

Review of precision calculations for the measurement of electroweak boson production and properties at hadron colliders

Guido Montagna

Dipartimento di Fisica Nucleare e Teorica, Università di Pavia
Istituto Nazionale Fisica Nucleare, Sezione di Pavia
guido.montagna@pv.infn.it

EPS HEP 2007
Manchester, July 19 – 25, 2007

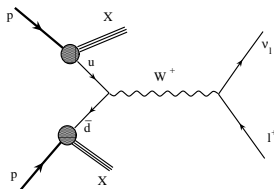
with G. Balossini, C.M. Carloni Calame, M. Moretti, O. Nicrosini, F. Piccinini,
M. Treccani, A. Vicini

and also based on work and collaboration with
A. Arbuzov, D. Bardin, U. Baur, M. Bellomo, S. Dittmaier, S. Jadach,
M. Krämer, G. Polesello, W. Placzek, V. Vercesi, D. Wackerth...



At Fermilab today and at CERN, in the near future

Single W/Z boson production, with $W \rightarrow \ell\nu_\ell, Z \rightarrow \ell^+\ell^-$ decays \implies **clean processes with a large cross section**. They are useful



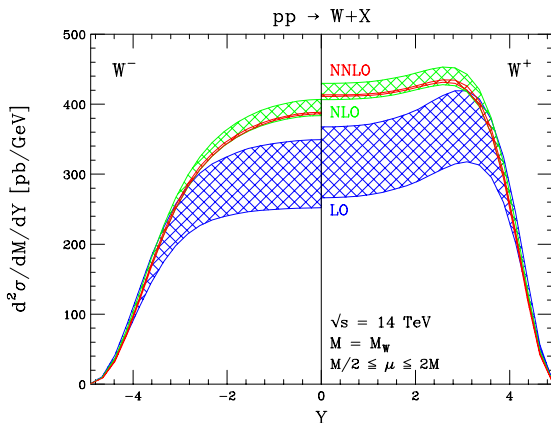
- to derive **precise measurements of the electroweak parameters** $M_W, \Gamma_W, \sin^2 \theta_{\text{eff}}^\ell$. Relevant observables: leptons' transverse momentum p_\perp^ℓ , W transverse mass M_\perp^W , ratio of W/Z distributions, forward-backward asymmetry A_{FB}^Z ...
- to monitor the **collider luminosity** and constrain the **parton distribution functions** (PDFs). Relevant observables: total cross section, W rapidity y_W and charge asymmetry $A(y_\ell)$, lepton pseudorapidity η_ℓ ...
- to search for **new physics**. Relevant observables: Z invariant mass distribution $M_{\ell\ell}^Z$ and W transverse mass M_\perp^W in the high tail...

Higher-order QCD & QCD generators

- NLO/NNLO corrections to W/Z total production rate
G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. **B157** (1979) 461
R. Hamberg, W.L. van Neerven, T. Matsuura, Nucl. Phys. **B359** (1991) 343
- NLO calculations for $W, Z + 1, 2$ jets (**DYRAD**, **MCFM** ...)
W.T. Giele, E.W.N. Glover and D.A. Kosower, Nucl. Phys. **B403** (1993) 633
J.M. Campbell and R.K. Ellis, Phys. Rev. **D65** (2002) 113007
- soft-gluon resummation of leading/next-to-leading logs (**ResBos**)
C. Balazs and C.P. Yuan, Phys. Rev. **D56** (1997) 5558
- NLO corrections merged with **HERWIG Parton Shower** (**MC@NLO**)
S. Frixione and B.R. Webber, JHEP **0206** (2002) 029
- Multi-parton matrix elements Monte Carlo (**ALPGEN**, **HELAC**, **MADEVENT**, **SHERPA**...) matched with vetoed Parton Showers
M.L. Mangano *et al.*, JHEP **0307** (2003) 001
A. Kanaki and C.G. Papadopoulos, Comput. Phys. Commun. **132** (2000) 306
F. Maltoni and T. Stelzer, JHEP **02** (2003) 027
F. Krauss *et al.*, JHEP **0507** (2005) 018
- fully differential NNLO corrections to W/Z production (**FEWZ**)
C. Anastasiou *et al.*, Phys. Rev. **D69** (2004) 094008
K. Melnikov and F. Petriello, Phys. Rev. Lett. **96** (2006) 231803, Phys. Rev. **D74** (2006) 114017

High-precision QCD: W/Z rapidity @ NNLO

C. Anastasiou *et al.*, Phys. Rev. Lett. **91** (2003) 182002
C. Anastasiou *et al.*, Phys. Rev. **D69** (2004) 094008



- NNLO QCD corrections to W/Z rapidity at $\sim 2\%$ at the LHC and residual scale dependence below 1%

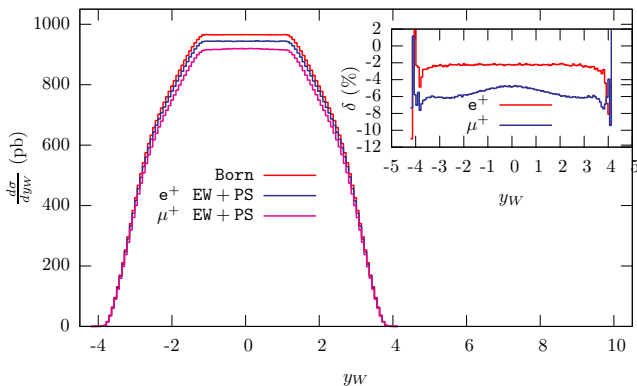
★ $\mathcal{O}(\alpha_S^2) \approx \mathcal{O}(\alpha_{em}) \rightarrow$ need to worry about electroweak corrections!

Electroweak corrections to W rapidity

C.M. Carloni Calame *et al.*, JHEP **0612** (2006) 016

$pp \rightarrow W^+ \rightarrow \ell^+ \nu_\ell (+\gamma)$ at LHC

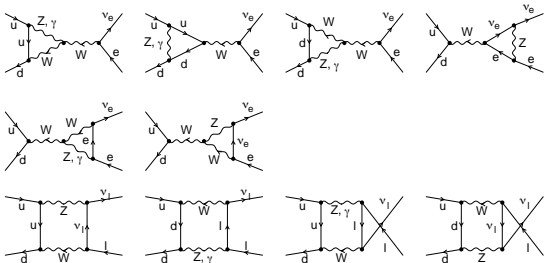
G_μ scheme and including detector effects



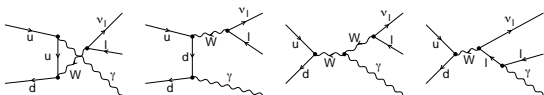
- NLO electroweak corrections to W rapidity are of the same order of NNLO QCD and PDFs uncertainty → **relevant for precision luminosity and PDFs constraints!**

Electroweak Feynman diagrams

★ virtual one-loop corrections (→ electroweak Sudakov logs)



★ bremsstrahlung corrections (→ collinear singularities: universal initial-state singularities reabsorbed into PDFs, as in NLO QCD calculations)



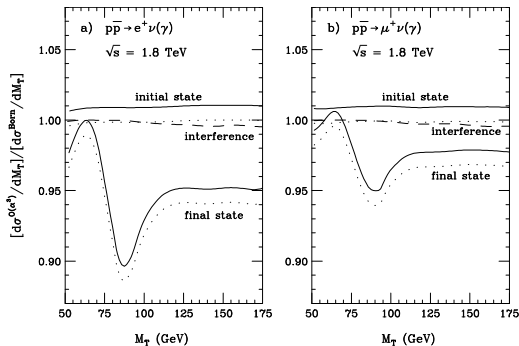
NLO electroweak calculations & tools

- $\mathcal{O}(\alpha)$ QED corrections to W/Z lepton decays
F.A. Berends *et al.* Z. Physik **C27** (1985) 155,365
- Electroweak corrections to W production
 - ★ Pole approximation ($\sqrt{\hat{s}} = M_W$)
D. Wackeroth and W. Hollik, Phys. Rev. **D55** (1997) 6788
U. Baur, S. Keller, D. Wackeroth, Phys. Rev. **D59** (1999) 013002 WGRAD
 - ★ Complete $\mathcal{O}(\alpha)$ corrections
V.A. Zykunov, Eur. P. J. **C3** (2001) 9, Phys. Atom. Nucl. **69** (2006) 1522
S. Dittmaier and M. Krämer, Phys. Rev. **D65** (2002) 073007 DK
U. Baur and D. Wackeroth, Phys. Rev. **D70** (2004) 073015 WGRAD2
A. Arbuzov *et al.*, Eur. Phys. J. **C46** (2006) 407 SANC
C.M. Carloni Calame *et al.*, JHEP **12** (2006) 016 HORACE
- Electroweak corrections to Z production
 - ★ $\mathcal{O}(\alpha)$ photonic corrections
U. Baur, S. Keller, W.K. Sakumoto, Phys. Rev. **D57** (1998) 199 ZGRAD
 - ★ Complete $\mathcal{O}(\alpha)$ corrections
U. Baur *et al.*, Phys. Rev. **D65** (2002) 033007 ZGRAD2

Electroweak corrections & W mass

U. Baur, S. Keller, D. Wackerath, Phys. Rev. **D59** (1999) 013002

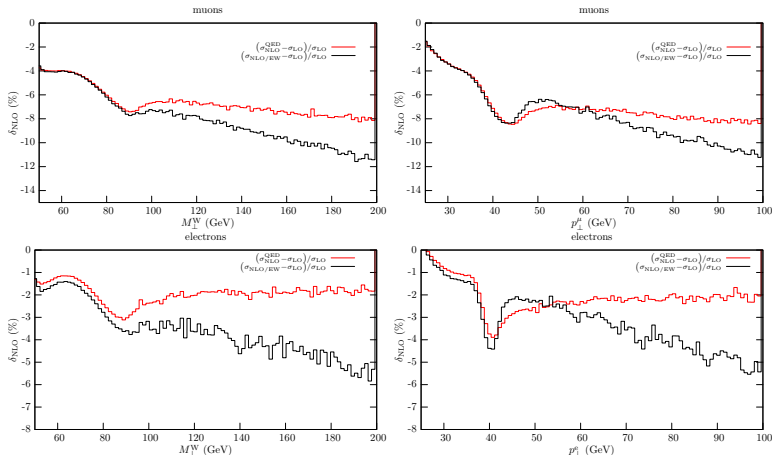
Pole approximation



- Around the W peak, electroweak corrections amount to **several per cents** and are dominated by **final-state photon radiation (FSR)** $\rightarrow \Delta M_W^{\text{FSR}} \sim 100 \text{ MeV}$ at the Tevatron
- **FSR** modifies the shape of the distributions and is sizeable because it contains mass logarithms of the form $\log(\hat{s}/m_\ell^2)$ \rightarrow **need to exponentiate FSR!**

