QCD-resummation for slepton pair production at hadron colliders

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Outline

1. Fixed-order results
   - Slepton-pair production at hadron colliders
   - Next-to-leading order calculations
   - The need for resummation
2. Resummation formalisms
   - Main features of the resummation
   - The resummed component
   - The matching
3. Numerical results
   - q_T-distribution at the LHC
   - Invariant-mass distributions
   - Total cross section
4. Summary and outlook

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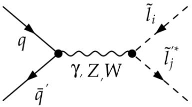
Slepton-pair production at hadron colliders

- Drell-Yan like process



$$q\bar{q} \to \gamma, Z^0 \to \tilde{l}_i \tilde{l}_j^* \qquad \text{and} \qquad q\bar{q}' \to W^\mp \to \tilde{l}_i \tilde{\nu}_l^* + c.c.$$

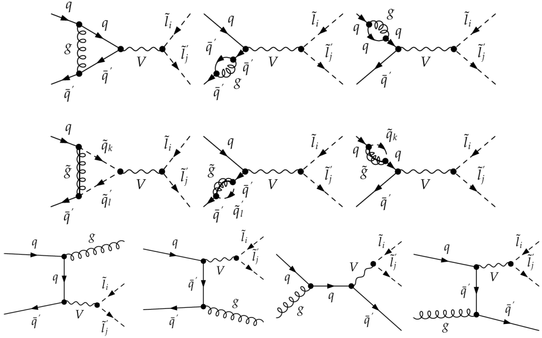
- Sleptons are often light $\Rightarrow$ decays into LSP + SM lepton $\Rightarrow$ clean signal.
- Cross sections given by

$$(\Delta)\sigma = \sum_{a,b} \int_\tau^1 \mathrm{d}x_a \int_{\tau/x_a}^1 \mathrm{d}x_b \, (\Delta) f_{a/h_1}(x_a, \mu_F) \; (\Delta) f_{b/h_2}(x_b, \mu_F) \, (\Delta)\hat{\sigma}_{ab}(z, M; \alpha_s(\mu_R), \tfrac{M}{\mu_F}, \tfrac{M}{\mu_R})$$

where $(\Delta)\hat{\sigma}_{ab}$ is computed perturbatively

$$(\Delta)\hat{\sigma}_{ab}(z, M; \alpha_s(\mu_R), \tfrac{M}{\mu_F}, \tfrac{M}{\mu_R}) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n (\Delta)\sigma_{ab}^{(n)}(z, M; \tfrac{M}{\mu_F}, \tfrac{M}{\mu_R}) \ .$$

NLO calculations



- Partonic M and q_T distributions at $\mathcal{O}(\alpha_s)$:

$$\frac{\mathrm{d}\hat{\sigma}_{ab}}{\mathrm{d}M^2} = \hat{\sigma}^{(0)}_{ab}(M)\,\delta(1-z)+\frac{\alpha_s}{\pi}\,\hat{\sigma}^{(1)}_{ab}(M,z)+\mathcal{O}(\alpha_s^2),$$

$$\frac{\mathrm{d}^2\hat{\sigma}_{ab}}{\mathrm{d}M^2\,\mathrm{d}q_T^2} = \hat{\sigma}^{(0)}_{ab}(M)\,\delta(q_T^2)\delta(1-z)+\frac{\alpha_s}{\pi}\,\hat{\sigma}^{(1)}_{ab}(M,z,q_T)+\mathcal{O}(\alpha_s^2),$$

where $z = M^2/s$.

- Squark mixing included in the SUSY-loops.

The need for resummation

- Soft and collinear radiations:
  - $\frac{\alpha_s^n}{q_T^2} \ln^m \frac{M^2}{q_T^2}$ or $\alpha_s^n \left(\frac{\ln^m(1-z)}{1-z}\right)_+$ terms in the distributions $(m \leq 2n-1)$.
  - Large at small q_T or $z \lesssim 1$.
  - Fixed-order theory unreliable in these kinematical regions.
  - Resummation to all orders needed.

