Exciting the quark-gluon plasma with a relativistic jet

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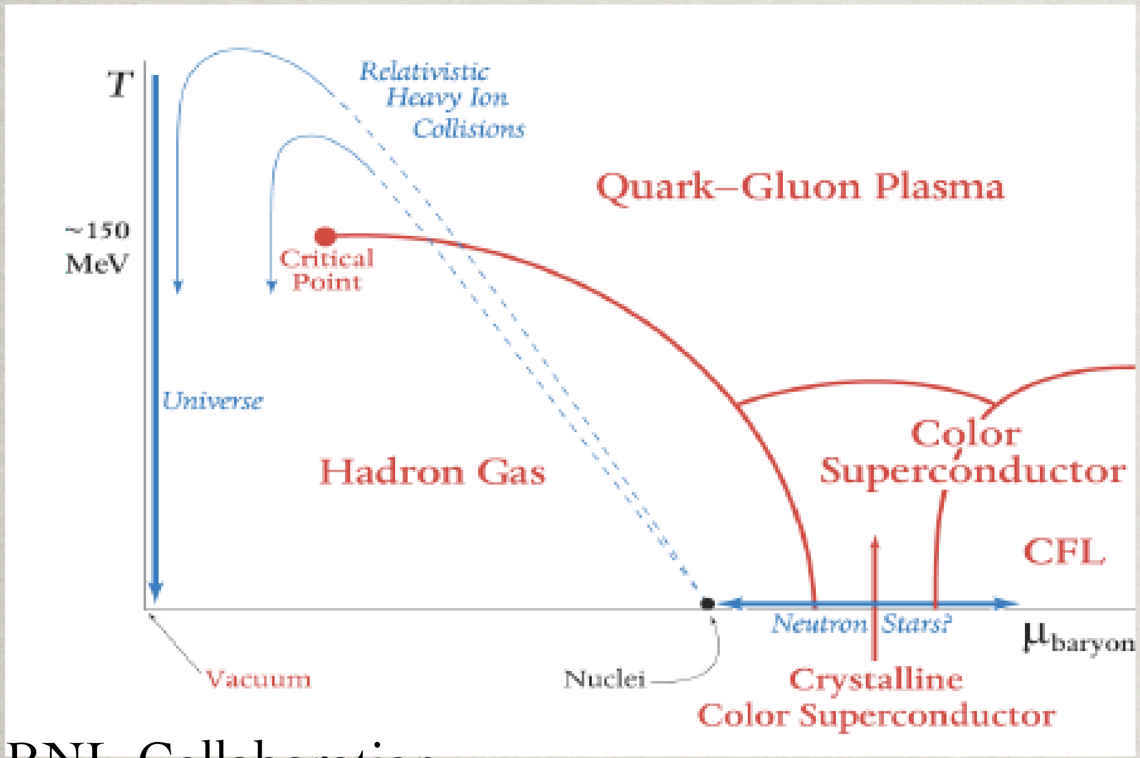
in collaboration with Cristina Manuel

arXiv:0705.1047 [hep-ph]

Outline

- Setting: QCD phase diagram
- Kinetic theory and stability analysis
- QCD kinetic theory and fluid approach
- Jet traversing an equilibrated plasma
- Conclusions and Outlook

