Generalized Chern-Simons terms in $\mathcal{N} = 1$ supergravity

Jan Rosseel (ITF, K. U. Leuven)

Based on: J. De Rydt, T. Schmidt, A. Van Proeyen, M. Zagermann, J.R., **arXiv:0705.4216**

Manchester, HEP 2007, 20/07/2007

Outline

1. Introduction
2. Symplectic transformations in $\mathcal{N} = 1$ supergravity
3. The kinetic terms of the vector multiplets
4. Generalized Chern-Simons terms
5. Anomalies
6. Cancellation
7. Supergravity and extended supersymmetry
8. Conclusions

Introduction

- Generalized Chern-Simons terms are terms of the form

$$C^{(\mathrm{CS})}_{AB,C}\, W^C \wedge W^A \wedge F^B \,.$$

- They can appear in certain flux compactifications and Scherk-Schwarz compactifications, where they are associated with the gauging of certain axionic shift symmetries (Andrianopoli, d'Auria, Ferrara, Lledo, de Wit, Samtleben, Trigiante).
- Recently their importance has been stressed in anomaly cancellation in orientifold models with intersecting D-branes (Anastasopoulos, Bianchi, Dudas, Kiritsis).
- In extended supersymmetry and supergravity, their presence is well-known (de Wit, Lauwers, Van Proeyen).
- Goal of this talk : Clarify the role of generalized Chern-Simons terms and anomalies in obtaining gauge-invariant and supersymmetric $\mathcal{N} = 1$ actions.

Introduction

- Generalized Chern-Simons terms are terms of the form

$$C^{(\mathrm{CS})}_{AB,C}\, W^C \wedge W^A \wedge F^B \,.$$

- They can appear in certain flux compactifications and Scherk-Schwarz compactifications, where they are associated with the gauging of certain axionic shift symmetries (Andrianopoli, d'Auria, Ferrara, Lledo, de Wit, Samtleben, Trigiante).
- Recently their importance has been stressed in anomaly cancellation in orientifold models with intersecting D-branes (Anastasopoulos, Bianchi, Dudas, Kiritsis).
- In extended supersymmetry and supergravity, their presence is well-known (de Wit, Lauwers, Van Proeyen).
- Goal of this talk : Clarify the role of generalized Chern-Simons terms and anomalies in obtaining gauge-invariant and supersymmetric $\mathcal{N}=1$ actions.

Introduction

- Generalized Chern-Simons terms are terms of the form

$$C^{(\mathrm{CS})}_{AB,C}\, W^C \wedge W^A \wedge F^B \,.$$

- They can appear in certain flux compactifications and Scherk-Schwarz compactifications, where they are associated with the gauging of certain axionic shift symmetries (Andrianopoli, d'Auria, Ferrara, Lledo, de Wit, Samtleben, Trigiante).
- Recently their importance has been stressed in anomaly cancellation in orientifold models with intersecting D-branes (Anastasopoulos, Bianchi, Dudas, Kiritsis).
- In extended supersymmetry and supergravity, their presence is well-known (de Wit, Lauwers, Van Proeyen).
- Goal of this talk : Clarify the role of generalized Chern-Simons terms and anomalies in obtaining gauge-invariant and supersymmetric $\mathcal{N} = 1$ actions.

Introduction

- Generalized Chern-Simons terms are terms of the form

$$C^{(\mathrm{CS})}_{AB,C}\, W^C \wedge W^A \wedge F^B \,.$$

- They can appear in certain flux compactifications and Scherk-Schwarz compactifications, where they are associated with the gauging of certain axionic shift symmetries (Andrianopoli, d'Auria, Ferrara, Lledo, de Wit, Samtleben, Trigiante).
- Recently their importance has been stressed in anomaly cancellation in orientifold models with intersecting D-branes (Anastasopoulos, Bianchi, Dudas, Kiritsis).
- In extended supersymmetry and supergravity, their presence is well-known (de Wit, Lauwers, Van Proeyen).
- Goal of this talk : Clarify the role of generalized Chern-Simons terms and anomalies in obtaining gauge-invariant and supersymmetric $\mathcal{N}=1$ actions.

Introduction

- Generalized Chern-Simons terms are terms of the form

$$C^{(\mathrm{CS})}_{AB,C}\, W^C \wedge W^A \wedge F^B\,.$$

- They can appear in certain flux compactifications and Scherk-Schwarz compactifications, where they are associated with the gauging of certain axionic shift symmetries (Andrianopoli, d'Auria, Ferrara, Lledo, de Wit, Samtleben, Trigiante).
- Recently their importance has been stressed in anomaly cancellation in orientifold models with intersecting D-branes (Anastasopoulos, Bianchi, Dudas, Kiritsis).
- In extended supersymmetry and supergravity, their presence is well-known (de Wit, Lauwers, Van Proeyen).
- Goal of this talk : Clarify the role of generalized Chern-Simons terms and anomalies in obtaining gauge-invariant and supersymmetric $\mathcal{N} = 1$ actions.

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- Consider $\mathcal{N} = 1$ supergravity coupled to chiral multiplets and vector multiplets.
- The kinetic terms for the vector fields read:

$$e^{-1}\mathcal{L}_1 = -\tfrac{1}{4}\,\mathrm{Re} f_{AB} F_{\mu\nu}^{A} F^{\mu\nu\, B} + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im} f_{AB} F_{\mu\nu}^{A} \tilde{F}^{\mu\nu\, B}$$

  The gauge kinetic function $f_{AB}(z)$ depends holomorphically on the scalar fields z^i.
- Gauge transformation under which z^i transform non-trivially can induce a gauge transformation of $f_{AB}(z)$.
- E.g. : gauge kinetic function transforms as a symmetric two-tensor in the adjoint representation.

$$\delta(\Lambda) f_{AB} = \Lambda^C \delta_C f_{AB}\,, \qquad \delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD}\,,$$

  where $\Lambda^A(x)$ are the parameters of the gauge transformations and $f_{AB}{}^C$ are the structure constants.
  $\Rightarrow$ kinetic terms of the vectors are obviously gauge invariant.

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- Consider $\mathcal{N} = 1$ supergravity coupled to chiral multiplets and vector multiplets.
- The kinetic terms for the vector fields read:

$$e^{-1}\mathcal{L}_1 = -\tfrac{1}{4}\,\mathrm{Re} f_{AB} F^{A}_{\mu\nu} F^{\mu\nu\,B} + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im} f_{AB} F^{A}_{\mu\nu} \tilde{F}^{\mu\nu\,B}$$

  The gauge kinetic function $f_{AB}(z)$ depends holomorphically on the scalar fields z^i.
- Gauge transformation under which z^i transform non-trivially can induce a gauge transformation of $f_{AB}(z)$.
- E.g. : gauge kinetic function transforms as a symmetric two-tensor in the adjoint representation.

$$\delta(\Lambda) f_{AB} = \Lambda^{C} \delta_{C} f_{AB}\,, \qquad \delta_{C} f_{AB} = f_{CA}{}^{D} f_{BD} + f_{CB}{}^{D} f_{AD}\,,$$

  where $\Lambda^{A}(x)$ are the parameters of the gauge transformations and $f_{AB}{}^{C}$ are the structure constants.
  $\Rightarrow$ kinetic terms of the vectors are obviously gauge invariant.