    $\Rightarrow$ q_T-resummation.

    $\Rightarrow$ Threshold resummation.

    $\Rightarrow$ Joint resummation.
- Advantages of resummation:
  - Reliable perturbative results.
  - Correct quantification of these radiations (even far from critical regions).
  - Accurate invariant-mass and q_T spectra.

    q_T-distribution $\Rightarrow$ stransverse mass $\Rightarrow$ spin and mass determination.

    [Lester, Summers (1999); Barr (2006)]

    M-distribution and total cross section $\Rightarrow$ accurate mass determination.

    [Bozzi, Fuks, Klasen (2007)]

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Main features of the resummation

Reorganization of the cross section

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(\mathrm{res})} + \mathrm{d}\sigma^{(\mathrm{fin})} \; .$$

- $\mathrm{d}\sigma^{(\mathrm{res})}$
  - Contains all the logarithmic terms.
  - Resummed to all orders in α_s.
  - Exponentiation (Sudakov form factor).
- $\mathrm{d}\sigma^{(\mathrm{fin})}$
  - Remaining contributions.

The resummed component: conjugate spaces (1)

- Conjugate space(s) introduced $\Rightarrow$ kinematics naturally factorizes.
- N-moments defined by a Mellin transform

$$F(N) = \int_0^1 \mathrm{d}y \, y^{N-1} \, F(y).$$

- Inverse transform:

$$F(y) = \oint_{C_N} \frac{\mathrm{d}N}{2\pi i} \, y^{-N} F(N).$$

- N-moment of the hadronic cross section taken with respect to $\tau = M^2/s_h$.
- q_T-spectrum: impact-parameter b-space defined via a Fourier transform.
- The logarithms:

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \quad \longrightarrow \quad \ln^2 \overline{N} \quad \text{with} \quad \overline{N} = N \exp[\gamma_E]$$

$$\frac{1}{q_T^2} \ln \frac{M^2}{q_T^2} \quad \longrightarrow \quad \ln \overline{b}^2 \quad \text{with} \quad \overline{b} = \frac{b\,M}{2} \exp[\gamma_E]$$

The resummed component: conjugate spaces (2)

- Factorization of the hadronic cross sections:

$$\frac{d\sigma^{(\mathrm{res})}}{dM^2}(\tau,M)=\sum_{a,b}\int_\tau^1 dx_a\int_{\tau/x_a}^1 dx_b\, f_{a/h_1}(x_a,\mu_F)\, f_{b/h_2}(x_b,\mu_F)\,\hat{\sigma}^{(\mathrm{res})}_{ab}(z;\alpha_s(\mu_R),\tfrac{M}{\mu_F},\tfrac{M}{\mu_R})$$

$$\Downarrow$$

$$\frac{d\sigma^{(\mathrm{res})}}{dM^2}(N,M)=\sum_{a,b} f_{a/h_1}(N+1,\mu_F)\, f_{b/h_2}(N+1,\mu_F)\,\hat{\sigma}^{(\mathrm{res})}_{ab}(N;\alpha_s,\tfrac{M}{\mu_R},\tfrac{M}{\mu_F}),$$

and

$$\frac{d^2\sigma^{(\mathrm{res})}}{dM^2dq_T^2}(\tau,M,q_T)=\sum_{a,b}\int_\tau^1 dx_a\int_{\tau/x_a}^1 dx_b\, f_{a/h_1}(x_a,\mu_F)\, f_{b/h_2}(x_b,\mu_F)\,\hat{\sigma}^{(\mathrm{res})}_{ab}(z,q_T;\alpha_s(\mu_R),\tfrac{M}{\mu_F},\tfrac{M}{\mu_R})$$

$$\Downarrow$$

$$\frac{d^2\sigma^{(\mathrm{res})}}{dM^2dq_T^2}(N,M,q_T)=\sum_{a,b} f_{a/h_1}(N+1,\mu_F)\, f_{b/h_2}(N+1,\mu_F)\int \tfrac{b}{2}\, db\, J_0(b\,q_T)\,\mathcal{W}^F_{ab}(N,b;\alpha_s,\tfrac{M}{\mu_R},\tfrac{M}{\mu_F}).$$

- The logarithms are included in the functions $\hat{\sigma}^{(\mathrm{res})}$ and $\mathcal{W}^F$.