QCD Phase diagram



BNL-Collaboration

Phases of matter

H Hadronic phase

QGP Quark-Gluon Plasma

CSC Color Superconductor

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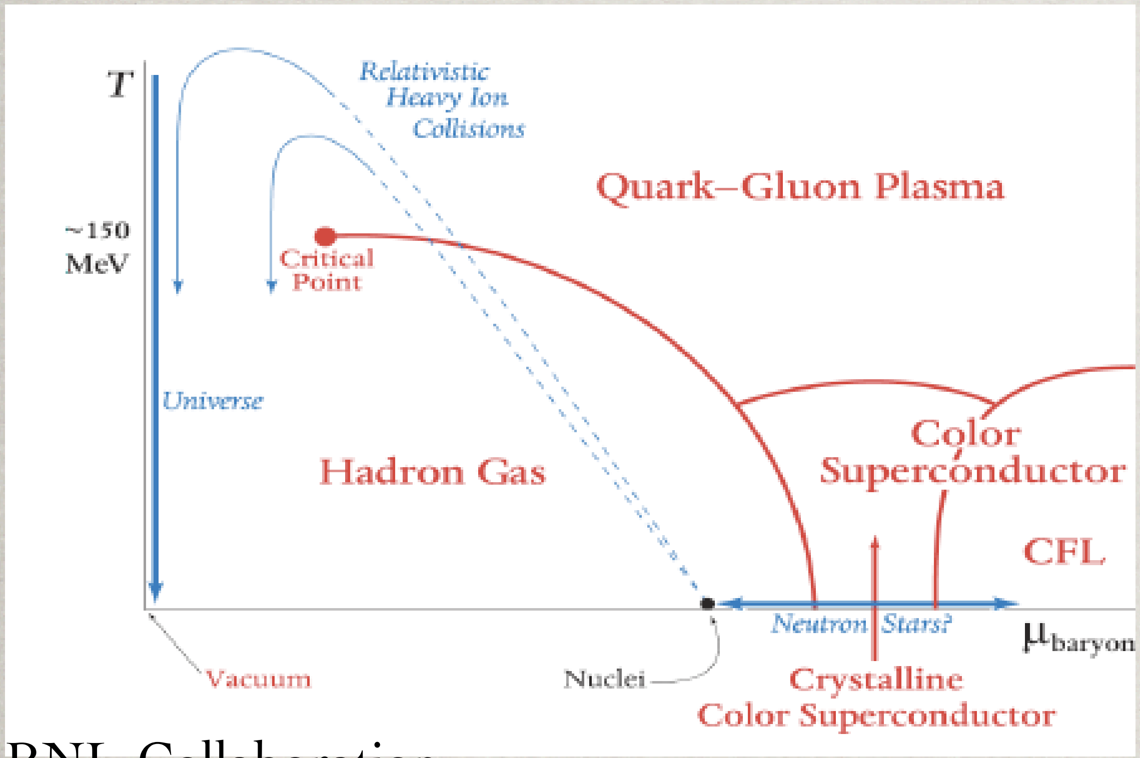

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✦ Heavy-ion collision experiments aim to produce (and detect) a new form of matter. Plasma of “liberated” quarks and gluons.

The method is very simple: smash heavy-ions to produce a system with **extended energy density.** The aim is to **understand QCD**

✦ Alternative methods: numerical **lattice simulations...**

Kinetic theory

Kinetic equations for the distribution function of charged particles

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \mathbf{F} \cdot \nabla_{\mathbf{p}} f = C$$

$$\partial_\mu F^{\mu\nu} = J^\nu$$

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Long-range forces may drive the system out of equilibrium

Stability analysis

Two methods for stability analysis:

1) Energy considerations:
Analyze the potential and see whether any perturbation destabilizes the system

2) Normal modes analysis:
Perturbe the equilibrium and study the (linearized) plasma equations. Fluctuations are $\sim exp(-i\omega t)$

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Ex. In thermonuclear fusion it is enough to have $t \gtrsim 1s$

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Ex. the plasma tends to expand

