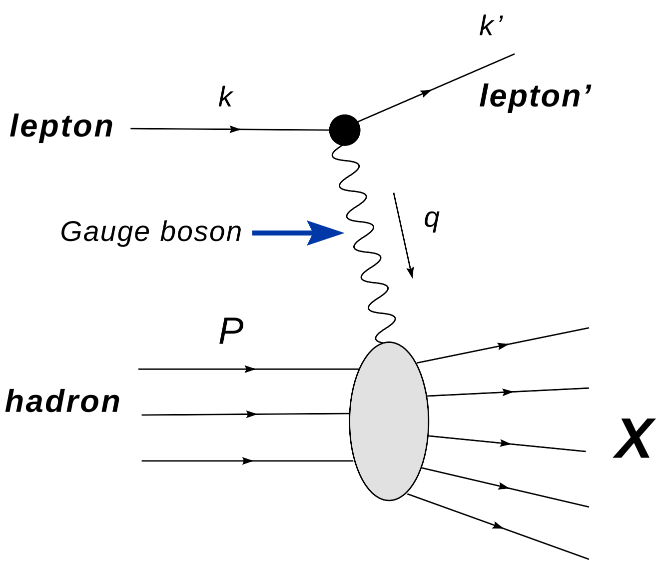
Mikhail.Rogal@desy.de

DESY, Zeuthen, Germany

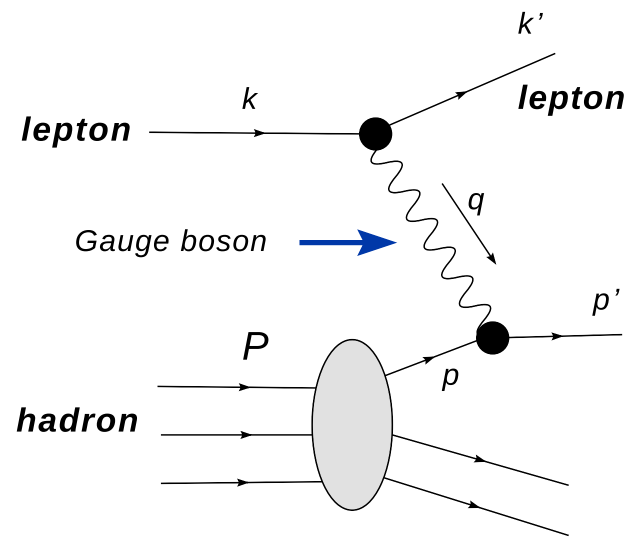
– HEP2007, Manchester, UK, 19-25 July, 2007

Introduction

- Deep-inelastic lepton-hadron scattering ($e^{\pm}p,\ e^{\pm}n,\ \nu p,\ \bar{\nu}p,\ \ldots$ - collisions)



