

---

# MPP inspired Two-Higgs Doublet Model

Roman Nevzorov

Glasgow University

*in collaboration with C.D.Froggatt and H.B.Nielsen*

# Outline

---

- Introduction
- MPP conditions
- MPP and custodial symmetries
- Higgs phenomenology
- Conclusions

Based on:

C. D. Froggatt, R. Nevzorov, H. B. Nielsen, D. Thompson, in preparation;

C. D. Froggatt, L. Laperashvili, R. Nevzorov, H. B. Nielsen and M. Sher,

Phys. Rev. D 73 (2006) 095005.

# Introduction

---

- In the SM particle content, gauge invariance and renormalizability imply the absence of FCNC transitions at the tree level.
- The CP violation in the SM arises from the phase of CKM matrix and from the "θ-term" in  $\mathcal{L}_{QCD}$ :

$$\mathcal{L}_\theta = \theta_{\text{eff}} \frac{\alpha_s}{8\pi} F^{\mu\nu a} \tilde{F}_{\mu\nu}^a, \quad \theta_{\text{eff}} = \theta + \arg \det M_q .$$

- $\mathcal{L}_\theta$  induces neutron electric dipole moment

$$|d_n| \sim |\theta_{\text{eff}}| \cdot 10^{-16} \text{ e cm} .$$

- Current experimental limit

$$d_n < 0.6 \cdot 10^{-25} \quad \Longrightarrow \quad |\theta_{\text{eff}}| \lesssim 10^{-9}$$

raises the question: why is CP violation so small?

- 
- The violation of CP invariance and FCNC transitions are generic features of  $SU(2)_W \times U(1)_Y$  theories with two and more Higgs doublets.
  - In particular, complex Higgs self-couplings in the 2HDM can result in relative phase between  $\langle H_1 \rangle$  and  $\langle H_2 \rangle$  that gives a non-trivial contribution to  $\theta_{\text{eff}}$ .
  - In the 2HDM one can eliminate the violation of CP invariance and tree-level FCNC transitions by imposing discrete  $Z_2$  symmetry.
  - However  $Z_2$  symmetry leads to the formation of domain walls in the early Universe which create unacceptably large anisotropies in the CMB radiation.

Y. B. Zeldovich, I. Y. Kobzarev and L. B. Okun, Sov. Phys. JETP 40 (1974) 1;  
A. Vilenkin, Phys. Rept. 121 (1985) 263.

# MPP conditions

---

- Here instead of the custodial  $Z_2$  symmetry we use multiple point principle (MPP) to suppress FCNC and CP-violation effects in the 2HDM.
- MPP postulates the existence of the maximal number of phases with the same energy density which are allowed by a given theory.
- Being applied to the 2HDM multiple point principle implies that at some high energy scale  $\Lambda$  there exists a large set of degenerate vacua.
  - According to the MPP these vacua and the physical one must have approximately the same energy densities.
  - This means that all couplings at the MPP scale must be adjusted so that the energy density of the MPP scale vacua vanishes with relatively high accuracy.

- The 2HDM effective potential can be written as

$$\begin{aligned}
 V_{eff}(H_1, H_2) = & m_1^2(\Phi)H_1^\dagger H_1 + m_2^2(\Phi)H_2^\dagger H_2 - \left[ m_3^2(\Phi)H_1^\dagger H_2 + h.c. \right] + \\
 & \frac{\lambda_1(\Phi)}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2(\Phi)}{2}(H_2^\dagger H_2)^2 + \lambda_3(\Phi)(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(\Phi)|H_1^\dagger H_2|^2 \\
 & + \left[ \frac{\lambda_5(\Phi)}{2}(H_1^\dagger H_2)^2 + \lambda_6(\Phi)(H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7(\Phi)(H_2^\dagger H_2)(H_1^\dagger H_2) + h.c. \right].
 \end{aligned}$$

- The most general vacuum configuration is given by

$$\langle H_1 \rangle = \Phi_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \Phi_2 \begin{pmatrix} \sin \theta \\ \cos \theta e^{i\omega} \end{pmatrix}.$$

- Here we assume that  $m_i^2$  and  $\lambda_i$  depend only on  $\Phi^2 = \Phi_1^2 + \Phi_2^2$  and near the MPP scale  $\Phi \simeq \Lambda$ .

