Adding Flavour to Twistor Strings

Work with C. Papageorgakis and K. Zoubos

To appear: arXiv:0707.XXXX

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21 July 2007

HEP Europhysics Conference, Manchester

Twistor String Theory

- Proposed correspondence between *weakly* coupled $\mathcal{N}=4$ SYM and the open-string topological B-model on (super)-twistor space ($\mathbb{CP}^{3|4}$) [Witten '03].

- Explains simplicity of Park-Taylor formula for n-gluon MHV amplitudes:

$$A_n = \frac{\langle i\, j \rangle}{\langle 1\, 2 \rangle \ldots \langle n\, 1 \rangle}$$

where $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ and $\langle k\, l \rangle = \epsilon^{\alpha\beta} \lambda^k_\alpha \lambda^l_\beta$

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Successes and Failures

- However, conformal supergravity spoils the picture at one-loop [Berkovits, Witten '04].

- But, the duality has inspired great progress in (non)-supersymmetric field theory, *e.g.*
  - CSW rules for gauge theory using MHV amplitudes as vertices [Cachazo, Svrček, Witten '04]
  - Extension of CSW to one-loop in $\mathcal{N}=4$ super-Yang-Mills [Brandhuber, Spence, Travaglini '04]
  - Also to $\mathcal{N}=2,1$ and non-SUSY gauge theory [J.B., Brandhuber, Spence, Travaglini; Quigley, Rozali '04; Badger, Glover, Risager '07].

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Possibilities

- One way to proceed: map out theories which *do* have a tree-level twistor dual. The most obvious candidates are those which preserve conformal invariance order-by-order in perturbation theory - UV finite theories.
- Marginal deformations of $\mathcal{N} = 4$ [Kulaxizi, Zoubos '04].
- Orbifolds giving $\mathcal{N} = 1, 2$ quiver gauge theories [Park, Rey; Giombi, Kulaxizi, Ricci, Robles-Llana, Trancanelli, Zoubos '04].
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1. Review of duality for $\mathcal{N} = 4$ SYM
2. Orientifolding: $\mathcal{N} = 2$ SYM with 4 flavours
3. Orbifolding: $\mathcal{N} = 2$ SYM with $2N_c$ flavours
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Penrose Transform

- Decomposition of a light-like momentum $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ plus non-linearity of conformal group suggests Penrose xfm:

$$\tilde{\lambda}_{\dot{\alpha}} \to i\frac{\partial}{\partial \mu^{\dot{\alpha}}} \quad ; \quad \mu_{\dot{\alpha}} \to -i\frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}}$$

  whereupon $Z^I = (\lambda^\alpha, \mu^{\dot{\alpha}})$ span a copy of $\mathbb{CP}^3$.

- Adding helicity gives fermionic directions ψ^I and then $(Z^I, \psi^I) \sim c(Z^I, \psi^I)$ describe the super-Calabi-Yau $\mathbb{CP}^{3|4}$, which can be considered as a good target space for the B-model.
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Holomorphic Chern Simons

- The topological B-model with "D5"-branes wrapping (Z^I, ψ^I) and with $\bar{\psi} = 0$ descends to holomorphic Chern-Simons theory:

$$S = \frac{1}{2} \int_{\mathrm{B6}} \mathbf{\Omega} \wedge \mathrm{Tr}(\mathcal{A} \cdot \bar{\partial} \mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A})$$

with $\mathbf{\Omega} \sim Z d^3 Z d^4 \psi$ the holomorphic volume form.

- $\mathcal{A}_{\bar{I}}(Z, \bar{Z}, \psi) d\bar{Z}^{\bar{I}}$ is the superfield

$$\mathcal{A} = A + \psi^I \lambda_I + \frac{1}{2!} \psi^I \psi^J \phi_{IJ} + \frac{1}{3!} \epsilon_{IJKL} \psi^I \psi^J \psi^K \tilde{\lambda}^L + \psi^1 \psi^2 \psi^3 \psi^4 G$$

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D1-Instantons

- Witten's solution was to add "D1"-instantons wrapping degree d holomorphic curves on which the gauge theory amplitudes localise.

