Higgs mechanism in five-dimensional gauge theories

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N. Irges, F.K. and M. Luz, arXiv:0706.3806 [hep-ph]

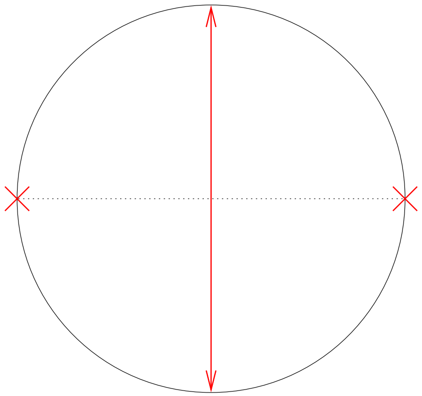
Perturbative computations in 5d gauge theories (1)

Motivation for gauge theories in dimensions $d > 4$

- Explain the origin of the Higgs field and electroweak spontaneous symmetry breaking (SSB)
- (Some of the) Extra dimensional components of the gauge field play the role of Higgs fields. The Higgs potential is generated by quantum corrections [Coleman and Weinberg, 1973]
- Extra dimensional space:
  - S^2 [Fairlie, 1979; Manton, 1979]
  - non-simply connected: S^1 or T^2 [Hosotani, 1983, 1989; Antoniadis, Benakli and Quiros, 2001]
- Finiteness of the Higgs potential to all orders in perturbation theory, without supersymmetry
- Triviality: cut-off cannot be removed, otherwise interactions vanish

Perturbative computations in 5d gauge theories (1)

Orbifold $S^1/\mathbb{Z}_2$



$S^1 : x_5 \in (-\pi R, \pi R]$; Reflection

$$\mathcal{R} : \; z = (x_\mu, x_5) \quad \longrightarrow \quad \bar{z} = (x_\mu, -x_5)$$

$$A_M(z) \quad \longrightarrow \quad \alpha_M A_M(\bar{z}) \,, \quad \alpha_\mu = 1 \,, \; \alpha_5 = -1$$

Fixed points $z = \bar{z} \Leftrightarrow x_5 = 0$ and $x_5 = \pi R$ define 4d boundaries

$\mathbb{Z}_2$ projection for gauge fields

$$\begin{aligned} \mathcal{R}\, A_M &= g\, A_M\, g^{-1} \,, \qquad g^2 \in \text{centre of } SU(N) \\ \mathcal{R}\, \partial_5 A_M &= g\, \partial_5 A_M\, g^{-1} \\ &\;\;\vdots \end{aligned}$$

Parities of $SU(N)$ generators

$$g\, T^a\, g^{-1} = T^a \text{ (unbroken)} \,, \quad g\, T^{\hat{a}}\, g^{-1} = -T^{\hat{a}} \text{ (broken)}$$

Perturbative computations in 5d gauge theories (1)

Dirichlet boundary conditions at $z = \bar{z}$

$$A_\mu \;=\; g\, A_\mu\, g^{-1} \qquad \text{and} \qquad A_5 \;=\; -g\, A_5\, g^{-1}$$

$\Rightarrow$ Only *even* components A_μ^a and $A_5^{\hat{a}}$ are $\neq 0$: breaking of the gauge symmetry

$$G \;=\; SU(p+q) \;\longrightarrow\; \mathcal{H} \;=\; SU(p)\times SU(q)\times U(1)$$

depending on the choice of g

- $SU(2) \longrightarrow U(1)$ with $g = \mathrm{diag}(-i, i)$: even fields
  A_μ^3: "photon/Z"
  $A_5^{1,2}$: complex "Higgs"

- $SU(3) \longrightarrow SU(2)\times U(1)$ with $g = \mathrm{diag}(-1,-1,1)$: even fields
  $A_\mu^{1,2,3,8}$: "photon,Z and $W^{\pm}$"
  $A_5^{4,5,6,7}$: doublet of complex "Higgs" in the fundamental representation of $SU(2)$

Perturbative computations in 5d gauge theories (1)