Electroweak corrections & W width

$p\bar{p} \rightarrow W^\pm \rightarrow \ell^\pm \nu_\ell (+\gamma)$ at the Tevatron, by HORACE
 G_μ scheme and including detector effects



- In the hard tails of M_\perp^W and p_\perp^ℓ predictions including QED FSR only differ at **some % level** from the complete NLO electroweak calculation
→ **important for precision W width measurement?**

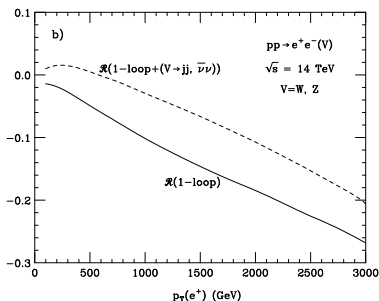
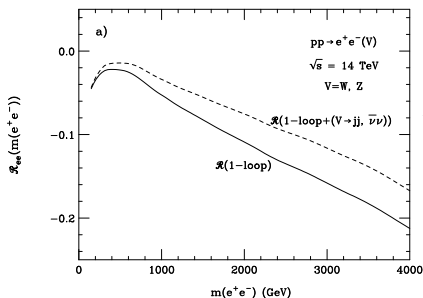
Electroweak Sudakov logs & new physics

S. Dittmaier and M. Krämer, Phys. Rev. **D65** (2002) 073007

U. Baur *et al.*, Phys. Rev. **D65** (2002) 033007

U. Baur and D. Wackerath, Phys. Rev. **D70** (2004) 073015

Complete NLO_{EW} calculations



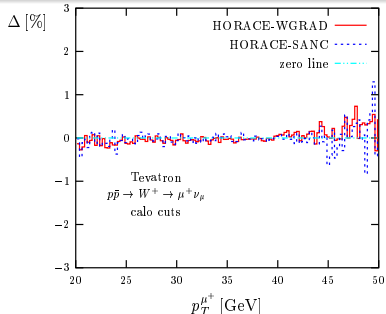
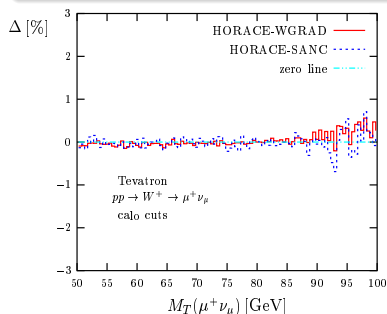
- Pole approximation fails for M_{\perp}^W or $M_{\ell^+\ell^-} \gg M_V$ $V = W, Z$, due to large Sudakov ew logs $-(\alpha/\pi) \log^2(\hat{s}/M_V^2) \rightarrow$ **important for new physics searches!**
- radiation of (undetected) real vector bosons partially cancels the Sudakov logs, e.g. $pp \rightarrow e^+e^-V + X$ $V = W, Z$ $V \rightarrow jj, \nu\bar{\nu}, \dots$

U. Baur, Phys. Rev. **D75** (2007) 013005

Courtesy of D. Wackeroth

Process and scheme – Detector modeling and lepton identification

- 1 $p\bar{p}(pp) \rightarrow W^+ \rightarrow \ell^+ \nu_\ell (+\gamma) - \alpha(0), G_\mu, M_Z \rightarrow M_W$ at two – loops
- 2 $\sqrt{s} = 1.96 \text{ TeV}, 14 \text{ TeV} \quad p_\perp^\ell > 20 \text{ GeV} \quad p_\perp^{\not{e}} > 20 \text{ GeV} \quad |\eta_\ell| < 2.5$
- 3 Bare (w/o recombination and smearing) and Calo (with recombination and smearing) event selection $\Delta R(e, \gamma) = \sqrt{(\Delta\eta(e, \gamma))^2 + (\Delta\phi(e, \gamma))^2} < 0.1$



- Electroweak generators agree within their statistical precision \rightarrow **NLO electroweak corrections to W production well under control!**
- **Comparisons on electroweak corrections to Z production in progress**

Multiple photon corrections & tools

- Higher-order (real+virtual) QED corrections to W/Z production
→ **HORACE** (Pavia): **QED Parton Shower** + NLO electroweak corrections to W/Z production (Z production available soon)

C.M. Carloni Calame *et al.*, Phys. Rev. **D69** (2004) 037301

C.M. Carloni Calame *et al.*, JHEP **05** (2005) 019; JHEP **12** (2006) 016

- **WINHAC** (Cracow): **YFS exponentiation** + electroweak corrections to W decay

S. Jadach and W. Placzek, Eur. Phys. J. **C29** (2003) 325

- Perfect agreement between **HORACE** and **WINHAC** on multiphoton corrections to all W observables

C.M. Carloni Calame *et al.*, Acta Phys. Pol. **B35** (2004) 1643

- Recent effort to improve the treatment of multiphoton radiation in HERWIG (with **SOPHTY** via YFS) and **PHOTOS** (via QED Parton Shower)

K. Hamilton and P. Richardson, JHEP **0607** (2006) 010

P. Golonka and Z. Was, Eur. Phys. J. **C45** (2006) 97

- ★ W -mass shift due to multiphoton radiation is about **10%** of that caused by one photon emission → **non-negligible for precision W mass measurements!** ★

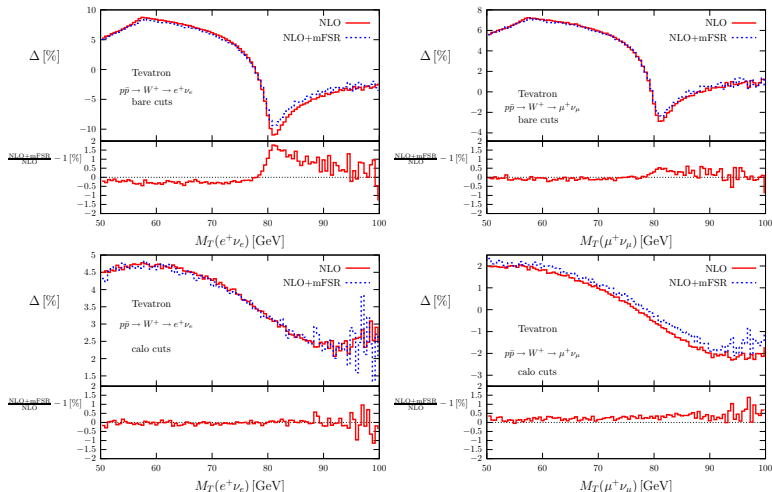
C.M. Carloni Calame *et al.*, Phys. Rev. **D69** (2004) 037301

Multiple photon corrections by HORACE: M_{\perp}^W

C. E. Gerber *et al.*, FERMILAB-CONF-07-052

arXiv.0705.3251 [hep-ph]

Courtesy of D. Wackerath

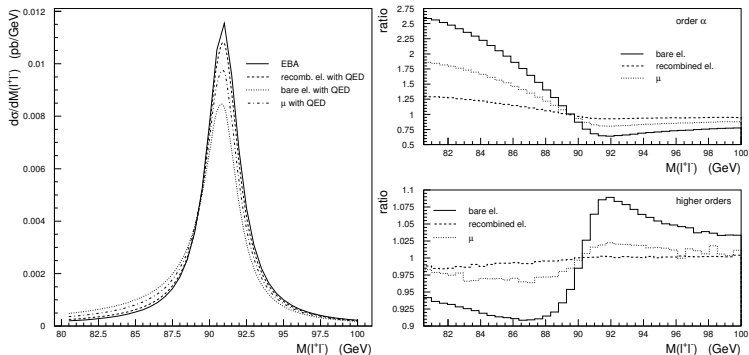


- For **bare** $e - \mu$ multiple photon corrections enhance the NLO electroweak corrections by $\sim 1.5\% - 0.5\%$. For **calo** $e - \mu$ they survive for μ only.

Multiple photon corrections to Z production: $M_{\ell^+\ell^-}$

C.M. Carloni Calame *et al.*, JHEP **05** (2005) 019

$p\bar{p} \rightarrow Z \rightarrow \ell^+\ell^- (+\gamma)$ at the Tevatron, by HORACE



- Multiple photon corrections to Z production are also needed, because important W mass measurement systematics (e.g. energy and momentum scale calibration) are related to Z mass extraction

★ $\Delta M_Z^{\text{h.o.}} \sim 10\% \Delta M_Z^{(\alpha)}$ with e.g. $\Delta M_Z^{\text{h.o.}} \sim 40$ MeV for muons ★

Combining electroweak and QCD corrections

- First attempt: combination of soft-gluon resummation with NLO final-state QED corrections

Q.-H. Cao and C.-P. Yuan, Phys. Rev. Lett. **93** (2004) 042001
ResBos-A

- QCD and electroweak corrections can be combined in factorized form to arrive at

$$\left[\frac{d\sigma}{d\mathcal{O}} \right]_{\text{QCD} \otimes \text{EW}} = \left\{ \frac{d\sigma}{d\mathcal{O}} \right\}_{\text{QCD}} + \left\{ \left[\frac{d\sigma}{d\mathcal{O}} \right]_{\text{EW}} - \left[\frac{d\sigma}{d\mathcal{O}} \right]_{\text{LO}} \right\}_{\text{HERWIG PS}}$$

- QCD \Rightarrow ResBos, MCFM, MC@NLO, ALPGEN (with MLM Parton Shower matching and standard matching parameters), ...
- EW \Rightarrow Electroweak + multiphoton corrections from HORACE convoluted with HERWIG QCD Parton Shower
 - ★ NLO electroweak corrections are interfaced to QCD Parton Shower evolution $\Rightarrow \mathcal{O}(\alpha\alpha_s)$ corrections not reliable when hard non-collinear QCD radiation is important
 - ★ Beyond this approximation, a full two-loop $\mathcal{O}(\alpha\alpha_s)$ calculation is needed (unavailable yet)

