Recent results in 4-dimensional non-perturbative string theory

Frank Saueressig

Institute for Theoretical Physics & Spinoza Institute

Utrecht University

D. Robles-Llana,F. S. , U. Theis, S. Vandoren, arXiv:0707:0838 [hep-th]

F. S., S. Vandoren, JHEP 07 (2007) 018

D. Robles-Llana, M. Roček, F. S., U. Theis, S. Vandoren, PRL 98 (2007) 211602

2007 Europhysics Conference on high energy physics

Manchester, July 20th, 2007

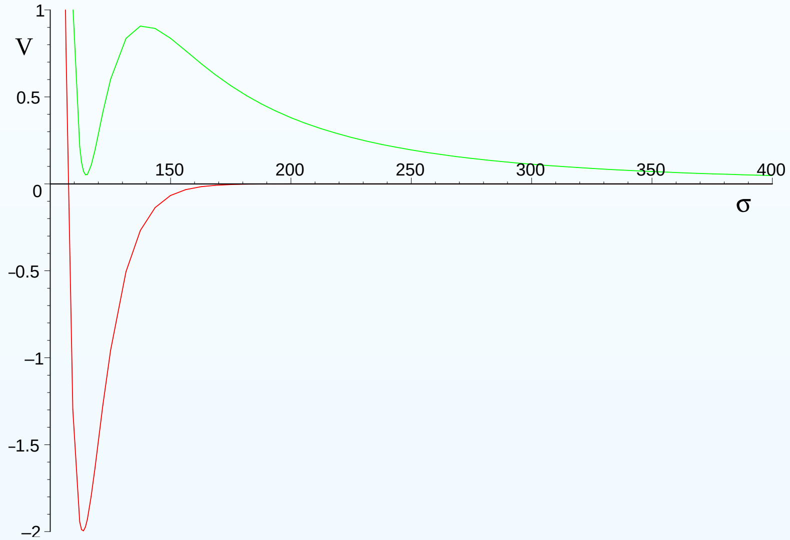
Non-perturbative effects in string theory

- source of mathematical insights for long time (e.g., mirror symmetry)
- recently: increasing role in string phenomenology
  - particle physics: generate “forbidden” couplings (neutrino masses)
  - moduli stabilization and flux vacua in type IIB strings

Non-perturbative effects in string theory

- source of mathematical insights for long time (e.g., mirror symmetry)
- recently: increasing role in string phenomenology
  - particle physics: generate “forbidden” couplings (neutrino masses)
  - moduli stabilization and flux vacua in type IIB strings

A positive cosmological constant a là KKLT

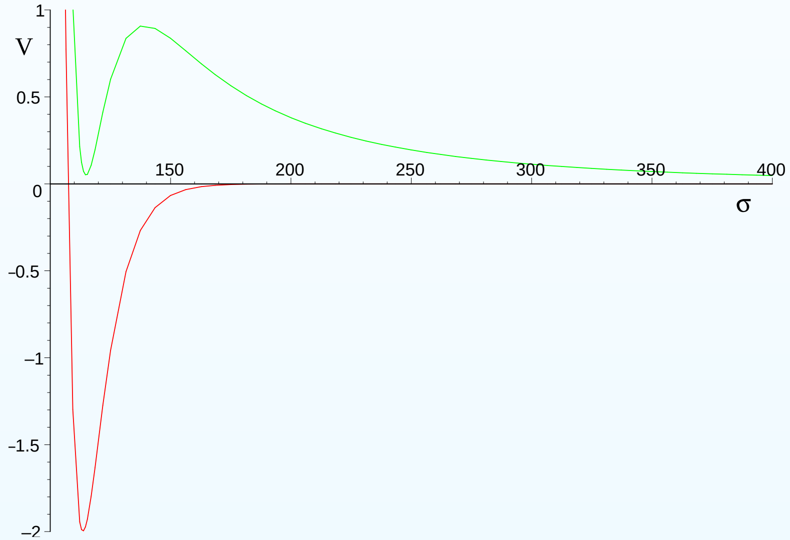


“class. fluxes + non-perturbative corrections + uplift = semirealistic physics”

Non-perturbative effects in string theory

- source of mathematical insights for long time (e.g., mirror symmetry)
- recently: increasing role in string phenomenology
  - particle physics: generate “forbidden” couplings (neutrino masses)
  - moduli stabilization and flux vacua in type IIB strings

A positive cosmological constant a là KKLT



“class. fluxes + non-perturbative corrections + uplift = semirealistic physics”

Non-perturbative corrections are far from understood!

Study in a “controlled” setup

Setup: Type II strings on Calabi-Yau threefolds

Consider type IIA/IIB string theory on $\mathrm{M}_4 \times \mathrm{CY}_3$

- Classical effective action: Compactify $d = 10$ type II supergravity

  $\Longrightarrow$ low energy physics: $d = 4, N = 2$ supergravity

  $\Longrightarrow$ special properties: symmetries (e.g. SL(2, $\mathbb{Z}$) in IIB)

Setup: Type II strings on Calabi-Yau threefolds

Consider type IIA/IIB string theory on $\mathrm{M}_4 \times \mathrm{CY}_3$

- Classical effective action: Compactify $d = 10$ type II supergravity

  $\Longrightarrow$ low energy physics: $d = 4, N = 2$ supergravity

  $\Longrightarrow$ special properties: symmetries (e.g. SL(2, $\mathbb{Z}$) in IIB)

- Quantum corrections:

  Worldsheet conformal field theory α'
  - Perturbative world sheet corrections
  - World sheet instantons

  String coupling constant g_s
  - String loop corrections
  - Instantons
    Euclidean branes wrapping cycles