- 
- When  $\Lambda \gg v$  the mass terms in the 2HDM effective potential can be safely ignored.
  - The degeneracy of vacua with respect to  $\omega$  requires that
    - 2HDM scalar potential becomes independent of phase  $\omega$  at the MPP scale;
    - its partial derivatives vanish for any choice of phase  $\omega$  near the scale  $\Lambda$ .
  - These conditions can be fulfilled only if

$$\lambda_5(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0,$$
$$\beta_{\lambda_5}(\Lambda) = \beta_{\lambda_6}(\Lambda)\Phi_1^2 + \beta_{\lambda_7}(\Lambda)\Phi_2^2 = 0.$$

where  $\beta_{\lambda_i}(\Phi) = \frac{d\lambda_i(\Phi)}{d \ln \Phi}$  is the beta function for  $\lambda_i(\Phi)$ .

- 
- As a result near the MPP scale the 2HDM effective potential takes a form

$$V(H_1, H_2) \approx \frac{1}{2} \left( \sqrt{\lambda_1(\Phi)} \Phi_1^2 - \sqrt{\lambda_2(\Phi)} \Phi_2^2 \right)^2 + \tilde{\lambda}(\Phi) \Phi_1^2 \Phi_2^2,$$
$$\tilde{\lambda}(\Phi) = \sqrt{\lambda_1(\Phi) \lambda_2(\Phi)} + \lambda_3(\Phi) + \lambda_4(\Phi),$$

where  $\lambda_4(\Lambda) < 0$ .

- The vacuum energy density and partial derivatives of  $V(H_1, H_2)$  vanish at the MPP scale vacua only if

$$\tilde{\lambda}(\Lambda) = \beta_{\tilde{\lambda}}(\Lambda) = 0.$$

- Then Higgs potential reaches its minimal value at

$$\Phi_1 = \Lambda \cos \gamma, \quad \Phi_2 = \Lambda \sin \gamma, \quad \tan \gamma = \left( \frac{\lambda_1(\Lambda)}{\lambda_2(\Lambda)} \right)^{1/4}.$$



- 
- MPP conditions give rise to the set of MPP scale vacua

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ \Phi_1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ \Phi_2 e^{i\omega} \end{pmatrix}.$$

- MPP conditions are satisfied identically in the MSSM at any scale lying higher than the masses of sparticles.
- To ensure the stability of physical and MPP scale vacua the following conditions must be satisfied everywhere between the MPP and EW scales:

$$\lambda_1(\Phi) > 0, \quad \lambda_2(\Phi) > 0, \quad \tilde{\lambda}(\Phi) > 0.$$

- Otherwise another minimum of the Higgs effective potential arises at some intermediate scale destabilising the physical and MPP scale vacua.

# MPP and custodial symmetries

---

- MPP conditions constrain the couplings of  $H_1$  and  $H_2$  to quarks and leptons.
- Lagrangian describing the interactions of Higgs fields with the third generation fermions is given by

$$\mathcal{L}_{Yuk} \simeq h_t(H_2 \varepsilon Q) \bar{t}_R + g_b(H_2^\dagger Q) \bar{b}_R + g_\tau(H_2^\dagger L) \bar{\tau}_R + \\ + g_t(H_1 \varepsilon Q) \bar{t}_R + h_b(H_1^\dagger Q) \bar{b}_R + h_\tau(H_1^\dagger L) \bar{\tau}_R + h.c.,$$

- One can redefine Higgs fields so that  $g_t = 0$ .
- The MPP conditions

$$\beta_{\lambda_5}(\Lambda) = \beta_{\lambda_6}(\Lambda) \sqrt{\lambda_2(\Lambda)} + \beta_{\lambda_7}(\Lambda) \sqrt{\lambda_1(\Lambda)} = 0$$

are fulfilled simultaneously only if

$$\begin{array}{ll} (I) & h_b(\Lambda) = h_\tau(\Lambda) = 0; \\ (II) & g_b(\Lambda) = g_\tau(\Lambda) = 0; \\ (III) & h_b(\Lambda) = g_\tau(\Lambda) = 0; \\ (IV) & g_b(\Lambda) = h_\tau(\Lambda) = 0. \end{array}$$