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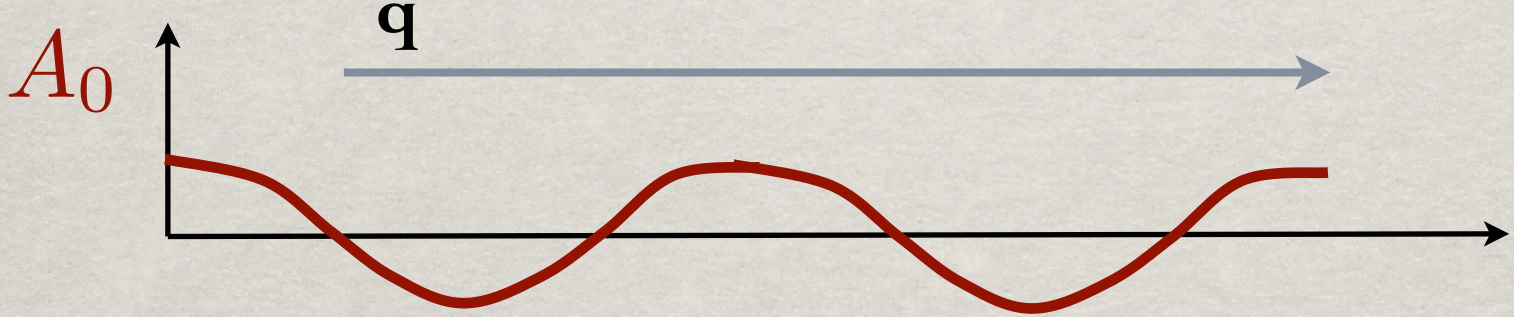

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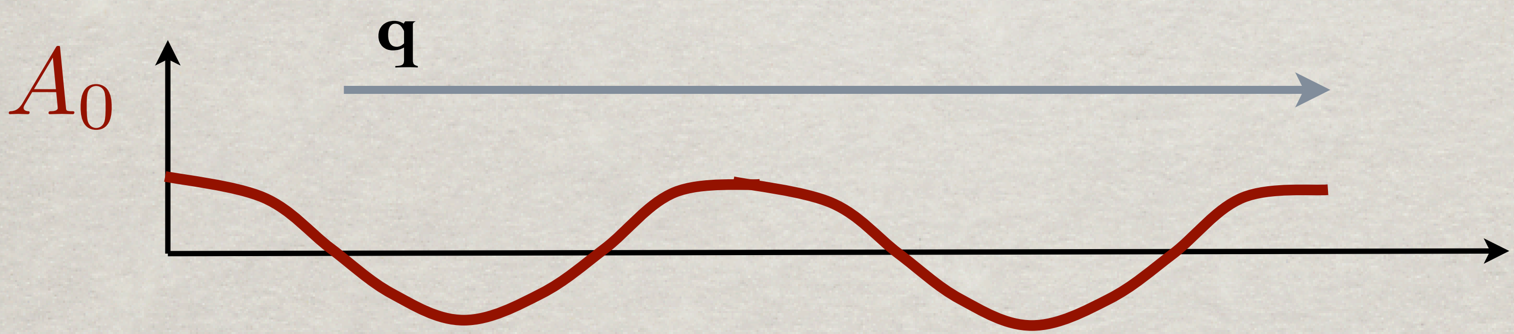

Electromagnetic instabilities: growing accumulation of current density (pinching)

"Weibel" instability

A mechanism to explain the rapid thermalization (or isotropization) by Mrowczynski Phys. Lett. B 214, 587 (1988) employs Weibel instabilities known in QED since the '50