Symplectic transformations in $\mathcal{N}=1$ supergravity

- Consider $\mathcal{N}=1$ supergravity coupled to chiral multiplets and vector multiplets.
- The kinetic terms for the vector fields read:

$$e^{-1}\mathcal{L}_1 = -\tfrac{1}{4}\,\mathrm{Re} f_{AB} F^A_{\mu\nu} F^{\mu\nu\,B} + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im} f_{AB} F^A_{\mu\nu} \tilde{F}^{\mu\nu\,B}$$

  The gauge kinetic function $f_{AB}(z)$ depends holomorphically on the scalar fields z^i.
- Gauge transformation under which z^i transform non-trivially can induce a gauge transformation of $f_{AB}(z)$.
- E.g. : gauge kinetic function transforms as a symmetric two-tensor in the adjoint representation.

$$\delta(\Lambda) f_{AB} = \Lambda^C \delta_C f_{AB}\,, \qquad \delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD}\,,$$

  where $\Lambda^A(x)$ are the parameters of the gauge transformations and $f_{AB}{}^C$ are the structure constants.
  $\Rightarrow$ kinetic terms of the vectors are obviously gauge invariant.

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- Consider $\mathcal{N} = 1$ supergravity coupled to chiral multiplets and vector multiplets.
- The kinetic terms for the vector fields read:

$$e^{-1}\mathcal{L}_1 = -\tfrac{1}{4}\,\mathrm{Re} f_{AB} F^{A}_{\mu\nu} F^{\mu\nu\,B} + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im} f_{AB} F^{A}_{\mu\nu} \tilde{F}^{\mu\nu\,B}$$

  The gauge kinetic function $f_{AB}(z)$ depends holomorphically on the scalar fields z^i.
- Gauge transformation under which z^i transform non-trivially can induce a gauge transformation of $f_{AB}(z)$.
- E.g. : gauge kinetic function transforms as a symmetric two-tensor in the adjoint representation.

$$\delta(\Lambda) f_{AB} = \Lambda^{C}\delta_{C} f_{AB}\,, \qquad \delta_{C} f_{AB} = f_{CA}{}^{D} f_{BD} + f_{CB}{}^{D} f_{AD}\,,$$

  where $\Lambda^{A}(x)$ are the parameters of the gauge transformations and $f_{AB}{}^{C}$ are the structure constants.
  $\Rightarrow$ kinetic terms of the vectors are obviously gauge invariant.

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- In fact : a more general transformation law for the gauge kinetic function is allowed, as is suggested by the symplectic formulation of $\mathcal{N} = 1$ supergravity.
- Rewrite the kinetic terms of the vectors as:

$$e^{-1}\mathcal{L}_1 = -\frac{1}{2}\mathrm{Im}\left(F^{-A}_{\mu\nu}G^{\mu\nu-}_{A}\right), \quad G^{\mu\nu\,-}_{A} = -2\mathrm{i}\frac{\partial e^{-1}\mathcal{L}_1}{\partial F^{-A}_{\mu\nu}} = \mathrm{i}f_{AB}F^{\mu\nu\,-B}.$$

The combined set of field equations and Bianchi identities

$$\begin{aligned} \partial^\mu \,\mathrm{Im}\, F^{A\,-}_{\mu\nu} &= 0 \quad \text{Bianchi identities,} \\ \partial_\mu \,\mathrm{Im}\, G^{\mu\nu\,-}_{A} &= 0 \quad \text{Equations of motion.} \end{aligned}$$

is then invariant under the symplectic transformations

$$\begin{pmatrix} F'^{-} \\ G'^{-} \end{pmatrix} = \mathcal{S}\begin{pmatrix} F^{-} \\ G^{-} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} F^{-} \\ G^{-} \end{pmatrix}, \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(2n, \mathbb{R}).$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- In fact : a more general transformation law for the gauge kinetic function is allowed, as is suggested by the symplectic formulation of $\mathcal{N} = 1$ supergravity.
- Rewrite the kinetic terms of the vectors as:

$$e^{-1}\mathcal{L}_1 = -\frac{1}{2}\mathrm{Im}\Big(F_{\mu\nu}^{-A} G_A^{\mu\nu -}\Big)\,, \quad G_A^{\mu\nu\,-} = -2\mathrm{i}\frac{\partial e^{-1}\mathcal{L}_1}{\partial F_{\mu\nu}^{-A}} = \mathrm{i}f_{AB}F^{\mu\nu\,-B}\,.$$

The combined set of field equations and Bianchi identities

$$\begin{aligned} \partial^\mu \,\mathrm{Im}\, F_{\mu\nu}^{A\,-} &= 0 \quad \text{Bianchi identities,} \\ \partial_\mu \,\mathrm{Im}\, G_A^{\mu\nu\,-} &= 0 \quad \text{Equations of motion.} \end{aligned}$$

is then invariant under the symplectic transformations

$$\begin{pmatrix} F'^- \\ G'^- \end{pmatrix} = \mathcal{S}\begin{pmatrix} F^- \\ G^- \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} F^- \\ G^- \end{pmatrix}\,, \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(2n,\mathbb{R})\,.$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- Under these symplectic transformations, the gauge kinetic function transforms as:

$$\mathrm{i}f' = (C + D\mathrm{i}f)(A + B\mathrm{i}f)^{-1}\,.$$

- Symmetries of the action then correspond to transformations with $B = 0$. If $C \neq 0$:

$$\begin{aligned} e^{-1}\mathcal{L}'_1 &= -\tfrac{1}{2}\,\mathrm{Im}(F'^{-A}_{\mu\nu} G'^{\mu\nu -}_{A}) \\ &= -\tfrac{1}{2}\,\mathrm{Im}(F^{-A}_{\mu\nu} G^{\mu\nu -}_{A} + F^{-A}_{\mu\nu}(C^T A)_{AB} F^{B\mu\nu -})\,. \end{aligned} \qquad (2.1)$$

  For rigid symmetries, the last term represents a total derivative.

- In order to promote rigid symmetries to gauge symmetries, the $F^A_{\mu\nu}$ have to transform in adjoint representation of the gauge group. For these transformations, the symplectic matrix reads

$$\mathcal{S} = \mathbb{1} - \Lambda^C \mathcal{S}_C\,, \qquad \mathcal{S}_C = \begin{pmatrix} f_{CB}{}^A & 0 \\ C_{AB,C} & -f_{CA}{}^B \end{pmatrix},$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- Under these symplectic transformations, the gauge kinetic function transforms as:

$$if' = (C + Dif)(A + Bif)^{-1} .$$

- Symmetries of the action then correspond to transformations with $B = 0$. If $C \neq 0$:

$$\begin{aligned} e^{-1}\mathcal{L}'_1 &= -\tfrac{1}{2}\,\mathrm{Im}(F'^{-A}_{\mu\nu} G'^{\mu\nu -}_A) \\ &= -\tfrac{1}{2}\,\mathrm{Im}(F^{-A}_{\mu\nu} G^{\mu\nu -}_A + F^{-A}_{\mu\nu}(C^T A)_{AB} F^{B\mu\nu -}) . \end{aligned} \quad (2.1)$$

For rigid symmetries, the last term represents a total derivative.

