Phenomenological applications of non-perturbative heavy quark effective theory

Mauro Papinutto

ALPHA Collaboration

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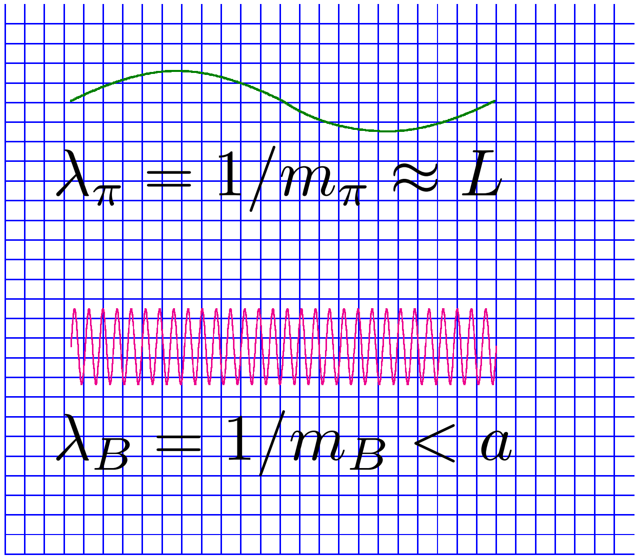
Outline:

- Motivations
- NP HQET
- M_b static
- M_b at order $1/m$
- B_B static

Motivations (lattice QCD)

only first-principles approach to study non-perturbative properties of QCD (hadron spectrum, matrix elements, . . .). Many systematics:

1. continuum limit extrapolation.
2. UV (lattice spacing a) and IR (volume V) cutoffs constrains quark masses $\Rightarrow$ extrapolations to the chiral/heavy quark regime.
3. dynamical light quark effects numerically expensive $\Rightarrow$ neglect them (quenched approximation $\Rightarrow$ pin down systematics, develop new methods. all the following results are quenched)



light quarks are too light $\Rightarrow$ extrapolate by matching with chiral effective theory.

b-quark is too heavy ($m_{\rm b}a > 1$) $\Rightarrow$ need an effective theory for the b quark: HQET

Non-perturbative HQET

$$P_+\psi_\mathrm{h} = \psi_\mathrm{h}\,, \quad \overline{\psi}_\mathrm{h} P_+ = \overline{\psi}_\mathrm{h}\,, \quad P_+ = \tfrac{1}{2}(1+\gamma_0)$$

action of the effective theory on a lattice [Eichten & Hill]

$$S_\mathrm{HQET} = a^4 \quad \sum_x \quad \{\overline{\psi}_\mathrm{h}(x)[D_0 + \delta m]\psi_\mathrm{h}(x) + \omega_\mathrm{spin}\underbrace{\overline{\psi}_\mathrm{h}(-\sigma\cdot\mathbf{B})\psi_\mathrm{h}}_{\mathcal{O}_\mathrm{spin}} + \omega_\mathrm{kin}\underbrace{\overline{\psi}_\mathrm{h}(-\tfrac{1}{2}\mathbf{D}^2)\psi_\mathrm{h}}_{\mathcal{O}_\mathrm{kin}} + \mathrm{O}(1/m^2)\}$$

also effective fields: time component of axial current in the effective theory

$$A_0^\mathrm{HQET}(x) \quad = \quad Z_\mathrm{A}^\mathrm{HQET}\overline{\psi}_\mathrm{l}(x)\gamma_0\gamma_5\psi_\mathrm{h}(x) + c_\mathrm{A}^\mathrm{HQET}\overline{\psi}_\mathrm{l}\gamma_j\overleftarrow{D}_j\psi_\mathrm{h} + \mathrm{O}(1/m^2)$$

where

$$\omega_\mathrm{kin} = \mathrm{O}(1/m)\,, \qquad \omega_\mathrm{spin} = \mathrm{O}(1/m)\,, \qquad c_\mathrm{A}^\mathrm{HQET} = \mathrm{O}(1/m)$$

under the path integral: expand the action in $1/m \to \mathcal{L}^{(\nu)}(x) = \mathrm{O}(1/m^{\nu})$ only as insertions

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathrm{D}\phi \, \mathrm{e}^{-S_{\mathrm{light}} - a^4 \sum_x \overline{\psi}_{\mathrm{h}}(x)[D_0 + \delta m]\psi_{\mathrm{h}}(x)} \, \mathcal{O} \left\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots \right\}$$

With this definition of the effective theory we have (at a given order in $1/m$)

- renormalizability $\equiv$ existence of the continuum limit due to universality (independence of details of the regularization)
- continuum asymptotic expansion in $1/m$

Note that these properties are not automatic for an effective field theory. ChPT shares these properties; NRQCD does not.

