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Exploring M theory symmetries through D=11 supergravity

Based on S.V. JHEP03 (2007) 010

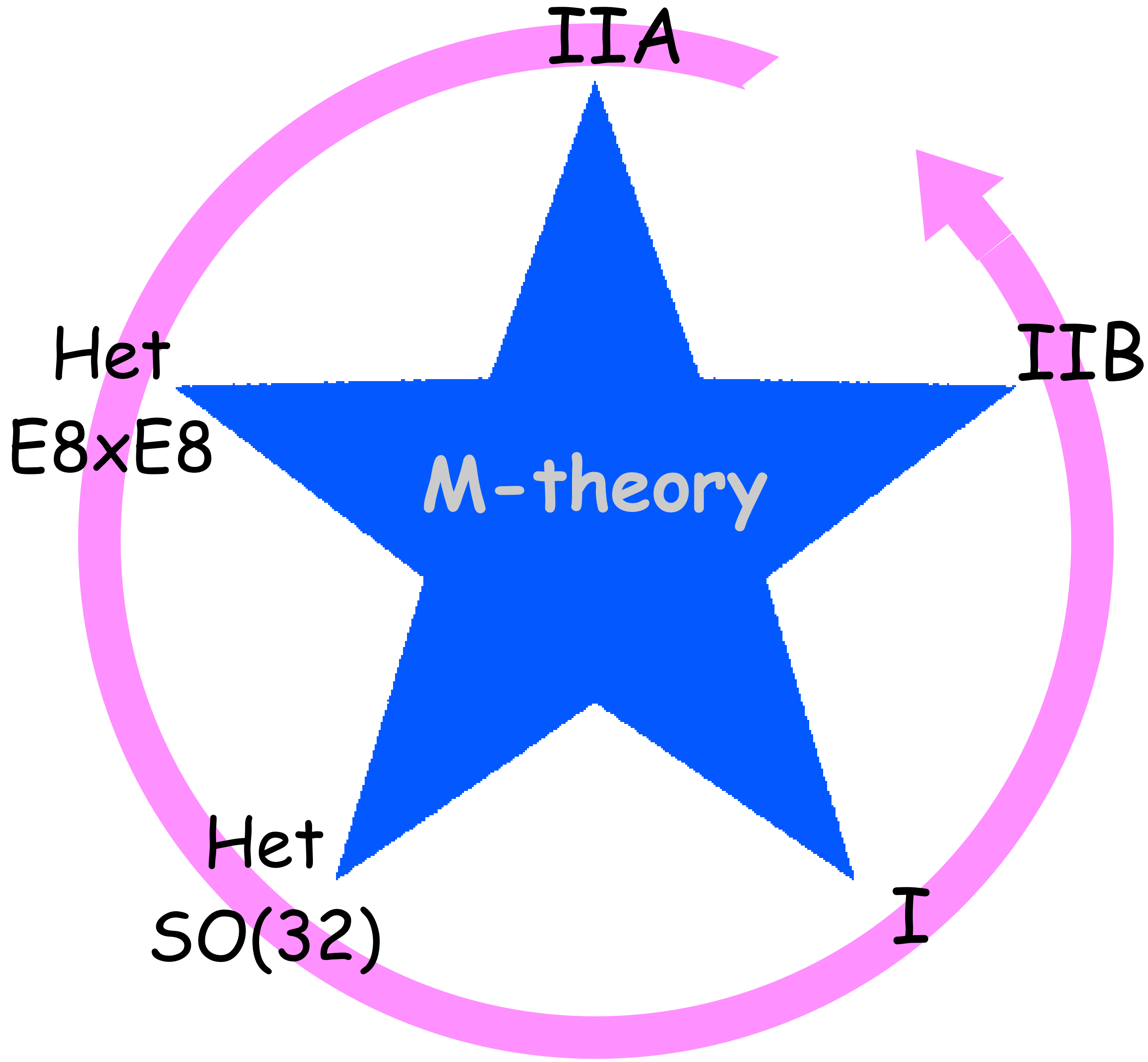
Silvia Vaulà, IFT/CSIC Madrid

Outline

- Goal: to infer some properties of M-Theory through the study of 11-dim Sugra.
  - Symmetries
  - Extended objects
- Review of the relevant aspects of String Theory
  - U-duality
  - D-branes
- 11-dim Sugra Free Differential Algebra
- Conclusions