- In order to promote rigid symmetries to gauge symmetries, the $F^A_{\mu\nu}$ have to transform in adjoint representation of the gauge group. For these transformations, the symplectic matrix reads

$$\mathcal{S} = \mathbb{1} - \Lambda^C \mathcal{S}_C , \qquad \mathcal{S}_C = \begin{pmatrix} f_{CB}{}^A & 0 \\ C_{AB,C} & -f_{CA}{}^B \end{pmatrix} ,$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- Under these symplectic transformations, the gauge kinetic function transforms as:

$$if' = (C + Dif)(A + Bif)^{-1}\,.$$

- Symmetries of the action then correspond to transformations with $B = 0$. If $C \neq 0$:

$$\begin{aligned} e^{-1}\mathcal{L}'_1 &= -\tfrac{1}{2}\,\mathrm{Im}(F'^{-A}_{\mu\nu} G'^{\mu\nu -}_{A}) \\ &= -\tfrac{1}{2}\,\mathrm{Im}(F^{-A}_{\mu\nu} G^{\mu\nu -}_{A} + F^{-A}_{\mu\nu}(C^T A)_{AB} F^{B\mu\nu -})\,. \end{aligned} \tag{2.1}$$

  For rigid symmetries, the last term represents a total derivative.
- In order to promote rigid symmetries to gauge symmetries, the $F^A_{\mu\nu}$ have to transform in adjoint representation of the gauge group. For these transformations, the symplectic matrix reads

$$\mathcal{S} = \mathbb{1} - \Lambda^C \mathcal{S}_C\,, \qquad \mathcal{S}_C = \begin{pmatrix} f_{CB}{}^A & 0 \\ C_{AB,C} & -f_{CA}{}^B \end{pmatrix}\,,$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- This reasoning suggests that we can allow for a more general transformation rule for the gauge kinetic function:

$$\delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD} + \mathrm{i} C_{AB,C} \,.$$

- Example:

$$f_{AB} = h_{ABi} z^i \,, \quad \delta z^i = \mathrm{i} M^i_C \Lambda^C \quad \Rightarrow \quad C_{AB,C} = h_{ABi} M^i_C \,,$$

- Note that the kinetic terms of the vectors are no longer gauge invariant:

$$\delta_{\text{gauge}} e^{-1} \mathcal{L}_1 = \frac{1}{4} \mathrm{i} C_{AB,C} \Lambda^C \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B} \,.$$

For rigid symmetries : total derivative. For gauge symmetries : no longer gauge invariant.

- $C_{AB,C}$ is symmetric in its first two indices.

$$C_{AB,C} \quad = \quad C^{(s)}_{AB,C} \quad + \quad C^{(m)}_{AB,C}$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- This reasoning suggests that we can allow for a more general transformation rule for the gauge kinetic function:

$$\delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD} + \mathrm{i} C_{AB,C} \,.$$

- Example:

$$f_{AB} = h_{ABi} z^i \,, \quad \delta z^i = \mathrm{i} M^i_C \Lambda^C \quad \Rightarrow \quad C_{AB,C} = h_{ABi} M^i_C \,,$$

- Note that the kinetic terms of the vectors are no longer gauge invariant:

$$\delta_{\text{gauge}} e^{-1} \mathcal{L}_1 = \frac{1}{4} \mathrm{i} C_{AB,C} \Lambda^C \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B} \,.$$

For rigid symmetries : total derivative. For gauge symmetries : no longer gauge invariant.

- $C_{AB,C}$ is symmetric in its first two indices.

$$C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- This reasoning suggests that we can allow for a more general transformation rule for the gauge kinetic function:

$$\delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD} + \mathrm{i}C_{AB,C}\,.$$

- Example:

$$f_{AB} = h_{ABi} z^i\,, \quad \delta z^i = \mathrm{i}M^i_C \Lambda^C \quad \Rightarrow \quad C_{AB,C} = h_{ABi} M^i_C\,,$$

- Note that the kinetic terms of the vectors are no longer gauge invariant:

$$\delta_{\mathrm{gauge}} e^{-1}\mathcal{L}_1 = \frac{1}{4}\mathrm{i}C_{AB,C}\Lambda^C \mathcal{F}^A_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu B}\,.$$

  For rigid symmetries : total derivative. For gauge symmetries : no longer gauge invariant.

- $C_{AB,C}$ is symmetric in its first two indices.

$$C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- This reasoning suggests that we can allow for a more general transformation rule for the gauge kinetic function:

$$\delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD} + \mathrm{i} C_{AB,C}\,.$$

- Example:

$$f_{AB} = h_{ABi} z^i\,, \quad \delta z^i = \mathrm{i} M^i_C \Lambda^C \quad \Rightarrow \quad C_{AB,C} = h_{ABi} M^i_C\,,$$

- Note that the kinetic terms of the vectors are no longer gauge invariant:

$$\delta_{\text{gauge}} e^{-1} \mathcal{L}_1 = \frac{1}{4} \mathrm{i} C_{AB,C} \Lambda^C \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B}\,.$$

For rigid symmetries : total derivative. For gauge symmetries : no longer gauge invariant.

- $C_{AB,C}$ is symmetric in its first two indices.

$$C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$$

Symplectic transformations in $\mathcal{N} = 1$ supergravity

- This reasoning suggests that we can allow for a more general transformation rule for the gauge kinetic function:

$$\delta_C f_{AB} = f_{CA}{}^D f_{BD} + f_{CB}{}^D f_{AD} + \mathrm{i} C_{AB,C} \,.$$

- Example:

$$f_{AB} = h_{ABi} z^i \,, \quad \delta z^i = \mathrm{i} M^i_C \Lambda^C \quad \Rightarrow \quad C_{AB,C} = h_{ABi} M^i_C \,,$$

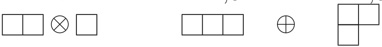
- Note that the kinetic terms of the vectors are no longer gauge invariant:

$$\delta_{\text{gauge}} e^{-1} \mathcal{L}_1 = \frac{1}{4} \mathrm{i} C_{AB,C} \Lambda^C \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B} \,.$$

  For rigid symmetries : total derivative. For gauge symmetries : no longer gauge invariant.

- $C_{AB,C}$ is symmetric in its first two indices.

$$C_{AB,C} \quad = \quad C^{(s)}_{AB,C} \quad + \quad C^{(m)}_{AB,C}$$



The kinetic terms of the vector multiplet

- Consider the full kinetic terms of the vector multiplet in $\mathcal{N}=1$ rigid supersymmetry:

$$\begin{aligned} S_f &= \int \mathrm{d}^4x\mathrm{d}^2\theta f_{AB}(X) W_\alpha^A W_\beta^B \varepsilon^{\alpha\beta} + \ c.c. \\ &= \int \mathrm{d}^4x \Big[-\tfrac{1}{4}\operatorname{Re} f_{AB} \mathcal{F}_{\mu\nu}^A \mathcal{F}^{\mu\nu B} - \tfrac{1}{2}\operatorname{Re} f_{AB} \bar{\lambda}^A \not{\mathcal{D}} \lambda^B \\ &\qquad + \tfrac{1}{4}\mathrm{i}\operatorname{Im} f_{AB} \mathcal{F}_{\mu\nu}^A \tilde{\mathcal{F}}^{\mu\nu B} + \tfrac{1}{4}\mathrm{i}(\mathcal{D}_\mu \operatorname{Im} f_{AB}) \bar{\lambda}^A \gamma^5 \gamma^\mu \lambda^B \Big], \end{aligned}$$

$$\mathcal{D}_\mu f_{AB} = \partial_\mu f_{AB} - 2W_\mu^C f_{C(A}{}^D f_{B)D}\,.$$

- To covariantize with respect to the more general transformation rule of f_{AB}

$$\hat{\mathcal{D}}_\mu f_{AB} = \partial_\mu f_{AB} - 2W_\mu^C f_{C(A}{}^D f_{B)D} - \mathrm{i}W_\mu^C C_{AB,C}\,.$$

From now on we will consider the action $\hat{S}_f$, where $\mathcal{D}_\mu \to \hat{\mathcal{D}}_\mu$.