Difference to ChPT: as $1/m \to 0$ interactions are not turned off $\Rightarrow$ need lattice formulation to evaluate it non-perturbatively in g^2

Matching between QCD and HQET

bare couplings of HQET $(m_{\rm bare}, \omega_{\rm kin}, \omega_{\rm spin}, c_{\rm A}^{\rm HQET}, Z_{\rm A}^{\rm HQET}, \ldots)$ computable by matching with QCD $\Rightarrow$ transfer of predictivity QCD $\rightarrow$ HQET

this has to be done non-perturbatively: $\mathcal{O}$ generical field

$$\text{e.g.} \qquad {\mathcal{O}_{\rm R}}^{\rm d=5} = Z_{\mathcal{O}} \left[\mathcal{O}^{\rm d=5} + \sum_k c_k {\mathcal{O}_k}^{\rm d=4} \right] \qquad\qquad c_k = \frac{c_k^{(0)} + c_k^{(1)} g_0^2 + \ldots}{a}$$

if c_k computed at a finite order in g^2, there is no continuum limit!

$$\Delta c_k \sim \frac{g_0^{2(l+1)}}{a} \sim \frac{1}{a\,[\ln(a\Lambda)]^{l+1}} \xrightarrow{a\to 0} \infty$$

parameters computed to l-loops with l arbitrary

Non-pertubative matching: $\Phi_k^{\rm QCD} = \Phi_k^{\rm HQET}, \quad k = 1, 2, \ldots, N_{\rm HQET}$

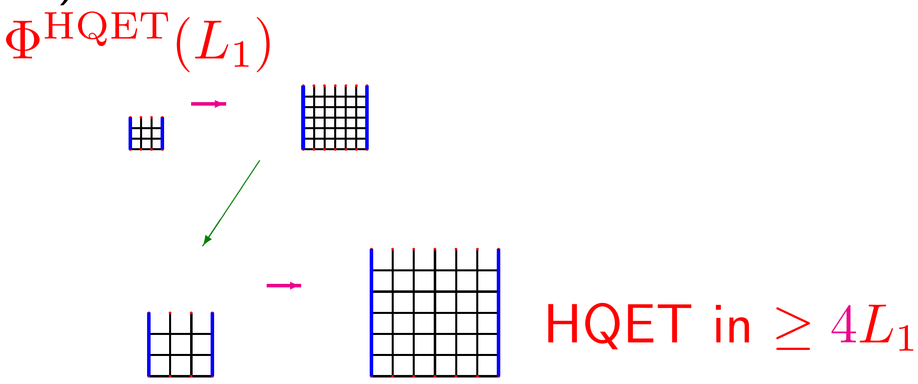
requires to be able to simulate the b-quark with finite mass!

Non-perturbative matching between QCD and HQET

The trick: start in small volume, $L = L_1 \approx 0.4\,\mathrm{fm} \Rightarrow m_{\mathrm{b}} a \ll 1$ & $1/(m_b L) \ll 1$

HQET-parameters from QCD observables in small volume at small lattice spacing (using the Schrödinger Functional)

Physical observables (e.g. $B_{\mathrm{B_s}}$, $F_{\mathrm{B_s}}$) need a large volume, such that the B-meson fits comfortably: $L \approx 4L_1 \approx 1.6\,\mathrm{fm}$



Connection achieved by recursive method:
[Lüscher *et al.*, 91; ALPHA 1993-2003]

$$\Phi_k^{\mathrm{HQET}}(2L) = F_k\left(\left\{\Phi_j^{\mathrm{HQET}}(L), j = 1, \ldots, N\right\}\right)$$

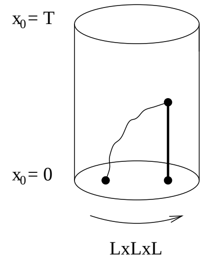
fully non-perturbative formulation of HQET (including matching) [Heitger & Sommer, 2004]

continuum limit can be taken in all steps

Example: $M_{\rm b}$ static (at order $1/m^0$)

[Heitger & Sommer, 2004; M. Della Morte, N. Garron, M. Papinutto and R. Sommer, 2005-2006]

finite volume B-meson "mass":

$f_{\rm A} \sim$ 

$$\Gamma = -\partial_0 \log[f_{\rm A}(x_0)]_{x_0=L/2, T=L}$$

$$\Phi_2^{\rm QCD}(L,M) = L\Gamma(L,M)\,, \quad \Phi^{\rm HQET}(L,M) = L\left[\Gamma^{\rm stat}(L) + m_{\rm bare}\right]$$