The resummed component: the partonic cross section

- The process-dependence is factorized outside the exponent:

$$\begin{aligned} \mathcal{W}^{F}_{ab}(N,b) &= \mathcal{H}^{F}_{ab}(N)\,\exp\left\{\mathcal{G}(N,b)\right\}, \\ \hat{\sigma}^{(\mathrm{res})}_{ab}(N) &= \sigma^{(LO)}\,\tilde{C}_{ab}(N;\alpha_s)\,\exp\left\{\mathcal{G}(N,L)\right\}. \end{aligned}$$

- $\mathcal{H}^F$- and $\tilde{C}$-functions:
  - * Can be computed perturbatively.
  - * Are process-dependent.
  - * Contain all the finite terms in the limit $N \to \infty$ and $b \to \infty$ (real and virtual collinear radiation, hard contributions).

The resummed component: the Sudakov form factor

- The Sudakov form factor contains the soft-collinear radiation:

$$\mathcal{G}(N,L;\tfrac{M^2}{\mu_R^2})=L\,g^{(1)}(\alpha_s\,L)+\sum_{n=2}^{\infty}\left(\tfrac{\alpha_s}{\pi}\right)^{n-2}g_N^{(n)}(\alpha_s\,L;\tfrac{M^2}{\mu_R^2})$$

- The logarithm L is

	q_T	Joint	Threshold
$L=\ln(\dots)$	$1+\bar{b}^2$	$\bar{b}+\frac{\bar{N}}{1+\frac{\bar{b}}{4\,\bar{N}}}$	$\bar{N}$

- q_T-resummation not justified at small b (large q_T) $\Rightarrow$ $+1$ in the log (no change at large b).
- Argument of the log in joint resummation:
  $\Rightarrow$ no subleading terms in perturbative expansions of $\sigma^{(\mathrm{res})}$.
- At NLL accuracy: $g^{(1)}$ and $g^{(2)}$ needed
  $\equiv \alpha_s^n L^{2n}$ and $\alpha_s^n L^{2n-1}$.

The resummed component: improvements and remarks

- q_T-resummation [Catani, de Florian, Grazzini (2001); Bozzi, Catani, de Florian, Grazzini (2006)]
  - Universal formalism.
  - Process-independent Sudakov form factor.
  - Resummation impact only in the relevant kinematical region.
- Threshold resummation [Sterman (1987); Catani, Trentadue (1989, 1991)]
  - Consistent inclusion of the collinear radiation in the $\tilde{C}$-function.

    [Krämer, Laenen, Spira (1998); Catani, de Florian, Grazzini (2001)]
- Joint resummation [Laenen, Sterman, Vogelsang (2001); Kulesza, Sterman, Vogelsang (2002, 2004)]
  - Process-independent and universal Sudakov form factor.

    [Bozzi, Fuks, Klasen (*in prep.*)]

The finite component: matching procedure

- Fixed-order theory
  - Reliable far from the critical kinematical regions ($z \ll 1$, $q_T \gg 0$).
  - Spoiled in the critical regions ($z \sim 1$, $q_T \sim 0$).
- Resummation
  - Needed in the critical regions.
  - Not justified far from the critical regions.
- Both contributions important in the intermediate kinematical regions.