- In the considered models the quartic part of  $V(H_1, H_2)$  and  $\mathcal{L}_{Yuk}$  are invariant under  $Z_2$  and Peccei–Quinn symmetry transformations.
- Using the MPP assumption one can derive global custodial symmetries even when all three generations of fermions have non–zero couplings to the Higgs fields.
- The independence of Higgs effective potential

$$V_{eff}(H_1, H_2) = \sum_{n=0}^{\infty} V_n(H_1, H_2), \quad V_1 = \frac{1}{64\pi^2} Str |M|^4 \left[ \log \frac{|M|^2}{\mu^2} - C \right]$$

on  $\omega$  at the MPP scale implies that  $\mathcal{L}_{Yuk}$  is invariant under the following symmetry transformations:

$$\begin{aligned} H_1 &\rightarrow e^{i\alpha} H_1, & u'_{R_i} &\rightarrow e^{i\alpha} u'_{R_i}, & d'_{R_i} &\rightarrow e^{-i\alpha} d'_{R_i}, & e'_{R_i} &\rightarrow e^{-i\alpha} e'_{R_i}, \\ H_2 &\rightarrow e^{i\beta} H_2, & u''_{R_i} &\rightarrow e^{i\beta} u''_{R_i}, & d''_{R_i} &\rightarrow e^{-i\beta} d''_{R_i}, & e''_{R_i} &\rightarrow e^{-i\beta} e''_{R_i}. \end{aligned}$$

- 
- Global custodial symmetries allow to avoid CP–violation entirely eliminating the  $\theta$ –term and prevent the appearance of FCNC at the tree level.
  - The term  $m_3^2(H_1^\dagger H_2)$  in  $V(H_1, H_2)$ , which is not forbidden by the MPP, breaks  $Z_2$  and  $U(1)$  global symmetries.
  - It spoils the solution of the strong CP problem but it does not give rise to CP–violation or FCNC transitions.
  - Because MPP does not imply that vacua should be exactly degenerate the couplings which violate custodial symmetries are allowed to have non–zero values.
  - Requiring that energy densities of the considered vacua should be the same with the accuracy  $v^2 \Lambda^2$  we get

$$\lambda_5(\Lambda) \simeq \lambda_6(\Lambda) \simeq \lambda_7(\Lambda) \lesssim \frac{v^2}{\Lambda^2}, \quad g_i(\Lambda) \lesssim (4\pi)^2 \frac{v^2}{\Lambda^2}.$$

# Higgs phenomenology

---

- The Higgs spectrum of the MPP inspired 2HDM involves
  - one pseudoscalar  $m_A$ ,
  - two charged states  $m_{H^\pm}^2 = m_A^2 - \frac{\lambda_4}{2}v^2$ ,
  - two scalars  $m_H^2 \simeq m_A^2$ ,
$$m_h^2 \lesssim \left( \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{1}{2}(\lambda_3 + \lambda_4) \sin^2 2\beta \right) v^2 .$$
- The direct searches for the rare B–meson decays ( $B \rightarrow X_s \gamma$ ) set a stringent lower limit on  $m_{H^\pm}^2$  in some MPP inspired models.
- In the MPP inspired 2HDM the Higgs masses and couplings can be parametrised in terms of

$$m_A, \quad \tan \beta = v_2/v_1, \quad \lambda_1, \quad \lambda_2, \quad \lambda_3, \quad \lambda_4 .$$

- However MPP conditions allow to express  $\lambda_3(\Lambda)$  and  $\lambda_4(\Lambda)$  in terms of other couplings

$$\lambda_3(\Lambda) = -\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)} - \lambda_4(\Lambda),$$

$$\lambda_4^2(\Lambda) = \frac{6h_t^4(\Lambda)\lambda_1(\Lambda)}{(\sqrt{\lambda_1(\Lambda)} + \sqrt{\lambda_2(\Lambda)})^2} - 2\lambda_1(\Lambda)\lambda_2(\Lambda) - \frac{3}{8} \left( 3g_2^4(\Lambda) + 2g_2^2(\Lambda)g_1^2(\Lambda) + g_1^4(\Lambda) \right),$$

where  $\lambda_4(\Lambda) < 0$ .