The kinetic terms of the vector multiplet

- Consider the full kinetic terms of the vector multiplet in $\mathcal{N} = 1$ rigid supersymmetry:

$$
\begin{aligned}
S_f &= \int \mathrm{d}^4x\mathrm{d}^2\theta\, f_{AB}(X) W_\alpha^A W_\beta^B \varepsilon^{\alpha\beta} + \ c.c. \\
&= \int \mathrm{d}^4x \Big[-\tfrac{1}{4}\,\mathrm{Re}\, f_{AB}\mathcal{F}_{\mu\nu}^A \mathcal{F}^{\mu\nu B} - \tfrac{1}{2}\,\mathrm{Re}\, f_{AB}\bar{\lambda}^A \not{\mathcal{D}}\lambda^B \\
&\qquad + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im}\, f_{AB}\mathcal{F}_{\mu\nu}^A \tilde{\mathcal{F}}^{\mu\nu B} + \tfrac{1}{4}\mathrm{i}(\mathcal{D}_\mu\,\mathrm{Im}\, f_{AB})\bar{\lambda}^A\gamma^5\gamma^\mu\lambda^B \Big],
\end{aligned}
$$

$$
\mathcal{D}_\mu f_{AB} = \partial_\mu f_{AB} - 2W_\mu^C f_{C(A}{}^D f_{B)D}\,.
$$

- To covariantize with respect to the more general transformation rule of f_{AB}

$$
\hat{\mathcal{D}}_\mu f_{AB} = \partial_\mu f_{AB} - 2W_\mu^C f_{C(A}{}^D f_{B)D} - \mathrm{i}W_\mu^C C_{AB,C}\,.
$$

From now on we will consider the action $\hat{S}_f$, where $\mathcal{D}_\mu \to \hat{\mathcal{D}}_\mu$.

The kinetic terms of the vector multiplet

- Consider the full kinetic terms of the vector multiplet in $\mathcal{N} = 1$ rigid supersymmetry:

$$
\begin{aligned}
S_f &= \int \mathrm{d}^4x\mathrm{d}^2\theta f_{AB}(X) W^A_\alpha W^B_\beta \varepsilon^{\alpha\beta} + \ c.c. \\
&= \int \mathrm{d}^4x \Big[-\tfrac{1}{4}\,\mathrm{Re} f_{AB} \mathcal{F}^A_{\mu\nu}\mathcal{F}^{\mu\nu B} - \tfrac{1}{2}\,\mathrm{Re} f_{AB} \bar{\lambda}^A \not{\mathcal{D}} \lambda^B \\
&\qquad + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im} f_{AB} \mathcal{F}^A_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu B} + \tfrac{1}{4}\mathrm{i}(\mathcal{D}_\mu\, \mathrm{Im} f_{AB})\bar{\lambda}^A \gamma^5\gamma^\mu \lambda^B \Big],
\end{aligned}
$$

$$\mathcal{D}_\mu f_{AB} = \partial_\mu f_{AB} - 2W^C_\mu f_{C(A}{}^D f_{B)D}\,.$$

- To covariantize with respect to the more general transformation rule of f_{AB}

$$\hat{\mathcal{D}}_\mu f_{AB} = \partial_\mu f_{AB} - 2W^C_\mu f_{C(A}{}^D f_{B)D} - \mathrm{i} W^C_\mu C_{AB,C}\,.$$

From now on we will consider the action $\hat{S}_f$, where $\mathcal{D}_\mu \to \hat{\mathcal{D}}_\mu$.

The kinetic terms of the vector multiplet

- Consider the full kinetic terms of the vector multiplet in $\mathcal{N} = 1$ rigid supersymmetry:

$$
\begin{aligned}
S_f &= \int \mathrm{d}^4x\mathrm{d}^2\theta\, f_{AB}(X) W^A_\alpha W^B_\beta \varepsilon^{\alpha\beta} + \, c.c. \\
&= \int \mathrm{d}^4x \Big[-\tfrac{1}{4}\,\mathrm{Re}\, f_{AB} \mathcal{F}^A_{\mu\nu} \mathcal{F}^{\mu\nu B} - \tfrac{1}{2}\,\mathrm{Re}\, f_{AB} \bar{\lambda}^A \not{\mathcal{D}} \lambda^B \\
&\qquad + \tfrac{1}{4}\mathrm{i}\,\mathrm{Im}\, f_{AB} \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B} + \tfrac{1}{4}\mathrm{i}(\mathcal{D}_\mu\,\mathrm{Im}\, f_{AB}) \bar{\lambda}^A \gamma^5 \gamma^\mu \lambda^B \Big] ,
\end{aligned}
$$

$$
\mathcal{D}_\mu f_{AB} = \partial_\mu f_{AB} - 2W^C_\mu f_{C(A}{}^D f_{B)D} \,.
$$

- To covariantize with respect to the more general transformation rule of f_{AB}

$$
\hat{\mathcal{D}}_\mu f_{AB} = \partial_\mu f_{AB} - 2W^C_\mu f_{C(A}{}^D f_{B)D} - \mathrm{i} W^C_\mu C_{AB,C} \,.
$$

From now on we will consider the action $\hat{S}_f$, where $\mathcal{D}_\mu \to \hat{\mathcal{D}}_\mu$.

The kinetic terms of the vector multiplet

- In this way, the gauge non-invariance only originates from one term:

$$\delta_{\text{gauge}} e^{-1} \mathcal{L}_1 = \frac{1}{4} \mathrm{i} C_{AB,C} \Lambda^C \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B} \,.$$

- Note the relation between gauge non-invariance and supersymmetry non-invariance:

$$\left\{ Q_\alpha, Q^\dagger_{\dot\alpha} \right\} = \sigma^\mu_{\alpha\dot\alpha} \mathcal{D}_\mu = \sigma^\mu_{\alpha\dot\alpha} (\partial_\mu - W^A_\mu \delta_A) \,. \tag{3.1}$$

- Indeed, the action is not invariant under supersymmetry either:

$$\delta(\epsilon) \hat{S}_f = \int \mathrm{d}^4 x \, \mathrm{Re} \left(\tfrac{1}{2} C_{AB,C} \varepsilon^{\mu\nu\rho\sigma} W^C_\mu \mathcal{F}^A_{\nu\rho} \bar\epsilon_R \gamma_\sigma \lambda^B_L - \tfrac{3}{2} \mathrm{i} C_{(AB,C)} \bar\epsilon_R \lambda^C_R \bar\lambda^A_L \lambda^B_L \right) \,.$$

  Note that this expression only depends on the fields of the vector multiplets.
- In the following we will attempt to construct an action that is invariant under gauge and supersymmetry, by means of *generalized Chern-Simons terms* and *anomalies*.