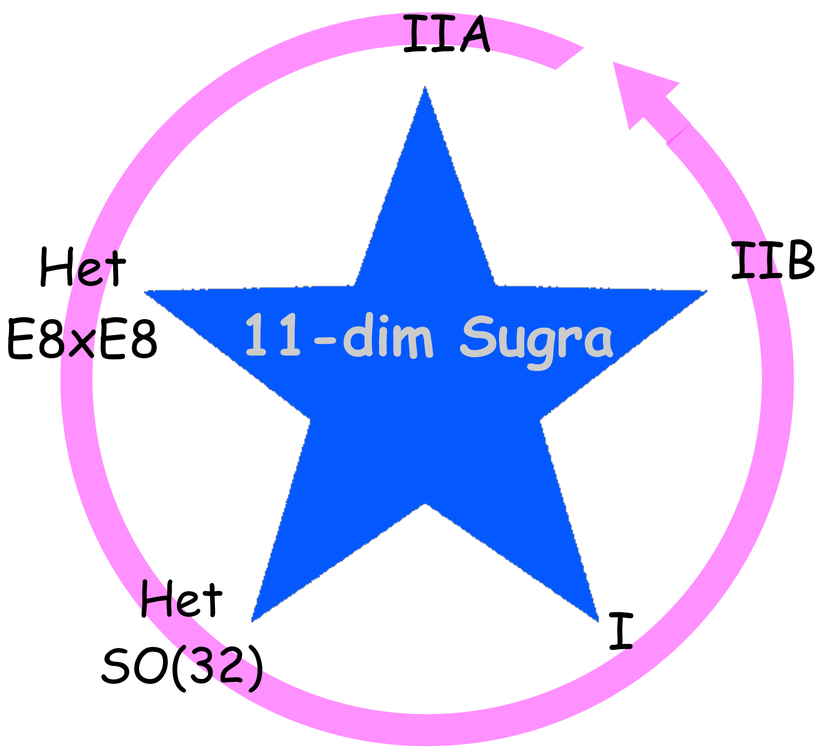
String dualities

M-Theory is a more fundamental unifying theory



S and T duality relate different String Theories

U-Duality



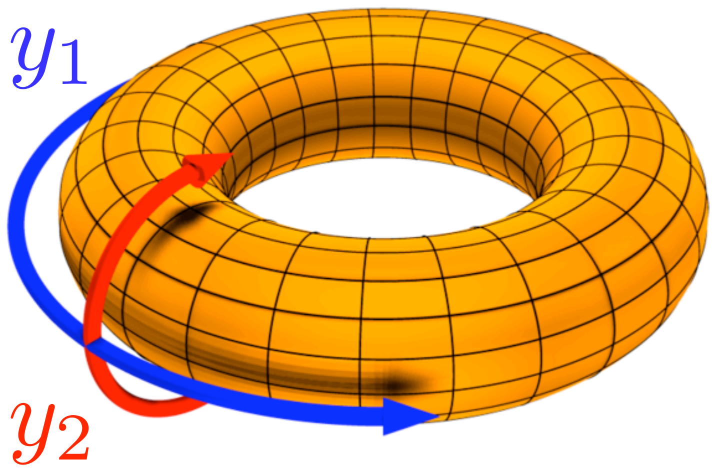
- 11-dim Sugra (M-Theory) is proposed as unifying theory
- In the Supergravity limit, type IIA arises form compactification of 11-dim Sugra on a circle
- 10-dim S and T duality are unified in lower dimensions