- Thus the RG flow of  $h_t(\mu)$  and  $\lambda_i(\mu)$  is determined by

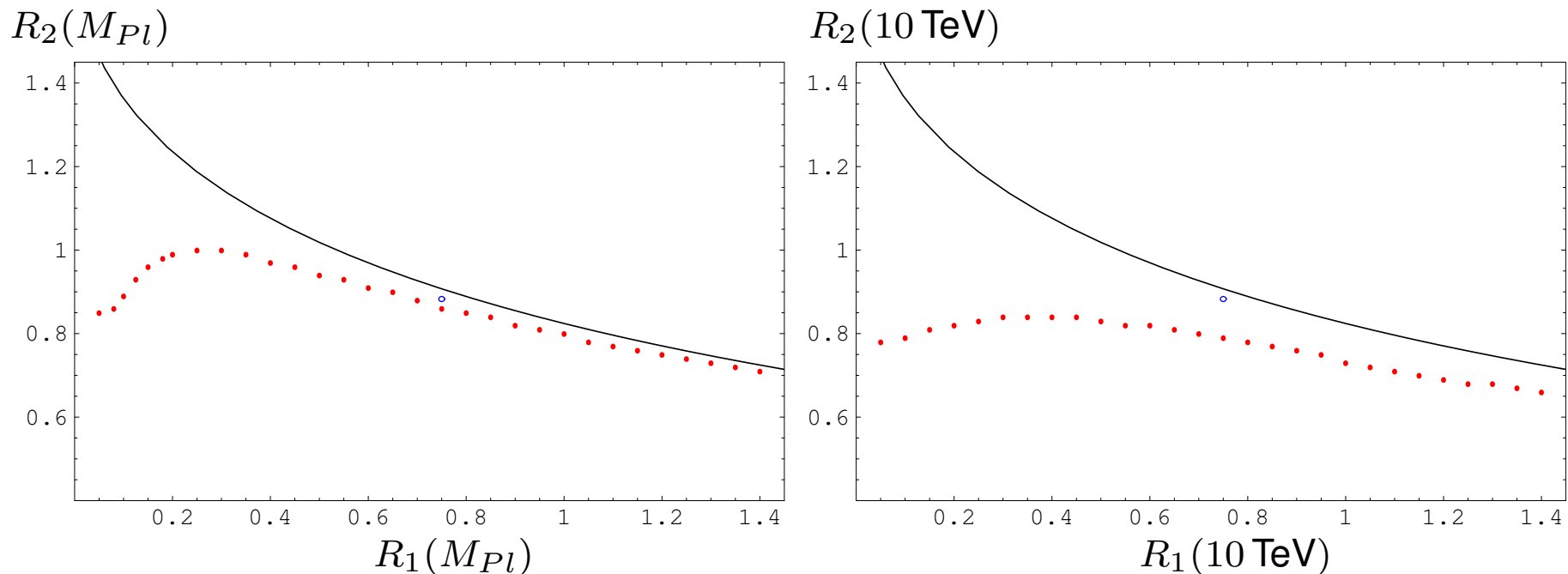
$$\Lambda, \quad h_t(\Lambda) \quad (\text{or } \tan \beta), \quad \lambda_1(\Lambda), \quad \lambda_2(\Lambda).$$