Compactification: Kaluza–Klein (KK) expansion on $S^1\mathbb{Z}_2$

$$E(x,x_5) = \frac{1}{\sqrt{2\pi R}} E^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} E^{(n)}(x) \cos(n x_5/R) \quad \text{even fields}$$

$$O(x,x_5) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} O^{(n)}(x) \sin(n x_5/R) \quad \text{odd fields}$$

Dimensional reduction for energies $E \ll 1/R$ (cf. finite temperature field theory): low-energy 4d effective theory of zero modes $E^{(0)}(x)$

Lagrangean

$$\mathcal{L} = -\frac{1}{2g_5^2} \mathrm{tr}\{F_{MN}F_{MN}\} - \frac{1}{g_5^2\xi} \mathrm{tr}\{(\bar{D}_M A_M)^2$$

with $\bar{D}_M F = \partial_M F + [\langle A_M \rangle, F]$ and we set $\xi = 1$. A_5 is a scalar from 4d point of view and can have a vacuum expectation value (vev)

$$\langle A_5 \rangle \neq 0$$

The Higgs mass and potential in perturbation theory

Mass eigenvalues [Kubo, Lim and Yamashita, 2002] Take for example $SU(2)$ and define

$$\alpha \quad = \quad g_5 \langle A_5^1 \rangle R$$

Zero modes: $m_{Z^0} R = \alpha \; ; \quad m_{A_5} R = 0 \, , \alpha$
$n \neq 0$: for gauge bosons and scalars

$$(m_n R)^2 = n^2 \, , \; (n \pm \alpha)^2$$

Hosotani mechanism: α is determined by minimizing the Coleman–Weinberg (CW) potential V. We first compute it at "infinite cut-off"

- 1-loop definition in D-dimensional Euclidean space for a real scalar field

$$\int [\mathrm{D}\phi] \mathrm{e}^{-S_{\mathrm{E}}} \quad \sim \quad \mathrm{e}^{-V} \equiv \frac{1}{\sqrt{\det[-\partial_\mu \partial_\mu + M^2]}}$$

- Take $D = 4$, use KK masses m_n; after a Poisson resummation

$$V \quad = \quad -\frac{9}{64\pi^6 R^4} \sum_{m=1}^{\infty} \frac{\cos(2\pi m \alpha)}{m^5}$$

The Higgs mass and potential in perturbation theory

- The CW potential is periodic and has degenerate minima at

$$\alpha = \alpha_{\rm min} = 0 \bmod \mathbb{Z}$$

- No spontaneous symmetry breaking $U(1) \longrightarrow$ nothing: integer shift of KK index n
- Polyakov line P around the extra dimension is the physical meaning of A_5

$$[\langle P\rangle, \text{Cartan generator}] = [e^{-i\pi\alpha\sigma^1}, \sigma^3] \quad = \quad 0 \quad \text{for } \alpha \in \mathbb{Z}$$

- Higgs mass

$$(m_H R)^2_{\rm pert} \;\equiv\; R^2 \frac{{\rm d}^2 V}{{\rm d}v^2}\bigg|_{\alpha=\alpha_{\rm min}} \;=\; \frac{9\zeta(3)}{16\pi^4} g_4^2\bigg|_{\alpha=0}$$

in terms of dimensionless effective 4d coupling

$$g_4^2 \;=\; \frac{g_5^2}{2\pi R} \quad \left(= \frac{N}{N_5\beta} \right)$$

- "Infinite cut-off" means close to the trivial point $g_4 \to 0$: Z^0 massless and Higgs $\to$ massless

Lattice simulations of gauge group $SU(2)$

- Lattice cut-off: $\Lambda = 1/a$, a is the lattice spacing
- Orbifold geometry: $\frac{T}{a} \times (\frac{L}{a})^3 \times (N_5 = \frac{\pi R}{a})$, compact: $\frac{T}{a}\,,\ \frac{L}{a} \gg N_5$
- Parameter space

$$N_5 \;=\; \pi R \Lambda \quad \text{and} \quad \beta = 2N/(g_5^2 \Lambda)$$

Trivial point: $\beta \to \infty$ *and* $N_5 \to \infty$

- Lattice gauge field on the orbifold

$$A_M(z,M) \in su(N) \quad \longrightarrow \quad U(z,M) \;=\; \exp\{a A_M(z)\} \in SU(N)$$