Setup: Type II strings on Calabi-Yau threefolds

Consider type IIA/IIB string theory on $\mathrm{M}_4 \times \mathrm{CY}_3$

- Classical effective action: Compactify $d = 10$ type II supergravity

  $\Longrightarrow$ low energy physics: $d = 4, N = 2$ supergravity

  $\Longrightarrow$ special properties: symmetries (e.g. SL(2, $\mathbb{Z}$) in IIB)

- Quantum corrections:

  Worldsheet conformal field theory α'
  - Perturbative world sheet corrections
  - World sheet instantons

  String coupling constant g_s
  - String loop corrections
  - Instantons
    Euclidean branes wrapping cycles

Exploiting supersymmetry: Basic facts about $N = 2$ supergravity

Basic building blocks are supermultiplets

- supergravity multiplet: $e_\mu{}^a$, A_μ
- vector multiplets (VM): X, $\bar{X}$, A_μ
- hypermultiplets (HM): q^1, q^2, q^3, q^4

Exploiting supersymmetry: Basic facts about $N = 2$ supergravity

Basic building blocks are supermultiplets

- supergravity multiplet: $e_\mu{}^a$, A_μ
- vector multiplets (VM): X, $\bar{X}$, A_μ
- hypermultiplets (HM): q^1, q^2, q^3, q^4

$N = 2$ supersymmerty $\Longrightarrow$ factorization of scalar manifolds

$$\mathcal{M} = \mathcal{M}_{\rm VM} \otimes \mathcal{M}_{\rm HM}$$

- profound consequences for quantum corrections
  - α' corrections: factor containing volume of CY$_3$
  - g_s corrections: factor containing dilaton $\tau_2 = g_s^{-1}$

Exploiting supersymmetry: Basic facts about $N = 2$ supergravity

Basic building blocks are supermultiplets

- supergravity multiplet: ${e_\mu}^a$, A_μ
- vector multiplets (VM): X, $\bar{X}$, A_μ
- hypermultiplets (HM): q^1, q^2, q^3, q^4

$N = 2$ supersymmerty $\Longrightarrow$ factorization of scalar manifolds

$$\mathcal{M} = \mathcal{M}_{\text{VM}} \otimes \mathcal{M}_{\text{HM}}$$

- profound consequences for quantum corrections
  - α' corrections: factor containing volume of CY_3
  - g_s corrections: factor containing dilaton $\tau_2 = g_s^{-1}$

Vector multiplet coulings:

- completely determined by holomorphic prepotential $F(X)$
- kinetic terms of scalars given by Kähler potential $K = i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)$

Off-shell/superspace formulation for hypermultiplets

Generically: hypermultiplet sector is very complicated

- simplification: q^i with suitable shift symmetries
  - dualize scalar $q^i \longrightarrow$ second rank tensor $E^i_{\mu\nu}$
  - multiplets: hypermultiplet $\longrightarrow$ tensor multiplet (TM)
    $q^1, q^2, q^3, q^4 \longrightarrow v, \bar{v}, x, E_{\mu\nu}$
- brings in new tools:
  - off-shell projective superspace description
  - superconformal calculus linking this to Poincaré supergravity

Off-shell/superspace formulation for hypermultiplets

Generically: hypermultiplet sector is very complicated

- simplification: q^i with suitable shift symmetries
  - dualize scalar $q^i \longrightarrow$ second rank tensor $E^i_{\mu\nu}$
  - multiplets: hypermultiplet $\longrightarrow$ tensor multiplet (TM)
    $q^1, q^2, q^3, q^4 \longrightarrow v, \bar{v}, x, E_{\mu\nu}$
- brings in new tools:
  - off-shell projective superspace description
  - superconformal calculus linking this to Poincaré supergravity
- HM couplings determined by single function $\mathcal{L}$ or χ

$\mathcal{L}, \chi$ for HM have similar role as F, K for VM

Off-shell/superspace formulation for hypermultiplets

- Basic building block: $N = 2$ tensor superfield

$$\eta^I = \frac{v^I}{\zeta} + x^I - \bar{v}^I \zeta$$

- construct superspace density $\mathcal{L}$ (of conformal tensor multiplets)

$$\mathcal{L} = \mathrm{Im} \oint_{\mathcal{C}} \frac{d\zeta}{2\pi i \zeta} H(\eta^I)$$

  - restricted to scalars $\longrightarrow$ function $\mathcal{L}(v, \bar{v}, x)$ encoding HM sector

Off-shell/superspace formulation for hypermultiplets

- Basic building block: $N=2$ tensor superfield

$$\eta^I = \frac{v^I}{\zeta} + x^I - \bar{v}^I \zeta$$

- construct superspace density $\mathcal{L}$ (of conformal tensor multiplets)

$$\mathcal{L} = \mathrm{Im} \oint_{\mathcal{C}} \frac{d\zeta}{2\pi i \zeta} H(\eta^I)$$

  - restricted to scalars $\longrightarrow$ function $\mathcal{L}(v, \bar{v}, x)$ encoding HM sector

- Defines a tensor potential χ:

$$\chi = -\mathcal{L} + x^I \mathcal{L}_{x^I}$$

  - χ also specifies hypermultiplet sector completely
  - Symmetries of the Poincaré theory $\Longleftrightarrow$ invariances of χ

CY_3 compactifications of type II supergravity

fields in effective action come from two sources

- metric: Kähler and complex structure moduli $=$ shape, size of internal space
- reduction of other p-form fields

CY_3 compactifications of type II supergravity

fields in effective action come from two sources

- metric: Kähler and complex structure moduli $=$ shape, size of internal space
- reduction of other p-form fields

IIA $\left\{ e_\mu{}^a, \mathrm{e}^{-\phi}, \hat{B}_2, \hat{C}_1, \hat{C}_3 \right\}$