- For the purposes of RG studies it is convenient to introduce

$$R_i(\mu) = \frac{\lambda_i(\mu)}{h_t^2(\mu)}.$$

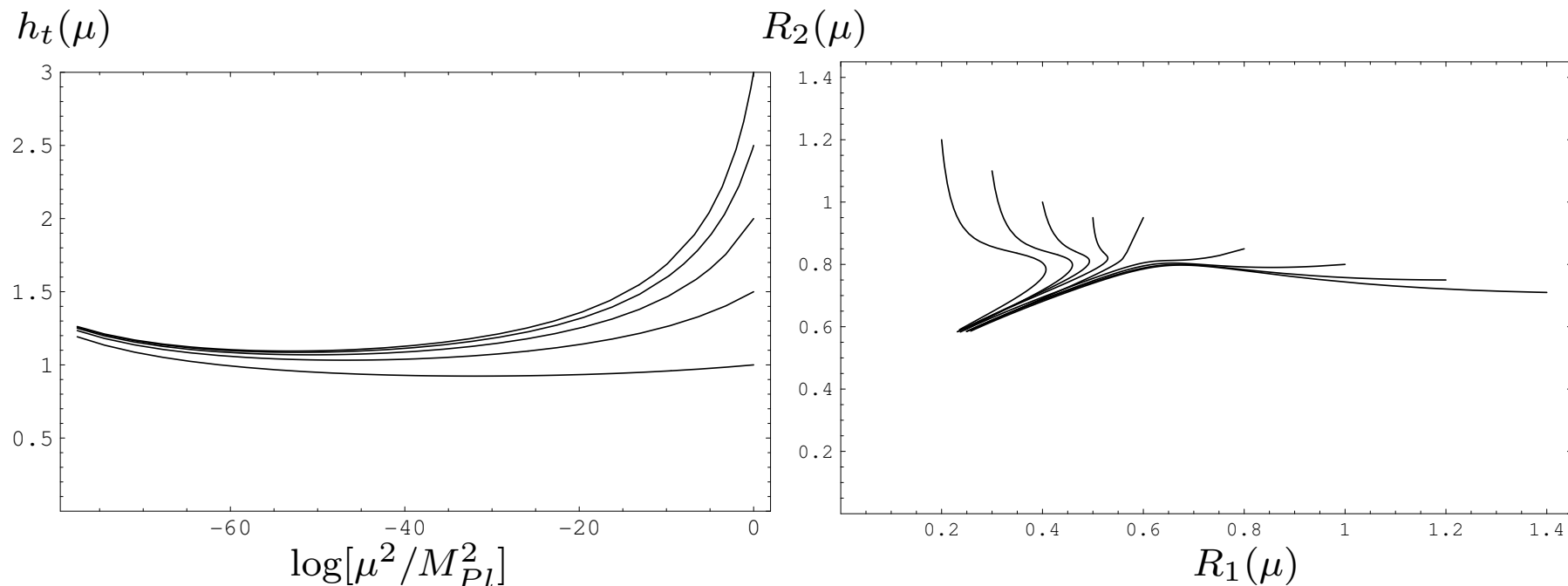
- MPP conditions and vacuum stability requirements constrain  $R_1(\Lambda)$  and  $R_2(\Lambda)$  very strongly.
- As a consequence the mass of the SM-like Higgs boson does not exceed **180 GeV** for a wide set of the MPP scales ( $\Lambda \gtrsim 10 \text{ TeV}$ ) and  $\tan \beta$  ( $2 \lesssim \tan \beta \ll 60$ ).

The allowed range of  $R_1(\Lambda)$  and  $R_2(\Lambda)$  for  $h_t(\Lambda) = 3$



- When MPP scale is high enough and  $h_t(\Lambda)$  is rather large ( $h_t^2(\Lambda) \gtrsim 1$ ) the solutions of RG equations are focused in a narrow interval near the quasi-fixed point.
- If  $\Lambda \gtrsim 10^{10}$  GeV the quasi-fixed point scenario results in the stringent restriction on  $m_{h_1} \lesssim 125$  GeV.