- Lattice action

$$S_{\mathrm{W}}^{\mathrm{orb}}[U] \;=\; \frac{\beta}{2N} \sum_p w(p)\,\mathrm{tr}\{1 - U(p)\}\,, \quad w(p) \;=\; \begin{cases} \frac{1}{2} & p \text{ in the boundary} \\ 1 & \text{in all other cases.} \end{cases}$$

Sum over oriented plaquettes p in the strip $\{z = a(n_\mu, n_5)|\ 0 \le n_5 \le N_5\}$.
Periodic boundary conditions (b.c.) in 4d. In the 5th dimension, *only* Dirichlet b.c.

$$A_\mu \;=\; g\, A_\mu\, g^{-1} \quad \longrightarrow \quad U(z,\mu) \;=\; g\, U(z,\mu)\, g^{-1} \qquad \text{at } n_5 = 0\,,\ N_5$$

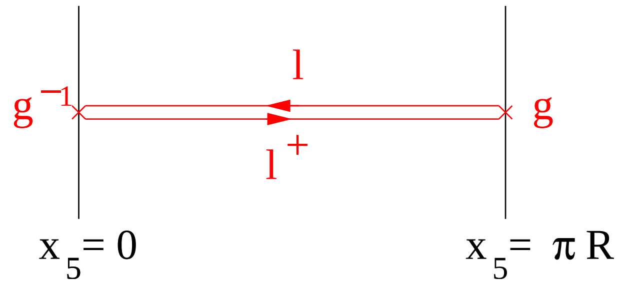
Lattice simulations of gauge group $SU(2)$

- on S^1: can gauge transform $U((n_\mu, n_5), 5) \longrightarrow V(n_\mu) = \exp\{a(A_5)_{\mathrm{lat}}\}$, related to the Polyakov line $P = V^{2N_5}$

$$a(A_5)_{\mathrm{lat}} \quad = \quad \frac{1}{4N_5}(P - P^\dagger) + \mathrm{O}(a^3)$$

- Polyakov line P on $S^1/\mathbb{Z}_2$

$$P(n_\mu) = lgl^\dagger g^{-1}$$



- Lattice Higgs field

$$\Phi(n_\mu) \quad = \quad [\frac{1}{4N_5}(P - P^\dagger), g] \quad \sim \quad \phi^{\hat{a}} T^{\hat{a}}$$

  Φ transforms under $U(1)$ like field strength and has charge equal to 2

- Since $\mathrm{tr}\{\Phi\} \equiv 0$ take

$$H(n_0) \quad = \quad \sum_{n_1, n_2, n_3} \mathrm{tr}\{\Phi(n_\mu)\Phi(n_\mu)^\dagger\}$$

Lattice simulations of gauge group $SU(2)$

- Spectrum of scalars = Higgs/glueball states

$$
\begin{aligned}
C(t) &= \sum_{n_0} \{ < H(n_0) H(n_0+t) > - < H(n_0) >< H(n_0+t) > \} \\
&\overset{T\to\infty}{=} \sum_{\alpha} | < \alpha | H(0) | 0 > |^2 \mathrm{e}^{-m_\alpha t} \quad \xrightarrow{t\to\infty} \quad | < 1 | H(0) | 0 > |^2 \mathrm{e}^{-m_H t}
\end{aligned}
$$

- effective masses $am_{\mathrm{eff}}(t + a/2) = \ln(C(t)/C(t+a))$, plateau at am_H for large t
- Can do better: variational basis of (smeared) Higgs operators, masses m_α can be extracted with method of [Lüscher and Wolff, 1990]
- Photon field is defined using the Higgs field: for $SU(2)$, with $\varphi = \Phi/\sqrt{\det(\Phi)}$

$$
\begin{aligned}
W_k(n_\mu) &= -i \mathrm{tr}\{\sigma^3 V_k\}\,, \quad k = 1,2,3 \\
V_k(n_\mu) &= U(n_\mu, k)\varphi(n_\mu + \hat{k}) U^\dagger(n_\mu, k)\varphi(n_\mu)
\end{aligned}
$$