$L_2 m_{\rm B} = L_2 E_{\rm stat} + L_2 m_{\rm bare}$ B-meson mass ($E_{\rm stat} = \lim_{L\to\infty}\Gamma^{\rm stat}(L)$)

$$= L_2 E_{\rm stat} - L_2\Gamma^{\rm stat}(L_1) + \frac{L_2}{L_1}\Phi^{\rm HQET}(L_1, M_{\rm b})$$

$$= L_2 E_{\rm stat} - L_2\Gamma^{\rm stat}(L_1) + \frac{L_2}{L_1}\Phi_2^{\rm QCD}(L_1, M_{\rm b})$$

$$= L_2 E_{\rm stat} - L_2\Gamma^{\rm stat}(L_2) + \underbrace{L_2\Gamma^{\rm stat}(L_2) - L_2\Gamma^{\rm stat}(L_1)}_{=\sigma_{\rm m}(\bar{g}^2(L_1))} + \frac{L_2}{L_1}\Phi_2^{\rm QCD}(L_1, M_{\rm b})$$

experiment

Lattice with $am_{\mathrm{q}} \ll 1$

$m_{\mathrm{B}} = 5.4\,\mathrm{GeV}$ ↓ $\Gamma^{\mathrm{stat}}(L_2)$

$\Gamma(L_1, M)$ ↓ $\Gamma^{\mathrm{stat}}(L_1)$

$L_2 = 2L_1$

$u_i = \bar{g}^2(L_i)$

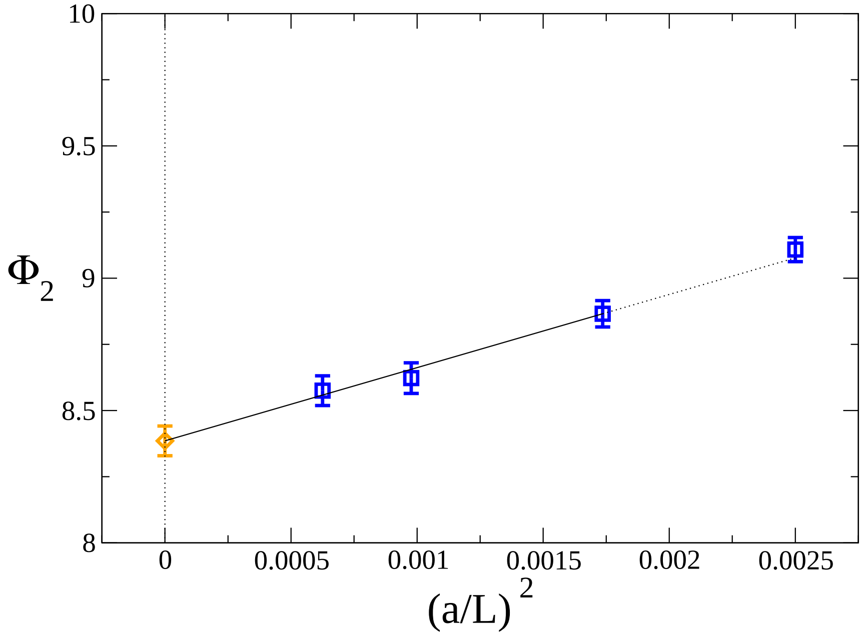
$\Gamma^{\mathrm{stat}}(L_2) \xleftarrow{\sigma_{\mathrm{m}}(u_1)} \Gamma^{\mathrm{stat}}(L_1)$