- Information from both fixed-order and resummation needed.
- Need to avoid double-counting.
- Consistent matching procedure required:

$$\mathrm{d}\sigma^{(\mathrm{fin})} = \mathrm{d}\sigma^{(\mathrm{f.o.})} - \mathrm{d}\sigma^{(\mathrm{exp})}.$$

Summary: complete resummation formulae

- Invariant-mass spectrum

$$\begin{aligned}\frac{d\sigma}{dM^2}(\tau,M) &= \frac{d\sigma^{(\mathrm{F.O.})}}{dM^2}(\tau,M)\\ &+ \oint_{C_N}\frac{dN}{2\pi i}\,\tau^{-N}\left[\frac{d\sigma^{(\mathrm{res})}}{dM^2}(N,M)-\frac{d\sigma^{(\mathrm{exp})}}{dM^2}(N,M)\right].\end{aligned}$$

- Transverse-momentum spectrum

$$\begin{aligned}\frac{d^2\sigma}{dM^2\,dq_T^2}(\tau,M,q_T) &= \frac{d^2\sigma^{(\mathrm{F.O.})}}{dM^2\,dq_T^2}(\tau,M,q_T)\\ &+ \oint_{C_N}\frac{dN}{2\pi i}\,\tau^{-N}\int\frac{b\,db}{2}J_0(q_T\,b)\left[\frac{d^2\sigma^{(\mathrm{res})}}{dM^2\,dq_T^2}(N,b)-\frac{d^2\sigma^{(\mathrm{exp})}}{dM^2\,dq_T^2}(N,b)\right].\end{aligned}$$

  * Far from the critical regions, $d\sigma^{(\mathrm{res})}\approx d\sigma^{(\mathrm{exp})}$ $\Rightarrow$ Perturbative theory.
  * In the critical regions, $d\sigma^{(\mathrm{F.O.})}\approx d\sigma^{(\mathrm{exp})}$ $\Rightarrow$ Pure resummation.
  * In the intermediate regions $\Rightarrow$ Consistent matching.

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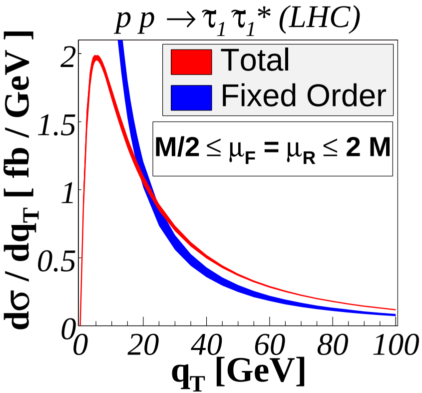
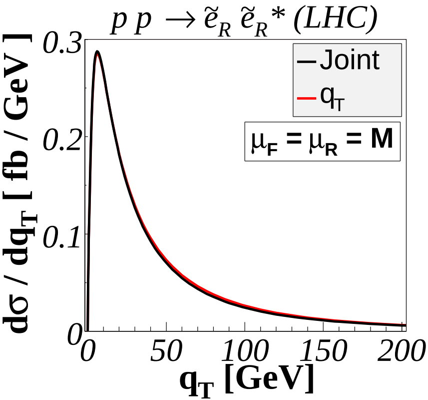
q_T-distribution at the LHC

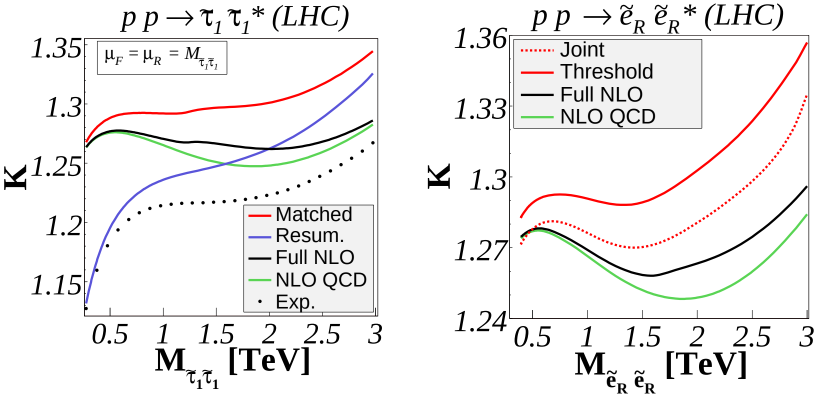




[Bozzi, Fuks, Klasen (2006; *in prep.*)]

- SPS1a scenario ($m_{\tilde{l}} \approx$ 100-200 GeV).
- Finite results at small-q_T, enhancement at intermediate-q_T, finite σ.
- Improvement of scale dependences: (NLL+LO $\lesssim$ 5%; LO 10%).
- Effects of the threshold-enhanced contributions in the intermediate-q_T region.