$\Longrightarrow$

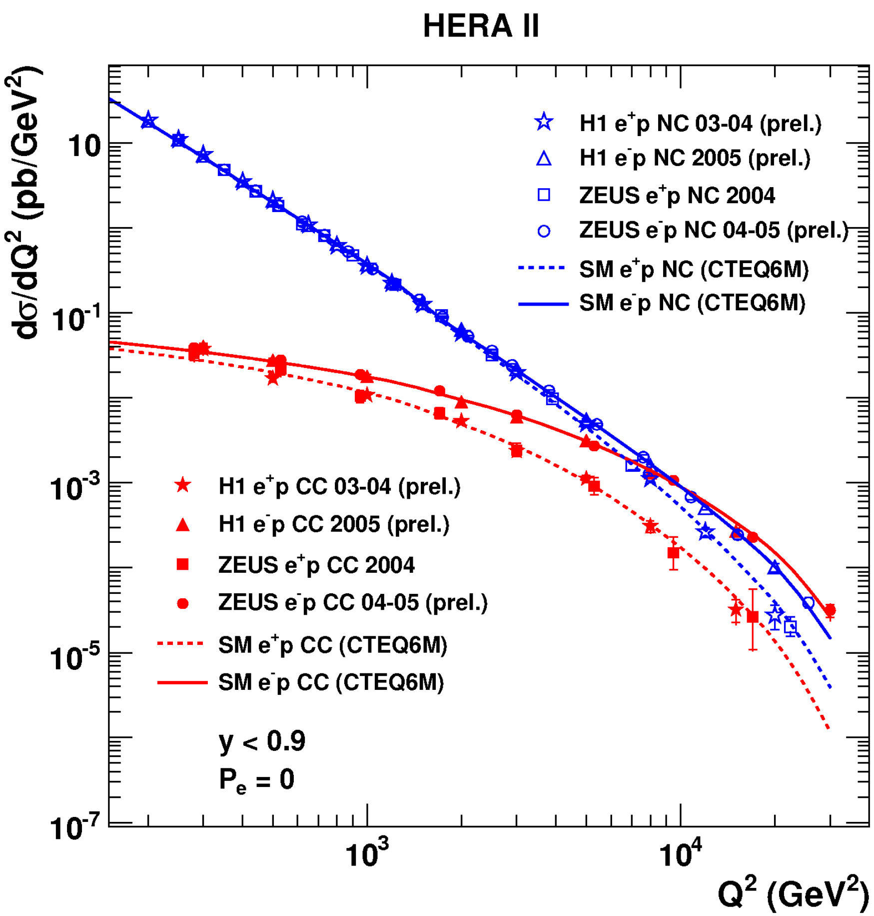


- *Gauge boson*:
  - $\gamma,\ Z^0$ - NC
  - $W^{\pm}$ - CC

- **Kinematic variables**
  - momentum transfer $Q^2 = -q^2 > 0$
  - Bjorken variable $x = Q^2/(2P \cdot q)$
  - Inelasticity $y = (P \cdot q)/(P \cdot k)$

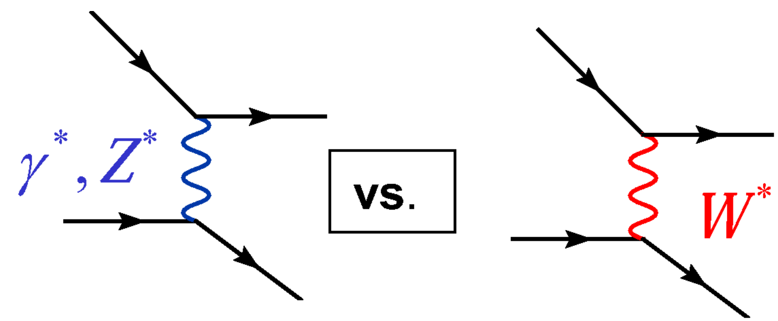
DIS experiments

- **EW unification at HERA:**
  neutral vs . charged current



Charged and neutral deep inelastic scattering cross sections become comparable when Q^2 reaches the electroweak scale

$\Longleftarrow$



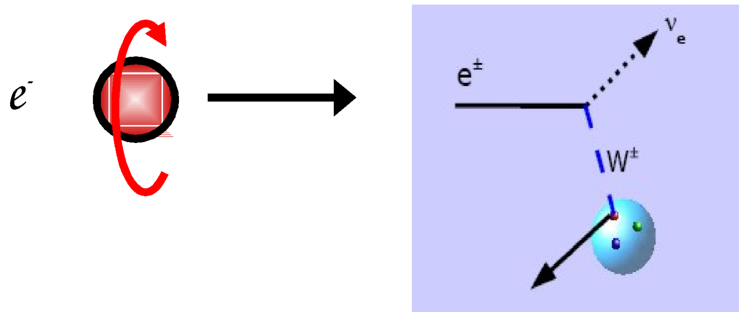
- Polarized charged current DIS at HERA

CC cross section modified by polarization:

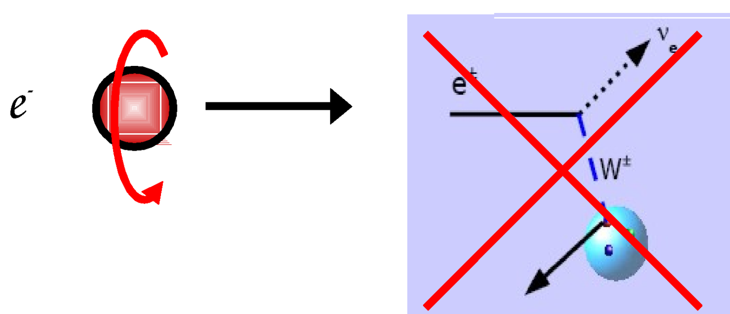
$$\sigma_{CC}^{e^{\pm}p}(P_e) = (1 \pm P_e) \cdot \sigma_{CC}^{e^{\pm}p}(P_e = 0) \qquad P_e = \frac{N_R - N_L}{N_R + N_L}$$

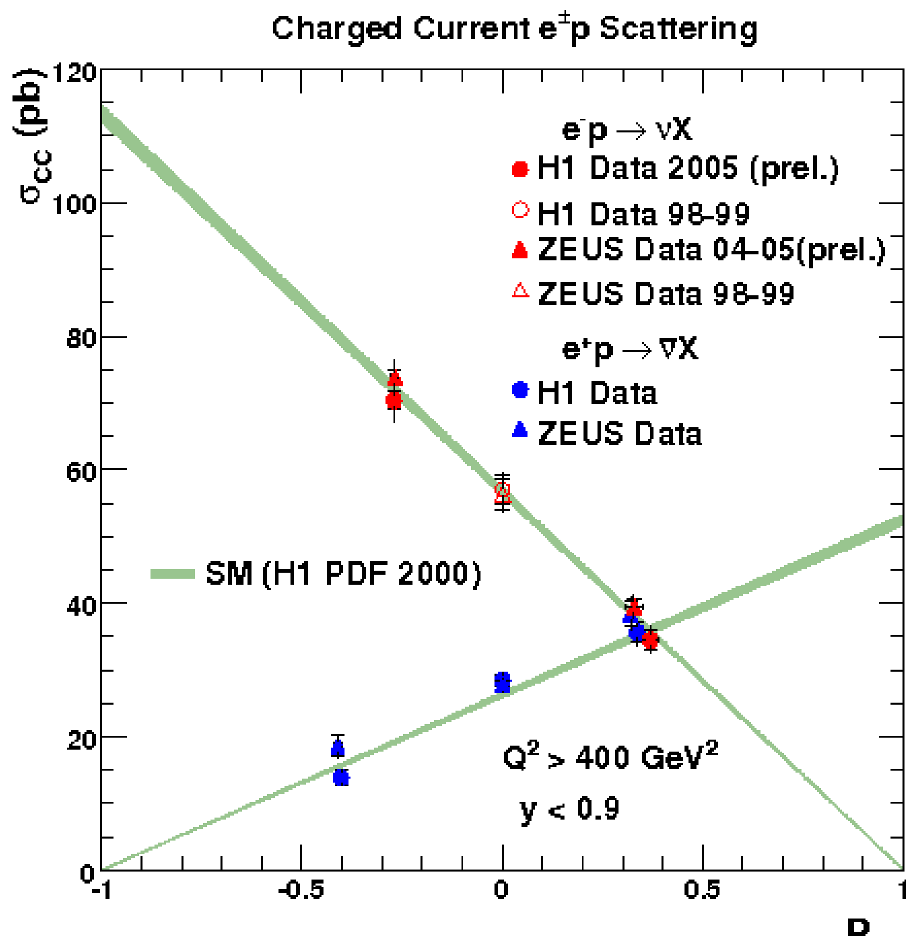
- Cross section is linearly proportional to polarization P_e
- Standard model prediction: vanishing cross section for $P_e = +1(-1)$ in $e^{-(+)}$ scattering

lefthanded electrons interact (CC)