The kinetic terms of the vector multiplet

- In this way, the gauge non-invariance only originates from one term:

$$\delta_{\text{gauge}} e^{-1} \mathcal{L}_1 = \frac{1}{4} \mathrm{i} C_{AB,C} \Lambda^C \mathcal{F}^A_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu B} \,.$$

- Note the relation between gauge non-invariance and supersymmetry non-invariance:

$$\left\{ Q_\alpha, Q^\dagger_{\dot\alpha} \right\} = \sigma^\mu_{\alpha\dot\alpha} \mathcal{D}_\mu = \sigma^\mu_{\alpha\dot\alpha} (\partial_\mu - W^A_\mu \delta_A) \,. \tag{3.1}$$

- Indeed, the action is not invariant under supersymmetry either:

$$\delta(\epsilon) \hat{S}_f = \int \mathrm{d}^4 x \, \mathrm{Re} \left(\tfrac{1}{2} C_{AB,C} \varepsilon^{\mu\nu\rho\sigma} W^C_\mu \mathcal{F}^A_{\nu\rho} \bar\epsilon_R \gamma_\sigma \lambda^B_L - \tfrac{3}{2} \mathrm{i} C_{(AB,C)} \bar\epsilon_R \lambda^C_R \bar\lambda^A_L \lambda^B_L \right) \,.$$

  Note that this expression only depends on the fields of the vector multiplets.

- In the following we will attempt to construct an action that is invariant under gauge and supersymmetry, by means of *generalized Chern-Simons terms* and *anomalies*.

The kinetic terms of the vector multiplet

- In this way, the gauge non-invariance only originates from one term:

$$\delta_{\text{gauge}} e^{-1}\mathcal{L}_1 = \frac{1}{4}\mathrm{i}C_{AB,C}\Lambda^C \mathcal{F}^A_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu B}\,.$$

- Note the relation between gauge non-invariance and supersymmetry non-invariance:

$$\left\{Q_\alpha, Q^\dagger_{\dot\alpha}\right\} = \sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu = \sigma^\mu_{\alpha\dot\alpha}(\partial_\mu - W^A_\mu\delta_A)\,. \tag{3.1}$$

- Indeed, the action is not invariant under supersymmetry either:

$$\delta(\epsilon)\hat{S}_f = \int \mathrm{d}^4x\,\mathrm{Re}\left(\tfrac{1}{2}C_{AB,C}\varepsilon^{\mu\nu\rho\sigma}W^C_\mu\mathcal{F}^A_{\nu\rho}\bar\epsilon_R\gamma_\sigma\lambda^B_L - \tfrac{3}{2}\mathrm{i}C_{(AB,C)}\bar\epsilon_R\lambda^C_R\bar\lambda^A_L\lambda^B_L\right)\,.$$

  Note that this expression only depends on the fields of the vector multiplets.

- In the following we will attempt to construct an action that is invariant under gauge and supersymmetry, by means of *generalized Chern-Simons terms* and *anomalies*.

The kinetic terms of the vector multiplet

- In this way, the gauge non-invariance only originates from one term:

$$\delta_{\text{gauge}} e^{-1}\mathcal{L}_1 = \frac{1}{4}\mathrm{i}C_{AB,C}\Lambda^C \mathcal{F}^A_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu B}\,.$$

- Note the relation between gauge non-invariance and supersymmetry non-invariance:

$$\left\{Q_\alpha, Q^\dagger_{\dot\alpha}\right\} = \sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu = \sigma^\mu_{\alpha\dot\alpha}(\partial_\mu - W^A_\mu\delta_A)\,. \tag{3.1}$$

- Indeed, the action is not invariant under supersymmetry either:

$$\delta(\epsilon)\hat{S}_f = \int \mathrm{d}^4x\,\mathrm{Re}\left(\tfrac{1}{2}C_{AB,C}\varepsilon^{\mu\nu\rho\sigma}W^C_\mu\mathcal{F}^A_{\nu\rho}\bar\epsilon_R\gamma_\sigma\lambda^B_L - \tfrac{3}{2}\mathrm{i}C_{(AB,C)}\bar\epsilon_R\lambda^C_R\bar\lambda^A_L\lambda^B_L\right)\,.$$

  Note that this expression only depends on the fields of the vector multiplets.
- In the following we will attempt to construct an action that is invariant under gauge and supersymmetry, by means of *generalized Chern-Simons terms* and *anomalies*.

Generalized Chern-Simons terms

- Generalized Chern-Simons terms are (for the general non-abelian case)

$$S_{\mathrm{CS}} = \int \mathrm{d}^4x \, \tfrac{1}{2} C^{(\mathrm{CS})}_{AB,C} \varepsilon^{\mu\nu\rho\sigma} \left(\tfrac{1}{3} W_\mu^C W_\nu^A F_{\rho\sigma}^B + \tfrac{1}{4} f_{DE}{}^A W_\mu^D W_\nu^E W_\rho^C W_\sigma^B \right) .$$

  Proportional to a three-index tensor $C^{(\mathrm{CS})}_{AB,C}$, not necessarily equal to $C_{AB,C}$.

- We can put

$$C^{(\mathrm{CS})}_{(AB,C)} = 0 \,,$$

  They thus correspond to .

- For semi-simple algebras, GCS terms do not bring anything new (de Wit, Hull, Rocek). In that case, one can find a constant, real, symmetric matrix Z_{AB}, such that:

$$C^{(\mathrm{CS})}_{AB,C} = 2 f_{C(A}{}^D Z_{B)D} \,,$$

  In this case, the GCS action can be reabsorbed in the original action S_f by redefining f_{AB}:

$$f'_{AB} = f_{AB} + \mathrm{i} Z_{AB} \,.$$

Generalized Chern-Simons terms

- Generalized Chern-Simons terms are (for the general non-abelian case)

$$S_{\mathrm{CS}} = \int \mathrm{d}^4x \, \tfrac{1}{2} C^{(\mathrm{CS})}_{AB,C} \varepsilon^{\mu\nu\rho\sigma} \left(\tfrac{1}{3} W^C_\mu W^A_\nu F^B_{\rho\sigma} + \tfrac{1}{4} f_{DE}{}^A W^D_\mu W^E_\nu W^C_\rho W^B_\sigma \right) .$$

  Proportional to a three-index tensor $C^{(\mathrm{CS})}_{AB,C}$, not necessarily equal to $C_{AB,C}$.

- We can put

$$C^{(\mathrm{CS})}_{(AB,C)} = 0 \, ,$$

  They thus correspond to .

- For semi-simple algebras, GCS terms do not bring anything new (de Wit, Hull, Rocek). In that case, one can find a constant, real, symmetric matrix Z_{AB}, such that:

$$C^{(\mathrm{CS})}_{AB,C} = 2 f_{C(A}{}^D Z_{B)D} \, ,$$

  In this case, the GCS action can be reabsorbed in the original action S_f by redefining f_{AB}:

$$f'_{AB} = f_{AB} + \mathrm{i} Z_{AB} \, .$$

Generalized Chern-Simons terms

- Generalized Chern-Simons terms are (for the general non-abelian case)

$$S_{\text{CS}} = \int \mathrm{d}^4x \, \tfrac{1}{2} C^{(\text{CS})}_{AB,C} \varepsilon^{\mu\nu\rho\sigma} \left(\tfrac{1}{3} W^C_\mu W^A_\nu F^B_{\rho\sigma} + \tfrac{1}{4} f_{DE}{}^A W^D_\mu W^E_\nu W^C_\rho W^B_\sigma \right) .$$

  Proportional to a three-index tensor $C^{(\text{CS})}_{AB,C}$, not necessarily equal to $C_{AB,C}$.