J.H. Kühn *et al.*, hep-ph/0703283
NLO/NNLO_{EW} to $pp \rightarrow Wj$

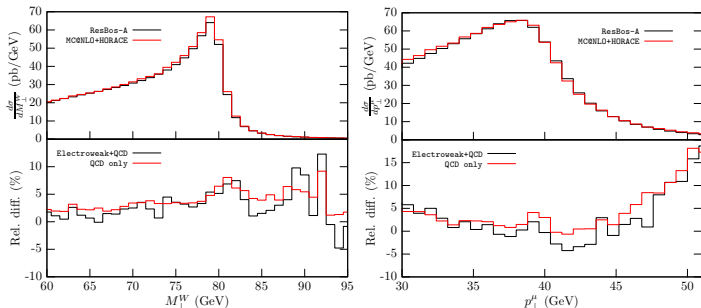


Electroweak \otimes QCD @ the Tevatron

Process and scheme – Detector modeling and lepton identification

- 1 $p\bar{p} \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$ $\sqrt{s} = 1.96$ TeV – G_μ scheme + $\alpha(0)$ for real γ emission
- 2 $p_\perp^\mu > 25$ GeV $p_\perp^W > 25$ GeV $|\eta_\mu| < 1.2$ $p_\perp^W \leq 50$ GeV $M_{\mu\nu} \in [50 - 200]$ GeV
- 3 PDF set: NLO CTEQ6M with $\mu_R = \mu_F = \sqrt{x_1 x_2 s}$

★ **Absolute comparison: ResBos-A vs MC@NLO + HORACE** HERWIG PS
(using the ResBos-A grids publicly available on the web)

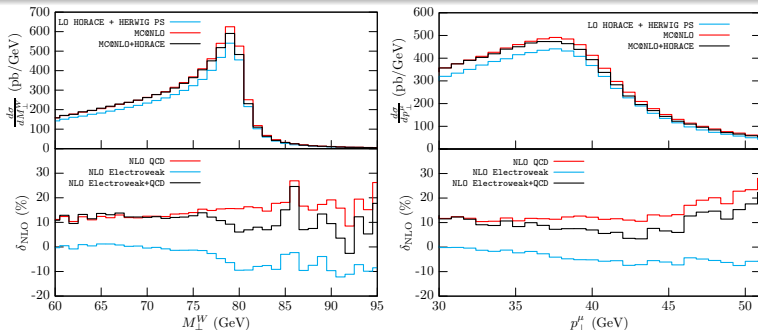


- The relative differences between the two tools are at $\sim 5\%$ level around the jacobian peak and can reach the $\sim 10 \div 15\%$ level in the hard tails. It would be interesting to compare with ResBos-A including the Y perturbative term.

Electroweak \otimes QCD @ the LHC

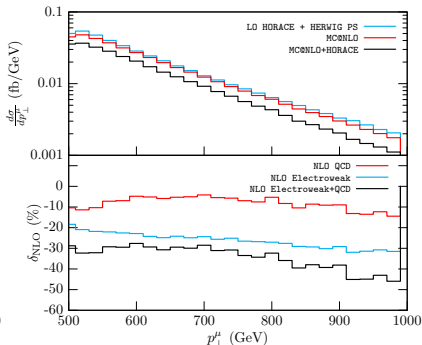
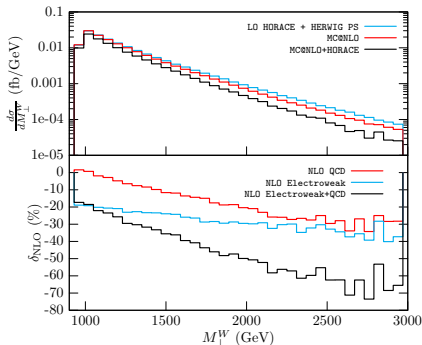
Process and scheme – Detector modeling and lepton identification

- 1 $pp \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$ $\sqrt{s} = 14$ TeV – G_μ scheme + $\alpha(0)$ for real γ emission
- 2 $p_\perp^\mu > 25$ GeV $p_\perp^\nu > 25$ GeV $|\eta_\mu| < 2.5$ \oplus (in case) $M_\perp^W > 1$ TeV
- 3 PDF set: NLO MRST2004QED with $\mu_R = \mu_F = \sqrt{p_{\perp,W}^2 + M_W^2}$



- Around the W peak, for both M_\perp^W and p_\perp^ℓ **NLO QCD corrections are positive** and **tend to compensate negative electroweak contributions**
- Convolution with QCD Parton Shower modifies the relative size and broadens the shape of electroweak corrections

- ★ To what extent large electroweak Sudakov logs compare with QCD corrections in the region relevant for the search of new physics at the LHC? ★



- In the high M_{\perp}^W and p_{\perp}^{ℓ} tails, NLO QCD corrections are negative and sum up to large negative electroweak Sudakov logs
- Their sum is $\sim -40(-70)\%$ for $M_{\perp}^W \simeq 1.5(3) \text{ TeV}$ and $\sim -30(-50)\%$ for $p_{\perp}^{\ell} \simeq 0.5(1) \text{ TeV}$ \rightarrow need to include two-loop electroweak Sudakov logs!

A. Denner, B. Jantzen and S. Pozzorini, Nucl. Phys. **D761** (2007) 1
 B. Jantzen *et al.*, Nucl. Phys. **D731** (2005) 188

W/Z transverse mass ratio: scaled observables method

$pp \rightarrow W^\pm \rightarrow \mu^\pm \nu (+\gamma) / pp \rightarrow Z \rightarrow \mu^+ \mu^- (+\gamma)$ at LHC, by HORACE

- The ratio $\frac{d\sigma}{dM_\perp^W} / \frac{d\sigma}{dM_\perp^Z}$ can be conveniently used to measure M_W , being slightly sensitive to experimental systematics and pQCD corrections

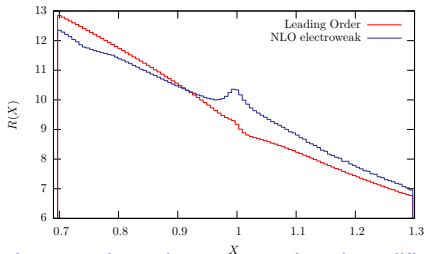
W. Giele and S. Keller, Phys. Rev. **D57** (1998) 4433

V. Büge *et al.*, CMS AN 2006/033

- defining $X_V \equiv \frac{M_\perp^V}{M_V}$:

$$\left. \frac{d\sigma}{dM_\perp^W} \right|_{\text{predicted}} = \frac{M_Z}{M_W} \times R \times \left. \frac{d\sigma}{dM_\perp^Z} \right|_{\text{measured}}$$

where $R \equiv \frac{d\sigma}{dX_W} / \frac{d\sigma}{dX_Z}$, the predicted M_\perp^W distribution can be used to extract M_W , but ... (preliminary analysis!)



★ NLO electroweak corrections do not cancel and modifies the R shape! ★

Conclusions

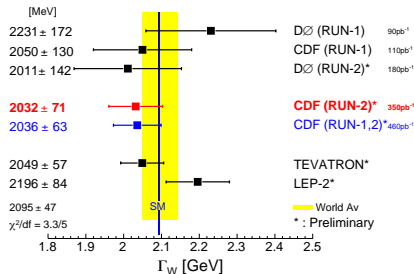
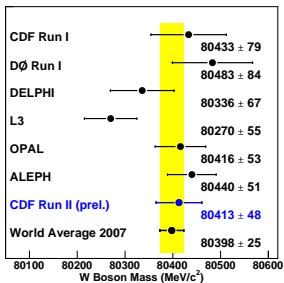
- Recent big theoretical effort towards high-precision predictions for Drell-Yan-like processes, including higher-order QCD and electroweak corrections, to keep under control theoretical systematics
- All these calculations are essential ingredients for precision studies at the Tevatron RunII and LHC
- Multiple photon corrections are a reducible source of systematic uncertainty in W parameters measurement and should be included in the experimental analysis
- It would be advisable to use the state-of-the-art of electroweak calculations and useful to cross-check the effects of QCD corrections
- **Electroweak precision measurements at hadron colliders are very challenging!**
- Our work in progress to
 - ★ make HORACE publicly available for electroweak corrections to Z production and compare with independent calculations
 - ★ scrutinize the electroweak and QCD systematics to the so-called “scaled observables method”
 - ★ Long term: combine HORACE with a precise QCD program into a single EW \otimes QCD generator

Backup slides

The quest for precision: W mass and width

T. Aaltonen *et al.*, CDF Coll., arXiv:0707.0085 [hep-ex]

- Present experimental status: **at CDF RunII the world's single most precise measurements of M_W and Γ_W**



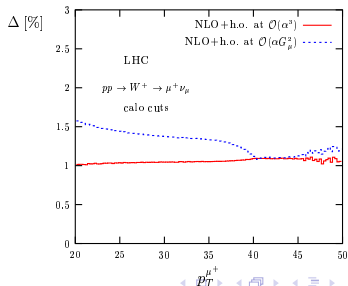
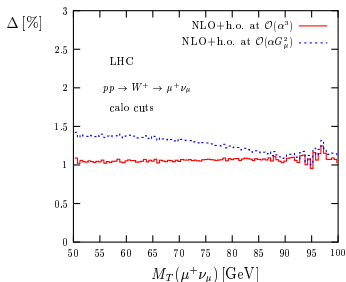
- Target ΔM_W precision → Tevatron RunII: ~ 20 MeV LHC: 15-20 MeV
- Target $\Delta \Gamma_W$ precision → Tevatron RunII: ~ 30 MeV LHC: ≤ 30 MeV
- ★ At the Tevatron, NLO QED corrections shift M_W by ~ 100 MeV ★
 electron channel: -65 ± 20 MeV
 muon channel: -168 ± 20 MeV

From TeV4LHC report: theoretical uncertainty $\sim 1\%$

- NLO at $\mathcal{O}(\alpha^3)$: $\alpha(0)$, G_μ , $M_Z \rightarrow M_W$ at two – loops
- NLO at $\mathcal{O}(\alpha^3)$ incl. h.o.: same input scheme + h.o. corrections to the ρ parameter
- NLO at $\mathcal{O}(\alpha G_\mu^2)$ incl. h.o.: change of the input scheme

$$\alpha(0) \rightarrow \frac{\sqrt{2}G_\mu M_W^2}{\pi} \left(1 - \frac{M_W^2}{M_Z^2} \right) + \text{same h.o. corrections}$$