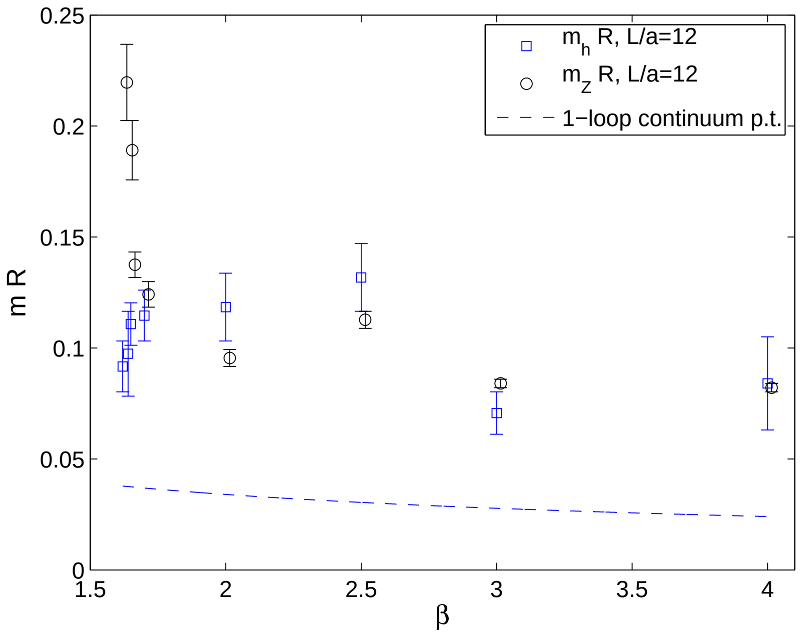
- Vector spectrum is extracted with the same method as for the scalar one

Lattice simulations of gauge group $SU(2)$

Simulation results for $SU(2) \xrightarrow{\mathbb{Z}_2} U(1)$, with $N_5 = 6$, $\frac{L}{a} = 12$ and $\frac{T}{a} = 96$

Phase transition at $\beta_c \simeq 1.607$
$\beta < \beta_c$: confinement phase, masses in lattice units $\gg 1$ cannot be measured
$\beta > \beta_c$: deconfinement Coulomb phase (no SSB) or Higgs phase $\Leftarrow$

- Higgs mass is larger than result from 1-loop CW
- Massive Z^0 $\Leftrightarrow$ SSB!, contrary to CW result at "infinite cut-off"
- 4d $U(1)$ Higgs model for *charge* $q = 2$ and infinite quartic coupling has a Higgs phase [Fradkin and Shenker 1979]



Perturbative computations with a cut-off (2)

The lattice action can be described by a continuum effective lagrangean [Symanzik, 1981; 1983]

$$-\mathcal{L} \quad = \quad \frac{1}{2g_5^2}\mathrm{tr}\{F_{MN}F_{MN}\} + \sum_{p_i} c^{(p_i)}(N_5,\beta)\; a^{p_i-4}\;\mathcal{O}^{(p_i)} + \ldots$$

$\mathcal{O}^{(p_i)}$ is an operator of dimension $p_i > 4$.

For example, for the Wilson plaquette action

$$c\,\mathcal{O}^{(6)} \quad = \quad \sum_{M,N}\frac{c}{2}\mathrm{tr}\{F_{MN}(D_M^2 + D_N^2)F_{MN}\}\,, \qquad c \equiv c^{(6)}(N_5,\beta) = \frac{1}{12} + \ldots$$

$\Rightarrow$ in the interior of the (N_5,β) parameter space higher derivative operators contribute to the mass matrix of A_μ

On the orbifold, there are additional boundary counterterms

$$c_0\,\mathcal{O}^{(5)} \quad = \quad \frac{\pi a c_0}{4}F_{5\mu}^{\hat{a}}F_{5\mu}^{\hat{a}}\,[\delta(x_5) + \delta(x_5 - \pi R)]\;, \qquad c_0 \equiv c^{(5)}(N_5,\beta)$$