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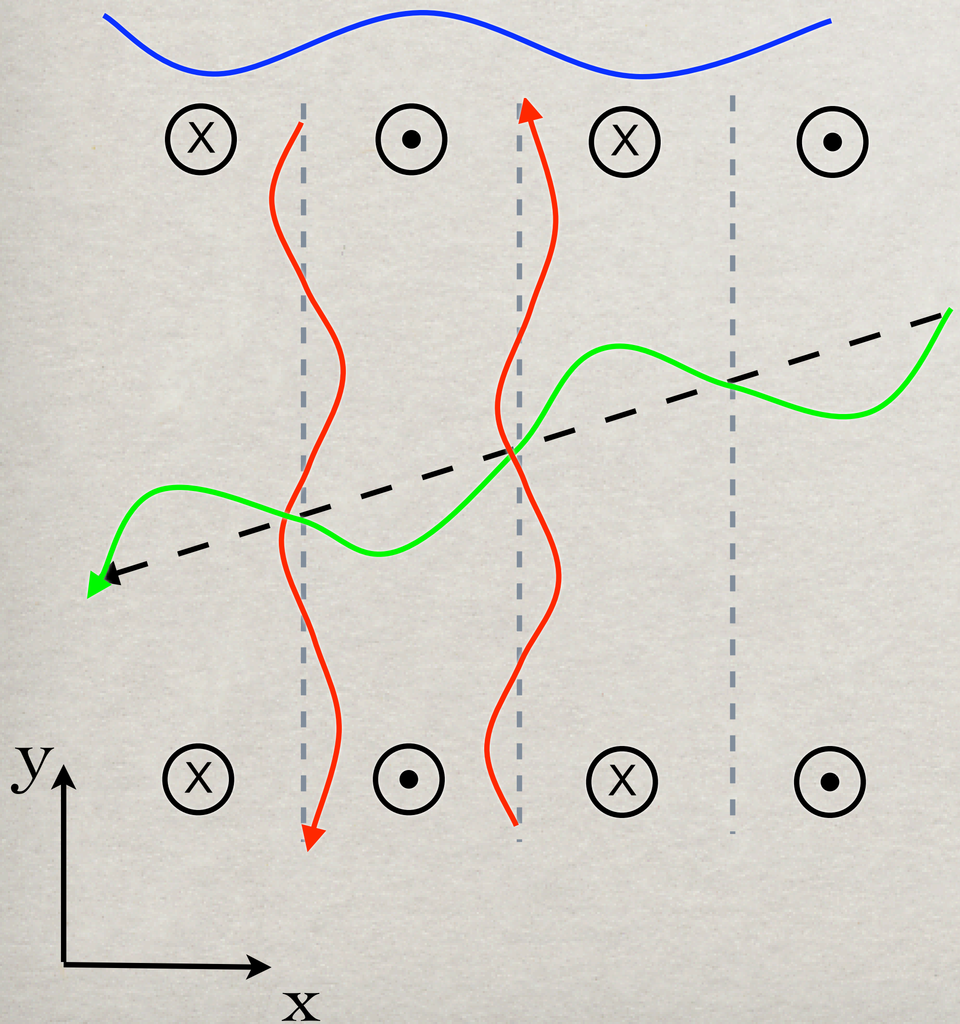

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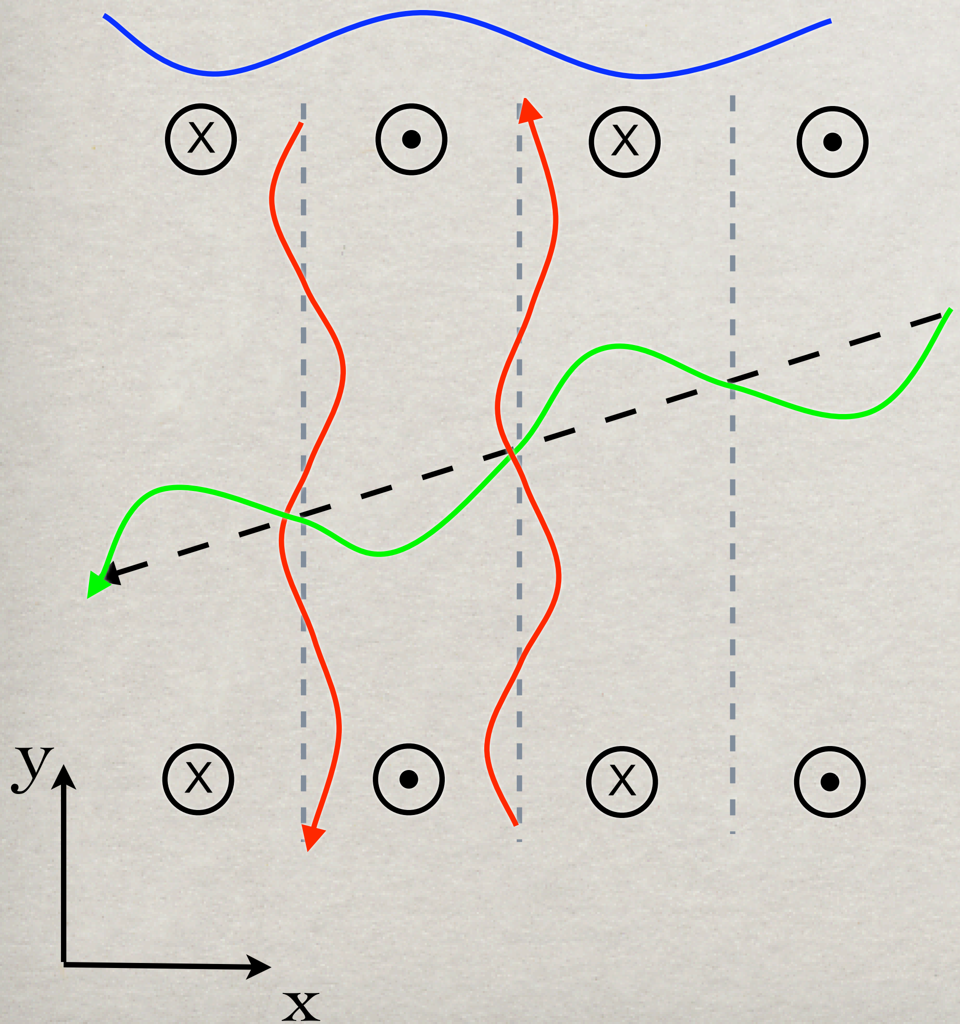