Invariant-mass distribution at the LHC

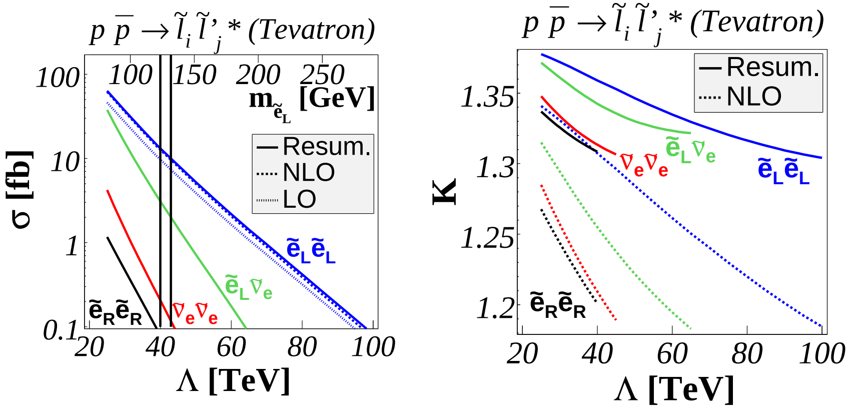


[Bozzi, Fuks, Klasen (2007; *in prep.*)]

- SPS1a scenario ($m_{\tilde{l}} \approx$ 100-200 GeV).
- Normalization to LO cross section.
- Small M: $\mathrm{d}\sigma^{(\mathrm{res})} \approx \mathrm{d}\sigma^{(\mathrm{exp})}$; Large M: $\mathrm{d}\sigma^{(\mathrm{F.O.})} \approx \mathrm{d}\sigma^{(\mathrm{exp})}$.
- Reduced SUSY-loop effects.
- Joint-exponent reproduces q_T-exponent.

  $\Rightarrow$ some differences with threshold-resummation (however under control).

Threshold-resummed total cross sections at the Tevatron



[Bozzi, Fuks, Klasen (2007)]

- SPS7 slope.
- $\sigma \sim 0.1 - 100$ fb ($\Rightarrow$ 1 to 1000 events).
- NLO and threshold-resummation effects important.
- Resummation more important for heavier sleptons.
- Shift in $m_{\tilde{e}_L}$ if deduced from total σ measurement.

Outline

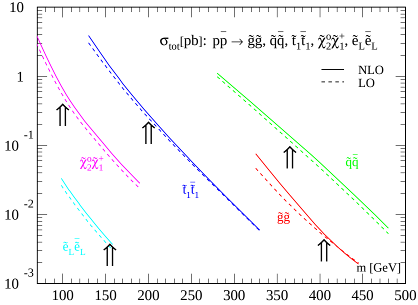
Conclusion and outlook

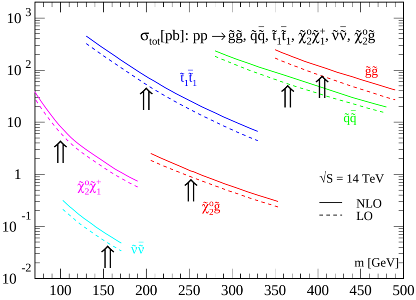
- Done
  - * Full NLO SUSY-QCD calculations, including squark mixing.
  - * Threshold, q_T and joint resummations.
- To do
  - * Comparison with the Monte Carlo approach.
  - * Study of other SUSY particle production processes.
  - * Resummation vs. Monte Carlo for other BSM theories (Z').