$\Rightarrow$ contribution to the mass matrix of the even gauge fields A_μ^a

Perturbative computations with a cut-off (2)

CW potential with cut-off effects

1. Compute the cut-off corrections to the KK masses m_n of A_μ
2. Insert the cut-off dependent KK masses into the 1-loop CW formula

$$V^{\text{gauge}} = -\frac{1}{2}\sum_n \int_0^\infty \frac{\mathrm{d}l}{l} \mathrm{e}^{-\frac{1}{l}(m_n^2 a^2 + 2D)} \frac{1}{a^D} \mathrm{I}_0^D\left(\frac{2}{l}\right)$$

   and compute it including up to O(a^2) corrections in V^{gauge}
3. Add scalar and ghost contributions (no cut-off corrections)

$$V = \underbrace{4V^{\text{gauge}}}_{A_\mu} \underbrace{+V^{\text{scalar}}}_{A_5} \underbrace{-2V^{\text{scalar}}}_{\text{ghosts}}$$

1. For $SU(2) \stackrel{\mathbb{Z}_2}{\to} U(1)$ the KK masses are corrected to $(N_5 = \frac{\pi R}{a})$

$$(m_n R)^2 = \quad n^2\,, \qquad \text{for } n > 0$$

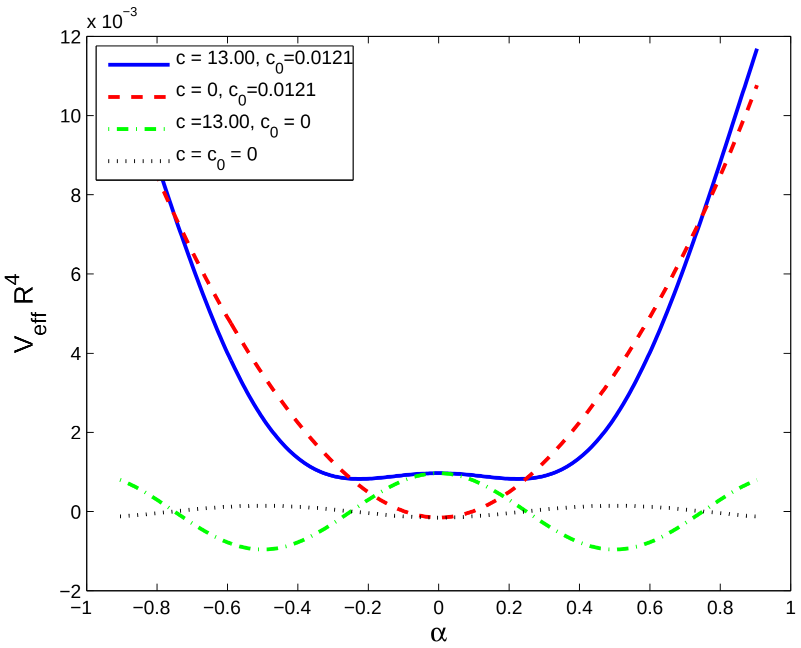
$$(n \pm \alpha)^2 + \frac{c_0 \alpha^2}{2} \frac{\pi}{N_5} + c\,(n \pm \alpha)^4 \frac{\pi^2}{N_5^2} \qquad \text{for } n \geq 0$$

Perturbative computations with a cut-off (2)

CW potential for $N_5 = 6$ with cut-off effects described by c and c_0. Requiring the vev $v < 1/a$ implies the constraint

$$|\alpha| < \sqrt{\frac{NN_5}{\pi^2\beta}}$$

- On the circle ($c_0 = 0$) the potential is periodic. If $c > 1.72$ there is SSB, $\alpha_{\min} : 0 \longrightarrow 1/2$
- On the orbifold ($c_0 \neq 0$) no periodicity and $0 < \alpha_{\min} < 1$
- It is not possible to have SSB with $c_0 > 0$ and $c = 0$
- Large values of c are not unexpected due to quantum corrections, which can be power-like in $1/a$



