Bubbles of True Vacuum and Duality in M-Theory

Sebastian de Haro
King's College, London

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Based on:

- SdH and A. Petkou, hep-th/0606276, **JHEP** 12 (2006) 76
- SdH, I. Papadimitriou and A. Petkou, hep-th/0611315, **PRL** 98 (2007) 231601
- SdH and Peng Gao, hep-th/0701144, submitted to **Phys. Rev. D**

Motivation

1. Tunneling effects in string theory/M-theory

Instantons describe tunneling across potential barrier

Important **quantum** effects on gravity

Difficulties:

- Need **exact solutions** of non-interacting theories
- **String theory**/M-theory on these backgrounds not well understood beyond supergravity approximation

$\Rightarrow$ Explicit exact solutions of M-theory compactified down to 4 dimensions

$\Rightarrow$ Holographic description

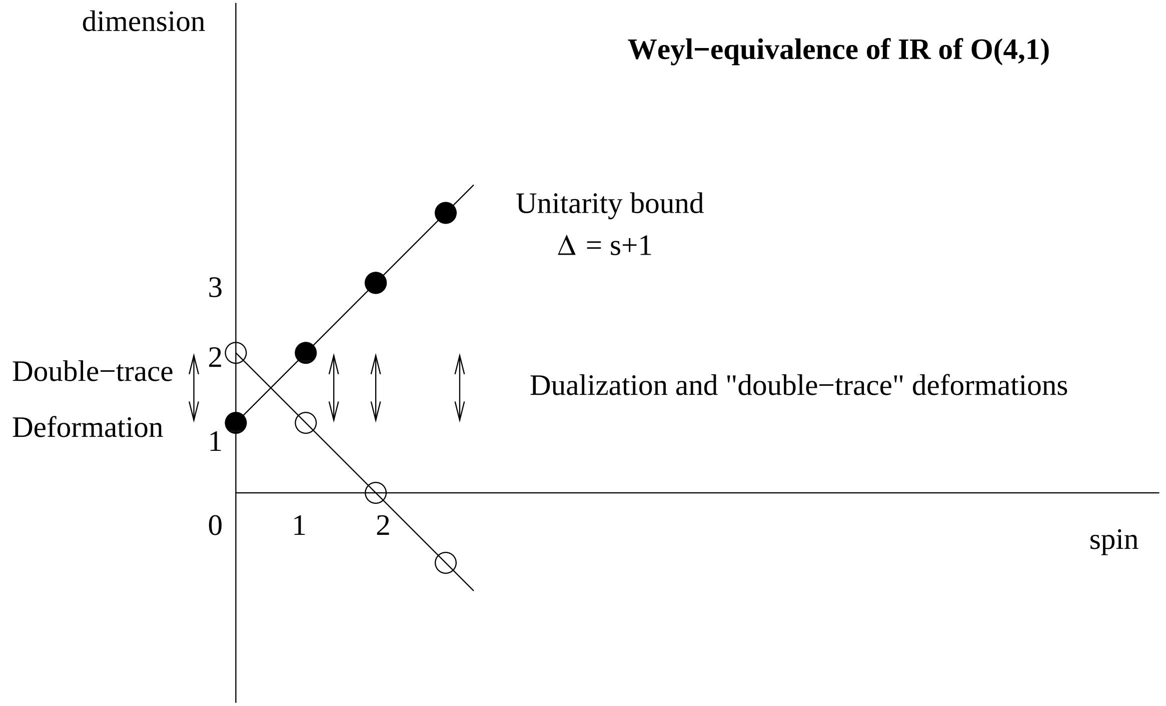
2. M-theory and Duality

The holographic dual of these theories describes world-volume theory of N coincident M2-branes. This is a 3d $\mathcal{N} = 8$ SCFT that arises as the IR limit of 3d $\mathcal{N} = 8$ SYM and it is currently unknown.

I will consider

$$\begin{aligned} \mathrm{AdS}_4 \times S^7 &\simeq \text{3d SCFT on } \partial(\mathrm{AdS}_4) \\ \ell_{\mathrm{Pl}}/\ell &\sim N^{-3/2} \end{aligned}$$

There is a **duality conjecture** for these theories, relating IR and UV theories: a generalization of **electric-magnetic duality** for higher spins [Leigh and Petkou, '03]



Duality conjecture relating IR and UV CFT_3's.