Appendix

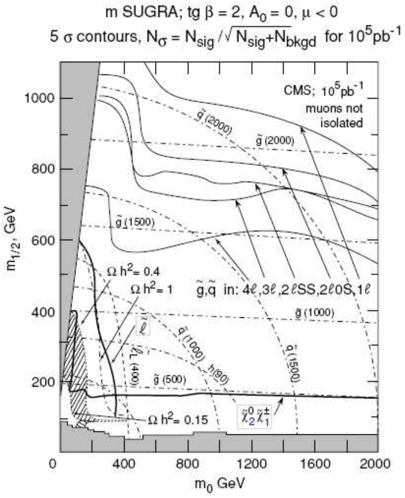
Appendix

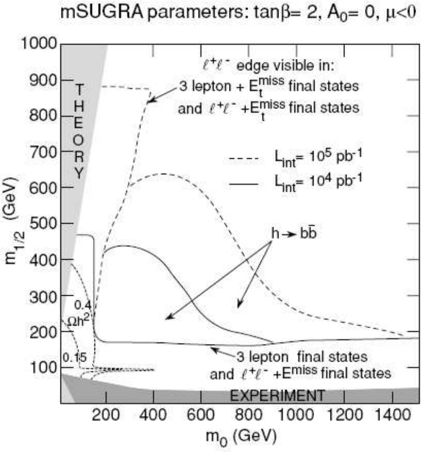
SUSY cross sections





SUSY cross sections





[Abdullin *et al.* (2002)]

Inverse transforms

- There are singularities in the integrand in the (b,N) spaces

$$\oint_{C_N} \frac{\mathrm{d}N}{2\pi i}\,\tau^{-N}\int \frac{b\mathrm{d}b}{2} J_0(q_T\, b)\left[\frac{\mathrm{d}^2\sigma^{(\mathrm{res})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b) - \frac{\mathrm{d}^2\sigma^{(\mathrm{exp})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b)\right].$$

  - * e.g. argument of the logarithm, Landau pole, PDFs,...
  - * Must be avoided when getting back to physical space.
  - * Prescription required.
- Inverse *b*-transform:
  - * Integration contour diverted in the complex plane.
  - * Bessel function replaced by more convenient auxiliary functions.

  [Laenen, Sterman, Vogelsang (2000)]
- Inverse Mellin transform:
  - * Specific contour avoiding all the poles.
  - * Minimal prescription and principal value resummation.

  [Catani, Mangano, Nason, Trentadue (1996); Contopanagos, Sterman (1994)]

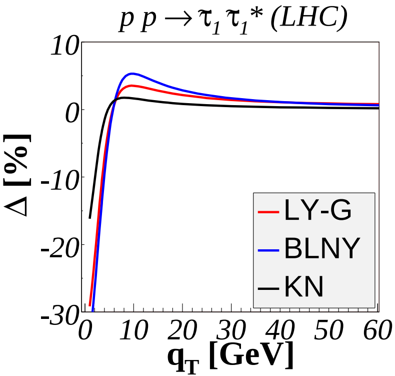
The resummed component: non perturbative effects

- Important non-perturbative (NP) effects for q_T-distributions (large-b region).
  $\equiv$ intrinsic q_T of the partons, inside the hadrons.
- Resummation formula

$$\mathcal{W}^F_{ab}(N,b) \quad = \quad \mathcal{H}^F_{ab}(N) \exp\left\{\mathcal{G}(N,b)+F^{\mathrm{NP}}_{ab}\right\}.$$

- NP form factor obtained from experimental data:
  - * Ladinsky-Yuan (LY-G) [Ladinsky, Yuan (1994)].
  - * Brock-Landry-Nadolsky-Yuan (BLNY) [Landry, Brock, Nadolsky, Yuan (2003)].
  - * Konyshev-Nadolsky (KN) [Konyshev, Nadolsky (2006)].