- In the case of the MHV amplitudes these are copies of $\mathbb{CP}^1 \subset \mathbb{CP}^{3|4}$ embedded via the relations

$$\mu_{\dot{\alpha}} + x_{\alpha\dot{\alpha}}\lambda^{\alpha} = 0 \quad ; \quad \psi^{A} + \theta^{A}_{\alpha}\lambda^{\alpha} = 0$$

- $x_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} x_{\mu}$ are just co-ordinates of Minkowski space!

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Amplitudes

- Scattering amplitudes are computed by evaluating the correlator

$$A_{(n)} = \int d^4x \, d^8\theta \, \langle \int_{\mathrm{D1}} J_1 w_1 \cdots \int_{\mathrm{D1}} J_n w_n \rangle$$

where the J_i are currents on the D1s and the w_i are wavefunctions of external particles.

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$\mathcal{N} = 2$ Sp(N) Field Content and Action

- We wish to obtain a certain $\mathcal{N} = 2$ SCFT with:
  1. one vector multiplet (V, Φ) in the *adjoint* of Sp(N)
  2. one hypermultiplet $(Z, Z'^{\dagger})$ in the *antisymmetric*
  3. 4 *fundamental* hypermultiplets $(Q^I, Q'^{\dagger I})$.
- This field content is just right to make the theory quantum-mechanically conformally-invariant.
- The action can be obtained from the following superspace formulation in terms of $\mathcal{N} = 1$ superfields

$$\mathcal{L} = \frac{1}{8\pi}\mathrm{Im}\ \mathrm{Tr}\left[\tau\left(\int d^2\theta\ W^\alpha W_\alpha + 2\int d^2\theta d^2\bar{\theta}\ e^{2V}\Phi^\dagger e^{-2V}\Phi\right)\right] + \int d^2\theta d^2\bar{\theta}\ Q^{\dagger I} e^{-2V} Q_I$$
$$+ \int d^2\theta d^2\bar{\theta}\ Q'^I e^{2V} Q'^{\dagger}_I + \mathrm{Tr}\left(\int d^2\theta d^2\bar{\theta}\ e^{2V} Z^\dagger e^{-2V} Z + \int d^2\theta d^2\bar{\theta}\ e^{-2V} Z' e^{2V} Z'^\dagger\right)$$
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- There is a stringy description for this gauge theory in terms of F-theory on $K3 \sim T^4/\mathbb{Z}_2$.
  [Sen; Banks, Douglas, Seiberg; Douglas, Lowe, Schwartz '96]
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  - An O7-plane in $x^1 \dots x^7$
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  [Sen; Banks, Douglas, Seiberg; Douglas, Lowe, Schwartz '96]

- Reduces to orientifold of type IIB with
  - An O7-plane in $x^1 \ldots x^7$
  - 4 D7-branes in $x^1 \ldots x^7$
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Symmetries

In this picture one can quickly see that the symmetries of the theory are:

Component	SO(1,3)	$SU(2)_a$	$SU(2)_{A'}$	U(1)	Sp(N)	SO(8)
A, G	(2,2)	1	1	0	$N(2N+1)$	1
ϕ	(1,1)	1	1	+2	$N(2N+1)$	1
$\phi^\dagger$	(1,1)	1	1	−2	$N(2N+1)$	1
$\lambda_{\alpha,a}$	(2,1)	2	1	+1	$N(2N+1)$	1
$\bar{\lambda}_{\dot{\alpha},a}$	(1,2)	2	1	−1	$N(2N+1)$	1
$z_{aA'}$	(1,1)	2	2	0	$N(2N-1)$	1
$\zeta_{\alpha,A'}$	(2,1)	1	2	−1	$N(2N-1)$	1
$\bar{\zeta}_{\dot{\alpha},A'}$	(1,2)	1	2	+1	$N(2N-1)$	1
q_a^M	(1,1)	2	1	0	$2N$	8
$\eta_{\alpha M}$	(2,1)	1	1	−1	$2N$	8
$\bar{\eta}_{\dot{\alpha}}^M$	(1,2)	1	1	+1	$2N$	8