Spontaneous Compactification

- Expand in Fourier serie along the compact directions

$$\{X\} \to \{x, y\}\,; \quad \Phi(X) = \Phi(x, y)$$

$$\Phi(x, y) = \sum_n \Phi_n(x) e^{i\frac{y}{R}n}$$



- Integrate out the massive modes

$$\Box_D \Phi(x, y) = 0$$

$$\Box_{D-1}\Phi_n(x) - m_n^2 \Phi(x)_n = 0 \qquad m_n = \frac{n}{R}$$

- The process generates more scalars and vectors

$$A(X)_M dX^M = A(x, y)_\mu dx^\mu + A(x, y)_y dy$$

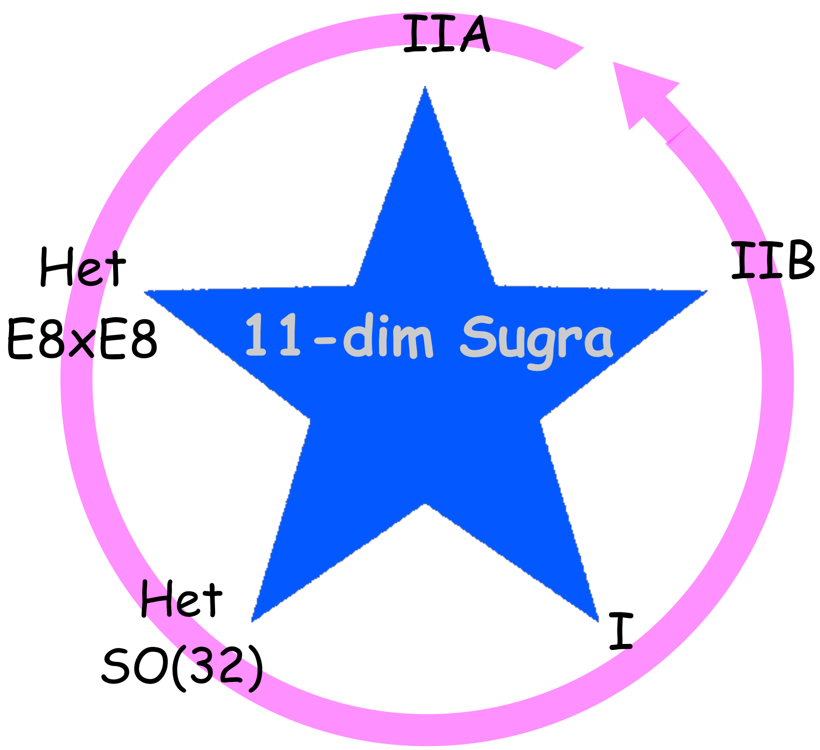
$$A(X)_M \Rightarrow A(x)_\mu + A(x)_y$$

10 dim Supergravity

$$g_{\mu\nu} \quad B_{\mu\nu} \quad \Phi$$

$$C_{\mu_1 \ldots \mu_p}$$

U-Duality



- 11-dim Sugra is the low energy limit of M-Theory
- In the Supergravity limit, type IIA arises form compactification of 11-dim Sugra on a circle
- S and T duality are “unified” in lower dimensions

D≤5 $$\mathcal{L} = R + \mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma\,\mu\nu} + g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j$$

S and T Duality unify in U-Duality which is a manifest symmetry of supergravities in low dimensions.

Lower dimensional theories have bigger U-Duality group.

Are symmetries unified in lower or higher dimensions?

Proposals

In spite of the fact that 11-dim Sugra action is manifestly invariant under the Super Poincaré group, it is conjectured that it encodes the symmetries of the lower dimensional U-duality groups.