q_T-distribution at the LHC

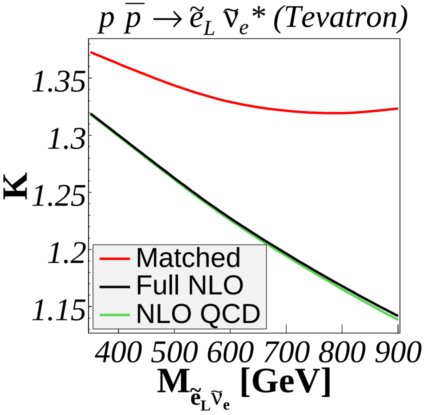


[Bozzi, BF, Klasen (2006)]

* SPS1a scenario (slepton masses ≈ 100-200 GeV).
* Importance of the NP effects:

$$\Delta = \frac{d\sigma^{(\mathrm{res.+NP})}(\mu = M) - d\sigma^{(\mathrm{res.})}(\mu = M)}{d\sigma^{(\mathrm{res.})}(\mu = M)}.$$

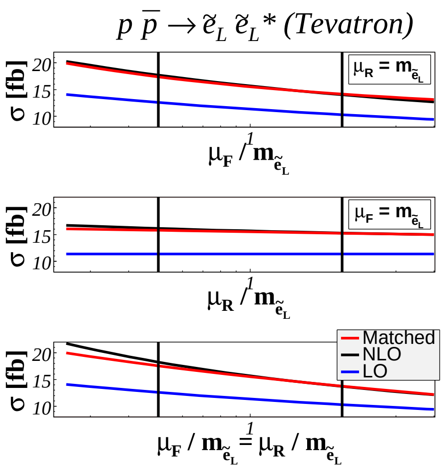
K-factors for associated $\tilde{e}_L \tilde{\nu}_e$ production at the Tevatron



[Bozzi, Fuks, Klasen (2007)]

- SPS1a' SUSY scenario.
- Normalization to LO cross section.
- Close to the threshold:
  - * Resummation effects important (even at low M).
  - * σ^{NLO} dominated by the logs.
  - * $\hat{\sigma} = \hat{\sigma}^{(res)}$ at the permille level.
  - * $\hat{\sigma}^{(f.o.)} = \hat{\sigma}^{(res)}\,|_{f.o.}$ at the same level.
- SUSY-loop effects reduced.

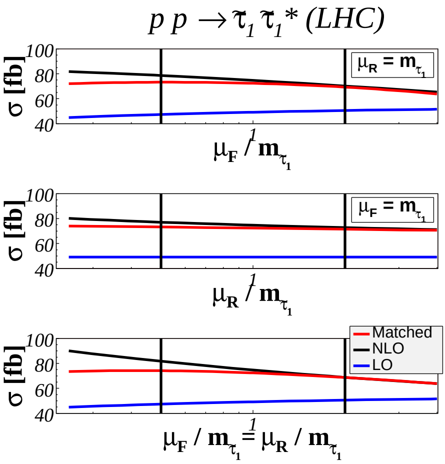
Dependence of the total σ on unphysical scales (1)



[Bozzi, Fuks, Klasen (2007)]

- SPS7 SUSY scenario.
- Theoretical uncertainty estimated by variations of μ_F and μ_R.
- μ_R dependence
  - Absent at LO.
  - Introduced at NLO.
  - Tamed by resummation.
- μ_F dependence
  - Reduced at NLO.
  - Further stabilized by resummation.
- Total theoretical uncertainty.
  - LO: 20%
  - NLO: 29%
  - Resummed: 23%

Dependence of the total σ on unphysical scales (2)



[Bozzi, Fuks, Klasen (2007)]

- SPS7 SUSY scenario.
- Theoretical uncertainty estimated by variations of μ_F and μ_R.
- μ_R dependence
  * Absent at LO.
  * Introduced at NLO.
  * Tamed by resummation.
- μ_F dependence
  * Overcompensated at NLO.
  * Stabilized by resummation.
- Total theoretical uncertainty.
  * LO: 7%
  * NLO: 17%
  * Resummed: 8%