An Alternative Approach

- Consider the following orientifold action on Witten's twistor string:

$$\begin{aligned} \psi^a &\rightarrow \psi^a\,, \quad a = 1,2 \\ \psi^A &\rightarrow -\psi^A\,, \quad A = 3,4 \\ \mathcal{A} &\rightarrow \gamma_c \mathcal{A}^T \gamma_c^{-1} \end{aligned}$$

- The $\mathcal{A}$ xfm acts on the colour indices of the $U(2N)$ theory and we take $\gamma_c = \mathbb{1}_{2N\times 2N}$.

- The invariant part of the superfield is

$$\begin{aligned} \hat{\mathcal{A}} &= (A+\psi^a\lambda_a+\psi^1\psi^2\phi+\psi^3\psi^4\phi^\dagger+\epsilon_{cd}\psi^3\psi^4\psi^c\tilde{\lambda}^d+\psi^1\psi^2\psi^3\psi^4 G) \\ &+ (\psi^A\zeta_A+\psi^a\psi^B z_{aB}+\epsilon_{CD}\psi^1\psi^2\psi^C\tilde{\zeta}^D) \\ &= \mathcal{V}+\mathcal{Z}\,, \end{aligned}$$

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Field Content

- $\mathcal{V}$ is a vector in the adjoint of Sp(N):

$$A_\mu \quad ; \quad (\lambda_\alpha \, , \bar{\lambda}_{\dot{\alpha}}) \quad ; \quad (\phi, \phi^\dagger)$$

- $\mathcal{Z}$ is a hypermultiplet in the antisymmetric:

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- What about the fundamentals?

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Flavour Branes

- Introduce *new* "flavour" (D_f) branes.
- Orientifold action on flavour indices (K, L) with

$$\gamma_f = -\mathbb{1}$$

- The $D_c - D_f$ state invariant under this is

$$\mathcal{Q}(Z,\bar{Z},\psi^a)^i{}_K = \psi^A Q^i{}_{AK} = \psi^A\left(\eta^i_{AK} + \psi^a q^i_{aAK} + \psi^1\psi^2\tilde{\eta}^i_{AK}\right)$$

- Get an extra term in the HCS action

$$\frac{1}{2}\int_{\mathrm{D_c}} \Omega \wedge \left(\mathcal{Q}^K \cdot \bar{\partial}\mathcal{Q}_K + \mathcal{Q}^K \wedge \hat{\mathcal{A}} \wedge \mathcal{Q}_K\right)$$

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Amplitudes

- Calculate tree amplitudes to check duality.
- Use Witten's prescription essentially unmodified.
- "Pre-analytic" amplitudes vanish

$$\langle \lambda^a \; \lambda^b \; \eta_A \; \eta_B \rangle \qquad \langle \lambda^a \; \eta_A \; \lambda^b \; \eta_B \rangle$$
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- "Analytic" (MHV) amplitudes matched:

$$\langle \phi \; \phi \; \phi^\dagger \; \phi^\dagger \rangle \qquad \langle \phi \; \phi^\dagger \; \phi \; \phi^\dagger \rangle$$
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- Flavour group realised is actually $SU(2) \times \mathrm{Sp}(2)$ subgroup, *not* full SO(8).
- Sp groups both on gauge *and* flavour branes.
- The SU(2) subgroup is realised *geometrically*.
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$\mathcal{N} = 2$ theory with $N_f = 2N_c$

- Proceeds in similarity with the $N_f = 4$ theory.
- Orbifold action only - no world-sheet parity operation.
- Realises $\mathrm{SU}(N) \times \mathrm{SU}(2)$ subgroup of full $\mathrm{SU}(2N)$ flavour.
- Amplitudes match. Many are similar to before, but others different *e.g.*

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Conclusions

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