O'Raifeartaigh models with spontaneous R-symmetry breaking

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Introduction

O'Raifeartaigh models: SUSY breaking with $\langle F_X \rangle \neq 0$

- Chiral superfields X_n, ϕ_i, R-charges $\boxed{R(X_n) = 2,\ R(\phi_i) = 0}$
- Superpotential $\boxed{W = \sum_n X_n\, g_n(\phi_i)}$
- Vacuum equations $g_n(\phi_i) = 0, \sum_n X_n\, \partial_j g_n(\phi_i) = 0$, SUSY spontaneously broken if $n_X > n_\phi$
- Space of flat directions parametrized by $X_1 \ldots X_{n_X - n_\phi}$. Complexified R-symmetry acts as $\boxed{X_n \to \alpha X_n \quad \alpha \in \mathbb{C}}$
- Coleman-Weinberg 1-loop effective potential: minimum at $X_n = 0$

ISS model (Intriligator,Seiberg,Shih)

Metastable non-SUSY vacua in $\mathcal{N} = 1$ SQCD!

Low energy theory (Seiberg dual)

$$W = \tilde{q}_{i\alpha} M_{ij} q_j^\alpha + \mu^2 M_{ii} \quad i = 1 \ldots N_F,\ \alpha = 1 \ldots N_F - N_C$$

$\to$ (Weakly gauged) O'Raifeartaigh-like model

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SUSY breaking and R-symmetry

R-symmetry plays an important role in O'Raifeartaigh models

- Nelson-Seiberg argument:
  Generic superpotential and SUSY breaking $\Rightarrow$ R-symmetry
- ISS argument:
  Metastable SUSY breaking $\leftrightarrow$ *approximate* R-symmetry

Gaugino masses $\gtrsim$ 100 GeV $\Rightarrow$ R-symmetry must be broken

Two possibilities:

- Explicit breaking: vacuum metastability
- Spontaneous breaking: R-axion problem

How to achieve spontaneous breaking?

- Gauge interactions ("inverted hierarchy")
- Perturbative dynamics of O'Raifeartaigh models (Shih model)

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The Shih model

Choose R-charges different from $R = 2, 0$

- Fields: X, $\phi_{(-1)}$, $\phi_{(1)}$, $\phi_{(3)}$
- R-charges: $R(X) = 2$, $R(\phi_{(k)}) = k$
- Superpotential

$$W = fX + NX\phi_{(1)}\phi_{(-1)} + M_3\phi_{(3)}\phi_{(-1)} + \frac{M_1}{2}\phi_{(1)}^2$$

- Vacuum equations $\Rightarrow$ SUSY breaking

$$f + N\phi_{(1)}\phi_{(-1)} = 0 \qquad \begin{aligned} M_3\phi_{(3)} + NX\phi_{(1)} &= 0 \\ M_1\phi_{(1)} + NX\phi_{(-1)} &= 0 \\ M_3\phi_{(-1)} &= 0 \end{aligned}$$

- For f small, there is a flat direction of non-SUSY minima at $\phi_{(-1)} = \phi_{(1)} = \phi_{(3)} = 0$ parametrized by X

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Coleman-Weinberg potential in the Shih model

Flat directions are lifted by 1-loop effective potential:

$$V_{eff}^{(1-loop)}(X) = \frac{1}{64\pi^2}\mathrm{Tr}\left(\mathcal{M}_B^4(X)\ln\frac{\mathcal{M}_B^2(X)}{\Lambda^2} - \mathcal{M}_F^4(X)\ln\frac{\mathcal{M}_F^2(X)}{\Lambda^2}\right)$$

This potential has the form

$$V_{eff}^{(1-loop)}(X) = V_0 + m_X^2|X|^2 + \lambda_X|X|^4 + \dots$$

where (for f small)

$$m_X^2 = \frac{f^2}{32\pi^2}\mathrm{Tr}\int_0^\infty dv\, v^3\left[\mathcal{M}_1(v)\mathcal{M}_1^\dagger(v) - \mathcal{M}_2(v)\mathcal{M}_2^\dagger(v)\right]$$

Note that m_X^2 is *not* positive-definite!

$$\mathcal{M}_1(v) = \frac{1}{\sqrt{v^2+\hat{M}^2}}\hat{N}\left(\frac{\sqrt{2}v}{v^2+\hat{M}^2}\right)\hat{N}\frac{1}{\sqrt{v^2+\hat{M}^2}}$$

$$\mathcal{M}_2(v) = \frac{1}{\sqrt{v^2+\hat{M}^2}}\hat{N}\left(\frac{2\hat{M}}{v^2+\hat{M}^2}\right)\hat{N}\frac{1}{\sqrt{v^2+\hat{M}^2}}$$

$$\hat{N} = \begin{pmatrix} 0 & 0 & 0 & 0 & N & 0 \\ 0 & 0 & 0 & N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N & 0 & 0 & 0 & 0 \\ N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad , \quad \hat{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & M_3 \\ 0 & 0 & 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_3 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 & 0 & 0 \\ M_3 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The two contribution to m_X^2 are of the same order $\quad \rightarrow \not{R}$ if $m_X^2 < 0$

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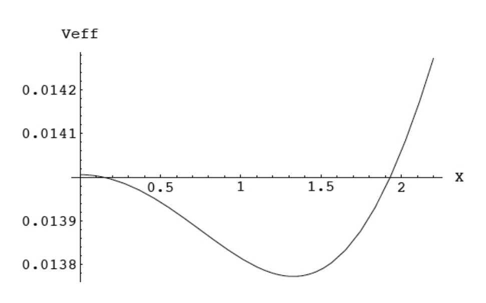
Choose the couplings such that $M_3 \lesssim 0.5\, M_1$