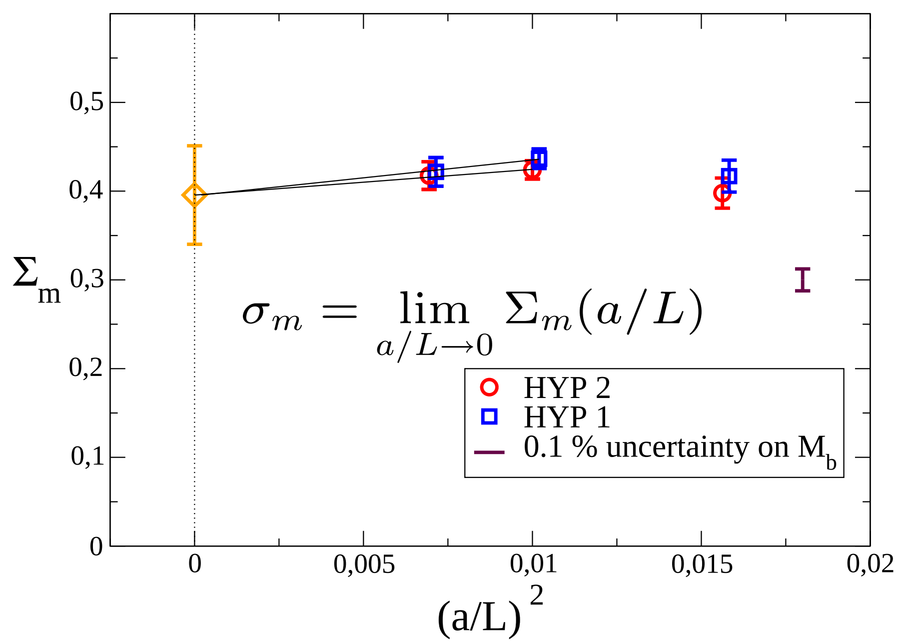
$$L_2 m_{\mathrm{B}} = \underbrace{L_2[E_{\mathrm{stat}} - \Gamma^{\mathrm{stat}}(L_2)]}_{a\to 0 \textbf{ in HQET}} + \underbrace{\sigma_{\mathrm{m}}(u_1)}_{a\to 0 \textbf{ in HQET}} + 2 \overbrace{\underbrace{L_1\Gamma(L_1, M_{\mathrm{b}})}_{a\to 0 \textbf{ for } M_{\mathrm{b}}L_1 \gg 1}}^{\equiv \Phi_2}$$

$\longrightarrow$ Solve the above equation for M_{b} (the RGI b-quark mass)

- fix $L_1 \approx 0.4\,\mathrm{fm}$ (from $\bar{g}^2(L_1)$)
- light quark mass = zero
- fix RGI quark masses of heavy quark (3 values around M_{b})
- In ∞ volume ($L_\infty = 4L_1$) light quark mass = strange quark mass

Examples of continuum extrapolations



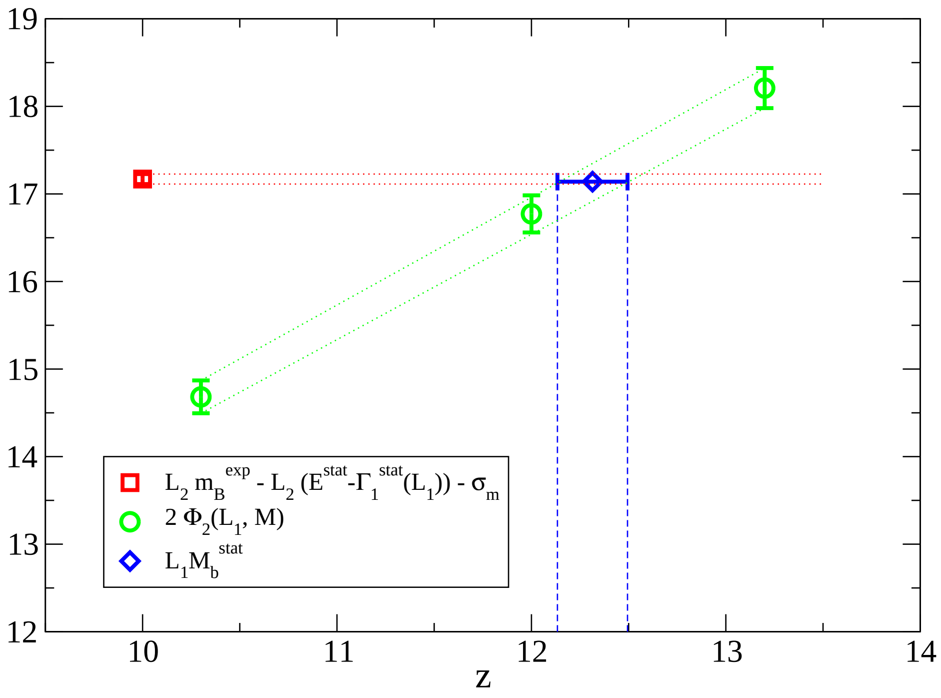


$$L_2 m_{\rm B} = \underbrace{L_2[E_{\rm stat} - \Gamma^{\rm stat}(L_2)]}_{a \to 0 \textbf{ in HQET}} + \underbrace{\sigma_{\rm m}(u_1)}_{a \to 0 \textbf{ in HQET}} + 2 \overbrace{\underbrace{L_1 \Gamma(L_1, M_{\rm b})}_{a \to 0 \textbf{ for } M_{\rm b} L_1 \gg 1}}^{\equiv \Phi_2}$$