→ $h^{1,1}$ VM
CY$_3$ Volume

→ 1 TM
$h^{1,2}$ HM
$\mathrm{e}^{-\phi}$, A^1, B^1, B_2
z^a, A^a, B^a
Cplx str. moduli

IIB $\left\{ e_\mu{}^a, \mathrm{e}^{-\phi}, \hat{B}_2, l, \hat{C}_2, \hat{C}_4 \right\}$

→ $h^{1,2}$ VM
Cplx str. moduli
class. exact

→ 1 DTM
$h^{1,1}$ TM
$\tau = l + i\mathrm{e}^{-\phi}$, B_2, C_2
$z^a = b^a + it^a$, c^a, E_2^a
CY$_3$ Volume

CY_3 compactifications of type II supergravity

fields in effective action come from two sources

- metric: Kähler and complex structure moduli $=$ shape, size of internal space
- reduction of other p-form fields

IIA $\left\{e_\mu{}^a, \mathrm{e}^{-\phi}, \hat{B}_2, \hat{C}_1, \hat{C}_3\right\}$

- $h^{1,1}$ VM
  - CY_3 Volume
- 1 TM
  - $h^{1,2}$ HM
  - $\mathrm{e}^{-\phi}$, A^1, B^1, B_2
  - z^a, A^a, B^a
  - Cplx str. moduli

IIB $\left\{e_\mu{}^a, \mathrm{e}^{-\phi}, \hat{B}_2, l, \hat{C}_2, \hat{C}_4\right\}$

- $h^{1,2}$ VM
  - Cplx str. moduli
  - class. exact
- 1 DTM
  - $h^{1,1}$ TM
  - $\tau = l + i\mathrm{e}^{-\phi}$, B_2, C_2
  - $z^a = b^a + it^a$, c^a, E_2^a
  - CY_3 Volume

IIA/IIB dilaton is in the hypermultiplet sector

determine g_s corrections $\Longleftrightarrow$ study $\mathcal{M}_{\mathrm{HM}}$

CY_3 compactifications of type II supergravity

IIA $\left\{ e_\mu{}^a, \mathrm{e}^{-\phi}, \hat{B}_2, \hat{C}_1, \hat{C}_3 \right\}$

- $h^{1,1}$ VM
  - CY_3 volume
- 1 TM
  - $h^{1,2}$ HM
  - $\mathrm{e}^{-\phi}$, A^1, B^1, B_2
  - z^a, A^a, B^a
  - Cplx str. moduli

IIB $\left\{ e_\mu{}^a, \mathrm{e}^{-\phi}, \hat{B}_2, l, \hat{C}_2, \hat{C}_4 \right\}$

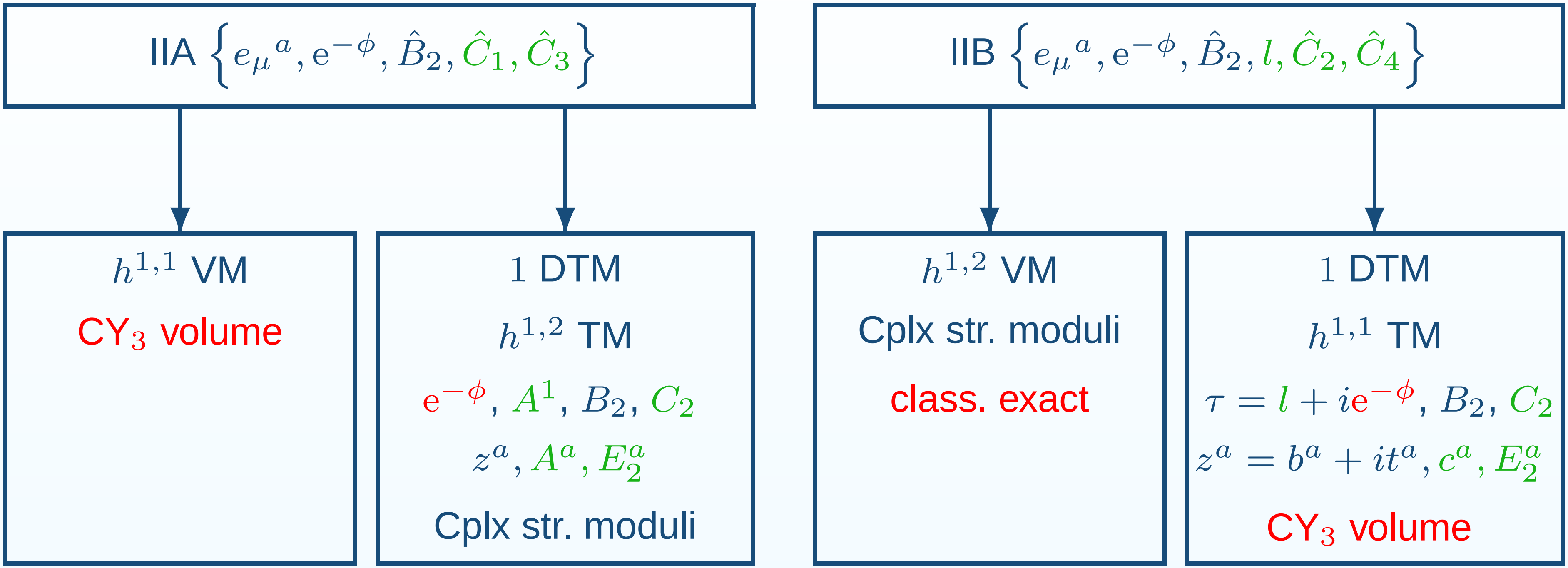
- $h^{1,2}$ VM
  - Cplx str. moduli
  - class. exact
- 1 DTM
  - $h^{1,1}$ TM
  - $\tau = l + i\mathrm{e}^{-\phi}$, B_2, C_2
  - $z^a = b^a + it^a$, c^a, E_2^a
  - CY_3 volume