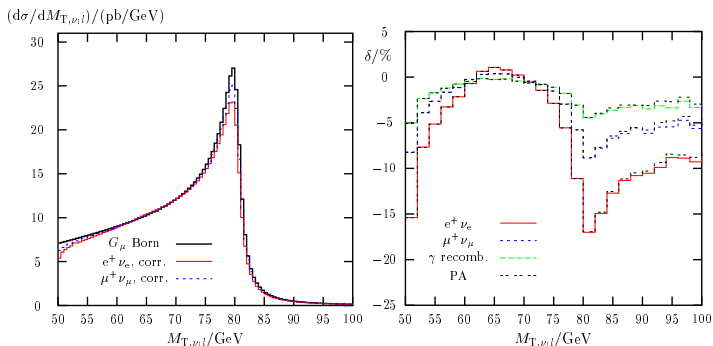
| | Tevatron, σ_W [pb] | LHC, σ_W [pb] |
|---|--|--|
| | $p\bar{p} \rightarrow W^+ \rightarrow \mu^+ \nu_\mu$ | $pp \rightarrow W^+ \rightarrow \mu^+ \nu_\mu$ |
| NLO at $\mathcal{O}(\alpha^3)$ | 738.00(1) | 4943.0(1) |
| NLO at $\mathcal{O}(\alpha^3)$ incl. h.o. | 745.80(1) | 4995.5(1) |
| NLO at $\mathcal{O}(\alpha G_\mu^2)$ incl. h.o. | 747.62(1) | 5006.5(1) |



Electroweak and multiple γ

Photon radiation and lepton identification

S. Dittmaier and M. Krämer, Phys. Rev. **D65** (2002) 073007



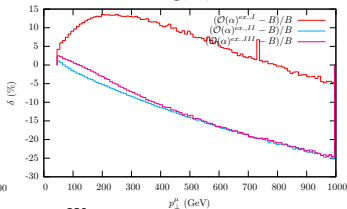
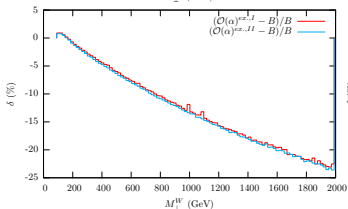
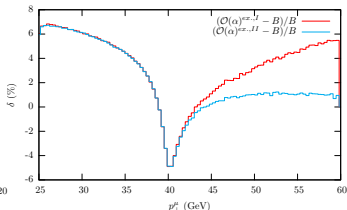
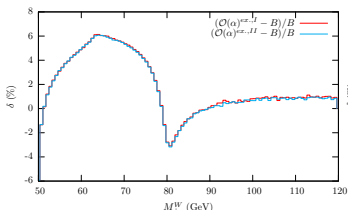
- Lepton identification requirements (and detector effects) strongly affect final-state photon radiation (“the KLN theorem at work”)
- Pole approximation agrees with the full calculation within a few 0.1% around the W resonance

γ -induced processes vs NLO electroweak (LHC)

★ Legenda

$pp \rightarrow W^\pm \rightarrow \mu^\pm \nu (+\gamma)$ with MRTS2004QED - $\alpha(0)$, M_W , M_Z scheme

- I. with γ induced processes, without jet cut
- II. without γ induced processes (pure NLO electroweak)
- III. with γ induced processes, with jet cut ($p_\perp^{\text{jet}} < 30$ GeV and $|\eta^{\text{jet}}| > 2.5$)



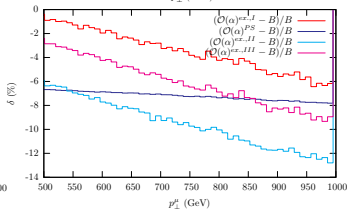
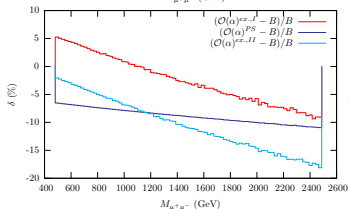
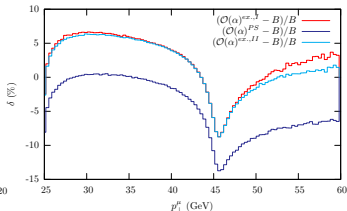
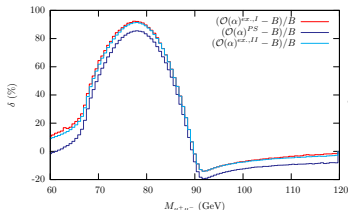
- γ induced processes are very small for M_\perp^W and important (at some % level) for p_\perp^ℓ at the LHC (everywhere negligible at the Tevatron)
- γ induced processes are strongly suppressed by jet cuts and overwhelmed by QCD effects at high p_\perp^ℓ

NLO electroweak corrections to Z observables (LHC)

★ Legenda

$pp \rightarrow \gamma/Z \rightarrow \mu^+ \mu^- (+\gamma)$ with MRTS2004QED - $\alpha(0)$, M_W , M_Z scheme

- I. with γ induced processes, without jet cut
- II. without γ induced processes (pure NLO electroweak)
- III. with γ induced processes, with jet cut ($p_{\perp}^{\text{jet}} < 30$ GeV and $|\eta^{\text{jet}}| > 2.5$)
- IV. QED Parton Shower approximation



- non-negligible γ induced effects both for M_{l+l-} and p_{\perp}^{ℓ} in the hard tails
- large corrections due to Sudakov logs not accounted for by QED PS

Fitting the W mass

χ^2 fits to Monte Carlo pseudo-data for the M_T^W spectrum with

- $\sqrt{s} = 2 \text{ TeV}$ $p_{\perp}(\ell) > 25 \text{ GeV}$ $|\eta(\ell)| < 1.2$ $p_{\perp} > 25 \text{ GeV}$
- lepton identification requirements based on Tevatron analyses (e.g., if $\Delta R_{e\gamma} = \sqrt{\Delta\eta^2 + \Delta\phi^2} < 0.2$, e and γ momenta are recombined)
- particles' momenta are smeared according to RunII DØ detector specifications

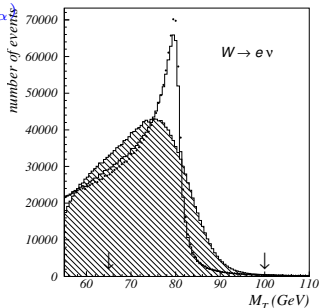
$$\chi^2(M_W) = \sum_{i=\text{bins}} (\sigma_{i,h.o.} - \sigma_{i,\alpha})^2 / (\Delta\sigma_{i,h.o.}^2 + \Delta\sigma_{i,\alpha}^2)$$

histogram: no lepton identification criteria, no detector effects

markers: with lepton identification criteria

shaded: with lepton identification criteria and detector effects

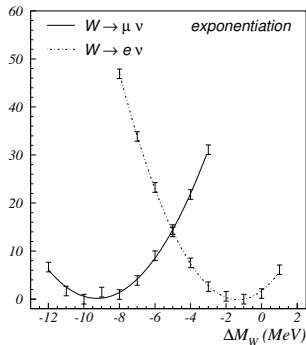
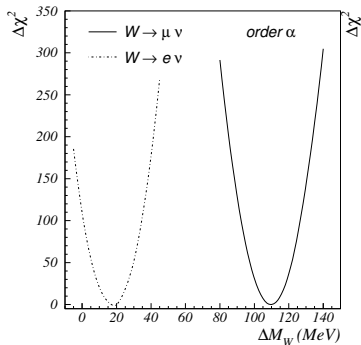
arrows: fitting region, $65 \text{ GeV} < M_T < 100 \text{ GeV}$



Why higher-order QED is important: W mass

C.M. Carloni Calame *et al.*, Phys. Rev. **D69** (2004) 037301

Including recombination and smearing



$$\Delta M_W^{\alpha,e} \sim 20 \text{ MeV}$$
$$\Delta M_W^{\alpha,\mu} \sim 110 \text{ MeV}$$

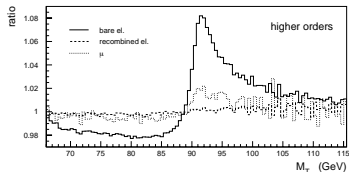
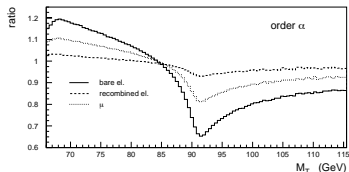
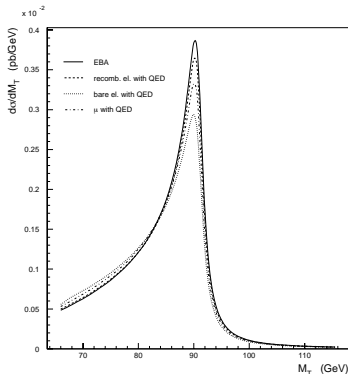
$$\Delta M_W^{\infty,e} \sim 2 \text{ MeV}$$
$$\Delta M_W^{\infty,\mu} \sim 10 \text{ MeV}$$

- W -mass shift due to multiphoton radiation is about **10%** of that caused by one photon emission \longrightarrow **non-negligible for W mass!**

Higher-order QED corrections to Z production: M_T^Z

C.M. Carloni Calame *et al.*, JHEP **05** (2005) 019

$p\bar{p} \rightarrow Z \rightarrow \ell^+ \ell^- (+\gamma)$ at the Tevatron, by HORACE

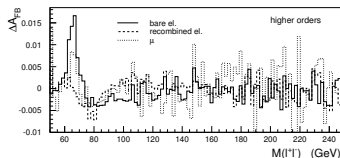
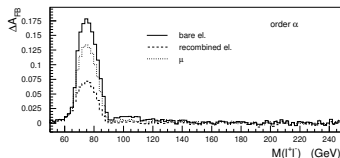
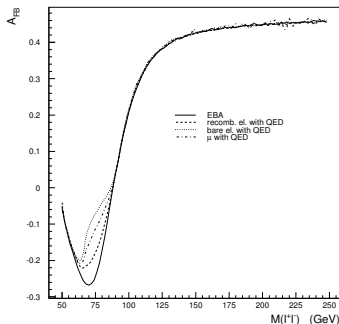


- Multiple photon corrections to Z transverse mass are $\sim 2\%$ for muons (*i.e.* a factor of four w.r.t the same corrections to M_T^W) and a few per mille level for recombined electrons.

