Geodesic motion, power-law cosmologies and pseudo-susy

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related to earlier work with J. Rosseel and D. Westra [hep-th/0610143]

Overview

1. Motivation: branes and moduli

2. Free scalars and time-dependent Einstein solutions

3. Massive scalars and pseudo supersymmetry

4. Outlook

Approach

D-branes, black holes, instantons and wormholes in string theory?

$\rightarrow$ first in supergravity

$\rightarrow$ this talk is pedagogical: mainly gravity...

Literature on branes and geodesics: Breitenlohner, Gibbons, Maison '88; Gal'tsov, Rytchkov '98; Cremmer, Lavrinenko, Lu, Pope, Stelle, Tran '98; Bergshoeff, Collinucci, Gran, Roest, Vandoren '04; Gunaydin, Neitzke, Pioline, Waldron '05; Polchinski,.....; Fré, Gili, Gargiulo, Sorin, Rulik, Trigiante '03 ;Karthauser, Saffin '06 ;Rosseel, VR, Westra '06; Chemissany, Ploegh, VR '07; Arkani-Hamed, Orgera, Polchinski '07;. . .

Motivation

Consider p-branes charged electrically under A_{p+1} or magnetically under A_{D-p-3}:

- Timelike p-branes: $\mathrm{d}s_D^2 = \mathrm{e}^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{e}^{2B(r)}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-p-2})$.
- Spacelike p-branes: $\mathrm{d}s_D^2 = \mathrm{e}^{2A(t)}\delta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{e}^{2B(t)}(-\mathrm{d}t^2 + t^2\mathrm{d}\Sigma^2_{D-p-2})$.

Special cases are

1. domain walls = timelike $(D-2)$-brane,
2. FLRW-cosmologies = spacelike (D-2)-brane,
3. instantons = Euclidean (-1)-brane [IIB D-instanton]
4. $(D-3)$-branes [IIB 7-branes]

$$\mathcal{L} = \sqrt{|g|}\Big(\mathcal{R} - \tfrac{1}{2}G_{ij}\partial\Phi^i\partial\Phi^j - V(\Phi)\Big) \tag{1}$$

Apart from the (D-3)-branes, solutions depend on one parameter

$$\mathrm{d}s_D^2 = \pm f(r)^2\mathrm{d}r^2 + g(r)^2\mathrm{d}s^2_{D-1}\,, \qquad \Phi^i(r)\,. \tag{2}$$

Motivation

Such solutions can be identified with curves $\Phi^i(r)$ on the scalar manifold.

What about the other brane-type solutions? $\longrightarrow$ dimensionally reduce over $p+1$-dimensional worldvolume (Killing directions):

- Timelike p-brane in D dimensions $\longrightarrow$ Euclidean "instanton" solution in $D-p-1$ dimensions. (e.g. the black hole-instanton correspondence)
- Spacelike p-brane in D dimensions $\longrightarrow$ cosmological solution in $D-p-1$ dimensions.

$\boxed{V=0}\,(\ldots)$

REVERSED REASONING: Reduce over transversal space

- Timelike p-brane in D dimensions $\longrightarrow$ domain wall in $p+2$ dimensions (e.g. DW/QFT correspondence).
- Spacelike p-brane in D dimensions $\longrightarrow$ Cosmology in $p+2$ dimensions (e.g. do we live on a S2-brane?).

$\boxed{V\neq 0}\,(\ldots)$

Motivation

Al brane-type solutions have description in terms of curves $\Phi^i(r)$ on a scalar manifold.

When reduction over worldvolume is considered: $V = 0 \rightarrow$ **geodesic motion** on the scalar manifold. In terms of an *affine* parameter $\rho(r)$:

$$\Phi''^i + \Gamma^i_{jk}\Phi'^j\Phi'^k = 0\,, \qquad \mathcal{R}_{rr} = \tfrac{1}{2}||v^2||g^{2-2D}f^2 \tag{3}$$

Affine velocity $||v^2||$ is constant $||v^2|| = G_{ij}\Phi'^i\Phi'^j$. If G_{ij} is indefinite then

- $||v||^2 > 0$ spacelike geodesics
- $||v||^2 = 0$ lightlike geodesics
- $||v||^2 < 0$ timelike geodesics

Program:

1. Dimensional reduce
2. Solve geodesic equations
3. Uplift the solutions

Motivation

Find geodesic curves on *which spaces?*
Consider *maximal supergravity* in $D = 10, 11$ and their corresponding reductions over timelike and spacelike tori:

	Minkowskian	Euclidean
$D = 10$	O(1,1)	O(1,1)
$D = 9$	$\frac{\mathrm{GL}(2,\mathbb{R})}{O(2)}$	$\frac{\mathrm{GL}(2,\mathbb{R})}{O(1,1)}$
$D = 8$	$\frac{\mathrm{SL}(3,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})}{O(3)\times O(2)}$	$\frac{\mathrm{SL}(3,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})}{O(2,1)\times O(1,1)}$
$D = 7$	$\frac{\mathrm{SL}(5,\mathbb{R})}{O(5)}$	$\frac{\mathrm{SL}(5,\mathbb{R})}{O(3,2)}$
$D = 6$	$\frac{O(5,5)}{O(5)\times O(5)}$	$\frac{O(5,5)}{O(5,C)}$
$D = 5$	$\frac{E_{6(+6)}}{USp(8)}$	$\frac{E_{6(+6)}}{USp(4,4)}$
$D = 4$	$\frac{E_{7(+7)}}{SU(8)}$	$\frac{E_{7(+7)}}{SU^*(8)}$
$D = 3$	$\frac{E_{8(+8)}}{SO(16)}$	$\frac{E_{8(+8)}}{SO^*(16)}$

Motivation

Simplest example moduli space: torus-reduction of pure gravity

1. $\mathrm{GL}(n,\mathbb{R})/\,\mathrm{SO}(n)$ for spacelike reductions.
2. $\mathrm{GL}(n,\mathbb{R})/\,\mathrm{SO}(n-1,1)$ for timelike reductions.

$$\mathrm{d}s^2_{D+n} = \mathrm{e}^{2\alpha\varphi}\mathrm{d}s^2_D + \mathrm{e}^{2\beta\varphi} e^n \otimes e_n \,. \tag{4}$$

- φ is breathing mode, controls overall volume.
- e^n is vielbein on n-torus: $e^n = L^n_a(\underline{x})\mathrm{d}y^a$ and since overall volume is constant det$L$=1.

Thus internal space

$$\mathrm{d}s^2_n = \mathcal{M}_{ab}\mathrm{d}y^a\mathrm{d}y^b \,, \qquad \mathcal{M}_{ab} = L^n_a L_{nb} \,. \tag{5}$$

- Local $\mathrm{SO}(n)$ $L \to L\,O(y)$ leaves $\mathcal{M}_{ab}\mathrm{d}y^a\mathrm{d}y^b$ *invariant*.
- Rigid $\mathrm{SL}(n,\mathbb{R})$ $L \to \Omega L$ *preserves* Ansatz:$\mathcal{M}_{ab}\mathrm{d}y'^a\mathrm{d}y'^b$ where $y' = \Omega y$.

Thus L is coset representative of $\mathrm{SL}(n,\mathbb{R})/\,\mathrm{SO}(n)$, the moduli group of the n-torus.

$$\sqrt{-g_{D+n}}\mathcal{R}_{D+n} \quad \to \quad \sqrt{-g_D}\{\mathcal{R}_D + \tfrac{1}{4}\mathrm{Tr}\partial\mathcal{M}\partial\mathcal{M}^{-1} - \tfrac{1}{2}\partial\varphi\partial\varphi\} \tag{6}$$

Time-dependent Einstein solutions

Geodesics on $\mathrm{SL}(n,\mathbb{R})/\mathrm{SO}(n)$?

$$\frac{\mathrm{d}}{\mathrm{d}t}[\mathcal{M}^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{M}] = 0\,. \tag{7}$$

Integrable problem! $\rightarrow \mathcal{M}^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{M} = Q$. And thus

$$\mathcal{M}(t) = \mathcal{M}(0)\mathrm{e}^{Qt}\,. \tag{8}$$

For geodesics through the origin $\mathcal{M}(0) = \mathbb{1}$ we have a solution if

$$\mathrm{Tr}Q = 0\,, \qquad Q^T = Q\,. \tag{9}$$

Isometry maps geodesic $\rightarrow$ new geodesic and Q transforms in the adjoint:$Q \rightarrow \Omega Q \Omega^{-1}$. For geodesics through origin $\Omega \in \mathrm{SO}(n) \subset \mathrm{SL}(n,\mathbb{R})$ and Q can be diagonalized! This is a straight line $\boxed{\vec{\phi} = \vec{v}t}$.
Geodesics not true origin can be obtained via a non-compact $\mathrm{SL}(n,\mathbb{R})$-transformation.
All geodesics on $\mathrm{SL}(n,\mathbb{R})/\mathrm{SO}(n)$ can be obtained via an isometry transformation of a straight line.

- This result extends to all coset spaces that are *maximally non-compact* [Chemissany, Ploegh, VR '07].