- **Instantons** describe the self-dual point of duality
- **Duality** plays an essential role in finding their holographic description
- Instantons probe CFT **off conformal vacuum**

Typically, the dual effective action is a **"topological" action**

This talk:

1. Conformally coupled scalar:

Bulk: scalar instantons $\longrightarrow$ instability

Boundary: effective action $\longrightarrow$ 3d conformally coupled scalar field with φ^6 potential

2. $U(1)$ gauge fields, RG flows and S-duality

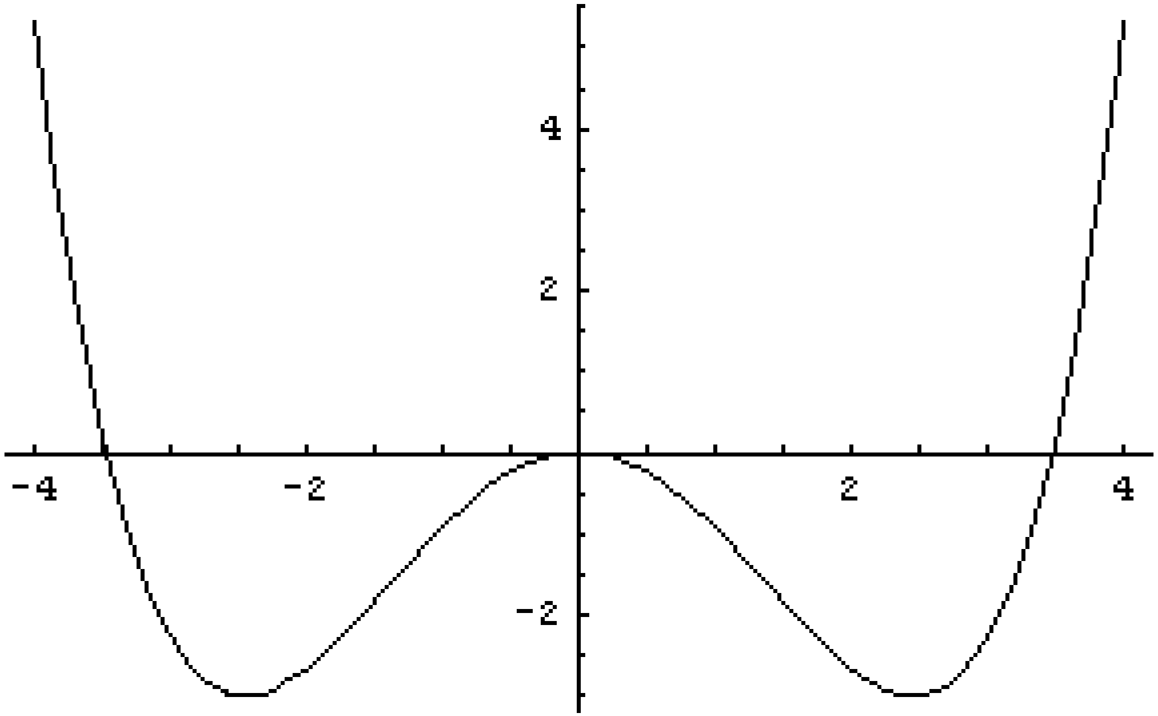
Conformally Coupled Scalars and AdS$_4 \times S^7$

The model: a conformally coupled scalar with quartic interaction:

$$S = \frac{1}{2} \int \mathrm{d}^4 x \, \sqrt{g} \left[\frac{-R + 2\Lambda}{8\pi G_N} + (\partial\phi)^2 + \frac{1}{6} R \, \phi^2 + \lambda \, \phi^4 \right] \tag{1}$$

• This model arises in **M-theory compactification** on S^7. It is a consistent truncation of the $\mathcal{N} = 8$ 4d sugra action where we only keep the **metric** and one **scalar field** [Duff,Liu 1999].

• The coupling is then given by $\lambda = \frac{8\pi G_N}{6\ell^2}$



Potential $\ell^2 V(\phi)$ in units where $8\pi G_N = 1$ for a background with negative cosmological constant.

• There are **two extrema**: $\phi = 0$ and $\phi = \sqrt{6/8\pi G_N}$.