QED corrections to forward-backward asymmetry

C.M. Carloni Calame *et al.*, JHEP **05** (2005) 019

$p\bar{p} \rightarrow Z \rightarrow \ell^+\ell^- (+\gamma)$ at the Tevatron, by HORACE

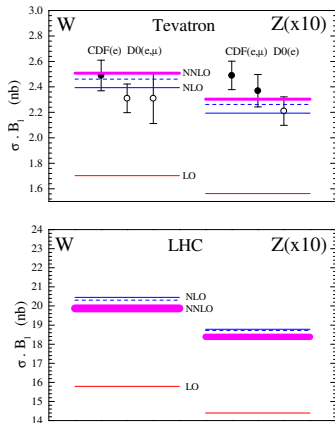


- $O(\alpha)$ QED corrections to forward-backward asymmetry are large below the Z peak and “small” around and above it. Multiple photon corrections are about a factor ten smaller, at 1% level below the peak.

QCD

QCD predictions for W/Z total rates

R. Hamberg, W.L. van Neerven, T. Matsuura, Nucl. Phys. **B359** (1991) 343
A.D. Martin *et al.*, Eur. Phys. J. **C19** (2001) 313



- Good convergence of α_s expansion. NLO-NNLO difference $\sim 2\%$ at LHC
- ★ New CTEQ and MRST(MSTW) parametrizations shift the W/Z cross sections of a few % at the Tevatron and of some % at the LHC!

W.K. Tung *et al.*, JHEP **0702** (2007) 053
A.D. Martin *et al.*, arXiv:0706.0459 [hep-ph]

PDFs and total rates

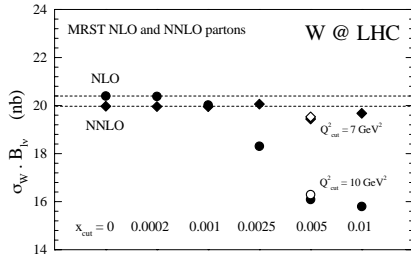
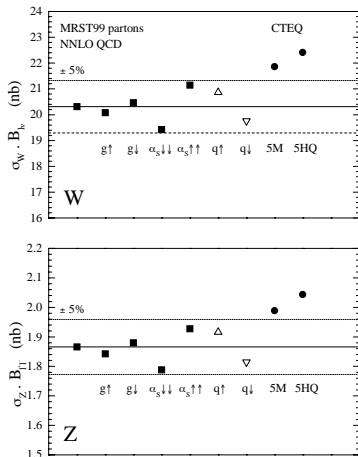
A.D. Martin *et al.*, Eur. Phys. J. **C35** (2004) 325

A.D. Martin *et al.*, Eur. Phys. J. **C14** (2000) 133

W.K. Tung *et al.*, JHEP **0702** (2007) 053

A.D. Martin *et al.*, arXiv:0706.0459 [hep-ph]

W and Z Cross Sections: LHC



- Present PDFs uncertainty at some per cent level at the LHC

Combination of ew and QCD corrections

Monte Carlo “tuning”: Tevatron and LHC

| Monte Carlo | ALPGEN | FEWZ | HORACE | ResBos-A |
|---------------------------|----------|------------|-----------|------------|
| σ_{LO} (pb) | 906.3(3) | 906.20(16) | 905.64(4) | 905.26(24) |

Table: MC tuning at the Tevatron for the LO cross section with cuts of the process $p\bar{p} \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$, using CTEQ6M with $\mu_R = \mu_F = \sqrt{x_1 x_2 s}$

| Monte Carlo | ALPGEN | FEWZ | HORACE |
|---------------------------|---------|---------|-----------|
| σ_{LO} (pb) | 8310(2) | 8304(2) | 8307.9(2) |

Table: MC tuning at the LHC for the LO cross section with cuts of the process $pp \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$, using MRST2004QED with $\mu_R = \mu_F = \sqrt{p_{\perp,W}^2 + M_W^2}$

| Monte Carlo | $\sigma_{\text{NLO}}^{\text{Tevatron}}$ (pb) | $\sigma_{\text{NLO}}^{\text{LHC}}$ (pb) |
|-------------|--|---|
| MC@NLO | 2638.8(4) | 20939(19) |
| FEWZ | 2643.0(8) | 21001(14) |

Table: MC tuning for MC@NLO and FEWZ NLO inclusive cross sections of the process $p\bar{p} \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$, with CTEQ6M (Tevatron) and MRST2004QED (LHC)

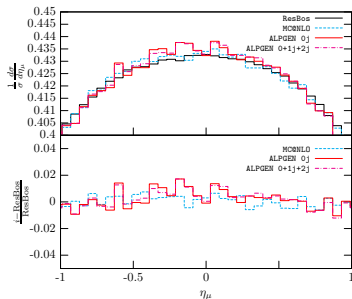
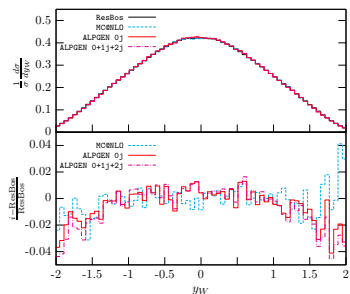
- ★ After appropriate “tuning”, and with same input parameters, cuts and PDFs, Monte Carlos **agree at $\sim 0.1\%$ level** (or better) ★

QCD @ the Tevatron (I)

Process and scheme – Detector modeling and lepton identification

- 1 $p\bar{p} \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$ $\sqrt{s} = 1.96$ TeV – G_μ scheme + $\alpha(0)$ for real γ emission
- 2 $p_\perp^\mu > 25$ GeV $\cancel{p}_\perp > 25$ GeV $|\eta_\mu| < 1.2$ $p_\perp^W \leq 50$ GeV $M_{\mu\nu} \in [50 - 200]$ GeV
- 3 PDF set: NLO CTEQ6M with $\mu_R = \mu_F = \sqrt{x_1 x_2 s}$

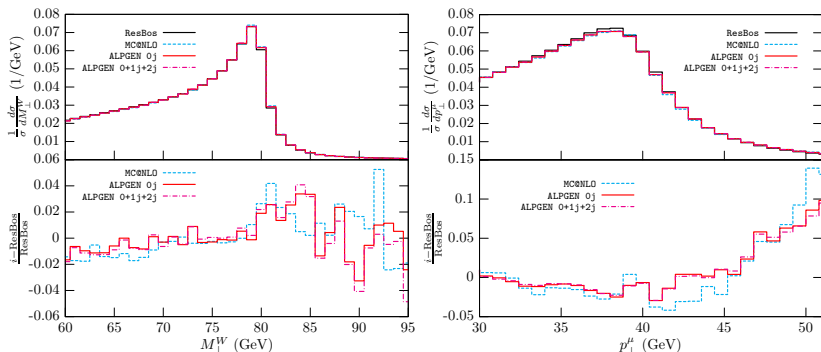
★ QCD generators are normalized to the corresponding integrated cross section, to point out the shape differences. Relative deviations w.r.t. ResBos ★



- For W rapidity and lepton pseudorapidity QCD generators agree at the $\sim 1\%$ level

QCD @ the Tevatron (II)

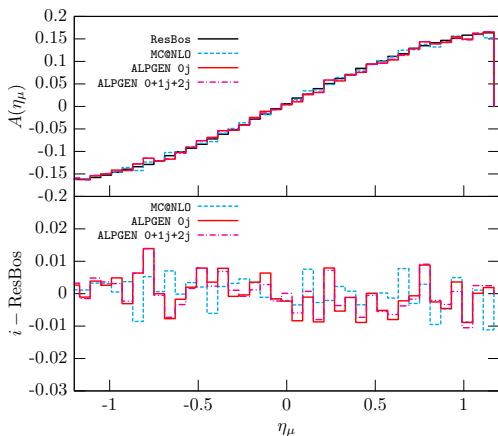
- ★ QCD generators are normalized to the corresponding integrated cross section, to point out the shape differences. Relative deviations w.r.t. ResBos ★



- For M_{\perp}^W and p_{\perp}^l QCD generators agree at **some % level** around the jacobian peak
- In the hard p_{\perp}^l tail the QCD differences can reach the **10 % level**
- It would be useful to compare with ResBos including the Y perturbative term

QCD @ the Tevatron (III)

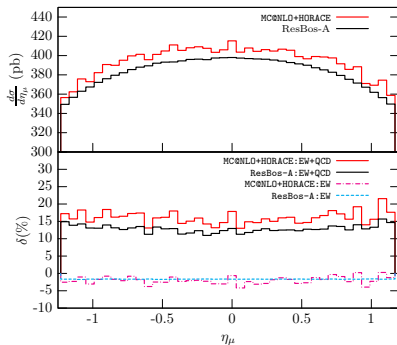
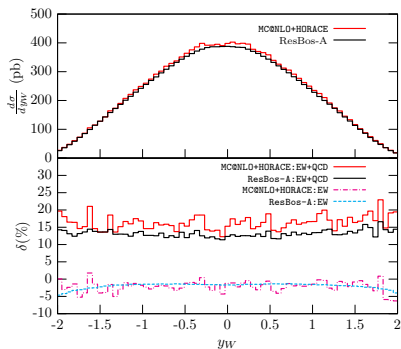
- ★ QCD generators are normalized to the corresponding integrated cross section, to point out the shape differences. Relative deviations w.r.t. ResBos ★



- For W charge asymmetry QCD programs agree at the **1% level**

Electroweak \otimes QCD @ the Tevatron

- ★ Absolute comparison: ResBos-A vs MC@NLO + HORACE^{HERWIG PS}
(using the ResBos-A grids publicly available on the web)

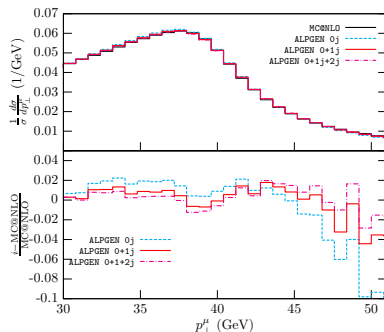
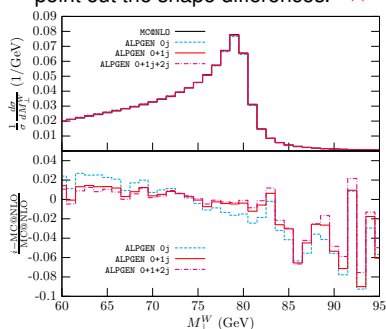


- For y_W and η_μ distributions ResBos-A and MC@NLO + HORACE^{HERWIG PS} **well agree** in the evaluation of pure electroweak corrections
- For the combination of electroweak and QCD corrections there are differences at **a few % level**. It would be interesting to compare with ResBos-A including the Y perturbative term.