- 11-dim Sugra is a non linear realisation of a bigger group (E_{10} or E_{11})

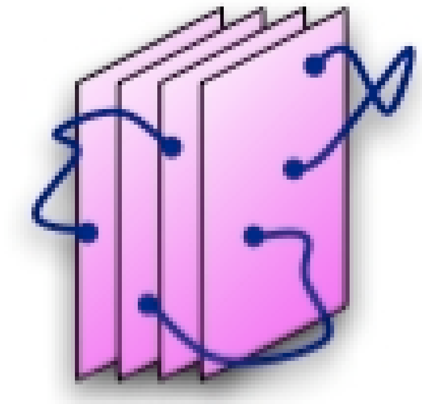
  Cremmer, Julia, Lu, Pope, West, Damour, Kleinschmidt, Nicolai (1998)

- 11-dim Sugra encodes a bigger group into its FDA

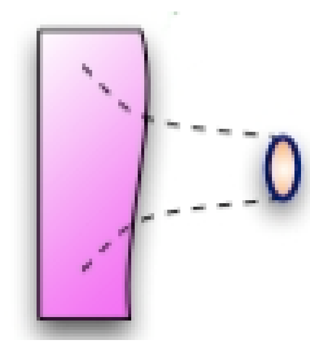
  D'Auria, Fre (1982) S.V. (2006)

Extended objects

- When strings are introduced, other extended objects (D-branes) naturally enter the game.



- D-branes are gravity sources and appear as massive extended objects in the classical solutions of 10-dim Sugra



- There exist M-brane solution also in 11-dim Sugra

- String Theory and M-theory as theories of extended objects

Extended objects

- 11-dim Sugra

$(g_{\mu\nu},\ \psi_\mu,\ C_{\mu\nu\rho})$

$$[P_a, P_b] = 0 \quad [Q_\alpha, M_{ab}] = \gamma_{ab\alpha}{}^{\beta} Q_\beta \quad [Q_\alpha, P_b] = 0$$

$$[M_{ab}, M_{cd}] = \eta_{ac} M_{cd} + \ldots \qquad [M_{ab}, P_c] = \eta_{c[a} P_{b]}$$

$$\{Q_\alpha, Q_\beta\} = i P_a \gamma^a_{\alpha\beta}$$

11-dim Super Poincaré Group

- 11-dim Sugra + M-branes

de Azcarraga, Gauntlett, Izquierdo, Townsend (1989)

$$\{Q_\alpha, Q_\beta\} = i P_a \gamma^a_{\alpha\beta} + Z_{ab} \gamma^{ab}_{\alpha\beta} + i Z_{a_1 \ldots a_5} \gamma^{a_1 \ldots a_5}_{\alpha\beta}$$

Central extension of the 11-dim Super Poincaré Group

- The M-Algebra

Sezgin (1996)

$$\{Q_\alpha, Q_\beta\} = i P_a \gamma^a_{\alpha\beta} + Z_{ab} \gamma^{ab}_{\alpha\beta} + i Z_{a_1 \ldots a_5} \gamma^{a_1 \ldots a_5}_{\alpha\beta}$$

$$[Q_\alpha, P_b] \neq 0 \qquad [Q_\alpha, Z_{ab}] \neq 0 \qquad \ldots$$

$$[P_a, P_b] \neq 0 \qquad \ldots \qquad \ldots$$

Our first results

The Minimal (zero field strengths) FDA
of 11-dim Supergravity
encodes M-Algebra symmetries

The FDA
of 11-dim Supergravity
encodes at least the lowest level of E_{11}

Free Differential Algebras

Commutators $\mathcal{T}\mathcal{G}$

Maurer-Cartan eqns $\mathcal{T}^*\mathcal{G}$

$$[T_A, T_B\} = C^C_{AB} T_C \qquad \mu^A(T_B) = \delta^A_B \qquad d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0$$