$M_{\rm b}$ static

in the static approximation solve:

$$\underbrace{L_2 m_{\rm B}}_{\rm experiment} - L_2[E_{\rm stat} - \Gamma^{\rm stat}(L_1)] - \sigma_{\rm m}(\bar{g}^2(L_1)) = 2\Phi_2^{\rm QCD}(L_1, M_{\rm b})$$



$$M_b^{\rm stat} = \begin{cases} 6771 \pm 99 \text{ MeV (HYP2)} \\ 6757 \pm 99 \text{ MeV (HYP1)} \end{cases}$$

and obtain the slope

$$S = \frac{1}{L_1} \frac{\partial \Phi_2^{\rm QCD}(L_1, M)}{\partial M} = 0.61(5)$$

error dominated by that on renorm. constant of the quark mass.

$M_{\rm b}$ at order $1/m$ [M. Della Morte, N. Garron, M. Papinutto and R. Sommer, 2005-2006]

coefficients in the action:

	$\mathrm{O}(1)$	$m_{\rm bare}(\text{or } \delta m)$	of $\overline{\psi}_{\rm h}\psi_{\rm h}$
	$\mathrm{O}(1/m)$	$\omega_{\rm kin}$	of $\overline{\psi}_{\rm h}(-\frac{1}{2}\mathbf{D}^2)\psi_{\rm h}$
	$\mathrm{O}(1/m)$	$\omega_{\rm spin}$	of $\overline{\psi}_{\rm h}(-\sigma\cdot\mathbf{B})\psi_{\rm h}$

$\omega_{\rm spin}$ cancels in spin averaged quantities

∞ volume $\quad m_{\rm B} = E^{\rm stat} + m_{\rm bare} + \omega_{\rm kin}E^{\rm kin}$

Matching 1 $\quad \Gamma^{\rm QCD}(L,M) = \Gamma^{\rm stat}(L) + m_{\rm bare} + \omega_{\rm kin}\Gamma^{\rm kin}(L) = \frac{\Phi_2^{\rm HQET}}{L}$

Matching 2 $\quad \Phi_1^{\rm QCD}(L) = \omega_{\rm kin}R_1^{\rm kin}(L) = \Phi_1^{\rm HQET}$

$$m_{\rm B} = \left[E^{\rm stat} - \Gamma^{\rm stat}(L)\right] + \Gamma^{\rm QCD}(L,M) + \left[\frac{\Phi_1^{\rm QCD}(L)}{R_1^{\rm kin}(L)}(E^{\rm kin} - \Gamma^{\rm kin}(L))\right]$$

($m_{\rm bare}$, $\omega_{\rm kin}$ eliminated). Set $L = L_2$ and use the SSF to relate L_2 with L_1:

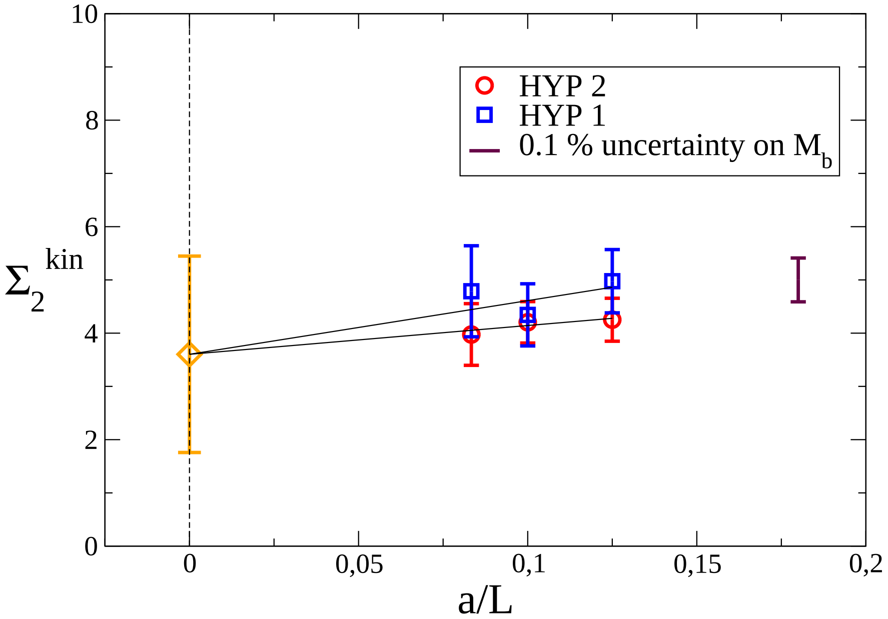
$\Phi_1(2L) = \sigma_1^{\rm kin}(u)\Phi_1(L)$, $\quad \Phi_2(2L) = 2\Phi_2(L) + \sigma_{\rm m}(u) + \sigma_2^{\rm kin}(u)\Phi_1(L)$