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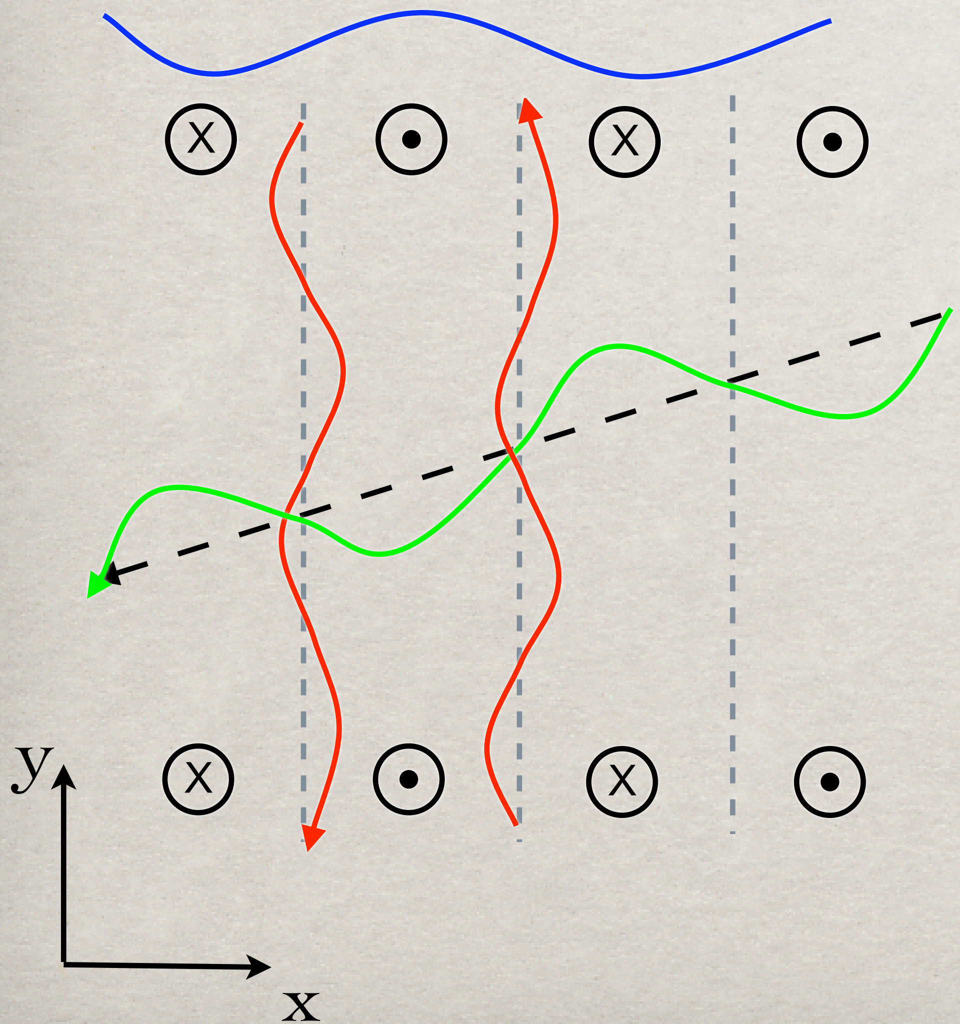