• Naively we would expect to roll down the hill by small perturbations. However, in the presence of gravity such a picture is misleading: one needs to take into account **kinetic terms** and **boundary terms** [E. Weinberg '86]

• In fact, the $\phi = 0$ point is known to be **stable**. It is the well-known **AdS_4 vacuum** with standard choice of boundary conditions [Breitenlohner and Freedman, '82]

- But it is **unstable** for generic choice of **boundary conditions**. One has to consider tunneling effects due to the presence of kinetic terms [E. Weinberg '82]. Instanton effects can mediate the decay.

- I will construct such **instantons** explicitly for one particular choice of boundary conditions and compute the **decay rate**.

- There is an interesting **holographic dual** description of the decay that I will also analyze

Instantons

Instanton solutions: exact solutions of the Euclidean equations of motion with **finite action**.

’t Hooft instantons have zero stress-energy tensor:

$$T_{00} \sim E^2 - B^2 = 0 \tag{2}$$

We will likewise look for solutions with

$$T_{\mu\nu} = 0 \tag{3}$$

- They are "ground states" of the Euclidean theory
- The problem of solving the eom is simplified because there is no **back-reaction** on the metric:

$$\mathrm{d}s^2 = \frac{\ell^2}{r^2}\left(\mathrm{d}r^2 + \mathrm{d}\vec{x}^2\right) \ , \ r > 0 \tag{4}$$

We need to solve the Klein-Gordon equation in an AdS_4 background:

$$\Box\phi - \frac{1}{6}R\,\phi - 2\lambda\,\phi^3 = 0 \tag{5}$$

Unique solution with vanishing stress-energy tensor:

$$\phi = \frac{2}{\ell\sqrt{|\lambda|}} \frac{br}{-\mathrm{sgn}(\lambda)b^2 + (r+a)^2 + (\vec{x}-\vec{x}_0)^2} \tag{6}$$

- $\lambda < 0 \Rightarrow$ solution is regular everywhere
- $\lambda > 0 \Rightarrow$ solution is regular everywhere provided

$$a > b \geq 0$$

- $\alpha = a/b$ labels different boundary conditions:

$$\phi(r,x) = r\,\phi_{(0)}(x) + r^2\,\phi_{(1)}(x) + \ldots \tag{7}$$

It gives the relation between $\phi_{(1)}$ and $\phi_{(0)}$:

$$\phi_{(1)}(x) = -\ell\alpha\,\phi^2_{(0)}(x) \tag{8}$$

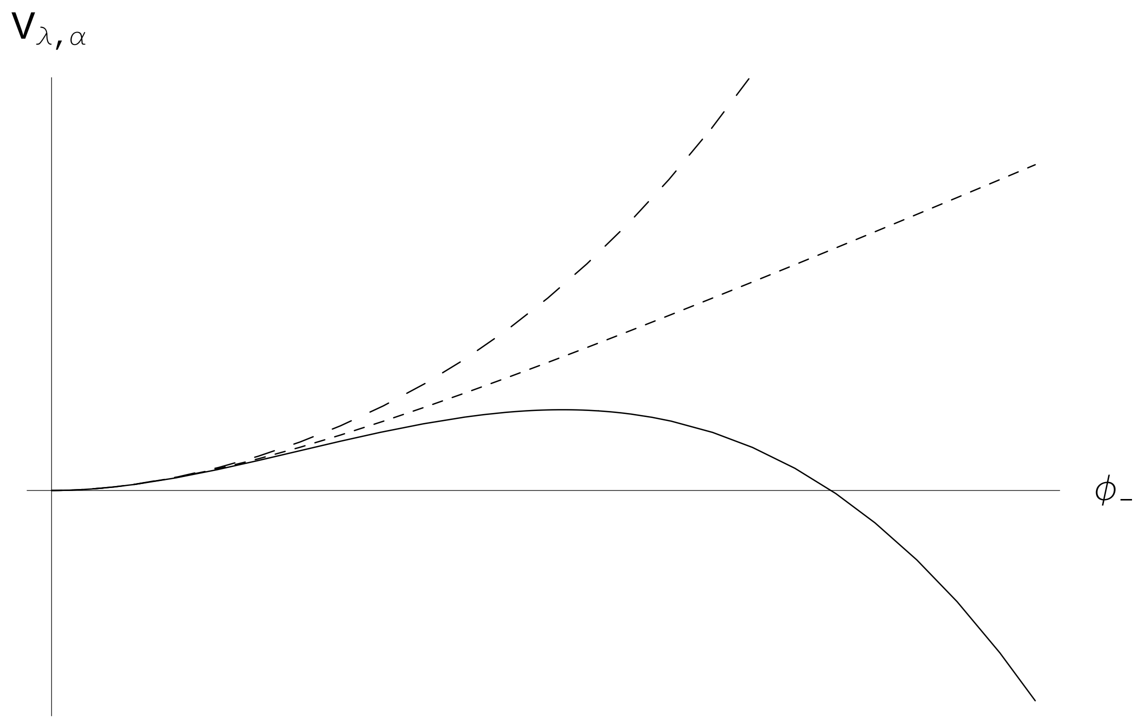
Holographic analysis

1) α is a **deformation** parameter of the dual CFT
2) $\vec{x}_0, a^2 - b^2$ parametrize 3d instanton **vacuum**

• For $\alpha > \sqrt{\lambda}$ ($a > b$) the effective potential becomes **unbounded** from below. This is the holographic image of the **vacuum instability** of AdS$_4$ towards dressing by a non-zero scalar field with mixed boundary conditions discussed above.

Similar conclusions were reached by Hertog & Horowitz [2005] (although only numerically).

Taking the boundary to be S^3 we plot the effective potential to be:

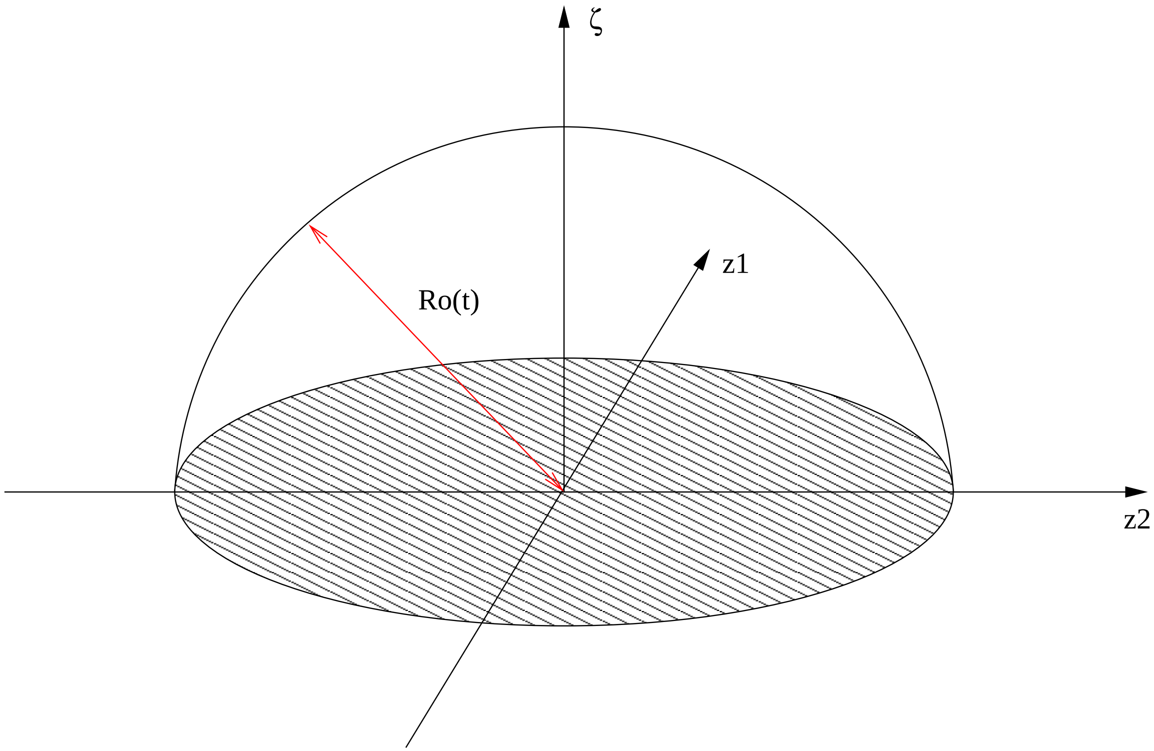


The global minimum $\phi_{(0)} = 0$ for $\alpha < \sqrt{\lambda}$ becomes local for $\alpha > \sqrt{\lambda}$. There is a potential barrier and the vacuum decays via tunneling of the field.