- We can put

$$C^{(\text{CS})}_{(AB,C)} = 0 \, ,$$

  They thus correspond to .

- For semi-simple algebras, GCS terms do not bring anything new (de Wit, Hull, Rocek). In that case, one can find a constant, real, symmetric matrix Z_{AB}, such that:

$$C^{(\text{CS})}_{AB,C} = 2 f_{C(A}{}^D Z_{B)D} \, ,$$

  In this case, the GCS action can be reabsorbed in the original action S_f by redefining f_{AB}:

$$f'_{AB} = f_{AB} + \mathrm{i} Z_{AB} \, .$$

Anomalies

- Anomalies indicate a non-invariance of the effective action $\Gamma[W_\mu]$:

$$e^{-\Gamma[W_\mu]} = \int \mathcal{D}\bar{\phi}\mathcal{D}\phi e^{-\mathcal{S}(W_\mu,\bar{\phi},\phi)} .$$

$$\delta(\Lambda)\Gamma[W] = -\int d^4x\, \Lambda^A \left(\mathcal{D}_\mu \frac{\delta\Gamma[W]}{\delta W_\mu}\right)_A \equiv \int d^4x\, \Lambda^A \mathcal{A}_A ,$$

- The anomaly satisfies the Wess-Zumino consistency conditions:

$$\delta(\Lambda_1)\left(\Lambda_2^A \mathcal{A}_A\right) - \delta(\Lambda_2)\left(\Lambda_1^A \mathcal{A}_A\right) = \Lambda_1^B \Lambda_2^C f_{BC}{}^A \mathcal{A}_A .$$

- For instance, for an arbitrary non-abelian gauge group, the consistent form of the anomaly is given by:

$$\mathcal{A}_A \sim \varepsilon^{\mu\nu\rho\sigma}\, \mathrm{Tr}\left(T_A \partial_\mu \left(W_\nu \partial_\rho W_\sigma + \tfrac{1}{2} W_\nu W_\rho W_\sigma\right)\right) ,$$

Anomalies

- Anomalies indicate a non-invariance of the effective action $\Gamma[W_\mu]$:

$$\mathrm{e}^{-\Gamma[W_\mu]} = \int \mathcal{D}\bar{\phi}\mathcal{D}\phi \mathrm{e}^{-\mathcal{S}(W_\mu,\bar{\phi},\phi)} .$$

$$\delta(\Lambda)\Gamma[W] = -\int \mathrm{d}^4x\,\Lambda^A \left(\mathcal{D}_\mu \frac{\delta\Gamma[W]}{\delta W_\mu}\right)_A \equiv \int \mathrm{d}^4x\,\Lambda^A \mathcal{A}_A ,$$

- The anomaly satisfies the Wess-Zumino consistency conditions:

$$\delta(\Lambda_1)\left(\Lambda_2^A \mathcal{A}_A\right) - \delta(\Lambda_2)\left(\Lambda_1^A \mathcal{A}_A\right) = \Lambda_1^B \Lambda_2^C f_{BC}{}^A \mathcal{A}_A .$$

- For instance, for an arbitrary non-abelian gauge group, the consistent form of the anomaly is given by:

$$\mathcal{A}_A \sim \varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(T_A\partial_\mu\left(W_\nu\partial_\rho W_\sigma + \tfrac{1}{2}W_\nu W_\rho W_\sigma\right)\right) ,$$

Anomalies

- Anomalies indicate a non-invariance of the effective action $\Gamma[W_\mu]$:

$$\mathrm{e}^{-\Gamma[W_\mu]} = \int \mathcal{D}\bar{\phi}\mathcal{D}\phi \mathrm{e}^{-\mathcal{S}(W_\mu,\bar{\phi},\phi)} \,.$$

$$\delta(\Lambda)\Gamma[W] = -\int \mathrm{d}^4x\, \Lambda^A \left(\mathcal{D}_\mu \frac{\delta\Gamma[W]}{\delta W_\mu}\right)_A \equiv \int \mathrm{d}^4x\, \Lambda^A \mathcal{A}_A \,,$$

- The anomaly satisfies the Wess-Zumino consistency conditions:

$$\delta(\Lambda_1)\left(\Lambda_2^A \mathcal{A}_A\right) - \delta(\Lambda_2)\left(\Lambda_1^A \mathcal{A}_A\right) = \Lambda_1^B \Lambda_2^C f_{BC}{}^A \mathcal{A}_A \,.$$

- For instance, for an arbitrary non-abelian gauge group, the consistent form of the anomaly is given by:

$$\mathcal{A}_A \sim \varepsilon^{\mu\nu\rho\sigma}\, \mathrm{Tr}\left(T_A \partial_\mu \left(W_\nu \partial_\rho W_\sigma + \tfrac{1}{2} W_\nu W_\rho W_\sigma\right)\right) ,$$

Anomalies

- Note that if Γ is non-invariant under gauge transformations, also its supersymmetry variation is non-vanishing:

$$\mathcal{A} = \delta\Gamma(W) = \delta(\Lambda)\Gamma[W] + \delta(\epsilon)\Gamma[W] = \int \mathrm{d}^4x\ \left(\Lambda^A \mathcal{A}_A + \bar{\epsilon}\mathcal{A}_\epsilon\right)\ .$$

- A full cohomological analysis of anomalies in supergravity has been made (Brandt):

$$\begin{aligned} \mathcal{A}_C &= -\tfrac{1}{4}\mathrm{i}\left[d_{ABC}F_{\mu\nu}^B + \left(d_{ABD}f_{CE}{}^B + \tfrac{3}{2}d_{ABC}f_{DE}{}^B\right)W_\mu^D W_\nu^E\right]\tilde{F}^{\mu\nu A}\,, \\ \bar{\epsilon}\mathcal{A}_\epsilon &= \mathrm{Re}\left[\tfrac{3}{2}\mathrm{i}d_{ABC}\bar{\epsilon}_R\lambda_R^C\bar{\lambda}_L^A\lambda_L^B + \mathrm{i}d_{ABC}W_\nu^C\tilde{F}^{\mu\nu A}\bar{\epsilon}_L\gamma_\mu\lambda_R^B\right. \\ &\qquad \left. + \tfrac{3}{8}d_{ABC}f_{DE}{}^A\varepsilon^{\mu\nu\rho\sigma}W_\mu^D W_\nu^E W_\sigma^C\bar{\epsilon}_L\gamma_\rho\lambda_R^B\right]\ . \end{aligned}$$