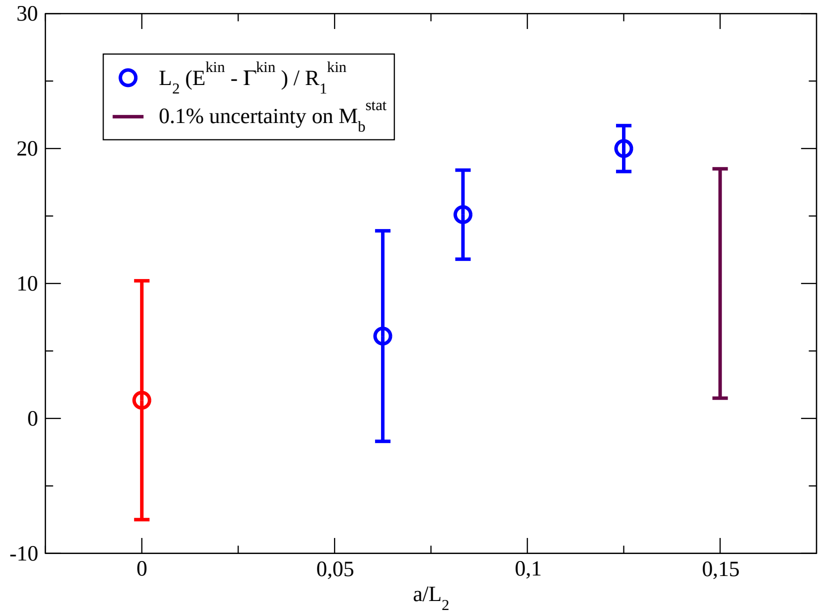
$\Rightarrow m_{\rm B} = m_{\rm B}^{\rm stat} + m_{\rm B}^{(1a)} + m_{\rm B}^{(1b)}$, $\quad m_{\rm B}(M_{\rm b}^{\rm stat} + M_{\rm b}^{(1a)} + M_{\rm b}^{(1b)}) = m_{\rm B}^{\rm exp}$

Continuum extrapolations at order $1/m$

Most difficult steps of the computation:

$$m_{\rm B}^{(1a)}(M) = \frac{1}{L_2}\sigma_2^{\rm kin}(u_1)\,\Phi_1(L_1,M) \qquad m_{\rm B}^{(1b)}(M) = \frac{(E^{\rm kin}-\Gamma_1^{\rm kin}(L_2))}{R_1^{\rm kin}}\Phi_1(L_2,M)$$





In $\sigma_2^{\rm kin}(u_1)$ and $(E^{\rm kin} - \Gamma_1^{\rm kin}(L_2))$ cancellation of $1/a^2$ power divergences (extrapolation linear in a)

Results at order $1/m$

then the $1/m$ correction to $M_{\rm b}$ is

$$M_{\rm b}^{(1)} = M_{\rm b}^{(1a)} + M_{\rm b}^{(1b)}$$

$$M_b^{(1{\rm a})} = -\frac{\sigma_2^{\rm kin}(\bar{g}^2(L_1))\Phi_1(L_1, M_b^{\rm stat})}{S\,L_2} = -30(15)\,{\rm MeV}$$

$$M_b^{(1{\rm b})} = -\frac{(E^{\rm kin} - \Gamma_1^{\rm kin}(L_2))\Phi_1(L_2, M)}{S R_1^{\rm kin}} = -5(33)\,{\rm MeV}$$

and in the $\overline{\rm MS}$ scheme:

$$m_{\rm b}(m_{\rm b}) = m_{\rm b}^{\rm stat} + m_{\rm b}^{(1)}$$

$$m_{\rm b}^{\rm stat} = 4.35(6)\,{\rm GeV}\,, \quad m_{\rm b}^{(1)} = -0.02(2)\,{\rm GeV}\,.$$

agrees with PDG, despite quenched approximation.

check by using different matching conditions: $f_{\rm A}$ needs ${\rm O}(1/m)$-correction to $A_0^{\rm stat}$ $\Rightarrow$ more step scaling functions $\Rightarrow$ final result agrees up to ${\rm O}(1/m^2)$-corrections

B_B static [F. Palombi, M. Papinutto, C. Pena and H. Wittig, 2005-2007]