- type IIB: tensor multiplets appear naturally
- type IIA: reduction of p-form gauge-fields gives shift symmetries in B^Λ

  $\Longrightarrow$ B^Λ can be dualized to tensors E_2^Λ

  $\Longrightarrow$ gain: 1DTM + $h^{1,2}$ TM

CY_3 compactifications of type II supergravity



- type IIB: tensor multiplets appear naturally
- type IIA: reduction of p-form gauge-fields gives shift symmetries in B^Λ
  $\Longrightarrow$ B^Λ can be dualized to tensors E_2^Λ
  $\Longrightarrow$ gain: 1DTM + $h^{1,2}$ TM

1DTM + $h^{1,2}$ TM = field content for off-shell description $(\mathcal{L}, \chi)$

Non-perturbative corrections from D-branes

Dp-branes as higher dimensional objects with $p+1$ dimensions

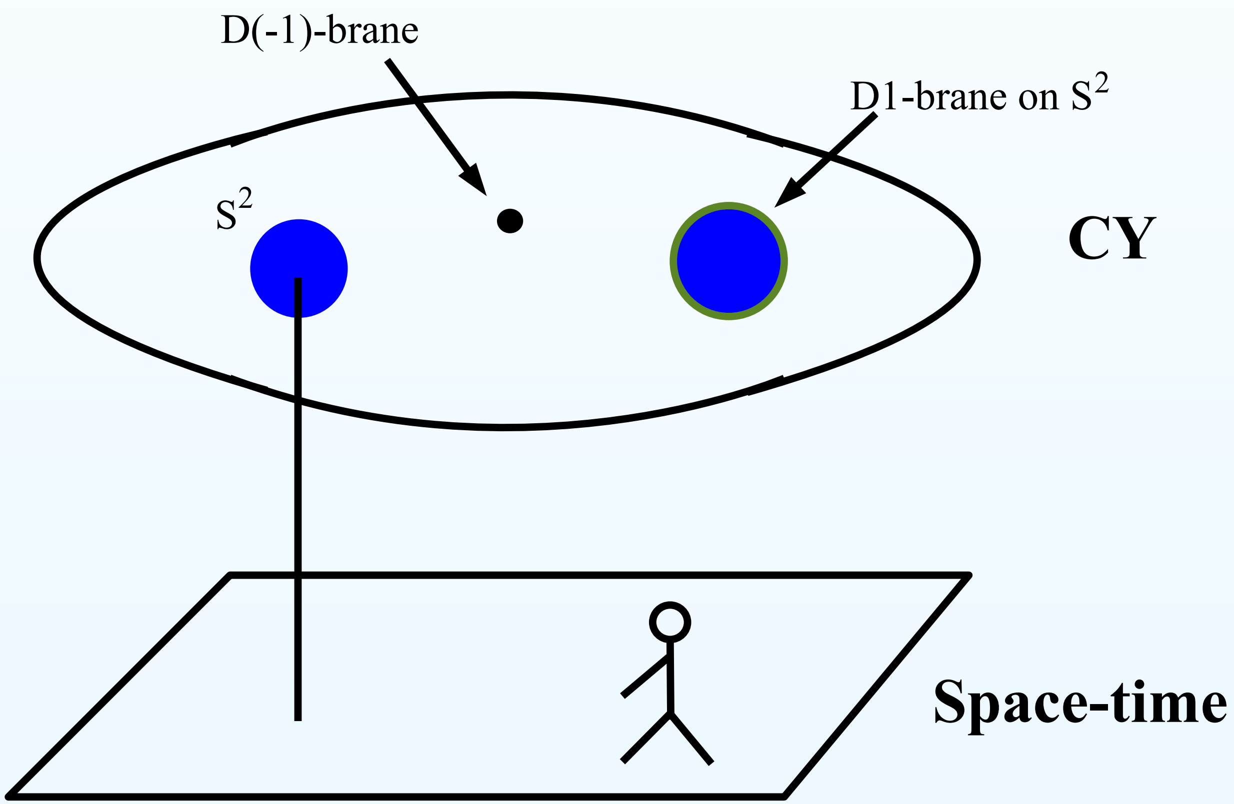
- IIA: p even: D0, D2 (membranes), D4, . . .
- IIB: p odd: D(-1) (point), D1 (D-string), D3, D5, . . .

Non-perturbative corrections from D-branes

Dp-branes as higher dimensional objects with $p+1$ dimensions

- IIA: p even: D0, D2 (membranes), D4, . . .
- IIB: p odd: D(-1) (point), D1 (D-string), D3, D5, . . .

Instanton configurations in type IIB

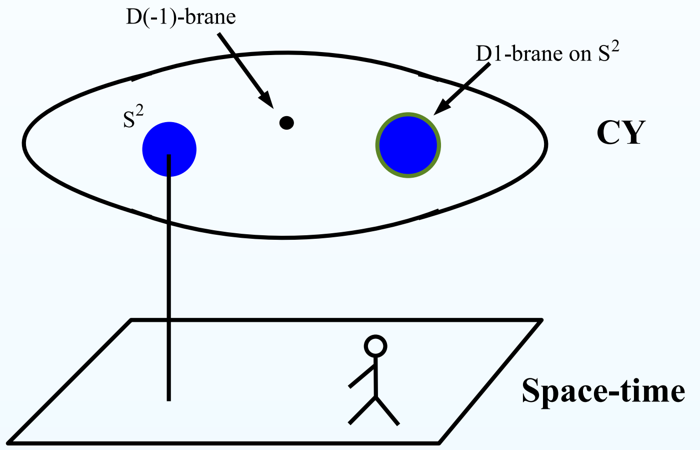


Instanton corrections from D-branes

Dp-branes as higher dimensional objects with $p+1$ dimensions

- IIA: p even: D0, D2 (membranes), D4, . . .
- IIB: p odd: D(-1) (point), D1 (D-string), D3, D5, . . .