• The instability region is $\phi \to \sqrt{6/8\pi G_N}$, which corresponds to the total squashing of an S^2 in the corresponding 11d geometry. This signals a breakdown of the supergravity description in this limit.

• In the Lorentzian Coleman-De Luccia picture, our solution describes an **expanding bubble** centered at the boundary. Outisde the bubble, the metric is AdS_4 (the false vacuum).

Inside, the metric is currently unknown (the true vacuum). One needs to go beyond sugra to find the true vacuum metric.



Expansion of the bubble towards the bulk in the Lorentzian.

$R_0(t)$: radius of the bubble

z_1, z_2: boundary coordinates

ζ: radial AdS coordinate

The **tunneling probability** can be computed and equals

$$\mathcal{P} \sim e^{-\Gamma_{\rm eff}} \quad , \qquad\qquad \Gamma_{\rm eff} = \frac{4\pi^2\ell^2}{\kappa^2}\left(\frac{1}{\sqrt{1-\frac{\kappa^2}{6\ell^2\alpha^2}}} - 1\right) \tag{9}$$

The deformation parameter α drives the theory from regime of marginal instability $\alpha = \kappa/\sqrt{6}\ell$ ($\mathcal{P} \to 0$) to total instability at $\alpha \to \infty$ ($\mathcal{P} \to 1$).

Boundary description of the instability

- We have seen that the bulk instability is mirrored by the unboundedness of the CFT effective potential

- According to the usual **AdS/CFT** recipe, the boundary generator of correlation functions $W[\phi_{(0)}]$ at large N is obtained from the bulk on-shell sugra action:

$$
\begin{aligned}
\phi(r,x) &= r^{\Delta_-}\,\phi_{(0)}(x) + \dots \\
e^{S^{\text{bulk}}_{\text{on-shell}}[\phi]} &= e^{W[\phi_{(0)}]} \equiv \langle e^{\int \mathrm{d}^3x\,\phi_{(0)}(x)\,\mathcal{O}(x)}\rangle_{\text{CFT}}
\end{aligned} \tag{10}
$$

• The **Wilsonian effective action** $\Gamma[\langle\mathcal{O}(x)\rangle]$ can be obtained from $W[\phi_{(0)}]$ but this is in practice a complicated procedure. In the current example, dim $\mathcal{O} = 2$

• Bulk analysis implies there is a duality between $\phi_{(0)} \leftrightarrow \phi_{(1)}$, hence $\mathcal{O} \leftrightarrow \tilde{\mathcal{O}}$

• Therefore we can use duality to obtain the effective action where operator $\tilde{\mathcal{O}}$ of dimension 1 is turned on:

$$\begin{aligned}(\phi_{(0)}, \phi_{(1)}) = (J, \langle\mathcal{O}\rangle) \; &= \; (\langle\tilde{\mathcal{O}}\rangle, \tilde{J}) \\ W[J] \; &= \; \tilde{\Gamma}[\langle\tilde{\mathcal{O}}\rangle] \\ \Gamma[\langle\mathcal{O}\rangle] \; &= \; \tilde{W}[\tilde{J}] \end{aligned} \tag{11}$$

The **effective action** can be computed:

$$\Gamma_{\text{eff}} = \frac{1}{\sqrt{\lambda}} \int d^3x \sqrt{g_{(0)}} \Big(\phi_{(0)}^{-1} \partial_i \phi_{(0)} \partial^i \phi_{(0)} + \frac{1}{2} R[g_{(0)}] \phi_{(0)} + 2\sqrt{\lambda}(\sqrt{\lambda} - \alpha)\, \phi_{(0)}^3 \Big) \quad (12)$$

Redefining $\phi_{(0)} = \varphi^2$, we get

$$\Gamma_{\text{eff}}[\varphi, g_{(0)}] = \frac{1}{\sqrt{\lambda}} \int d^3x \sqrt{g_{(0)}} \Big(\frac{1}{2} (\partial\varphi)^2 + \frac{1}{2} R[g_{(0)}]\, \varphi^2 + 2\sqrt{\lambda}(\sqrt{\lambda} - \alpha)\, \varphi^6 \Big) \quad (13)$$

$\Rightarrow$ **3d conformally coupled scalar field with** φ^6 **interaction.**

[SdH,AP 0606276; SdH, AP, IP 0611315]

[Hertog, Horowitz hep-th/0503071]

This describes the large N limit of the strongly coupled 3d $\mathcal{N} = 8$ SCFT where an operator of dimension 1 is turned on. This CFT describes N **coincident M2-branes** for large N away from the conformal fixed point.