$\Delta B = 2$ oscillations: $\langle \bar{B}^0_q | \mathcal{O}^{\Delta B=2}_{\rm LL} | B^0_q \rangle = \frac{8}{3} B_{B_q} f^2_{B_q} m^2_{B_q}$ relevant for UT analysis

Combine relativistic simulations with $m_{\rm q} \approx m_{\rm c}$ and the static limit of HQET to interpolate at $m_{\rm b}$[Becirevic, Gimenez, Martinelli, Papinutto, Reyes 2002] or compute $1/m$ correction to static HQET (to be done)

$$\langle \bar{B}^0_q | \mathcal{O}^{\Delta B=2}_{\rm LL}(m_{\rm b}) | B^0_q \rangle = C_1(m_{\rm b}, \mu)\, \langle \bar{B}^0_q | \hat{Q}^+_1(\mu) | B^0_q \rangle_{\rm HQET} + C_2(m_{\rm b}, \mu)\, \langle \bar{B}^0_q | \hat{Q}^+_2(\mu) | B^0_q \rangle_{\rm HQET} + {\rm O}\left(1/m_{\rm b}\right)$$

for the moment: non-perturbative renormalization in quenched static HQET. Computation of bare matrix elements: on going.

Wilson like fermions are particularly suitable for unquenched simulations but break chirality $\Rightarrow$ renormalization pattern of composite operators complicates with respect to the continuum (mixing with operators of different naïve chirality)

$$\mathcal{O}^{\pm}_{\Gamma_1\Gamma_2} = \frac{1}{2}\left[(\bar{\psi}_{\rm h}\Gamma_1\psi_1)(\bar{\psi}_{\bar{\rm h}}\Gamma_2\psi_2) \pm (\bar{\psi}_{\rm h}\Gamma_1\psi_2)(\bar{\psi}_{\bar{\rm h}}\Gamma_2\psi_1) \right]$$

$$(Q_1^+, Q_2^+) = (\mathcal{O}^+_{\mathrm{VV+AA}}, \mathcal{O}^+_{\mathrm{SS+PP}}) \qquad (\mathcal{Q}_1^+, \mathcal{Q}_2^+) = (\mathcal{O}^+_{\mathrm{VA+AV}}, \mathcal{O}^+_{\mathrm{SP+PS}})$$

Heavy Quark Spin simmetry + $H(3)$ spatial rotations + Time Reversal $\Rightarrow$

$$(Q'^+_1, Q'^+_2) = (Q_1^+, Q_1^+ + 4Q_2^+) \qquad (\mathcal{Q}'^+_1, \mathcal{Q}'^+_2) = (\mathcal{Q}_1^+, \mathcal{Q}_1^+ + 4\mathcal{Q}_2^+)$$