Instanton configurations in type IIB



Instantons yield exponentially suppressed corrections to the effective action:

Example: D(-1)-instanton contributions $\propto \mathrm{e}^{-S_{\mathrm{D}(-1)}}$

$$S_{\mathrm{D}(-1)} = 2\pi\tau_2 + 2\pi i\tau_1$$

Computing instanton corrections

- Direct computation rather hopeless:
  - already very complicated in quantum field theory (e.g., YM-theories)
  - no non-perturbative formulation for string theory

Computing instanton corrections

- Direct computation rather hopeless:
  - already very complicated in quantum field theory (e.g., YM-theories)
  - no non-perturbative formulation for string theory
- Key idea: indirect computation using dualities of string theory:
  - duality: two string theories (or sectors) encode the same physics
  - find a sector which is classically exact
  - use duality maps to learn about quantum corrections

Dualities of type II string theory

- mirror symmetry (non-perturbative)
  - for X and Y mirror pair of Calabi-Yau manifolds

$$\text{LEEA for type IIA}/X \quad \simeq \quad \text{LEEA for type IIB}/Y$$

Dualities of type II string theory

- mirror symmetry (non-perturbative)
  - for X and Y mirror pair of Calabi-Yau manifolds

$$\text{LEEA for type IIA}/X \quad \simeq \quad \text{LEEA for type IIB}/Y$$

- T-duality (c-map) (string tree-level)
  - IIA/IIB string theory on S^1 (swap winding $\Leftrightarrow$ momentum modes):

$$\text{type IIA}/S^1_R \simeq \text{type IIB}/S^1_{1/R}$$

  - LEEA:

    type IIA/X vector multiplets $\Longleftrightarrow$ type IIB/X hypermultiples

Dualities of type II string theory

- mirror symmetry (non-perturbative)
  - for X and Y mirror pair of Calabi-Yau manifolds

$$\text{LEEA for type IIA}/X \quad \simeq \quad \text{LEEA for type IIB}/Y$$

- T-duality (c-map) (string tree-level)
  - IIA/IIB string theory on S^1 (swap winding $\Leftrightarrow$ momentum modes):

$$\text{type IIA}/S^1_R \simeq \text{type IIB}/S^1_{1/R}$$

  - LEEA:

    type IIA/X vector multiplets $\Longleftrightarrow$ type IIB/X hypermultiples

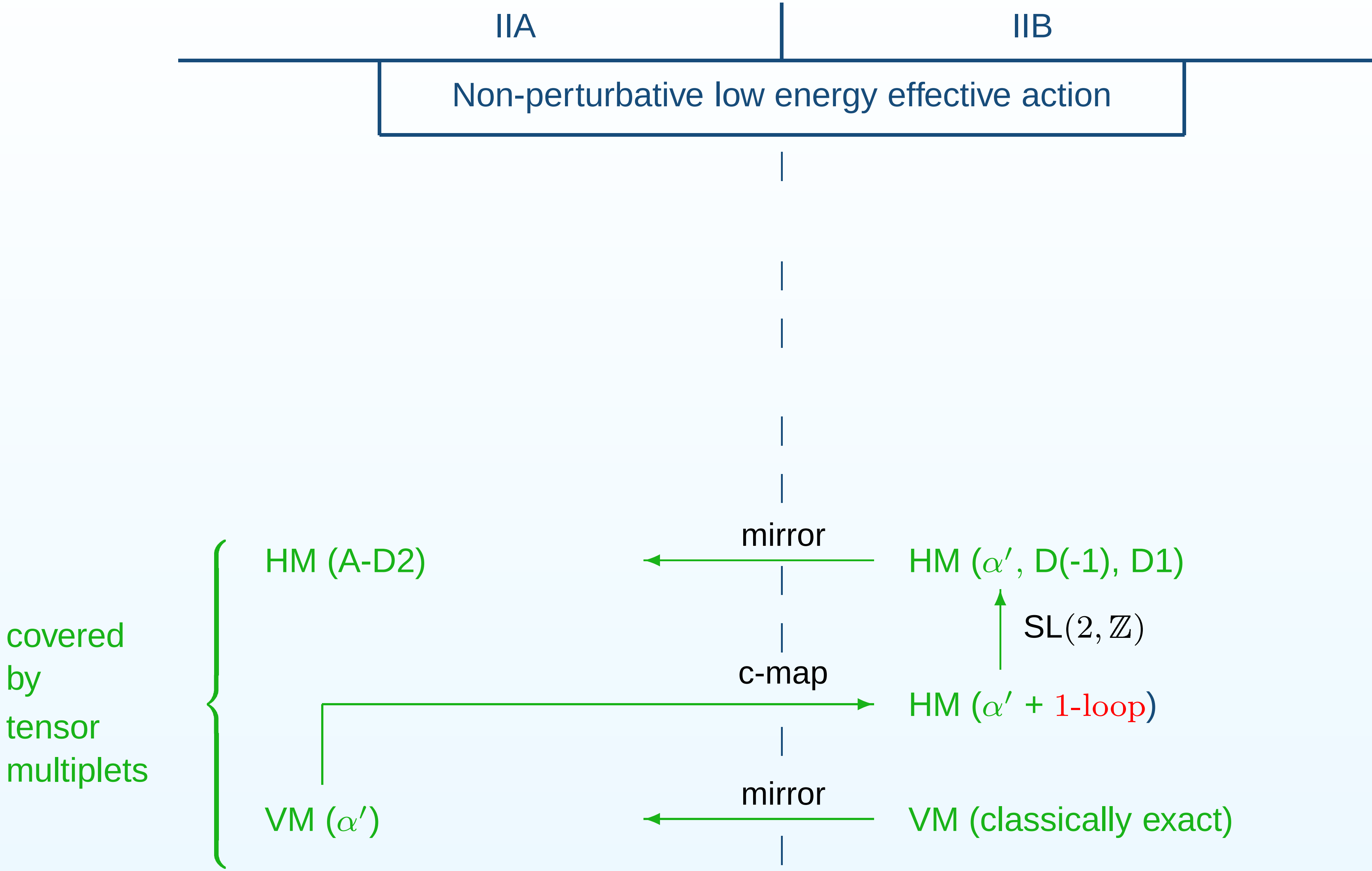
- SL(2,$\mathbb{Z}$)-invariance (non-perturbative)
  - type IIB string has invariance:

$$\tau_1 \to 1/\tau_1 \text{ and } \tau_2 \to \tau_2 + 1 \text{ (+ transformation of tensors)}$$

    relates e.g. fundamental strings and D-strings $\to$ (p, q)-strings
  - LEEA:

    invariant under transformations inherited by 4-dimensional fields

Building a duality chain



Duality cascade I: type IIB VM sector

Starting point: IIB vector multiplets (classically exact):

- fixed by prepotential $F^{\rm IIB}_{\rm VM}(X)$
- X related to the complex structure moduli z^a
- $F^{\rm IIB}_{\rm VM}(X)$ determined by period integrals of hol. 3-form Ω

Duality cascade II: IIA VM sector via mirror symmetry

(P. Candelas, et. al., Nucl. Phys. B359 (1991) 21)

The mirror map:

- complex structure moduli X^{cs} of $\mathrm{CY}_3 \simeq$ Kähler moduli X^{Kahler} of mirror CY_3
- Exist canonical coordinates in which $F^{\mathrm{IIB}}_{\mathrm{VM}}(X) = F^{\mathrm{IIA}}_{\mathrm{VM}}(X)$

Duality cascade II: IIA VM sector via mirror symmetry

(P. Candelas, et. al., Nucl. Phys. B359 (1991) 21)

The mirror map:

- complex structure moduli $X^{\rm cs}$ of CY$_3$ $\simeq$ Kähler moduli $X^{\rm Kahler}$ of mirror CY$_3$
- Exist canonical coordinates in which $F^{\rm IIB}_{\rm VM}(X) = F^{\rm IIA}_{\rm VM}(X)$

Interpretation of $F^{\rm IIA}_{\rm VM}(X) = F_{\rm cl}(X) + F_{\rm pt}(X) + F_{\rm ws}(X)$:

$$F_{\rm cl}(X) = \frac{1}{3!}\,\kappa_{abc}\,\frac{X^a X^b X^c}{X^1}$$

$$F_{\rm pt}(X) = \frac{i\,\zeta(3)}{2(2\pi)^3}\,\chi_E\,(X^1)^2$$

$$F_{\rm ws}(X) = -\,\frac{i}{(2\pi)^3}\,(X^1)^2 \sum_{k_a} n_{k_a}\,{\rm Li}_3\left({\rm e}^{2\pi i k_a X^a/X^1}\right)$$

- $F_{\rm cl}$ classical part (triple intersection numbers κ_{abc})
- $F_{\rm pt}$ and $F_{\rm ws}$ are perturbative and non-perturbative α' corrections (Euler number χ_E, Gopakumar-Vafa invariants n_{k_a})

Duality cascade III: IIB HM sector in string perturbation theory

(M. Roček, C. Vafa, S. Vandoren, hep-th/0512206)

(D. Robles-Llana, F.S., S. Vandoren, hep-th/0602164)

perturbative IIB HM sector (classical c-map + 1-loop correction):

$$\mathcal{L}(v,\bar{v},x) = \mathrm{Im} \oint_{\mathcal{C}} \frac{\mathrm{d}\zeta}{2\pi i \zeta} \left[\frac{F(\eta^\Lambda)}{\eta^0} + \frac{i\,\chi_E}{(2\pi)^3}\, 2\zeta(2)\, \eta^0 \, \ln(\eta^0) \right]$$

- $F(\cdot)$ prepotential of dual VM sector
- η^0 conformal compensator (additional TM)
- contour $\mathcal{C}$ taken “zero” of $\zeta\eta^0$

Duality cascade III: IIB HM sector in string perturbation theory

(M. Roček, C. Vafa, S. Vandoren, hep-th/0512206)

(D. Robles-Llana, F.S., S. Vandoren, hep-th/0602164)

perturbative IIB HM sector (classical c-map + 1-loop correction):

$$\mathcal{L}(v,\bar{v},x) = \mathrm{Im} \oint_{\mathcal{C}} \frac{\mathrm{d}\zeta}{2\pi i \zeta} \left[\frac{F(\eta^\Lambda)}{\eta^0} + \frac{i\,\chi_E}{(2\pi)^3}\, 2\zeta(2)\, \eta^0\, \ln(\eta^0) \right]$$

- $F(\cdot)$ prepotential of dual VM sector
- η^0 conformal compensator (additional TM)
- contour $\mathcal{C}$ taken "zero" of $\zeta\eta^0$