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Numerical simulations seem to indicate that such a mechanism is not able to produce the sudden thermalization of the system (Arnold, Lenaghan, Moore, Strickland, Yaffe …)

Criteria for Instability

A sufficient condition for instability (assuming parity invariant distribution) is given by the Penrose criterion (see e.g. Arnold et al. hep-ph/0307325):

- The mode with wave number $\mathbf{q}$ is unstable when the matrix

$$q^2\delta^{ij} + \Pi^{ij}(0, \mathbf{q})$$

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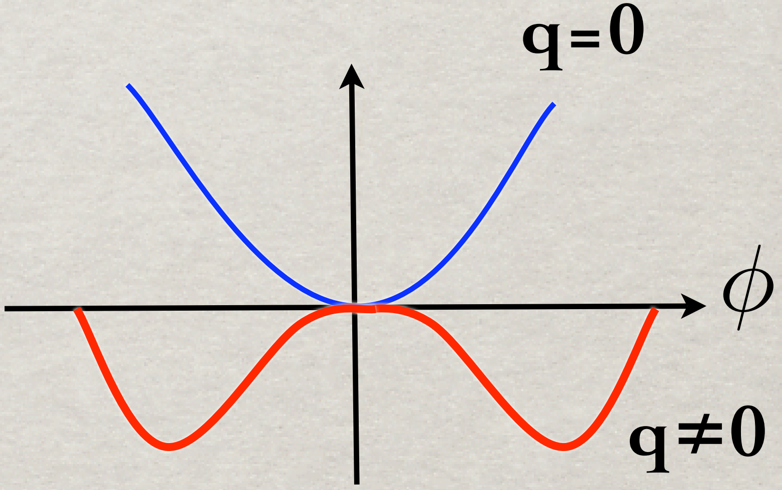

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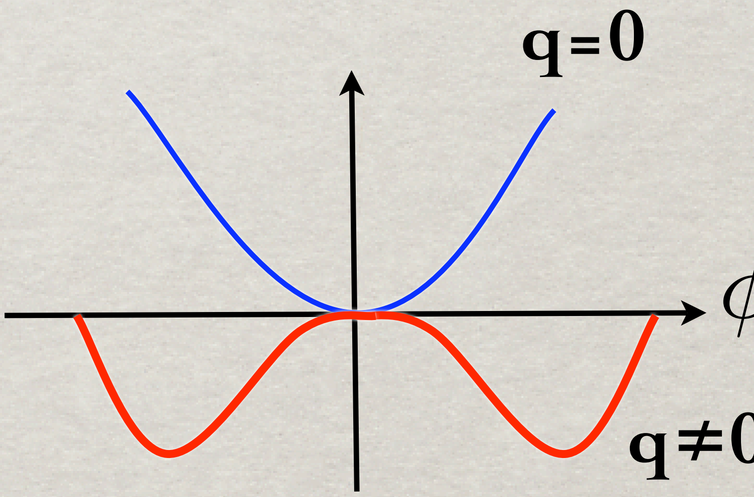

For **q=0** the system is stable. Dynamical instabilities do not correspond to tachyons.