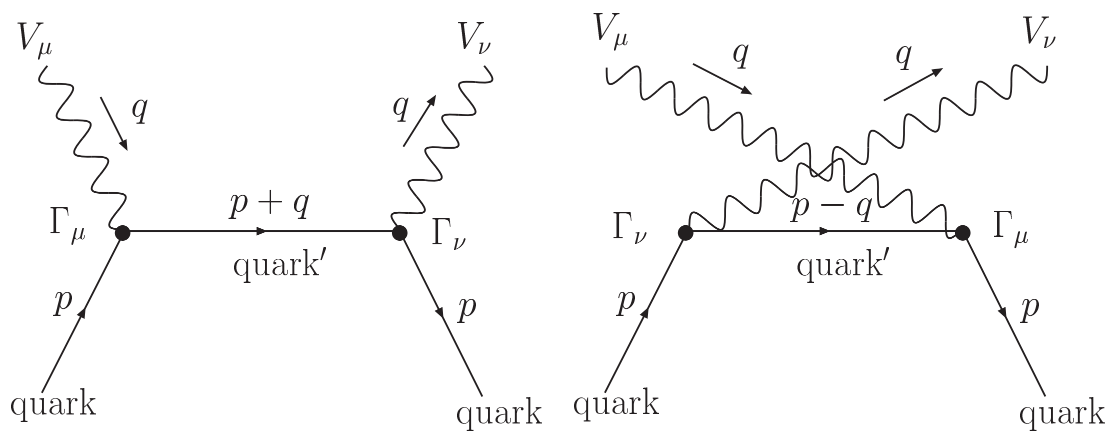
righthanded electrons do not!





Calculation

- Leading order diagrams at parton level
  - Vector and axial-vector interaction $a\gamma^\mu + b\gamma^\mu\gamma^5$



- Mellin moments with definite symmetry properties
  - process dependent distinction even/odd N (from OPE)

$$F_i(N,Q^2) = \int_0^1 dx\, x^{N-2} F_i(x,Q^2)\,, \quad i = 2, L$$

$$F_3(N,Q^2) = \int_0^1 dx\, x^{N-1} F_3(x,Q^2)$$

Known

- NC (exchange via γ gauge boson) $\longrightarrow F_2^{eP}$
  CC (exchange via $W^{\pm}$ gauge boson) $\longrightarrow F_2^{\nu p+\bar{\nu}p}$, $F_3^{\nu p+\bar{\nu}p}$

  even N for F_2, odd N for F_3

  - NLO Bardeen, Buras, Duke, Muta ‘78
  - N^{2}LO Zijlstra, van Neerven ‘92
  - N^{3}LO Moch, Vermaseren, Vogt ‘05/‘06

Known

- NC (exchange via γ gauge boson) $\longrightarrow F_2^{eP}$
  CC (exchange via $W^\pm$ gauge boson) $\longrightarrow F_2^{\nu p+\bar{\nu} p}$, $F_3^{\nu p+\bar{\nu} p}$

  even N for F_2, odd N for F_3

  - NLO Bardeen, Buras, Duke, Muta '78
  - N^{2}LO Zijlstra, van Neerven '92
  - N^{3}LO Moch, Vermaseren, Vogt '05/'06

New

- NC $\gamma - Z$ interference at N^{3}LO still missing
- CC (exchange via $W^\pm$ gauge boson) $\longrightarrow F_2^{\nu p-\bar{\nu} p}$, $F_3^{\nu p-\bar{\nu} p}$

  odd N for F_2, even N for F_3

  - order N^{3}LO *already known*
    Charged current deep-inelastic scattering at three loops.
    S. Moch and M. R. arXiv:0704.1740v1 [hep-ph]; Nuclear Physics, in press
    best use: difference "even-odd"
    Differences between CC coefficient functions, applications
    S. Moch, M. R. and A. Vogt. DESY 07-048

In practice

The calculation

- Big number of diagrams $\Rightarrow$ need of automatization
  e.g. DIS structure functions $F_{2,L}^{\nu p \pm \bar{\nu} p}$ - 1076 diagrams, $F_3^{\nu p \pm \bar{\nu} p}$ - 1314 diagrams up to 3 loops

- latest version of FORM and TFORM (multi-threaded version)
  Vermaseren, FORM version 3.2 (Apr 16 2007)

- QGRAF $\mapsto$ generation of diagrams for DIS structure functions
  Nogueira '93