Determine χ using physical fields ($z^a = b^a + it^a$, $t^a =$ Kähler modulus)

$$\chi_{\rm cl} = 4\, r^0\, \tau_2^2\, \frac{1}{3!}\, \kappa_{abc}\, t^a\, t^b\, t^c$$

$$\chi_{\rm pt} = \frac{1}{(2\pi)^3}\, r^0\, \chi_E\, \left[\zeta(3)\tau_2^2 + 2\zeta(2)\right]$$

$$\chi_{\rm ws} = -\frac{r^0 \tau_2^2}{(2\pi)^3} \sum_{k_a} n_{k_a} \left[\mathrm{Li}_3(\mathrm{e}^{2\pi i k_a z^a}) + 2\pi k_a t^a \mathrm{Li}_2(\mathrm{e}^{2\pi i k_a z^a}) + c.c. \right]$$

Duality cascade IV: SL(2, $\mathbb{Z}$) invariance of IIB LEEA

4-dimensional fields transform under SL(2,$\mathbb{Z}$) of type IIB string

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}\,,\ t^a \mapsto t^a|c\tau + d|\,,\ b^a \mapsto d\,b^a + c\,c^a\,,\ c^a \mapsto b\,b^a + a\,c^a\,;\ \ (r^0 \mapsto r^0|c\tau + d|)$$

Basic idea: IIB string has non-perturbative SL(2, $\mathbb{Z}$)-invariance

- observation: α', g_s-corrections break SL(2, $\mathbb{Z}$)-invariance of classical LEEA
- $\Longrightarrow$ restore SL(2, $\mathbb{Z}$)-invariance of LEEA
- $\Longleftrightarrow$ complete χ to a modular invariant function

Duality cascade IV: SL(2, $\mathbb{Z}$) invariance of IIB LEEA

4-dimensional fields transform under SL(2,$\mathbb{Z}$) of type IIB string

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}\,,\; t^a \mapsto t^a|c\tau + d|\,,\; b^a \mapsto d\,b^a + c\,c^a\,,\; c^a \mapsto b\,b^a + a\,c^a\,;\;\; (r^0 \mapsto r^0|c\tau + d|)$$

Basic idea: IIB string has non-perturbative SL(2, $\mathbb{Z}$)-invariance

- observation: α', g_s-corrections break SL(2, $\mathbb{Z}$)-invariance of classical LEEA
- $\Longrightarrow$ restore SL(2, $\mathbb{Z}$)-invariance of LEEA
- $\Longleftrightarrow$ complete χ to a modular invariant function

Classical contribution:

$$\chi_{\rm cl} = \frac{4}{3!}\,(r^0\,\sqrt{\tau_2})\,\kappa_{abc}\,(\sqrt{\tau_2}t^a)\,(\sqrt{\tau_2}t^b)\,(\sqrt{\tau_2}t^b)$$

- each bracket is an SL$(2, \mathbb{Z})$ invariant
- $\Longrightarrow$ $\chi_{\rm cl}$ is modular invariant!
- reflects SL(2, $\mathbb{Z}$) invariance of the classical LEEA

Duality cascade IV: Restoring SL(2, $\mathbb{Z}$) invariance of $\chi_{\rm pt}$

- Perturbative contributions are NOT modular invariant!

$$\chi_{\rm pt} = \frac{1}{2(2\pi)^3}\, r^0 \sqrt{\tau_2}\, \chi_E \left[\underbrace{2\,\zeta(3)\,\tau_2^{3/2}}_{\rm pert.\alpha'} + \underbrace{4\,\zeta(2)\,\tau_2^{-1/2}}_{\rm pert.g_s} \right]$$

Duality cascade IV: Restoring SL(2, $\mathbb{Z}$) invariance of $\chi_{\rm pt}$

- Perturbative contributions are NOT modular invariant!

$$\chi_{\rm pt} = \frac{1}{2(2\pi)^3}\, r^0 \sqrt{\tau_2}\, \chi_E \Big[\underbrace{2\,\zeta(3)\,\tau_2^{3/2}}_{\rm pert.\alpha'} + \underbrace{4\,\zeta(2)\,\tau_2^{-1/2}}_{\rm pert.g_s} \Big]$$

- compare to non-holomorphic Eisenstein series

$$Z_{3/2} = 2\zeta(3)\,\tau_2^{3/2} + 4\zeta(2)\,\tau_2^{-1/2} + 8\pi\sqrt{\tau_2} \sum_{m\neq 0, n>0} \left|\frac{m}{n}\right| {\rm e}^{2\pi i m n \tau_1}\, K_1(2\pi |mn| \tau_2)$$

- Sum over images provides modular completion:

$$\chi_{(-1)} = \frac{1}{2(2\pi)^3}\, r^0 \sqrt{\tau_2}\, \chi_E\, Z_{3/2}(\tau, \bar\tau)$$

- First two terms: perturbative α' and g_s corrections
- Sum: $D(-1)$ instanton contributions

Duality cascade IV: Restoring SL(2, $\mathbb{Z}$) invariance of $\chi_{\rm ws}$

- Worldsheet-instanton contribution also NOT modular invariant

$$\chi_{\rm ws} = -\frac{r^0\,\tau_2^2}{(2\pi)^3}\sum_{k_a} n_{k_a}\,{\rm Re}\Big[{\rm Li}_3\left({\rm e}^{2\pi i k_a z^a}\right) + 2\pi k_a t^a\,{\rm Li}_2\left({\rm e}^{2\pi i k_a z^a}\right)\Big]$$

  - SL(2, $\mathbb{Z}$) multiplet: fundamental string + D(1)-string + their bound states

Duality cascade IV: Restoring SL(2, $\mathbb{Z}$) invariance of $\chi_{\rm ws}$

- Worldsheet-instanton contribution also NOT modular invariant

$$\chi_{\rm ws} = -\frac{r^0\,\tau_2^2}{(2\pi)^3}\sum_{k_a} n_{k_a} \mathrm{Re}\Big[\mathrm{Li}_3\left(\mathrm{e}^{2\pi i k_a z^a}\right) + 2\pi k_a t^a \mathrm{Li}_2\left(\mathrm{e}^{2\pi i k_a z^a}\right)\Big]$$