$$[T_{[A}, [T_B, T_{C\}}\}\} = 0$$

Jacobi identities

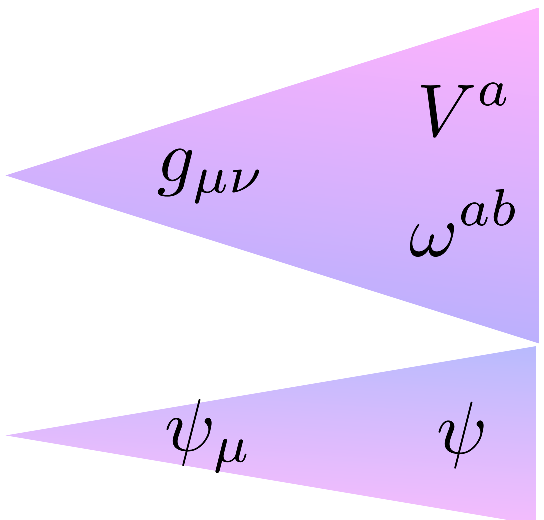
$$C^A{}_{B[C} C^B{}_{DE\}} = 0$$

$$d\left(d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0\right)$$

For the construction of sugra with the geometric approach we associate to every generator a 1-form potential of the theory: $\mu^A \longrightarrow \mathcal{A}^A$

11-dim Supergravity

11-dim Supermultiplet $(g_{\mu\nu},\, \psi_\mu,\, C_{\mu\nu\rho})$ $\mu,\, \nu,\, \rho = 0, \ldots 10$



$$V^a P_b = \delta^a_b \qquad \omega^{ab} M_{cd} = \delta^{ab}_{cd}$$

$$\psi^\alpha Q_\beta = \delta^\alpha_\beta$$

$(V^a,\, \omega^{ab},\, \psi) \in \mathcal{T}^* \mathcal{G}$ $\mathcal{G} =$ super-Poincaré in D=11

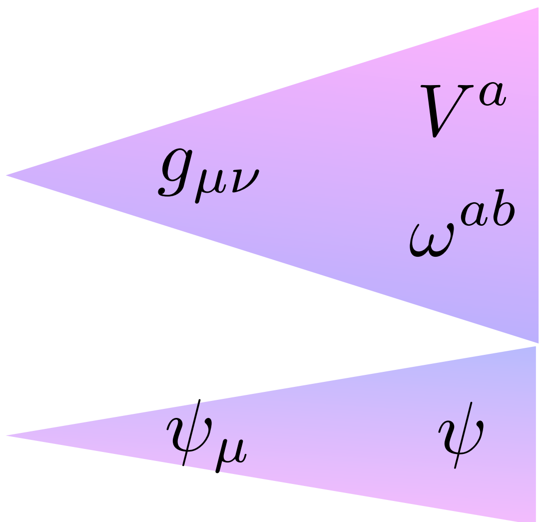
$$T^a \equiv DV^a - \frac{i}{2}\bar{\psi}\gamma^a\psi = 0$$

$$\mathcal{R}^{ab} \equiv d\omega^{ab} - \omega^a{}_c\omega^{cb} = 0$$

$$\rho \equiv D\psi = 0$$

11-dim Supergravity

11-dim Supermultiplet $(g_{\mu\nu},\, \psi_\mu,\, C_{\mu\nu\rho})$ $\mu,\, \nu,\, \rho = 0, \ldots 10$



$V^a P_b = \delta^a_b$ $\omega^{ab} M_{cd} = \delta^{ab}_{cd}$

$\psi^\alpha Q_\beta = \delta^\alpha_\beta$

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d

D

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$$\mathcal{R}^{ab} \equiv d\omega^{ab} - \omega^a{}_c\omega^{cb} = 0$$

$$\rho \equiv D\psi = 0$$

Minimal FDA

algebra

$$d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0 \qquad d\left(d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0\right)$$

μ^A 1-forms

FDA

$$d\pi^I + C^I_{J_1 \ldots J_n} \pi^{J_1} \wedge \ldots \pi^{J_n} = 0 \qquad d\left(d\pi^I + C^I_{J_1 \ldots J_n} \pi^{J_1} \wedge \ldots \pi^{J_n}\right) = 0$$

π^i p-forms

non Minimal FDA

algebra

$$d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0 \qquad d\left(d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0\right)$$