- Calculation of diagrams $\mapsto$ MINCER in FORM Larin, Tkachev, Vermaseren '91

$$
\begin{aligned}
C^{\mathrm{ns}}_{3,10} \quad = \quad & 1 + a_{\mathrm{s}} C_F \frac{1953379}{138600} + a_{\mathrm{s}}^2 C_F n_f \left(- \frac{537659500957277}{15975002736000} \right) + a_{\mathrm{s}}^2 {C_F}^2 \left(\frac{597399446375524589}{14760902528064000} \right. \\
& \left. + \frac{7202}{105} \zeta_3 \right) + a_{\mathrm{s}}^2 C_A C_F \left(\frac{583602058122267}{2904545952000} - \frac{99886}{1155} \zeta_3 \right) \\
& + a_{\mathrm{s}}^3 C_F {n_f}^2 \left(\frac{5133975667319461 7191}{9963609206443200 00} + \frac{48220}{18711} \zeta_3 \right) \\
& + a_{\mathrm{s}}^3 {C_F}^2 n_f \left(- \frac{1254838179460551213513 53}{209235793335307200000} - \frac{59829376}{3274425} \zeta_3 + \frac{24110}{693} \zeta_4 \right) \\
& + a_{\mathrm{s}}^3 {C_F}^3 \left(- \frac{744474223606695878525401307}{708890867820020793600000 0} + \frac{28630985464358}{24960941775} \zeta_3 \right. \\
& \qquad \left. + \frac{151796299}{8004150} \zeta_4 - \frac{53708}{99} \zeta_5 \right) \\
& + a_{\mathrm{s}}^3 C_A C_F n_f \left(- \frac{185221350045507487753}{2264456637828000 00} + \frac{8071097}{39690} \zeta_3 - \frac{24110}{693} \zeta_4 \right) \\
& + a_{\mathrm{s}}^3 C_A {C_F}^2 \left(\frac{1977007872933860773 2075449}{836943173341228800 0000} - \frac{619383700181}{5546875950} \zeta_3 \right. \\
& \qquad \left. - \frac{151796299}{5336100} \zeta_4 - \frac{37322}{99} \zeta_5 \right) \\
& + a_{\mathrm{s}}^3 {C_A}^2 C_F \left(\frac{93798719639056648125143}{362313062052480000 00} - \frac{43202630363}{20582100} \zeta_3 \right. \\
& \qquad \left. + \frac{151796299}{16008300} \zeta_4 + \frac{195422}{231} \zeta_5 \right) .
\end{aligned}
$$

Checks

- Known Mellin moments for $F_{2,L}^{\nu P+\bar{\nu}P}$ (even) and $F_3^{\nu P+\bar{\nu}P}$ (odd) recalculated
- All calculations with gauge parameter ξ for gluon propagator (Up to 10'th MM)

$$\mathrm{i}\frac{-g^{\mu\nu}+(1-\xi)q^{\mu}q^{\nu}}{q^2}$$

- Adler sum rule for DIS structure functions $\longrightarrow C_{2,1}^{\mathrm{ns}}=1$

$$\int_0^1 \frac{dx}{x}\left(F_2^{\nu P}(x,Q^2)-F_2^{\nu N}(x,Q^2)\right)=2$$

  - measures isospin of the nucleon in the quark-parton model
  - neither perturbative nor non-perturbative corrections in QCD

Applications

- Gottfried sum rule (charged lepton(l)-proton(P) or neutron(N) DIS)