Process and scheme – Detector modeling and lepton identification

- 1 $pp \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu$ $\sqrt{s} = 14$ TeV – G_μ scheme + $\alpha(0)$ for real γ emission
- 2 $p_\perp^\mu > 25$ GeV $\cancel{p}_\perp > 25$ GeV $|\eta_\mu| < 2.5$ \oplus (in case) $M_\perp^W > 1$ TeV
- 3 PDF set: NLO MRST2004QED with $\mu_R = \mu_F = \sqrt{p_{\perp,W}^2 + M_W^2}$

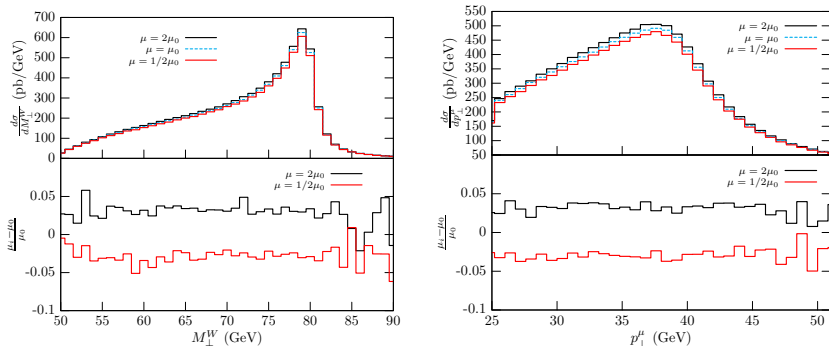
★ QCD generators are normalized to the corresponding integrated cross section, to point out the shape differences. ★



- For M_\perp^W and p_\perp^ℓ the relative differences are at a few % level around the jacobian peak and can reach the ~ 10 % level in the hard tails.

★ Varying the renormalization/factorization scale from its default value

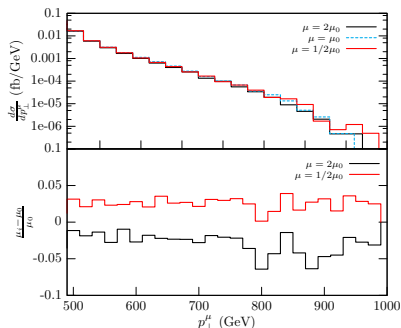
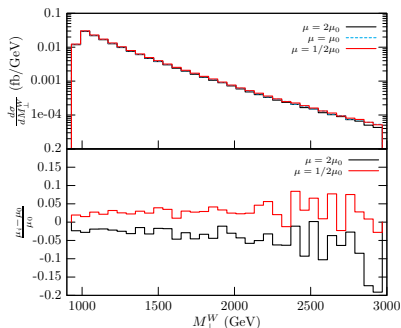
$\mu_0 = \mu_R = \mu_F = \sqrt{p_{\perp}^2 + M_W^2}$ to $1/2\mu_0$ and $2\mu_0$, with MC@NLO ★



- Around the W peak, for both M_{\perp}^W and p_{\perp}^{ℓ} the scale variations induce relative differences w.r.t. the default choice of $\sim \pm 3\%$. It can be seen as an estimate of the size of NNLO QCD corrections to such distributions.

★ Varying the renormalization/factorization scale from its default value

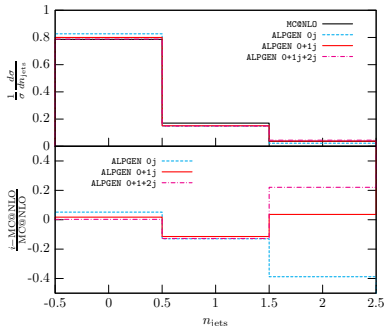
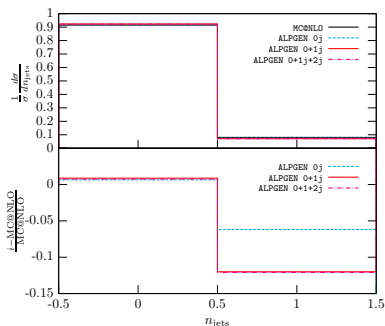
$\mu_0 = \mu_R = \mu_F = \sqrt{p_{\perp}^2 + M_W^2}$ to $1/2\mu_0$ and $2\mu_0$, with MC@NLO ★



- In the high transverse mass region at the LHC, the scale variations induce relative differences w.r.t. the default choice of $\sim \pm 5 - 10\%$ for M_{\perp}^W and of $\sim \pm 5\%$ for p_{\perp}^{ℓ} . In this region there's also a large uncertainty due to PDFs (gluon at large x).

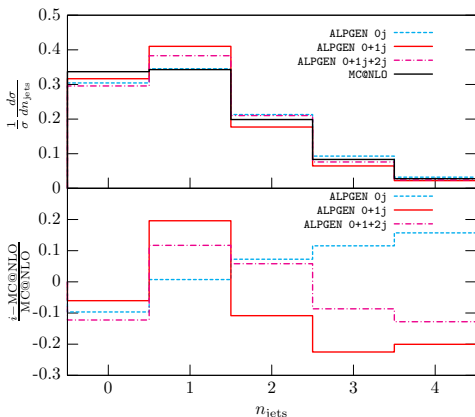
Jet multiplicity: Tevatron vs LHC

- ★ How many are the jets present in association with W production events for standard selection cuts? ★



- Tevatron: $\sim 91\%$ W events without extra jets; $\sim 8\%$ with one extra jet
- LHC: $\sim 79\%$ W events without extra jets; $\sim 17\%$ with one extra jet; $\sim 3\%$ with two extra jets

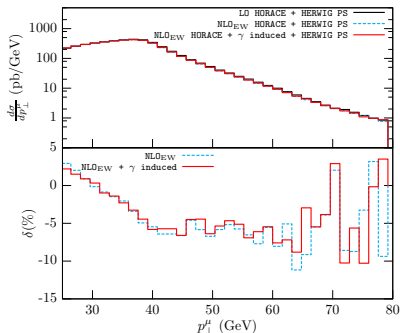
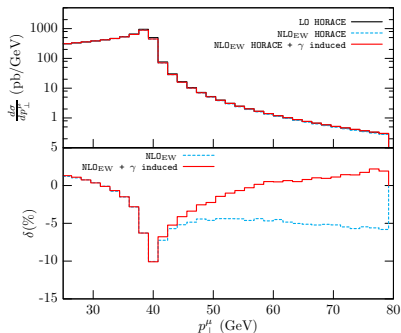
- ★ How many are the jets present in association with W production events in the region of interest for new physics searches? ★



- In the high transverse mass region at the LHC: $\sim 30\%$ W events without extra jets; $\sim 40\%$ with one extra jet; $\sim 20\%$ with two extra jets; $\sim 8\%$ with three extra jets; $\sim 2\%$ with four extra jets

Electroweak corrections vs QCD Parton Shower (LHC)

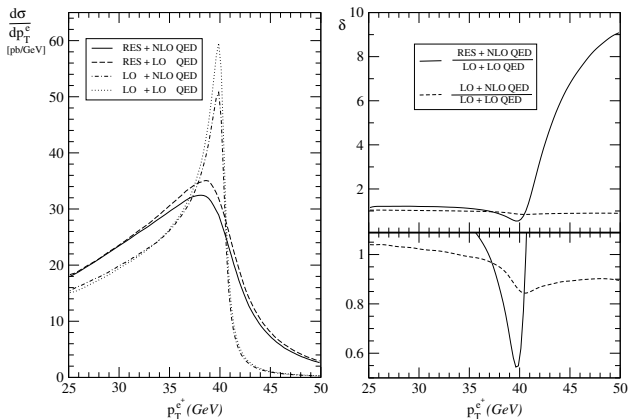
$pp \rightarrow W^\pm \rightarrow \mu^\pm \nu (+\gamma)$ at LHC by HORACE



- The relative size and shape of NLO electroweak corrections and γ -induced processes (left plot) is **significantly modified** by convolution with QCD Shower evolution (right plot)
- γ -induced processes also studied by A.B. Arbuzov and R.R. Sadykov, arXiv:0707.0423 [hep-ph].

Matching soft-gluon resummation with NLO QED: p_{\perp}^{ℓ}

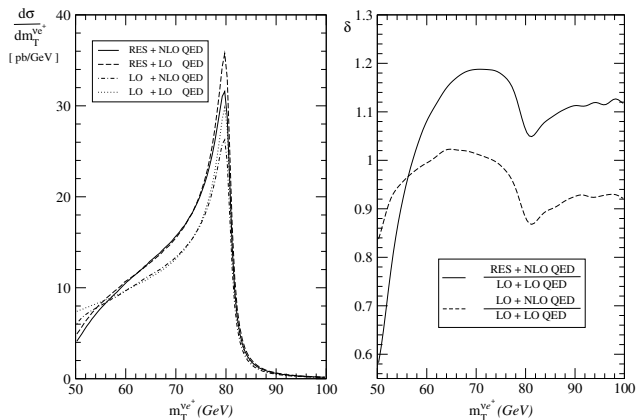
Q.-H. Cao and C.-P. Yuan, Phys. Rev. Lett. **93** (2004) 042001
ResBos-A



- QCD resummation and NLO QED differently modify the shape of p_{\perp}^{ℓ} and reach $\sim -45\%$ \rightarrow need to merge QCD and EW generators!