This action is the matter sector of the $U(1)$ $\mathcal{N} = 2$ **Chern-Simons** action. It has recently been proposed to be dual to AdS$_4$ [Schwarz, hep-th/0411077; Gaiotto, Yin, arXiv:0704.3740]

Quantum correspondence: a toy model

Φ^4 theory (with $\lambda < 0$) in asymptotically AdS_4 versus φ^6 theory on the boundary

Free theories agree: this follows largely from conformal symmetry and group theory

Bulk and boundary **instantons** and their **fluctuations** agree

$\Rightarrow$ This is non-trivial and relies on special property of associated Legendre functions:

$$P_{3/2}^{\ell+\frac{1}{2}}(z)P_{3/2}^{\ell'+\frac{1}{2}}(z)=\sum_{j=2}^{\ell+\ell'}d_{j\ell\ell'}\,\tilde{P}_2^{j+1}(z) \qquad (14)$$

This is a new mathematical result proven by Tom Koornwinder [de Haro, Petkou, Koornwinder '06].

Quantization of boundary φ^6 theory agrees with quantization of scalar field

$$\Phi_{\mathsf{hol}}(x)=:\varphi^2(x): \qquad (15)$$

S-duality for $U(1)$ gauge fields in AdS

• The duality between $\mathcal{O} \leftrightarrow \tilde{\mathcal{O}}$ in the scalar field case has a generalization to other fields: in the case of gauge fields, it is electric-magnetic duality

• The bulk **equations of motion** are invariant under e.m. transformations

• Bulk **action** is invariant only up to boundary terms. These boundary terms **perturb** the dual CFT and can be computed from the bulk

S-duality acts as follows:

$$\begin{aligned} E' &= B \\ B' &= -E \\ \tau' &= -1/\tau \quad , \quad \tau = \frac{\theta}{4\pi^2} + \frac{i}{g^2} \end{aligned} \tag{16}$$

The action transforms as follows:

$$S[A', E'] = S[A, E] + \int \mathrm{d}^3x \, (E - \theta\, B)\, A$$

In the usual theory, B is fixed at the boundary and corresponds to a **source** in the CFT. This source couples to a conserved current (operator). E corresponds this **conserved current**. S-duality interchanges the roles of the current and the source.

New source: $J' = E$

New current: $\langle \mathcal{O}' \rangle_{A'} = -B$

$$\Rightarrow \quad (J, \langle \mathcal{O} \rangle) \leftrightarrow (\langle \mathcal{O}' \rangle, J')$$

We can check how S-duality acts on **two-point functions** of the current $\mathcal{O}$:

$$\begin{aligned} \langle \mathcal{O}_i(p)\, \mathcal{O}_j(-p) \rangle_{A=0} &= \frac{1}{g^2}\, |p|\, \Pi_{ij} + \frac{\theta}{(4\pi^2)^2}\, i\, \epsilon_{ijk}\, p_k \\ \Pi_{ij} &= \delta_{ij} - \frac{p_i p_j}{|p|^2} \end{aligned} \qquad (17)$$

The action of S-duality on the two-point function can be computed from the bulk and is as expected:

$$\langle \mathcal{O}'_i(p)\mathcal{O}'_j(-p)\rangle_{A'} = \frac{g^2}{1+\frac{g^4\theta^2}{(4\pi^2)^2}}\,|p|\,\Pi_{ij} - \frac{\frac{g^4\theta}{4\pi^2}}{1+\frac{g^4\theta^2}{(4\pi^2)^2}}\,i\,\epsilon_{ijk}\,p_k \tag{18}$$

in other words

$$\tau \to -1/\tau$$

RG flow

• So far we have considered the CFT at the **conformal fixed point**, either IR (Dirichlet) or UV (Neumann). We will now consider how S-duality acts on **RG flows**

• Deforming the **boundary conditions** in a way that breaks conformal invariance (introducing mass parameter)
$\Leftrightarrow$ adding **relevant operator** that produces flow towards new IR fixed point

Take the following **massive boundary condition**:

$$A + \frac{1}{m}(E - \theta B) = J \tag{19}$$

In terms of this source, the boundary generating functional is:

$$W[J] = \frac{m}{2} \int J\,(m - \Box^{1/2}) a\, J + \theta\, \mathrm{d}J \wedge a\, J \tag{20}$$

where $a = a(\Box, m)$.