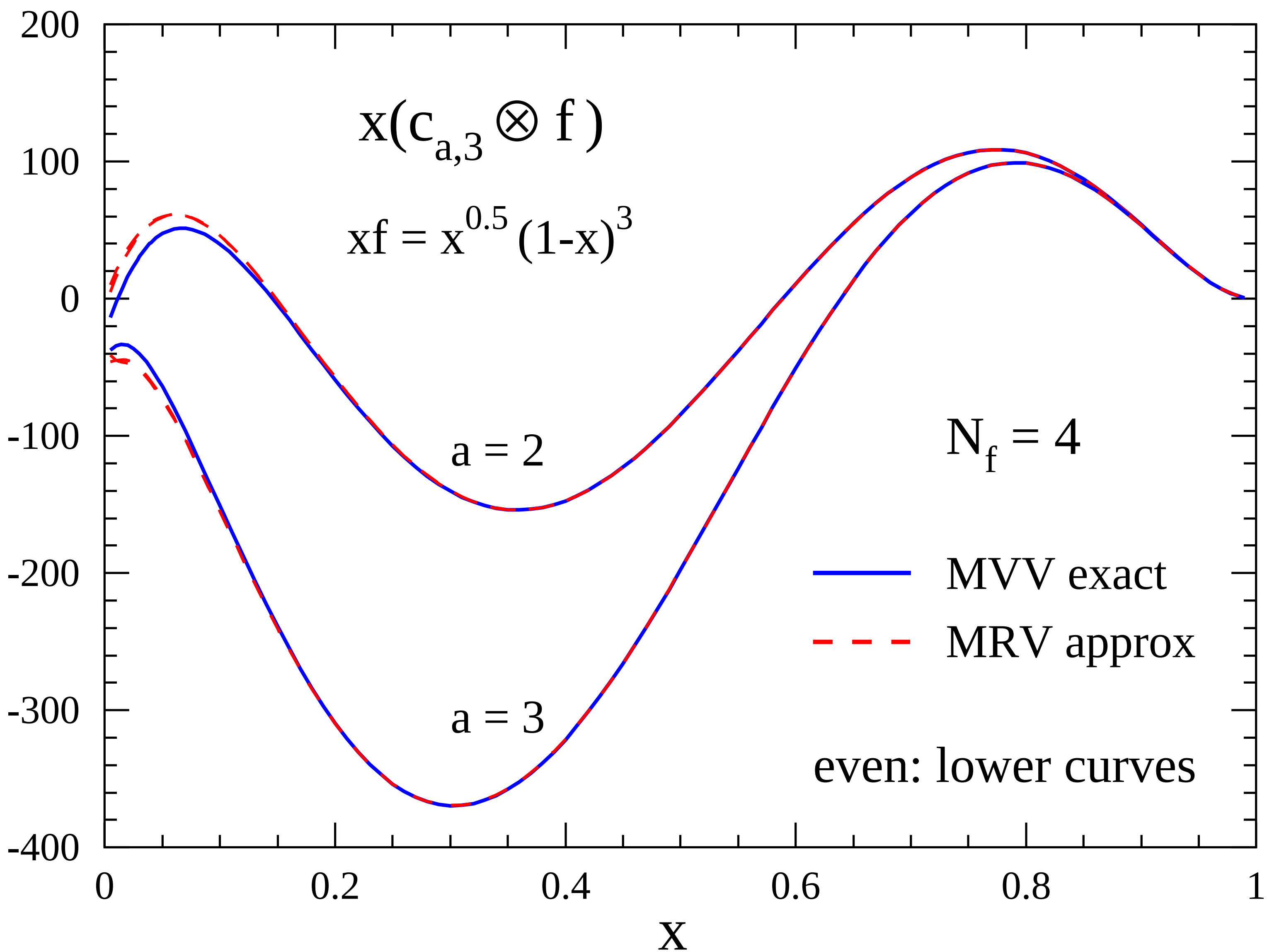
$$\int_0^1 \frac{dx}{x}\left(F_2^{lP}(x,Q^2) - F_2^{lN}(x,Q^2)\right)$$

  - Conjecture: difference of non-singlet coefficient functions for even and odd Mellin moments subleading in color $[C_F - C_A/2] \simeq 1/N_c$
    Broadhurst, Kataev, Maxwell ‘04

- $\delta C_{i,n}^{\rm ns} = C_{i,n}^{\nu P + \bar\nu P} - C_{i,n}^{\nu P - \bar\nu P}$ with color coefficient $[C_F - C_A/2]$
- e.g.