  - SL(2, $\mathbb{Z}$) multiplet: fundamental string + D(1)-string + their bound states

- Modular completion using sum over images:

$$\chi_{(1)} = -\frac{|r^0|\,\sqrt{\tau_2}}{(2\pi)^3}\sum_{k_a} n_{k_a} \sum_{m,n}{}' \frac{\tau_2^{3/2}}{|m\tau+n|^3}\,\left(1+2\pi|m\tau+n|\,k_a\,t^a\right)\,\mathrm{e}^{-S_{m,n}}\,.$$

$S_{m,n}$ is the instanton action for (p,q)-string

$$S_{m,n} = 2\pi\Big(|m\tau+n|\,k_a\,t^a - i\,k_a\,(n\,b^a + m\,c^a)\Big)\,.$$

Duality cascade V: A-type D2-instantons from mirror symmetry

Mirror map in the hypermultiplet sector:

$$\phi_{\rm IIA} = \phi_{\rm IIB}\ ,\ A^1 = \tau_1\ ,\ A^a = -(c^a - \tau_1 b^a)\ ,\ z^a_{\rm IIA} = z^a_{\rm IIB}$$

- map: IIB D(-1) + D1-instantons $\simeq$ (A-type) D2-instantons in IIA

Duality cascade V: A-type D2-instantons from mirror symmetry

Mirror map in the hypermultiplet sector:

$$\phi_{\rm IIA} = \phi_{\rm IIB}\,,\ A^1 = \tau_1\,,\ A^a = -(c^a - \tau_1 b^a)\,,\ z^a_{\rm IIA} = z^a_{\rm IIB}$$

- map: IIB D(-1) + D1-instantons $\simeq$ (A-type) D2-instantons in IIA

IIB HM	SL(2, $\mathbb{Z}$)-invariant	IIA HM	composed from IIB terms
$\chi_{\rm cl}$	= $\chi_{\rm cl}$	$\chi_{\rm tree}$	= $\chi_{\rm cl} + \chi_{\rm ws-pert} + \chi_{\rm ws-inst}$
$\chi_{(-1)}$	= $\chi_{\rm ws-pert} + \chi_{\rm loop} + \chi_{D(-1)}$	$\chi_{\rm loop}$	= $\chi_{\rm loop}$
$\chi_{(1)}$	= $\chi_{\rm ws-inst} + \chi_{\rm D1}$	$\chi_{\rm A-D2}$	= $\chi_{\rm D(-1)} + \chi_{\rm D1}$

Duality cascade V: A-type D2-instantons from mirror symmetry

Mirror map in the hypermultiplet sector:

$$\phi_{\rm IIA} = \phi_{\rm IIB}\,,\ A^1 = \tau_1\,,\ A^a = -(c^a - \tau_1 b^a)\,,\ z^a_{\rm IIA} = z^a_{\rm IIB}$$

- map: IIB D(-1) + D1-instantons $\simeq$ (A-type) D2-instantons in IIA

IIB HM	SL(2, $\mathbb{Z}$)-invariant	IIA HM	composed from IIB terms
$\chi_{\rm cl}$	$= \chi_{\rm cl}$	$\chi_{\rm tree}$	$= \chi_{\rm cl} + \chi_{\rm ws-pert} + \chi_{\rm ws-inst}$
$\chi_{(-1)}$	$= \chi_{\rm ws-pert} + \chi_{\rm loop} + \chi_{D(-1)}$	$\chi_{\rm loop}$	$= \chi_{\rm loop}$
$\chi_{(1)}$	$= \chi_{\rm ws-inst} + \chi_{\rm D1}$	$\chi_{\rm A-D2}$	$= \chi_{\rm D(-1)} + \chi_{\rm D1}$

- A-type D2-instanton contribution

$$\chi_{\rm A-D2} = -\frac{r^0\tau_2}{2\pi^2}\sum_{k_\Lambda} n_{k_\Lambda}\sum_{m\neq 0}\frac{1}{|m|}\,|k_\Lambda z^\Lambda|\,K_1\big(2\pi\tau_2\,|m\,k_\Lambda z^\Lambda|\big)\,{\rm e}^{-2\pi i m k_\Lambda A^\Lambda}$$

$$k_\Lambda = \{n, k_a\}\,,\quad n_{k_\Lambda} = \{-\frac{\chi_E}{2}, n_{k_a}\}\,,\quad z^\Lambda = \{1, z^a\}\,,\quad A^\Lambda = \{A^1, A^a\}$$

Summary . . .

Type II strings compactified on Calabi-Yau threefolds:

- hypermultiplet sector receives corrections from Euclidean D-branes
  - corrections encoded in functions $\mathcal{L}, \chi$
- determined and sumed up classes of instanton corrections to all others in g_s
  - D(-1), D1-instanton corrections in IIB
  - A-type D2-instanton corrections in IIA

Summary and Outlook

Type II strings compactified Calabi-Yau threefolds:

- LEEA receives instanton corrections in the hypermultiplet sector
  - $\mathcal{N} = 2$ hypermultiplet sector determined by functions $\mathcal{L}, \chi$
- determined classes of instanton contributions to all others in g_s
  - D(-1), D1-instanton corrections in IIB
  - A-type D2-instanton corrections in IIA

Outlook:

- Consequences for moduli stabilization and flux compactifications?
- Include other instantons classes (B-D2, NS5-branes)?

Duality chain for the LEEA of type II strings on CY_3