Matching soft-gluon resummation with NLO QED: M_T^W

Q.-H. Cao and C.-P. Yuan, Phys. Rev. Lett. **93** (2004) 042001
ResBos-A



- QCD resummation ($\sim +6\%$ at the peak) is compensated by NLO QED ($\sim -12\%$) \rightarrow **need to merge QCD and EW generators!**

Tuned comparisons

| Tevatron and LHC | |
|---|--|
| electrons | muons |
| combine e and γ momentum four vectors, if $\Delta R(e, \gamma) < 0.1$ | reject events with $E_\gamma > 2 \text{ GeV}$ for $\Delta R(\mu, \gamma) < 0.1$ |
| reject events with $E_\gamma > 0.1 E_e$ for $0.1 < \Delta R(e, \gamma) < 0.4$ | reject events with $E_\gamma > 0.1 E_\mu$ for $0.1 < \Delta R(\mu, \gamma) < 0.4$ |

Table: Summary of lepton identification requirements.

where

$$\Delta R(e, \gamma) = \sqrt{(\Delta\eta(e, \gamma))^2 + (\Delta\phi(e, \gamma))^2},$$

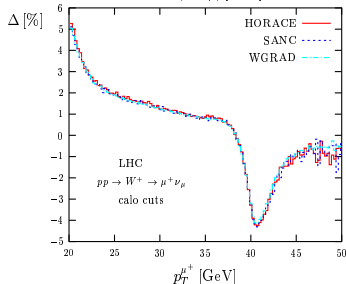
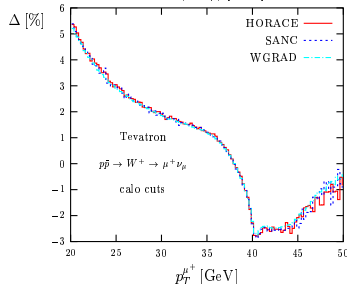
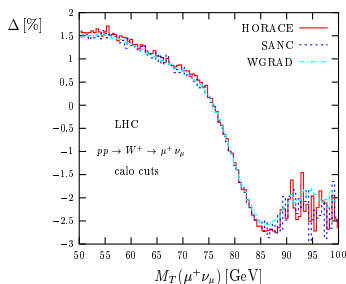
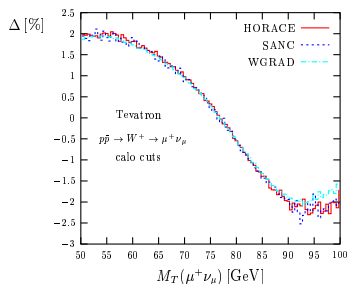
- ★ Uncertainties in the energy measurements of the charged leptons in the detector are simulated by Gaussian smearing of the particle four-momentum vector with standard deviation σ based on the DØ(upgrade) and ATLAS specifications.

TeV4LHC tuned comparisons

C. E. Gerber *et al.*, FERMILAB-CONF-07-052
arXiv.0705.3251 [hep-ph]

| Tevatron, $p\bar{p} \rightarrow W^+ \rightarrow e^+\nu_e$ | | | | | | | Courtesy of D. Wackerath |
|---|------------|-----------|--------------|------------|-----------|--------------|--------------------------|
| | bare cuts | | | calo cuts | | | |
| | LO [pb] | NLO [pb] | Δ [%] | LO [pb] | NLO [pb] | Δ [%] | |
| HORACE | 773.509(5) | 791.14(2) | 2.279(3) | 733.012(5) | 762.21(3) | 3.983(4) | |
| SANC | 773.510(2) | 791.04(8) | 2.27(1) | 733.024(2) | 762.03(9) | 3.96(1) | |
| WGRAD2 | 773.516(5) | 791.01(5) | 2.268(7) | 733.004(6) | 762.00(5) | 3.956(6) | |
| Tevatron, $p\bar{p} \rightarrow W^+ \rightarrow \mu^+\nu_\mu$ | | | | | | | |
| | bare cuts | | | calo cuts | | | |
| | LO [pb] | NLO [pb] | Δ [%] | LO [pb] | NLO [pb] | Δ [%] | |
| HORACE | 773.509(5) | 804.18(2) | 3.965(3) | 732.913(6) | 738.16(3) | 0.716(4) | |
| SANC | 773.510(2) | 804.07(6) | 3.951(7) | 732.908(2) | 738.01(5) | 0.696(7) | |
| WGRAD2 | 773.516(5) | 804.11(1) | 3.955(2) | 732.917(6) | 738.00(1) | 0.693(2) | |
| LHC, $pp \rightarrow W^+ \rightarrow e^+\nu_e$ | | | | | | | |
| | bare cuts | | | calo cuts | | | |
| | LO [pb] | NLO [pb] | Δ [%] | LO [pb] | NLO [pb] | Δ [%] | |
| HORACE | 5039.11(4) | 5140.6(1) | 2.014(2) | 4924.17(4) | 5115.5(2) | 3.886(4) | |
| SANC | 5039.21(1) | 5139.5(5) | 1.99(1) | 4925.31(1) | 5113.5(4) | 3.821(9) | |
| WGRAD2 | 5039.16(7) | 5139.6(6) | 1.99(1) | 4924.15(5) | 5114.1(6) | 3.86(1) | |
| LHC, $pp \rightarrow W^+ \rightarrow \mu^+\nu_\mu$ | | | | | | | |
| | bare cuts | | | calo cuts | | | |
| | LO [pb] | NLO [pb] | Δ [%] | LO [pb] | NLO [pb] | Δ [%] | |
| HORACE | 5039.11(4) | 5230.5(2) | 3.798(4) | 4925.16(5) | 4944.5(2) | 0.393(4) | |
| SANC | 5039.21(1) | 5229.4(3) | 3.775(7) | 4925.31(1) | 4942.5(5) | 0.349(9) | |
| WGRAD2 | 5039.16(7) | 5229.9(1) | 3.786(3) | 4925.30(7) | 4943.0(1) | 0.360(3) | |

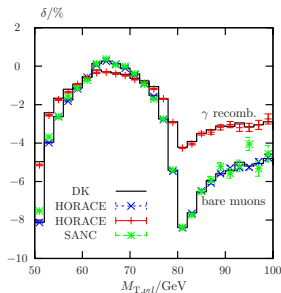
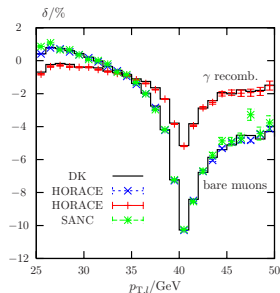
Courtesy of D. Wackerth



- Very good agreement between HORACE, SANC and WGRAD.

Process and scheme – Detector modeling and lepton identification

- 1 $pp \rightarrow W^+ \rightarrow \ell^+ \nu_\ell (+\gamma)$ – G_μ scheme + $\alpha(0)$ for real γ emission
- 2 $\sqrt{s} = 14$ TeV $p_\perp^\ell > 25$ GeV $p_\perp > 25$ GeV $|\eta_\ell| < 1.2$
- 3 $R_{l\gamma} = \sqrt{(\eta_l - \eta_\gamma)^2 + \phi_{l\gamma}^2} \leq 0.1 \Rightarrow$ electron/photon recombination
- 4 PDF set: MRST2004QED with $\mu_R = \mu_F = M_W$



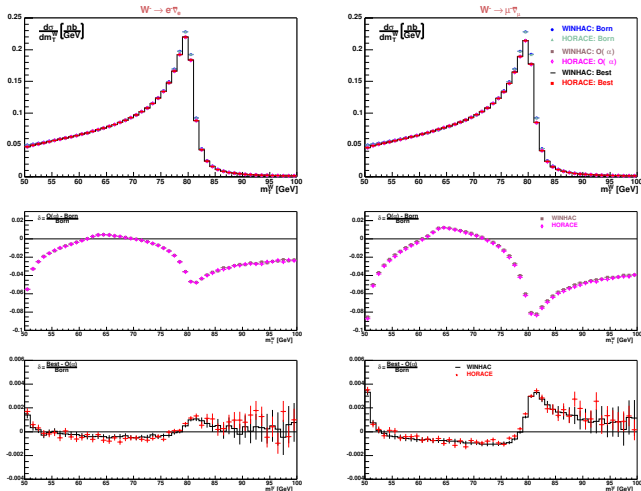
- Perfect agreement between independent calculations!

$$pp \rightarrow \nu_l l^+ (+\gamma) @ \sqrt{s} = 14 \text{ TeV (with MRSTQED04)}$$

| $p_{T,l}/\text{GeV}$ | 25- ∞ | 50- ∞ | 100- ∞ | 200- ∞ | 500- ∞ | 1000- ∞ |
|-----------------------------|--------------|--------------|---------------|---------------|---------------|----------------|
| σ_0/pb | | | | | | |
| DK | 2112.2(1) | 13.152(2) | 0.9452(1) | 0.11511(2) | 0.0054816(3) | 0.00026212(1) |
| HORACE | 2112.21(4) | 13.151(6) | 0.9451(1) | 0.11511(1) | 0.0054812(4) | 0.00026211(2) |
| SANC | 2112.22(2) | 13.1507(2) | 0.94506(1) | 0.115106(1) | 0.00548132(6) | 0.000262108(3) |
| WGRAD | 2112.3(1) | 13.149(1) | 0.94510(5) | 0.115097(5) | 0.0054818(2) | 0.00026209(2) |
| $\delta_{e+\nu_e}/\%$ | | | | | | |
| DK | -5.19(1) | -8.92(3) | -11.47(2) | -16.01(2) | -26.35(1) | -37.92(1) |
| HORACE | -5.23(1) | -8.98(1) | -11.49(1) | -16.03(1) | -26.36(1) | -37.92(2) |
| WGRAD | -5.10(1) | -8.55(5) | -11.32(1) | -15.91(2) | -26.1(1) | -38.2(2) |
| $\delta_{\mu+\nu_\mu}/\%$ | | | | | | |
| DK | -2.75(1) | -4.78(3) | -8.19(2) | -12.71(2) | -22.64(1) | -33.54(2) |
| HORACE | -2.79(1) | -4.84(1) | -8.21(1) | -12.73(1) | -22.65(1) | -33.57(1) |
| SANC | -2.80(1) | -4.82(2) | -8.17(2) | -12.67(2) | -22.63(2) | -33.50(2) |
| WGRAD | -2.69(1) | -4.53(1) | -8.12(1) | -12.68(1) | -22.62(2) | -33.6(2) |
| $\delta_{\text{recomb}}/\%$ | | | | | | |
| DK | -1.73(1) | -2.45(3) | -5.91(2) | -9.99(2) | -18.95(1) | -28.60(1) |
| HORACE | -1.77(1) | -2.51(1) | -5.94(1) | -10.02(1) | -18.96(1) | -28.65(1) |
| SANC | -1.89(1) | -2.56(1) | -5.97(1) | -10.02(1) | -18.96(1) | -28.61(1) |
| WGRAD | -1.71(1) | -2.32(1) | -5.94(1) | -10.11(2) | -19.08(3) | -28.73(6) |
| $\delta_{\gamma q}/\%$ | | | | | | |
| DK | +0.071(1) | +5.24(1) | +13.10(1) | +16.44(2) | +14.30(1) | +11.89(1) |