μ^A 1-forms

FDA

$$d\pi^I + C^I_{J_1 \dots J_n} \pi^{J_1} \wedge \dots \pi^{J_n} = F^I \qquad d\left(d\pi^I + C^I_{J_1 \dots J_n} \pi^{J_1} \wedge \dots \pi^{J_n}\right) = dF^I$$

contractible generators (field strengths)

Bianchi identities

non Minimal FDA

algebra $$d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0 \qquad d\left(d\mu^A - \frac{1}{2} C^A_{CB}\, \mu^B \wedge \mu^C = 0 \right)$$

μ^A 1-forms

FDA $$d\pi^I + C^I_{J_1 \dots J_n} \pi^{J_1} \wedge \dots \pi^{J_n} = F^I \qquad d\left(d\pi^I + C^I_{J_1 \dots J_n} \pi^{J_1} \wedge \dots \pi^{J_n} \right) = dF^I$$

contractible generators (field strengths)

Bianchi identities

$$dC - \frac{1}{2} \bar{\psi} \gamma_{ab} \psi V^a V^b = 0 \qquad \bar{\psi} \gamma^{ab} \psi \bar{\psi} \gamma_a \psi = 0$$

Minimal FDA

$$dV^a - \frac{i}{2}\bar{\psi}\gamma^a\psi = 0$$

$$d\psi = 0$$

symmetries of the solution $\mathcal{R}^{ab} = F = \rho = 0$
(plus $\mathrm{SO}(1,10)$)

$$\{Q_\alpha, Q_\beta\} = \gamma^a_{\alpha\beta} P_a \quad \{Q_\alpha, P_a\} = 0 \quad [P_a, P_b] = 0$$

$$dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b = 0$$

? Does this equation encode more symmetries?

Yes

Frè, D'Auria
Nucl.Phys.B201:1982

Group reduction of a Minimal FDA

$$dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b = 0 \quad \{V^a, \psi\} \in \mathcal{T}^*\mathcal{G}_0$$

$$C = K_{ijk}\sigma^i\sigma^j\sigma^k \qquad \{V^a, \psi\} \subset \{\sigma^i\} \in \mathcal{T}^*\tilde{\mathcal{G}}_0 \quad \mathcal{G}_0 \subset \tilde{\mathcal{G}}_0$$

constants

If $\tilde{\mathcal{G}}_0$ is a group manifold, then

$$d\sigma^i - \frac{1}{2}c^i{}_{kj}\,\sigma^j \wedge \sigma^k = 0$$

$$dV^a - \frac{i}{2}\bar{\psi}\gamma^a\psi = 0$$

$$d\psi = 0$$

$$dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b = 0$$

closed under d-differentiation

closed under d-differentiation

Group reduction of a Minimal FDA

? which is the algebra $d\sigma^i - \frac{1}{2} c^i{}_{jk} \sigma^j \wedge \sigma^k = 0$

There is not a "turn of the crank" procedure to determine the algebra

- Frè, D'Auria Nucl.Phys. B201:1982
- Bandos, de Azcarraga, Izquierdo, Picon, Varela Phys. Lett.B 596 (2004)
- Castellani, arXiv:hep-th/0508213

? It is possible to find "the most general solution"?
In case, which is it?

Sezgin's M-Algebra is the biggest extension of the Super Poincaré Algebra
If this is a solution, then it is the most general solution

The M-Algebra

$$\{Q_\alpha, Q_\beta\} = i\gamma^a_{\alpha\beta}\, P_a + i\gamma^a_{\alpha\beta}\, Z_a + \gamma^{ab}_{\alpha\beta}\, Z_{ab} + i\gamma^{a_1\ldots a_5}_{\alpha\beta}\, Z_{a_1\ldots a_5}$$

$$[Q_\alpha, P_a] = i\gamma_{a\alpha\beta}\, \Sigma^\beta + \gamma_{ab\alpha\beta}\, \Sigma^{b\beta} + \gamma_{ab_1\cdots b_4\alpha\beta}\, \Sigma^{b_1\cdots b_4\beta}$$

$$[Q_\alpha, Z^a] = -i\,(1-\lambda-\tau)\,\gamma^a_{\alpha\beta}\, \Sigma^\beta$$