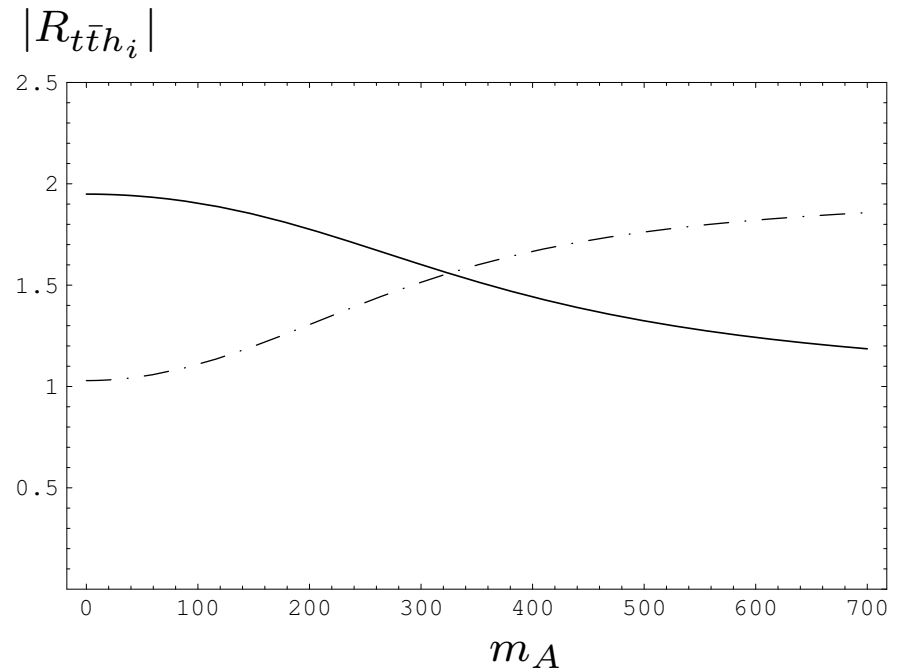
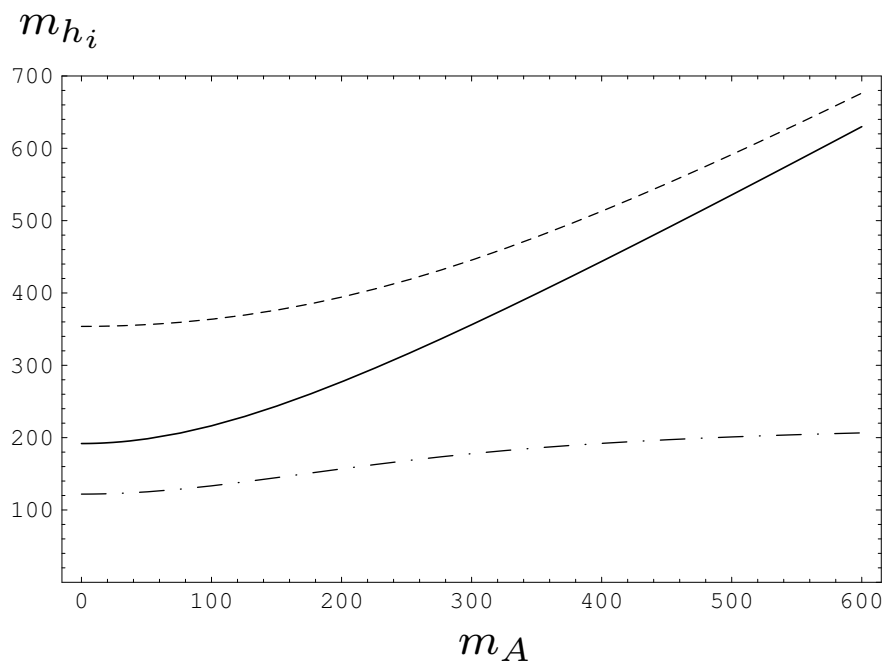
The RG flow of the top quark Yukawa and Higgs couplings





- For  $\Lambda \simeq 10 - 10^3 \text{ TeV}$  the quasi-fixed point solution leads to the large values of the coupling of the lightest CP-even Higgs boson to the  $t$ -quark  $R_{t\bar{t}h_1}$ .
- It gives rise to the enhanced production of the SM-like Higgs boson at the LHC.

Higgs masses and couplings for  $\Lambda = 10 \text{ TeV}$  and  $h_t^2(\Lambda) = 10$



# Conclusions

---

- We have argued that the multiple point principle (MPP) can be used as a mechanism for the suppression of FCNC and CP–violation effects in the 2HDM.
- In the supersymmetric two Higgs doublet extension of the SM the MPP conditions are fulfilled without any extra fine–tuning.
- In the MPP inspired 2HDM the Higgs self–couplings are severely constrained by the MPP conditions so that at large  $\tan \beta$  and  $\Lambda \gtrsim 10 \text{ TeV}$

$$m_{h_1} \lesssim 180 \text{ GeV} .$$

- In the quasi–fixed point scenario that corresponds to  $\Lambda \simeq 10 - 10^3 \text{ TeV}$  the Higgs couplings to the  $t$ –quark can be considerably larger than in the SM leading to the enhanced production of the Higgs bosons at the LHC.