HORACE vs WINHAC: M_{\perp}^W

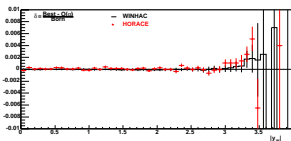
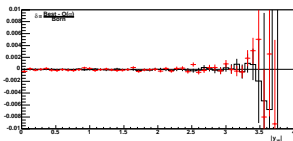
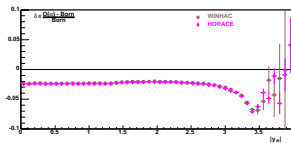
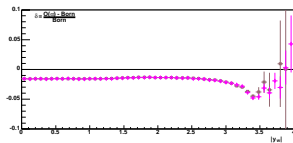
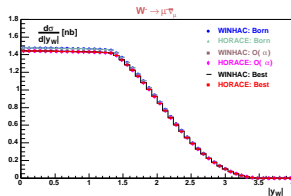
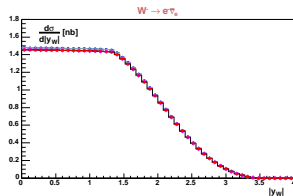
C.M. Carloni Calame *et al.*, Acta Phys. Pol. **B35** (2004) 1643



• Same effect of multiple photon radiation $\sim 0.2 - 0.5\%$ around W peak

HORACE vs WINHAC: W rapidity

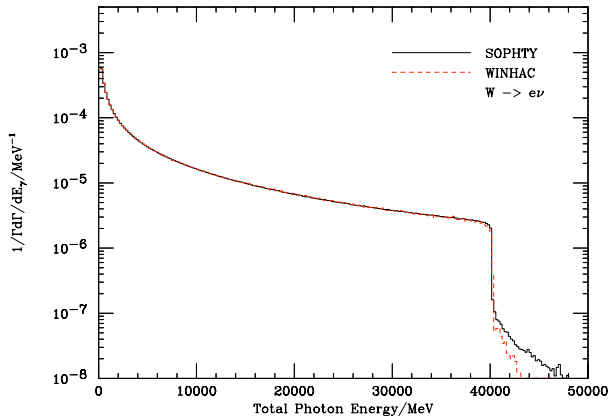
C.M. Carloni Calame *et al.*, Acta Phys. Pol. **B35** (2004) 1643



- $\mathcal{O}(\alpha)$ corrections at 2/5% level for recombined $e/\text{bare } \mu$. Negligible effect of multiple photon radiation.

HERWIG+SOPHTY vs WINHAC

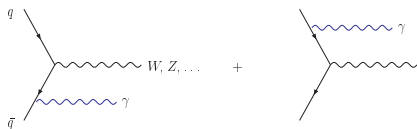
K. Hamilton and P. Richardson, JHEP **0607** (2006) 010



Theory

QED initial-state collinear singularities

- QED initial-state collinear singularities are universal \rightarrow can be absorbed into PDFs, as in QCD

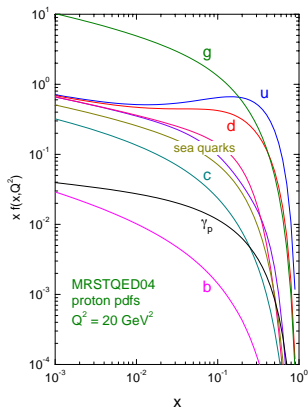


$$f(x) \rightarrow f(x, \mu_F^2) - \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}, \mu_F^2\right) \frac{\alpha}{2\pi} Q_q^2 \times \left\{ \ln\left(\frac{\mu_F^2}{m_q^2}\right) [P_{ff}(z)]_+ - [P_{ff}(z) (2 \ln(1-z) + 1)]_+ + C(z) \right\}$$

$$C(z) = \left\{ \begin{array}{l} 0 \\ [P_{ff}(z) (\ln(\frac{1-z}{z}) - \frac{3}{4}) + \frac{9+5z}{4}]_+ \end{array} \right. \overline{\text{MS}} \text{ DIS}$$

QED initial-state singularities & QED-improved PDFs

MRST2004QED



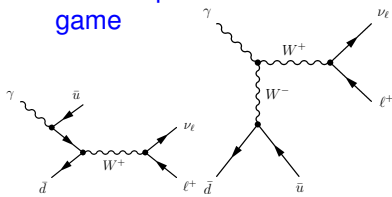
- QED initial-state collinear singularities are universal \longrightarrow can be absorbed into PDFs
- effect of QED evolution on PDFs through DGLAP equation is **small** ($\sim 0.1\%$ for $x < 1$)

H. Spiesberger, Phys. Rev. **D52** (1995) 4936

M. Roth and S. Weinzierl, Phys. Lett. **B590** (2004) 190

A.D. Martin *et al.*, Eur. Phys. J. **C39** (2005) 155

- dynamic generation of photon parton distribution \longrightarrow **photon induced processes enter the game**



The QED Parton Shower algorithm

C.M. Carloni Calame *et al.*, Nucl. Phys. **584** (2000) 459

- the Parton Shower (PS) is a Monte Carlo (MC) solution of the QED DGLAP equation

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D\left(\frac{x}{t}, Q^2\right)$$

- the solution can be cast in the form

$$D(x, Q^2) = \Pi_S(Q^2) \sum_{n=0}^{\infty} \int \frac{\delta(x-x_1 \cdots x_n)}{n!} \prod_{i=0}^n \left[\frac{\alpha}{2\pi} P(x_i) L dx_i \right]$$

★ $\Pi_S(Q^2) \equiv e^{-\frac{\alpha}{2\pi} LI_+}$ is the Sudakov form factor,

$I_+ \equiv \int_0^{1-\epsilon} P(x) dx$, $L \equiv \log \frac{Q^2}{m^2}$ and ϵ soft/hard separator

- the PS MC algorithm reproduces this solution
- at NLO, the resulting cross section has a **leading log accuracy**

Matching NLO electroweak with QED Parton Shower

C.M. Carloni Calame *et al.*, JHEP 12 (2006) 016

- NLO ($\mathcal{O}(\alpha)$) electroweak cross section

$$d\sigma_{\text{ew}}^{\alpha} \equiv d\sigma^{\alpha, \text{ex}} \equiv d\sigma_{\text{SV}}^{\alpha, \text{ex}} + d\sigma_{\text{H}}^{\alpha, \text{ex}}$$

- $\mathcal{O}(\alpha)$ Parton Shower (PS) cross section

$$d\sigma^{\alpha, \text{PS}} = [\Pi_S(Q^2)]_{\mathcal{O}(\alpha)} d\sigma_0 + \frac{\alpha}{2\pi} P_{ff}(x) I(k) dx dc d\hat{\sigma}_0 = \\ \equiv d\sigma_{\text{SV}}^{\alpha, \text{PS}} + d\sigma_{\text{H}}^{\alpha, \text{PS}}$$

- Resummed PS

$$d\sigma_{\text{PS}}^{\infty} = \Pi_S(Q^2) F_{\text{sv}} \sum_{n=0}^{\infty} d\hat{\sigma}_0 \frac{1}{n!} \prod_{i=0}^n \left[\frac{\alpha}{2\pi} P_{ff}(x_i) I(k_i) dx_i dc_i F_{\text{H},i} \right]$$

where $F_{\text{SV}} = 1 + \frac{d\sigma_{\text{SV}}^{\alpha, \text{ex}} - d\sigma_{\text{SV}}^{\alpha, \text{PS}}}{d\sigma_0}$ and $F_{\text{H},i} = 1 + \frac{d\sigma_{\text{H},i}^{\alpha, \text{ex}} - d\sigma_{\text{H},i}^{\alpha, \text{PS}}}{d\sigma_{\text{H},i}^{\alpha, \text{PS}}}$

- $[\sigma_{\text{matched}}^{\infty}]_{\mathcal{O}(\alpha)} = \sigma_{\text{exact}}^{\alpha}$, avoiding NLO double counting and preserving quark mass independence and exponentiation of QED leading logs

| W^+ cross section (pb) at LHC | $\mathcal{O}(\alpha)$ | matched |
|---------------------------------|-----------------------|--------------------|
| m_q | 4410.98 ± 0.20 | 4412.14 ± 0.26 |
| $m_q/10$ | 4410.92 ± 0.26 | 4411.89 ± 0.33 |
| $m_q/100$ | 4410.99 ± 0.29 | 4411.92 ± 0.50 |

Matching NLO electroweak with QED Parton Shower

C.M. Carloni Calame *et al.*, JHEP 12 (2006) 016

- NLO ($\mathcal{O}(\alpha)$) electroweak cross section

$$d\sigma_{\text{ew}}^{\alpha} \equiv d\sigma^{\alpha, \text{ex}} \equiv d\sigma_{SV}^{\alpha, \text{ex}} + d\sigma_H^{\alpha, \text{ex}}$$

- $\mathcal{O}(\alpha)$ Parton Shower (PS) cross section

$$d\sigma^{\alpha, PS} = [\Pi_S(Q^2)]_{\mathcal{O}(\alpha)} d\sigma_0 + \frac{\alpha}{2\pi} P_{ff}(x) I(k) dx dc d\hat{\sigma}_0 = \\ \equiv d\sigma_{SV}^{\alpha, PS} + d\sigma_H^{\alpha, PS}$$

- Resummed PS + NLO electroweak

$$d\sigma_{\text{matched}}^{\infty} = \Pi_S(Q^2) F_{sv} \sum_{n=0}^{\infty} d\hat{\sigma}_0 \frac{1}{n!} \prod_{i=0}^n \left[\frac{\alpha}{2\pi} P_{ff}(x_i) I(k_i) dx_i dc_i F_{H,i} \right]$$

$$\text{where } F_{SV} = 1 + \frac{d\sigma_{SV}^{\alpha, \text{ex}} - d\sigma_{SV}^{\alpha, PS}}{d\sigma_0} \text{ and } F_{H,i} = 1 + \frac{d\sigma_{H,i}^{\alpha, \text{ex}} - d\sigma_{H,i}^{\alpha, PS}}{d\sigma_{H,i}^{\alpha, PS}}$$

- $[\sigma_{\text{matched}}^{\infty}]_{\mathcal{O}(\alpha)} = \sigma_{\text{exact}}^{\alpha}$, avoiding NLO double counting and preserving quark mass independence and exponentiation of QED leading logs

| W^+ cross section (pb) at LHC | $\mathcal{O}(\alpha)$ | matched |
|---------------------------------|-----------------------|--------------------|
| m_q | 4410.98 ± 0.20 | 4412.14 ± 0.26 |
| $m_q/10$ | 4410.92 ± 0.26 | 4411.89 ± 0.33 |
| $m_q/100$ | 4410.99 ± 0.29 | 4411.92 ± 0.50 |