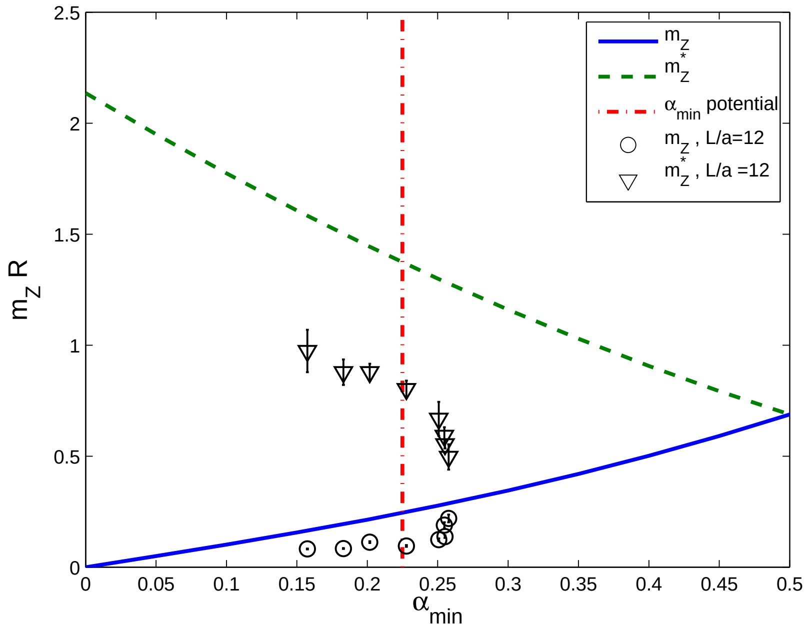
Comparing to the lattice

Compute m_Z, m_{Z^*} and

$$\alpha_{\min}(\beta) \quad = \quad \sqrt{\mathrm{tr}\{\Phi\Phi^\dagger\}N_5^2/(2\pi)}$$

from lattice simulations at $N_5 = 6$, $\frac{L}{a} = 12$ and $\frac{T}{a} = 96$

Compare with KK corrected masses using $c = 13.0$ and $c_0 = 0.0121$. CW has minimum at $\alpha_{\min} = 0.225$ (vertical line)



Comparing to the lattice

Ratio of the Higgs to the Z-boson mass

$$\rho_{HZ^0} = m_H/m_{Z^0}$$

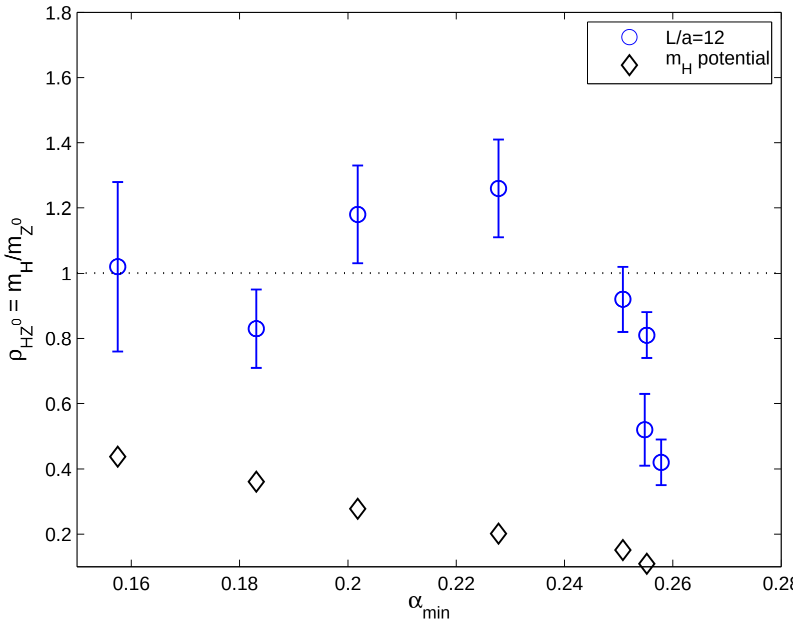
On the lattice it is possible to get $\rho_{HZ^0} \geq 1$