Coleman-Weinberg potential:



Minimum with $|\langle X \rangle| \sim M_3, M_1$

Non-hierarchical R-symmetry breaking

($\mathbb{Z}_2$ unbroken)

Same features for the class of models

$$W = fX + \frac{1}{2} N^{ij} X \phi_i \phi_j + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{2} Q_a^{ij} Y_a \phi_i \phi_j$$

Spontaneous R-symmetry breaking in the Shih models

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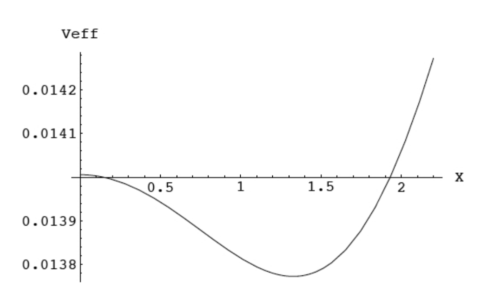

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Symmetries

Flavor and gauge symmetries needed for ultraviolet completion, SUSY-breaking mediation...
No relevant changes in Coleman-Weinberg potential

- Easy to introduce *real representations* in the Shih model, for example SO(N) fundamentals:

$$W = fX + NX\phi^{\alpha}_{(1)}\phi^{\alpha}_{(-1)} + M_3\phi^{\alpha}_{(3)}\phi^{\alpha}_{(-1)} + \frac{1}{2}M_1\phi^{\alpha}_{(1)}\phi^{\alpha}_{(1)}$$

- The simplest model with *complex representations*, for example U(N) fundamentals:

$$\begin{aligned} W &= fX + XN_5\phi^{\alpha}_{(5)}\phi_{(-5)\alpha} + XN_3\phi^{\alpha}_{(3)}\phi_{(-3)\alpha} + \\ &\quad + M_7\phi^{\alpha}_{(7)}\phi_{(-5)\alpha} + M_5\phi^{\alpha}_{(5)}\phi_{(-3)\alpha} + M_3\phi^{\alpha}_{(3)}\phi_{(-1)\alpha} \end{aligned}$$

This model also shows spontaneous R-symmetry breaking

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Runaway directions in the Shih model

Go back to the classical potential

$$V = \left|f + N\phi_{(1)}\phi_{(-1)}\right|^2 + \left|M_1\phi_{(1)} + NX\phi_{(-1)}\right|^2 + \left|M_3\phi_{(3)} + NX\phi_{(1)}\right|^2 + \left|M_3\phi_{(-1)}\right|^2$$

There is a *runaway direction*:

$$\phi_{(1)} = -\frac{f}{\lambda\phi_{(-1)}} \quad , \quad X = \frac{m_2 f}{\lambda^2\phi_{(-1)}^2} \quad , \quad \phi_{(3)} = \frac{m_2 f^2}{m_1\lambda^2\phi_{(-1)}^3} \quad , \quad \phi_{(-1)} \to 0$$

Along this direction $V \to 0$ (supersymmetric runaway vacuum)

- This direction corresponds to a complexified R-charge rescaling:

$$\boxed{\varphi(\epsilon) = \epsilon^{-R(\varphi)}\varphi \quad , \quad \epsilon \to 0}$$

- Vacua previously found are only metastable!

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Runaway directions

Many models $W(\varphi_j)$ with generic R-charges have runaway directions.
Argument:

- Vacuum equations $\partial_i W(\varphi_j) = 0$ can be classified by R-charges:

$$\begin{array}{ll} \partial_i W = 0,\ R(\varphi_i) < 2 & R > 0 \\ \partial_i W = 0,\ R(\varphi_i) = 2 & R = 0 \\ \partial_i W = 0,\ R(\varphi_i) > 2 & R < 0 \end{array}$$

- SUSY breaking $\Rightarrow$ this set of equations cannot be solved ...
  ... but it can be possible to solve the *subset* with $R \geq 0$
- The form of the potential $V = V_{R<0} = \sum_{R(\varphi_i)>2} |\partial_i W|^2$ does not change under complex R-symmetry transformations
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It can also happen that equations with $R = 0$ cannot be solved. In this case there could be non-SUSY runaway directions.

Example: modified Shih model

$$W = fX + NX\phi_{(1)}\phi_{(-1)} + M_3\phi_{(3)}\phi_{(-1)} + \frac{M_1}{2}\phi_{(1)}^2 + QY\phi_{(1)}\phi_{(-1)}$$

$$R(Y) = 2 \qquad V = \overbrace{\left|f + N\phi_{(1)}\phi_{(-1)}\right|^2}^{R=0} + \overbrace{\left|Q\phi_{(1)}\phi_{(-1)}\right|^2}^{R=0} + \dots$$

Runaway direction $V \to V_\infty = \min_\phi V_{R=0} = |f|^2(1 + |N|^2/|Q|^2)^{-1}$

Metastability

Small explicit R-symmetry breaking terms restore supersymmetry:

$$W_\varepsilon = W + \varepsilon_r W_r^{\not R} \qquad R(\varepsilon_r) \neq 0$$

SUSY vacua are pushed to infinity as $\varepsilon_r \to 0$
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- Flavor symmetries can be easily introduced
- These models often have runaway directions (both SUSY and non-SUSY)

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- Retrofitting
- Finding an ultraviolet completion
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