RG flow of the two-point function

IR: $\langle \mathcal{O}_i \mathcal{O}_j \rangle = |p| \Pi_{ij} + \theta \, i \, \epsilon_{ijk} \, p_k$

2-point function for **conserved current** of dim 2.

UV: $\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{m^2}{|p|^2 (1+\theta^2)^2} (|p| \Pi_{ij} - \theta \, i \, \epsilon_{ijk} \, p_k)$

S-**dual current** coming from dualizing gauge field.

See also [Leigh,Petkou hep-th/0309177;Kapustin]

Such behavior has also been found in quantum Hall systems [Burgess and Dolan, hep-th/0010246]

A Proposal for the Boundary Theory

Consider the model

$$I^g = C \int \mathrm{d}^3\vec{x} \left(\frac{1}{2}\, \partial_i \phi^a \partial^i \phi^a + \frac{g}{6N'^2} (\phi^a \phi^a)^3 \right), a = 1, \ldots, N' \tag{21}$$

For $g > 0$ large N effective action can be evaluated [Bardeen et al. '84]

$$\Gamma_{\mathrm{eff}}[\phi^\alpha_{\mathrm{cl}}] = \mathrm{Tr}\,\log(-\partial^2 + \sigma_*) + \int \left(\phi^\alpha_{\mathrm{cl}}(-\partial^2 + \sigma_*)\phi^\alpha_{\mathrm{cl}} + \frac{g}{3}\rho_*^3 - \sigma_* \rho_* \right) \tag{22}$$

Renormalized sadle point equations determine (σ_*, ρ_*).

$g > 16\pi^2 \Rightarrow$ effective potential unbounded from below. The model has instanton configurations for $g < 0$, responsible for non-perturbative instability:

$$\phi^a(\vec{x}) = (3N'^2/(-g))^{1/4}(c^a/\sqrt{b})(b^2 + (\vec{x} - \vec{x}_0)^2)^{-\frac{1}{2}} \ , \ c^a c^a = b^2 \tag{23}$$

The potential on S^3 for $\sigma \equiv \phi^a\phi^a/N$ coincides with holographic potential for small curvature:

$$\begin{aligned} V_g(\sigma) &= C\left(\frac{1}{16}R\,\sigma + \frac{g}{6}\,\sigma^3\right) \\ C &= \frac{4}{3\sqrt{\lambda}} \\ g &= \frac{3}{2}\sqrt{\lambda}(\sqrt{\lambda} - \alpha) \\ \sigma &= \phi_{(0)} \end{aligned} \tag{24}$$

Conclusions

• **Instanton configurations** in the bulk of AdS_4 are dual to CFT's whose effective action is given by some topological action, typically a relative of the Chern-Simons action. They describe **tunneling effects in M-theory**.

• We have checked the **duality conjecture** of [Leigh, Petkou '03] for CFT_3's for $s = 0, 1$ and found their bulk images.

Scalar fields

• Generalized b.c. that correspond to multiple trace operators **destabilize AdS_4 nonperturbatively** by dressing of the scalar field. The Lorentzian picture is in terms of tunneling to a **new vacuum**. The tunneling rate was computed.

• Boundary effective action was computed and it agrees with related proposals: 3d conformally coupled scalar with φ^6 interaction. Boundary instantons match bulk instantons and describe the decay.

Gauge fields

• **Electric-magnetic** duality interpolates between D & N. On the boundary it interchanges the source and the conserved current (**dual CFT$_3$'s**).

• Massive deformations generate **RG flow** of the two-point function. One finds the conserved current in the IR, but the S-**dual gauge field** in UV.

Outlook

- Linearized **gravity** also has a version of S-duality [Leigh, Petkou '07]. Duality should be helpful to deal the higher-spin case.

- Exact **instanton solutions** exist for gravity as well. The dual generating functional is the **gravitational Chern-Simons** action [de Haro-Petkou, to appear].

- Gravitational instanton describes AdS bubble inside a domain wall. Outside and close to domain wall the space looks locally de Sitter space [in progress].