- The coefficients d_{ABC} form a totally symmetric tensor, given in terms of generators of the gauge group:

$$d_{ABC} \sim \mathrm{Tr}(\{T_A, T_B\}T_C)\,.$$

Anomalies

- Note that if Γ is non-invariant under gauge transformations, also its supersymmetry variation is non-vanishing:

$$\mathcal{A} = \delta\Gamma(W) = \delta(\Lambda)\Gamma[W] + \delta(\epsilon)\Gamma[W] = \int \mathrm{d}^4x \, \left(\Lambda^A \mathcal{A}_A + \bar{\epsilon}\mathcal{A}_\epsilon\right) \, .$$

- A full cohomological analysis of anomalies in supergravity has been made (Brandt):

$$\begin{aligned}
\mathcal{A}_C &= -\tfrac{1}{4}\mathrm{i}\left[d_{ABC}F_{\mu\nu}^B + \left(d_{ABD}f_{CE}{}^B + \tfrac{3}{2}d_{ABC}f_{DE}{}^B\right)W_\mu^D W_\nu^E\right]\tilde{F}^{\mu\nu A} \, , \\
\bar{\epsilon}\mathcal{A}_\epsilon &= \mathrm{Re}\left[\tfrac{3}{2}\mathrm{i}d_{ABC}\bar{\epsilon}_R\lambda_R^C\bar{\lambda}_L^A\lambda_L^B + \mathrm{i}d_{ABC}W_\nu^C\tilde{F}^{\mu\nu A}\bar{\epsilon}_L\gamma_\mu\lambda_R^B \right. \\
&\qquad \left. + \tfrac{3}{8}d_{ABC}f_{DE}{}^A\varepsilon^{\mu\nu\rho\sigma}W_\mu^D W_\nu^E W_\sigma^C\bar{\epsilon}_L\gamma_\rho\lambda_R^B\right] \, .
\end{aligned}$$

- The coefficients d_{ABC} form a totally symmetric tensor, given in terms of generators of the gauge group:

$$d_{ABC} \sim \mathrm{Tr}(\{T_A, T_B\}T_C) \, .$$

Anomalies

- Note that if Γ is non-invariant under gauge transformations, also its supersymmetry variation is non-vanishing:

$$\mathcal{A} = \delta\Gamma(W) = \delta(\Lambda)\Gamma[W] + \delta(\epsilon)\Gamma[W] = \int \mathrm{d}^4x \, \left(\Lambda^A \mathcal{A}_A + \bar{\epsilon}\mathcal{A}_\epsilon\right) \, .$$

- A full cohomological analysis of anomalies in supergravity has been made (Brandt):

$$\begin{aligned}
\mathcal{A}_C &= -\tfrac{1}{4}\mathrm{i}\left[d_{ABC}F^B_{\mu\nu} + \left(d_{ABD}f_{CE}{}^B + \tfrac{3}{2}d_{ABC}f_{DE}{}^B\right) W^D_\mu W^E_\nu\right] \tilde{F}^{\mu\nu A} \, , \\
\bar{\epsilon}\mathcal{A}_\epsilon &= \mathrm{Re}\left[\tfrac{3}{2}\mathrm{i}d_{ABC}\bar{\epsilon}_R\lambda^C_R\bar{\lambda}^A_L\lambda^B_L + \mathrm{i}d_{ABC}W^C_\nu\tilde{F}^{\mu\nu A}\bar{\epsilon}_L\gamma_\mu\lambda^B_R \right. \\
&\quad \left. + \tfrac{3}{8}d_{ABC}f_{DE}{}^A\varepsilon^{\mu\nu\rho\sigma}W^D_\mu W^E_\nu W^C_\sigma\bar{\epsilon}_L\gamma_\rho\lambda^B_R\right] \, .
\end{aligned}$$

- The coefficients d_{ABC} form a totally symmetric tensor, given in terms of generators of the gauge group:

$$d_{ABC} \sim \mathrm{Tr}(\{T_A, T_B\}T_C) \, .$$

Cancellation

- In order to achieve cancellation, we set:

$$C^{(\mathrm{CS})}_{AB,C} = C^{(m)}_{AB,C} = C_{AB,C} - C^{(\mathrm{CS})}_{AB,C}\,.$$

- Using this identification, the sum of $\hat{S}_f + S_{\mathrm{CS}}$ is still not gauge- and supersymmetry-invariant. However:

$$\begin{aligned} \delta(\Lambda)\left(\hat{S}_f + S_{\mathrm{CS}}\right) &= -\int \mathrm{d}^4x\, \Lambda^A \mathcal{A}^{(s)}_A\,, \\ \delta(\epsilon)\left(\hat{S}_f + S_{\mathrm{CS}}\right) &= -\int \mathrm{d}^4x\, \bar{\epsilon}\mathcal{A}^{(s)}_\epsilon\,, \end{aligned}$$

where $\mathcal{A}^{(s)}$ represents the expression for the anomaly with $d_{ABC} \to C^{(s)}_{AB,C}$.
- In other words, when

$$C^{(\mathrm{CS})}_{AB,C} = C^{(m)}_{AB,C}\,, \qquad C^{(s)}_{AB,C} = d_{ABC}$$

gauge and supersymmetry invariance are restored.

Cancellation

- In order to achieve cancellation, we set:

$$C^{(\mathrm{CS})}_{AB,C} = C^{(m)}_{AB,C} = C_{AB,C} - C^{(\mathrm{CS})}_{AB,C} .$$

- Using this identification, the sum of $\hat{S}_f + S_{\mathrm{CS}}$ is still not gauge- and supersymmetry-invariant. However:

$$\begin{aligned} \delta(\Lambda)\left(\hat{S}_f + S_{\mathrm{CS}}\right) &= -\int \mathrm{d}^4x\, \Lambda^A \mathcal{A}^{(s)}_A , \\ \delta(\epsilon)\left(\hat{S}_f + S_{\mathrm{CS}}\right) &= -\int \mathrm{d}^4x\, \bar{\epsilon}\mathcal{A}^{(s)}_\epsilon , \end{aligned}$$

where $\mathcal{A}^{(s)}$ represents the expression for the anomaly with $d_{ABC} \to C^{(s)}_{AB,C}$.

- In other words, when

$$C^{(\mathrm{CS})}_{AB,C} = C^{(m)}_{AB,C} , \qquad C^{(s)}_{AB,C} = d_{ABC}$$

gauge and supersymmetry invariance are restored.

Cancellation

- In order to achieve cancellation, we set:

$$C^{(\mathrm{CS})}_{AB,C} = C^{(m)}_{AB,C} = C_{AB,C} - C^{(\mathrm{CS})}_{AB,C} \,.$$

- Using this identification, the sum of $\hat{S}_f + S_{\mathrm{CS}}$ is still not gauge- and supersymmetry-invariant. However:

$$\begin{aligned} \delta(\Lambda)\Big(\hat{S}_f + S_{\mathrm{CS}}\Big) &= -\int \mathrm{d}^4x\, \Lambda^A \mathcal{A}^{(s)}_A \,, \\ \delta(\epsilon)\Big(\hat{S}_f + S_{\mathrm{CS}}\Big) &= -\int \mathrm{d}^4x\, \bar{\epsilon}\mathcal{A}^{(s)}_\epsilon \,, \end{aligned}$$

where $\mathcal{A}^{(s)}$ represents the expression for the anomaly with $d_{ABC} \to C^{(s)}_{AB,C}$.

- In other words, when

$$C^{(\mathrm{CS})}_{AB,C} = C^{(m)}_{AB,C} \,, \qquad C^{(s)}_{AB,C} = d_{ABC}$$

gauge and supersymmetry invariance are restored.