$$[Q_\alpha, Z^{ab}] = \frac{\lambda}{10}\gamma^{ab}_{\alpha\beta}\, \Sigma^\beta + i\gamma^a_{\alpha\beta}\, \Sigma^{b\beta} - 6i\gamma^{cd}_{\alpha\beta}\Sigma^{abcd\beta}$$

$$[Q_\alpha, Z^{a_1\ldots a_5}] = i\frac{\tau}{720}\gamma^{a_1\ldots a_5}_{\alpha\beta}\, \Sigma^\beta + \gamma^{a_5}_{\alpha\beta}\, \Sigma^{\alpha_1\ldots a_4\beta}$$

$$[P_a, P_b] = 0$$

The M-Algebra

M2- and M5-brane central charges

$$\{Q_\alpha, Q_\beta\} = i\gamma^a_{\alpha\beta}\, P_a + i\gamma^a_{\alpha\beta}\, Z_a + \gamma^{ab}_{\alpha\beta}\, Z_{ab} + i\gamma^{a_1\ldots a_5}_{\alpha\beta}\, Z_{a_1\ldots a_5}$$

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The M-Algebra

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$$(Z_{a_1\ldots a_p},\, \Sigma^{\alpha\;\, a_1\ldots a_{p-1}}) \qquad p = 1, 2, 5$$

The M-Algebra

M2- and M5-brane central charges

$$\{Q_\alpha, Q_\beta\} = i\gamma^a_{\alpha\beta}\, P_a + i\gamma^a_{\alpha\beta}\, Z_a + \gamma^{ab}_{\alpha\beta}\, Z_{ab} + i\gamma^{a_1\ldots a_5}_{\alpha\beta}\, Z_{a_1\ldots a_5}$$

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$[P_a, P_b] \neq 0$ is not compatible with a flat background

Solutions (S.V. JHEP03 (2007) 010)

$$C = C(P^a,\, B^{a_1 \dots a_p},\, \psi,\, \eta^{a_1 \dots a_{p-1}}) \qquad dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b = 0$$

- $p = 1$ — No → it is impossible to satisfy the FDA
- $p = 2$ — Yes
- $p = 5$ — No
- $p = 1,\, 2$ — Yes
- $p = 1,\, 5$ — No
- $p = 2,\, 5$ — Yes
- $p = 1,\, 2,\, 5$ — Yes

($p = 5$ and $p = 1,\, 5$ →) non closure of the M-Algebra
No M5 without M2

Summary (1)

We started from a theory with "physical" fields

$$V^a_\mu,\ \psi_\mu,\ C_{\mu\nu\rho}$$

whose vacuum was invariant under the Super Poincaré Group

$$\Downarrow$$

We ended with a theory with "auxiliary" fields

$$V^a_\mu,\ \psi_\mu,\ B^a_\mu,\ B^{ab}_\mu,\ B^{a_1\ldots a_5}_\mu,\ \eta_\mu,\ \eta^a_\mu,\ \eta^{a_1\ldots a_4}_\mu$$

whose vacuum is invariant under the "flat" M-Algebra

Non-Minimal FDA

$$d\pi^i + \frac{1}{2} c^i_{jk} \pi^j \wedge \pi^k = F^i$$ ↔ Field strengths definition

$$c^i_{jk} (F^j - \frac{1}{2} c^j_{lm} \pi^l \wedge \pi^m) \wedge \pi^k = dF^i$$ ↔ Bianchi identities

minimal generators π^i ↔ vector potentials

contractible generators F^i ↔ field strengths

Frè, Class. Quant. Grav. **1** (1984)

? Is it possible to further enlarge the group $\tilde{\mathcal{G}}_0$ in order to find an expansion $F^i = K^i_{j_1 \dots j_n} \tilde{\sigma}^{j_1} \wedge \dots \tilde{\sigma}^{j_n}$ which satisfies the FDA?