$$
\begin{aligned}
\delta C_{2,3}^{\rm ns} \quad = \quad & + a_s^2 C_F [C_F - C_A/2] \Big(-\frac{4285}{96} - 122\zeta_3 + \frac{671}{9}\zeta_2 + \frac{128}{5}{\zeta_2}^2 \Big) \\
& + a_s^3 C_F [C_F - C_A/2]^2 \Big(\frac{1805677051}{466560} - \frac{2648}{9}\zeta_5 + \frac{10093427}{810}\zeta_3 - \frac{1472}{3}{\zeta_3}^2 \\
& \qquad - \frac{7787113}{1944}\zeta_2 + \frac{55336}{9}\zeta_2\zeta_3 - \frac{378838}{45}{\zeta_2}^2 - \frac{8992}{63}{\zeta_2}^3 \Big) \\
& + a_s^3 {C_F}^2 [C_F - C_A/2] \Big(-\frac{5165481803}{1399680} + \frac{40648}{9}\zeta_5 - \frac{9321697}{810}\zeta_3 + \frac{1456}{3}{\zeta_3}^2 \\
& \qquad + \frac{8046059}{1944}\zeta_2 - 4984\zeta_2\zeta_3 + \frac{798328}{135}{\zeta_2}^2 - \frac{56432}{315}{\zeta_2}^3 \Big) \\
& + a_s^3 n_f C_F [C_F - C_A/2] \Big(\frac{20396669}{116640} - \frac{1792}{9}\zeta_5 + \frac{405586}{405}\zeta_3 - \frac{139573}{486}\zeta_2 \\
& \qquad + \frac{1408}{9}\zeta_2\zeta_3 - \frac{50392}{135}{\zeta_2}^2 \Big) .
\end{aligned}
$$

α_s^3 part of structure functions



NuTeV experiment - Paschos-Wolfenstein relation

- Exact relation for massless quarks and isospin zero target

  Paschos,Wolfenstein'73, Llewelin Smith'83

$$R^- = \frac{\sigma(\nu_\mu N \to \nu_\mu X) - \sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \to \mu^- X) - \sigma(\bar{\nu}_\mu N \to \mu^+ X)} = \frac{1}{2} - \sin^2\theta_W$$

- measurement of $\sin^2\theta_W$ NuTeV '01 with large deviations from Standard model expectations

QCD corrections to the Paschos-Wolfenstein relation

- second moments of valence PDFs $q^- = \int dx\, x(q - \bar{q})$
- expansion in isoscalar combination $u^- + d^-$

  Davidson, Forte, Gambino, Rius, Strumia '01; Dobrescu, Ellis '03; Moch, McFarland '03

$$R^- = \frac{1}{2} - \sin^2\theta_W + \left[1 - \frac{7}{3}\sin^2\theta_W + \frac{8\alpha_s}{9\pi}\left\{1 + \alpha_s 1.689 + \alpha_s^2(3.661 \pm 0.002)\right\}\left(\frac{1}{2} - \sin^2\theta_W\right)\right] \times \left(\frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-}\right)$$

- main uncertainties in s^-

  Martin, Roberts, Stirling, Thorne '04; Lai, Nadolsky, Pumplin, Stump, Tung, Yuan '07

- QCD corrections under control, relevant Moch, M. R., Vogt '07

Summary

- New results for fixed Mellin moments at order α_s^3

  $F_{2,L}^{\nu p - \bar{\nu} p}$ (odd) and $F_3^{\nu p - \bar{\nu} p}$ (even)

- and differences "even-odd"

  ▲ interesting: OPE based moments

  [$F_{2,L}^{\nu p - \bar{\nu} p} - 1, 3, 5, \cdots$; $F_3^{\nu p - \bar{\nu} p} - 2, 4, 6, \cdots$]:

  weight w of zeta functions up to $2l - 1$ (l - number of loops)

  Intermediate integer N [$F_{2,L}^{\nu p + \bar{\nu} p} - 1, 3, 5, \cdots$; $F_3^{\nu p + \bar{\nu} p} - 2, 4, 6, \cdots$]:

  weight up to $2l$

- $1/N_c$ suppression of "even-odd"

  conjecture of Broadhurst, Kataev, Maxwell '04 verified at three loops

- Stability of QCD α_s expansion for Paschos-Wolfenstein relation

- Future plans $\mapsto$ NC $\gamma - Z$ interference, polarized process $\Delta G, \Delta q$.