Supergravity and extended supersymmetry

- So far, we considered rigid supersymmetry. What about supergravity?
  No extra GCS terms are needed to achieve cancellation.
- All extra contributions (e.g. gravitino contributions) that were not present in susy variation for rigid supersymmetry, vanish without need of extra terms.
- No new contributions to gauge non-invariance.
- In extended supersymmetry : Generalized Chern-Simons terms have been considered, for restoring gauge and supersymmetry invariance.
- Note that in extended supersymmetry, one can show that:

$$C_{(AB,C)} = 0\,.$$

  This is consistent with the fact that only $\mathcal{N} = 1$ theories have chiral fermions that lead to anomalies.

Supergravity and extended supersymmetry

- So far, we considered rigid supersymmetry. What about supergravity?
  No extra GCS terms are needed to achieve cancellation.
- All extra contributions (e.g. gravitino contributions) that were not present in susy variation for rigid supersymmetry, vanish without need of extra terms.
- No new contributions to gauge non-invariance.
- In extended supersymmetry : Generalized Chern-Simons terms have been considered, for restoring gauge and supersymmetry invariance.
- Note that in extended supersymmetry, one can show that:

$$C_{(AB,C)} = 0 \,.$$

  This is consistent with the fact that only $\mathcal{N} = 1$ theories have chiral fermions that lead to anomalies.

Supergravity and extended supersymmetry

- So far, we considered rigid supersymmetry. What about supergravity? No extra GCS terms are needed to achieve cancellation.
- All extra contributions (e.g. gravitino contributions) that were not present in susy variation for rigid supersymmetry, vanish without need of extra terms.
- No new contributions to gauge non-invariance.
- In extended supersymmetry : Generalized Chern-Simons terms have been considered, for restoring gauge and supersymmetry invariance.
- Note that in extended supersymmetry, one can show that:

$$C_{(AB,C)} = 0 \, .$$

This is consistent with the fact that only $\mathcal{N} = 1$ theories have chiral fermions that lead to anomalies.

Supergravity and extended supersymmetry

- So far, we considered rigid supersymmetry. What about supergravity? No extra GCS terms are needed to achieve cancellation.
- All extra contributions (e.g. gravitino contributions) that were not present in susy variation for rigid supersymmetry, vanish without need of extra terms.
- No new contributions to gauge non-invariance.
- In extended supersymmetry : Generalized Chern-Simons terms have been considered, for restoring gauge and supersymmetry invariance.
- Note that in extended supersymmetry, one can show that:

$$C_{(AB,C)} = 0\,.$$

This is consistent with the fact that only $\mathcal{N} = 1$ theories have chiral fermions that lead to anomalies.

Supergravity and extended supersymmetry

- So far, we considered rigid supersymmetry. What about supergravity? No extra GCS terms are needed to achieve cancellation.
- All extra contributions (e.g. gravitino contributions) that were not present in susy variation for rigid supersymmetry, vanish without need of extra terms.
- No new contributions to gauge non-invariance.
- In extended supersymmetry : Generalized Chern-Simons terms have been considered, for restoring gauge and supersymmetry invariance.
- Note that in extended supersymmetry, one can show that:

$$C_{(AB,C)} = 0\,.$$

This is consistent with the fact that only $\mathcal{N} = 1$ theories have chiral fermions that lead to anomalies.

Conclusions

- We considered gauge and supersymmetry invariance of matter coupled $\mathcal{N} = 1$ supergravity with Peccei-Quinn terms, generalized Chern-Simons terms and anomalies.
- 1. Gauge non-invariance of PQ terms is parametrized by $C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$.
  2. GCS terms are defined by a tensor $C^{(\mathrm{CS})}_{AB,C}$ of mixed symmetry.
  3. Anomalies are proportional to a symmetric tensor d_{ABC}.

  Invariance is restored when

$$C_{AB,C} = C^{(\mathrm{CS})}_{AB,C} + d_{ABC}\,.$$

- Presence of symmetric part in $C_{AB,C}$ and of anomalies is different from extended supersymmetry.
- Generalization to supergravity is straightforward.

Conclusions

- We considered gauge and supersymmetry invariance of matter coupled $\mathcal{N}=1$ supergravity with Peccei-Quinn terms, generalized Chern-Simons terms and anomalies.
- 1. Gauge non-invariance of PQ terms is parametrized by $C_{AB,C}=C^{(s)}_{AB,C}+C^{(m)}_{AB,C}$.
  2. GCS terms are defined by a tensor $C^{(\mathrm{CS})}_{AB,C}$ of mixed symmetry.
  3. Anomalies are proportional to a symmetric tensor d_{ABC}.

  Invariance is restored when

$$C_{AB,C}=C^{(\mathrm{CS})}_{AB,C}+d_{ABC}\,.$$

- Presence of symmetric part in $C_{AB,C}$ and of anomalies is different from extended supersymmetry.
- Generalization to supergravity is straightforward.

Conclusions

- We considered gauge and supersymmetry invariance of matter coupled $\mathcal{N} = 1$ supergravity with Peccei-Quinn terms, generalized Chern-Simons terms and anomalies.
- 1. Gauge non-invariance of PQ terms is parametrized by $C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$.
  2. GCS terms are defined by a tensor $C^{(\mathrm{CS})}_{AB,C}$ of mixed symmetry.
  3. Anomalies are proportional to a symmetric tensor d_{ABC}.

  Invariance is restored when

$$C_{AB,C} = C^{(\mathrm{CS})}_{AB,C} + d_{ABC}\,.$$

- Presence of symmetric part in $C_{AB,C}$ and of anomalies is different from extended supersymmetry.
- Generalization to supergravity is straightforward.

Conclusions

- We considered gauge and supersymmetry invariance of matter coupled $\mathcal{N}=1$ supergravity with Peccei-Quinn terms, generalized Chern-Simons terms and anomalies.
- 1. Gauge non-invariance of PQ terms is parametrized by $C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$.
  2. GCS terms are defined by a tensor $C^{(\mathrm{CS})}_{AB,C}$ of mixed symmetry.
  3. Anomalies are proportional to a symmetric tensor d_{ABC}.

  Invariance is restored when

$$C_{AB,C} = C^{(\mathrm{CS})}_{AB,C} + d_{ABC}\,.$$

- Presence of symmetric part in $C_{AB,C}$ and of anomalies is different from extended supersymmetry.
- Generalization to supergravity is straightforward.

Conclusions

- We considered gauge and supersymmetry invariance of matter coupled $\mathcal{N} = 1$ supergravity with Peccei-Quinn terms, generalized Chern-Simons terms and anomalies.
- 1. Gauge non-invariance of PQ terms is parametrized by $C_{AB,C} = C^{(s)}_{AB,C} + C^{(m)}_{AB,C}$.
  2. GCS terms are defined by a tensor $C^{(\mathrm{CS})}_{AB,C}$ of mixed symmetry.
  3. Anomalies are proportional to a symmetric tensor d_{ABC}.

  Invariance is restored when

$$C_{AB,C} = C^{(\mathrm{CS})}_{AB,C} + d_{ABC} \,.$$

- Presence of symmetric part in $C_{AB,C}$ and of anomalies is different from extended supersymmetry.
- Generalization to supergravity is straightforward.