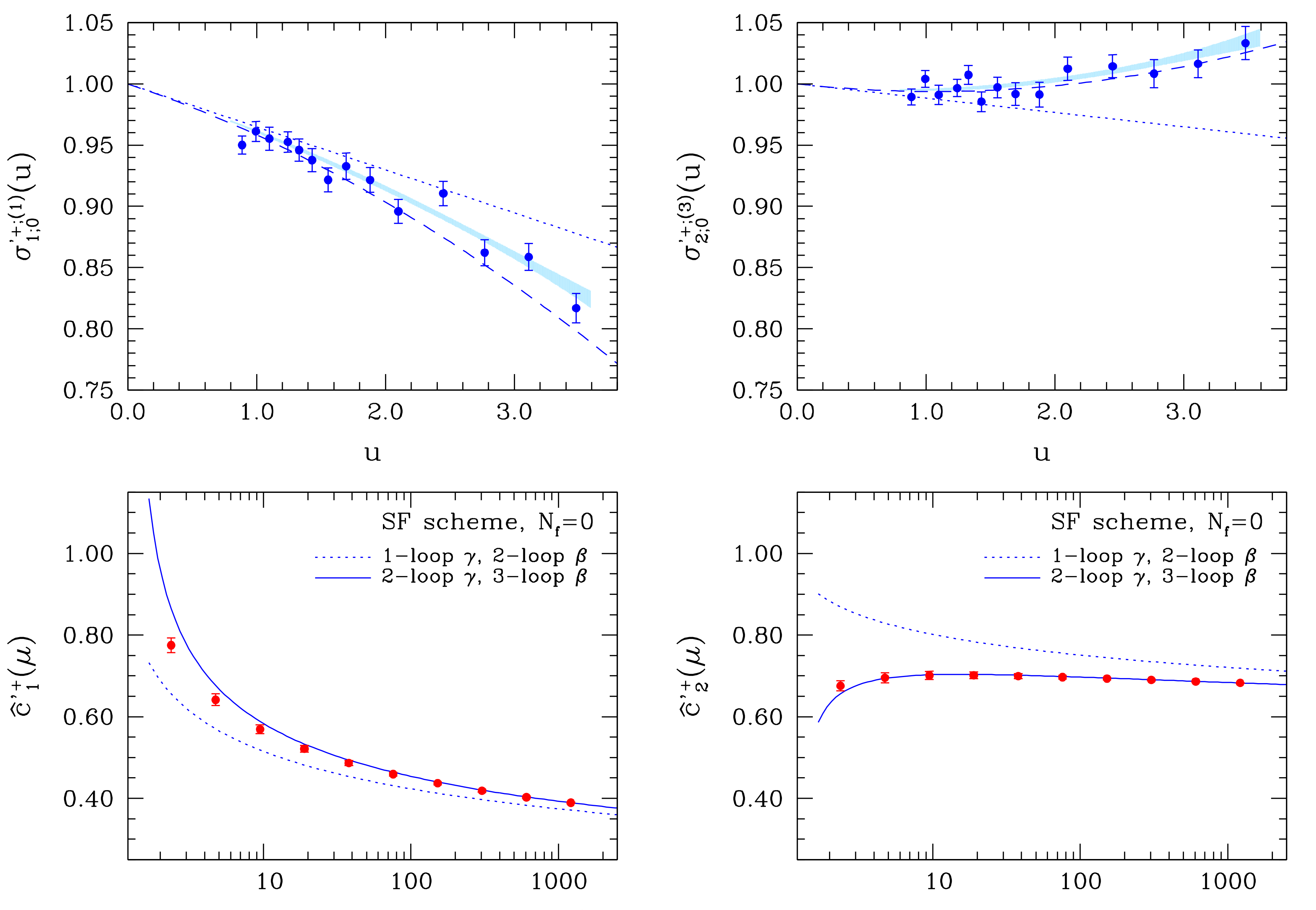
have simplified mixing pattern. The parity odd sector renormalizes multiplicatively

$\Rightarrow$ use HQET for the b quark and a Wilson-like regularization ("twisted mass QCD") for the light quarks. For the renormalized matrix elements it holds:

$$\langle \bar{B}^0_q | \hat{Q}'^+_k(\mu) | B^0_q \rangle_{\mathrm{HQET}} = \mathcal{Z}'^+_k(g_0, a\mu) \langle \bar{B}^0_q | \mathcal{Q}'^+_k(a) | B^0_q \rangle^{\alpha=\pi/2}_{\mathrm{tmQCD}}$$

$\mathcal{Z}'^+_k(g_0, a\mu)$ and its running $\sigma^+_k(u) = U'^+_k(\mu, \mu/2) = \lim_{a\to 0} \left. \frac{\mathcal{Z}'^+_k(g_0, a\mu/2)}{\mathcal{Z}'^+_k(g_0, a\mu)} \right|^{m=0}_{u \equiv \bar{g}^2(\mu)}$ computed non-perturbatively in the SF scheme (where $\mu = 1/L$)

$\Rightarrow$ $\hat{Z}'^+_{k,\mathrm{RGI}}(g_0) = U'^+_k(\infty, \mu_{\mathrm{had}}) \mathcal{Z}'^+_k(g_0, a\mu_{\mathrm{had}})$ computed non-perturbatively



$$\hat{c}'^{+}_{k}(\mu) \equiv U'^{+}_{k}(\infty, \mu)$$

Conclusions and outlook

- In HQET, renormalization of O(1) and O$(1/m)$ terms carried out non-perturbatively and continuum limit taken (for the first time: case of $M_{\rm b}$).

- next steps: $m_{\rm B^*} - m_{\rm B}$ ($\propto \omega_{\rm spin}$) and $F_{\rm B}$ at O$(1/m)$ (needed Φ_i, $i = 1, ..., 4$ to be matched in order to determine the HQET couplings). $F_{\rm B}$ static and interpolation using $F_{\rm PS}$ around $F_{\rm D}$ already performed[ALPHA 2003]

- extension to $N_{\rm f} > 0$: no new problems expected (recent progress in dynamical Wilson-like fermion algorithms [M. Lüscher, 2003-2007; Hasenbusch 2002; Urbach *et al.* 2005]).

- more complicates observables: B_B. non-perturbative renormalization performed. Matrix elements computation still on going. Matching to QCD perturbative. Next steps: non-perturbative matching, O$(1/m)$ terms.

- Further observables $B \to \pi l \nu$ ($\to V_{\rm ub}$),. . .