Time-dependent Einstein solutions

Uplift the general cosmological solution to $D+n$-dimensional vacuum solution.

$$\varphi = a\,t + b\,, \qquad \mathcal{M}(t) = \Omega D(t)\Omega^T \tag{10}$$

where $D(t)$ is diagonal and represents the trivial straight line solution. The Ω's can be absorbed as a coordinate transformation on the torus coordinates $\mathrm{d}y' = \Omega \mathrm{d}y$. Result is the Kasner solution:

$$\mathrm{d}s^2 = -t^{2p_0}\mathrm{d}t^2 + \sum_i t^{2p_i}\mathrm{d}z_i^2\,, \tag{11}$$

$$p_0 + 1 = \sum_i p_i\,, \qquad (p_0+1)^2 = \sum_i p_i^2\,. \tag{12}$$

- What if the D-dimensional FLRW Ansatz has $k \neq 0$? Kasner becomes a generalization of (flux-less) S-brane solutions for $k=-1$.
- This was only an exercise, should extend to supergravity. Uplift a bit more involved. For the result (but with a different technique), see [Fre, Gili, Gargiulo, Sorin, Rulik, Trigiante '03] .

Non-zero potential?

- If $V \neq 0$ geodesic motion is deformed, but not always [Karthauser, Saffin '06]
- Related to pseudo-supersymmetry [Sonner, Townsend '07, Chemissany, Ploegh, VR '07].

PSEUDO-SUSY?

Pseudo-supersymmetry and geodesics

=First-order formalism for cosmological solutions based on Domain wall/Cosmology correspondence [Skenderis, Townsend '06].

$$\mathrm{d}s_D^2 = g(y)^2 \mathrm{d}s_{D-1}^2 + \epsilon f(y)^2 \mathrm{d}y^2\,, \quad \mathrm{d}s_{D-1}^2 = (\eta_\epsilon)_{ab}\mathrm{d}x^a \mathrm{d}x^b\,, \tag{13}$$

where $\epsilon = \pm 1$ and $\eta_\epsilon = \mathrm{diag}(-\epsilon, 1, \dots, 1)$.
If $V(\Phi)$ can be written in terms of another function $W(\Phi)$ as follows

$$V = \epsilon \left\{ \tfrac{1}{2} G^{ij}\partial_i W \partial_j W - \tfrac{D-1}{4(D-2)} W^2 \right\}, \tag{14}$$

the action can be written as "a sum of squares" (plus a boundary term) :

$$S = \epsilon \int \mathrm{d}y\, f g^{D-1} \left\{ \tfrac{(D-1)}{4(D-2)} \left[W - 2(D-2)\frac{\dot g}{fg} \right]^2 - \tfrac{1}{2} || \frac{\dot\Phi^i}{f} + G^{ij}\partial_j W ||^2 \right\} \tag{15}$$

First-order (pseudo)-BPS equations:

$$\boxed{W = 2(D-2)\frac{\dot g}{fg}\,, \qquad \frac{\dot\Phi^i}{f} + G^{ij}\partial_j W = 0\,.} \tag{16}$$

Pseudo-supersymmetry and geodesics

For domain-walls corresponds to real supersymmetry.
Applications of pseudo-susy?$\rightarrow$ power-law cosmologies (scaling) :

$$a(\tau) \sim \tau^{P} \,. \tag{17}$$

Often dynamical attractors, cosmological “vacua”.

- It was shown that for power-law solutions the scalar flow $\dot{\Phi}^i$ is a **Killing Flow** [Tolley, Wesley '07] .
- Assume flow is pseudo-BPS, then consider $\ddot{\Phi}^i + \Gamma^i_{jk}\dot{\Phi}^j\dot{\Phi}^k = \nabla_{\dot{\Phi}}\dot{\Phi}^i$, in components:

$$\nabla_j \dot{\Phi}_i = \nabla_{[j}\dot{\Phi}_{i]} + \nabla_{(j}\dot{\Phi}_{i)} = \nabla_{[j}\dot{\Phi}_{i]} = \nabla_{[j}\nabla_{i]}W = 0 \,. \tag{18}$$

Therefore, *pseudo BPS powerlaw solutions describe a geodesic motion altough there is a non-zero potential.* (locally vice versa)

Summary & Outlook

Summary

1. There is a correspondence between brane solutions and geodesic curves using dimensional reduction.
2. The geodesic EOM can be solved using group theory. This was shown in a pedagogical example: time-dependent Einstein solutions.
3. We presented the multi-scalar pseudo-BPS equations.
4. When the scalars are massive a geodesic motion can occur: Power-law cosmological solutions that are pseudo-BPS have this property.

Future

1. Geodesic curves on cosets with non-compact isotropy and instantons [Bergshoeff, Chemissany, Ploegh, Van Riet, to appear]
2. Pseudo-supersymmetry extended to p-forms. Pseudo-susy of S-branes?
3. Pseudo-supersymmetry in holography?