For a given β

1. get $\alpha_{\min}(\beta)$ from simulations
2. tune c and c_0 such that the minimum of the CW potential is at $\alpha_{\min}(\beta)$
3. compute the Higgs mass

$$(m_H R)^2 = \frac{N}{N_5 \beta} R^4 \left. \frac{\mathrm{d}^2 V}{\mathrm{d}\alpha^2} \right|_{\alpha=\alpha_{\min}}$$

$N_5 = 6$



The $SU(3)$ case

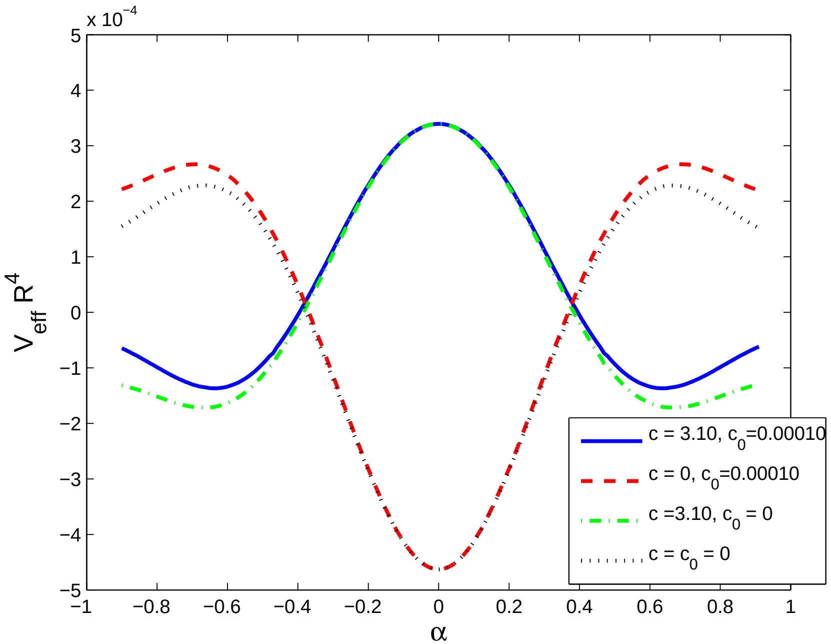
No lattice data yet, play with the CW potential. Is it possible to tune c and c_0 such that

$$\rho_{HZ^0} \equiv m_H/m_{Z^0} \geq 1.25 \qquad \text{and} \qquad \cos(\theta_{\mathrm{W}}) \equiv m_W/m_{Z^0} \simeq 0.877 \quad ?$$

With $c = c_0 = 0$: $\cos(\theta_{\mathrm{W}}) = \frac{1}{2}$

N_5	c	c_0	ρ_{HZ^0}	$\cos(\theta_{\mathrm{W}})$
2	3.1	0.02	0.54	0.899
2	50.0	0.2	1.25	0.888
3	3.1	0.0036	0.23	0.868
3	50.0	0.05	0.43	0.836
4	3.1	0.0007	0.12	0.877
4	50.0	0.01	0.21	0.883
6	3.1	0.0001	0.04	0.820
6	50.0	0.0017	0.07	0.888

$N_5 = 6$



Summary and outlook

- We have simulated a 5d pure gauge theory on an orbifold: $SU(2) \stackrel{\mathbb{Z}_2}{\longrightarrow} U(1)$ with $N_5 = 6$ and $\beta > \beta_c$
- The Z-boson mass is $m_{Z^0} \neq 0$, contrary to 1-loop predictions for the Coleman-Weinberg potential at "infinite cut-off"
- This puzzle is resolved by considering cut-off effects in the CW potential, in particular corrections to the KK masses from higher derivative operators
- There is good qualitative agreement comparing our new CW computation with the lattice data

- Outlook: there are two regions of the parameter space (N_5, β), where $g_4^2 = N/(N_5\beta)$ can be small and which are interesting for Standard Model physics:
  1. small N_5: compactification ($\longrightarrow$ anisotropic lattices)
  2. large N_5 and small β: localization [Dvali and Shifman]
- $SU(3)$, $SU(3) \times U(1)$, $\ldots$