The simplest example

$$d \quad 0 = DV^a - \frac{i}{2}\bar{\psi}\gamma^a\psi$$ no torsion

$$0 = d\omega^{ab} - \omega^{ac}\omega_c{}^b$$ flat space-time

$$D \quad 0 = D\psi$$ flat space-time

$$F = dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b$$

the FDA is consistent with any of the contractible generators set to zero

Keep the same parametrisation for C
and modify the underlying algebra

The "flat" M-Algebra is already the most general

The external automorphism group

$$C = C(P,\, B^{(p)},\, \psi,\, \eta^{(p-1)})$$

$$dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b = 0$$

$$F = dC - \frac{1}{2}\bar{\psi}\gamma_{ab}\psi V^a V^b$$

$$dB^{(p)} + \bar{\psi}\gamma^{(p)}\psi = 0$$

$$dB^{(p+q)} + A^{(q)} B^{(p)} + \bar{\psi}\gamma^{(p+q)}\psi = 0$$

$$R_{(r)} A^{(q)} = \delta^{(q)}_{(r)}$$

$$[R_{(r)}, Z_{(p)}] = Z_{(p+r)}$$

$$[R_{(r)}, R_{(s)}] = R_{(r+s)}$$

The external automorphism group

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Group generators

=

fields

Automorphism

generators

=

connections

$$R_{(r)}A^{(q)} = \delta^{(q)}_{(r)}$$

$$[R_{(r)}, Z_{(p)}] = Z_{(p+r)}$$

$$[R_{(r)}, R_{(s)}] = R_{(r+s)}$$

$r = 0$

$\mathrm{SO}(1,10)$

lowest levels of E_{11}

Summary

The "flat" M-Algebra

$$[P_a, P_b] = 0$$

$$\{Q_\alpha, Q_\beta\} = i\gamma^a_{\alpha\beta}\ P_a + \gamma^{(p)}_{\alpha\beta} Z_{(p)}$$

$$[Q_\alpha, P_a] = \gamma^{\cdots}_{\alpha\beta}\Sigma_{(p-1)}$$

$$[Q_\alpha, Z_{(p)}] = \gamma^{\cdots}_{\alpha\beta}\Sigma_{(p-1)}$$

fields

The lowest E11 levels

$$[R_{(r)}, Z_{(p)}] = Z_{(p+r)}$$

$$[R_{(r)}, R_{(s)}] = R_{(r+s)}$$

connections

Summary

The "flat" M-Algebra

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fields

M-Algebra generators can be interpreted as E11 generators

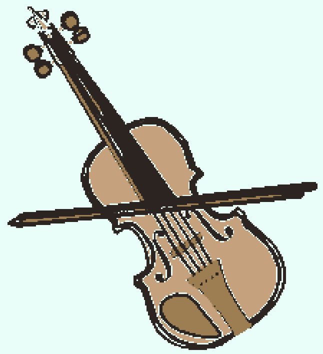
The lowest E11 levels

$$[R_{(r)}, Z_{(p)}] = Z_{(p+r)}$$

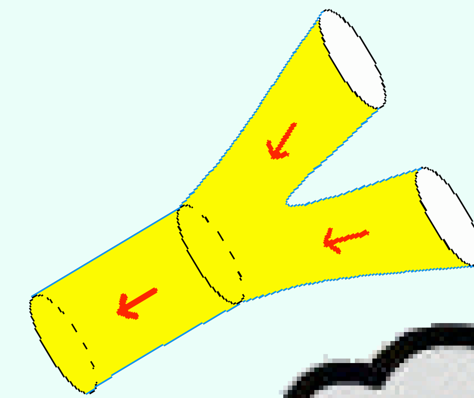
$$[R_{(r)}, R_{(s)}] = R_{(r+s)}$$

$R_{(r)}$ rotates one into another the brane charges and was shown (West) to be as well a symmetry of the system. Here we show that the D=11 action enjoys such a symmetry and that the brane charges are "hidden" in the structure of the 3-form.

Thank you!



String Theory



Our 4 dimensional World