Fluid equations in QCD

Derivation (Manuel-Mrowczynski arXiv:hep-ph/0606276**):**

$$n^{\mu}(x) = \int_p p^{\mu} f(p,x)$$

$$T^{\mu\nu}(x) = \int_p p^{\mu} p^{\nu} f(p,x)$$

Covariant continuity equation

$$\mathrm{D}_{\mu} n^{\mu} = 0$$

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$$\mathrm{D}_{\mu} T^{\mu\nu} - \tfrac{g}{2}\{F_{\mu}{}^{\nu}, n^{\mu}\} = 0$$

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$$n^\mu(x) = n(x)\, u^\mu(x)$$

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All quantities are matrices in color space e.g.

$$n_{\alpha\beta} = n_0 I_{\alpha\beta} + \tfrac{1}{2} n_a \tau^a_{\alpha\beta}$$

Fluctuations

Small colored fluctuations of density, energy density, pressure and plasma velocity, around their stationary and **colorless** values

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In the conformal limit $c_s^a = 1/\sqrt{3}$

however one can take this as a **parameter** for the fluid approach

Substituting the expression of the fluctuation we get

$$\begin{aligned}\bar{n}\, D_\mu \delta u^\mu + (D_\mu \delta n)\, \bar{u}^\mu &= 0\\ \bar{u}^\mu D_\mu \delta\epsilon + (\bar{\epsilon} + \bar{p}) D_\mu \delta u^\mu &= 0\\ (\bar{\epsilon} + \bar{p}) \bar{u}_\mu D^\mu \delta u^\nu - (D^\nu - \bar{u}^\nu \bar{u}_\mu D^\mu)\delta p - g\bar{n}\bar{u}_\mu F^{\mu\nu} &= 0\end{aligned}$$

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Colorless components: $\left(\frac{1}{c_s^2} - \frac{\mathbf{k}^2}{\omega^2} \right) \delta p_0 = 0$

The colorless components correspond to sound waves. The colored components are more involved.

Colored components

In the **stationary state** the color current vanishes

$$\bar{j}^{\mu} = -g\left(\bar{n}\,\bar{u}^{\mu} - \tfrac{1}{3}\mathrm{Tr}\left[\bar{n}\,\bar{u}^{\mu}\right]\right) = 0$$

whereas with **fluctuations**

$$\delta j_a^{\mu} = -\frac{g}{2}\left(\bar{n}\,\delta u_a^{\mu} + \delta n_a\,\bar{u}^{\mu}\right)$$

in **linear response theory**

$$\delta j_a^{\mu}(k) = -\Pi_{ab}^{\mu\nu}(k) A_{\nu,b}(k)\ ,$$

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Instead of studying the fluctuations of hydrodynamical quantities we study the fluctuations of the gauge fields.

As in QED we say that the system is unstable when the gauge field grows exponentially. The assumption is that the colored fluctuations will not be damped at short time scales.

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The gauge field obey the **equation of motion**

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For a **thermally equilibrated plasma**

Solutions:

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Instabilities (solutions with $\mathrm{Im}(\omega) > 0$) are absent

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Dielectric tensor of the total system in the "cold beam" approx.

$$\varepsilon_{\rm t}^{ij}(\omega,\mathbf{k}) = \left(1 - \frac{\omega_{\rm t}^2}{\omega^2}\right)\delta^{ij} - \frac{\omega_{\rm p}^2}{\omega^2}\frac{c_s^2 k^i k^j}{\omega^2 - c_s^2\mathbf{k}^2} - \frac{\omega_{\rm jet}^2}{\omega^2}\left(\frac{v^i k^j + v^j k^i}{\omega - \mathbf{k}\cdot\mathbf{v}} - \frac{(\omega^2-\mathbf{k}^2)v^i v^j}{(\omega-\mathbf{k}\cdot\mathbf{v})^2}\right)$$

where $\omega_{\rm t}^2 = \omega_{\rm p}^2 + \omega_{\rm jet}^2$ $\qquad b = \dfrac{\omega_{\rm jet}^2}{\omega_{\rm t}^2}$

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We look for unstable modes of the system as a function of the parameters $|\mathbf{k}|, |\mathbf{v}|, \cos\theta = \hat{\mathbf{k}}\cdot\hat{\mathbf{v}}, b$ and c_s

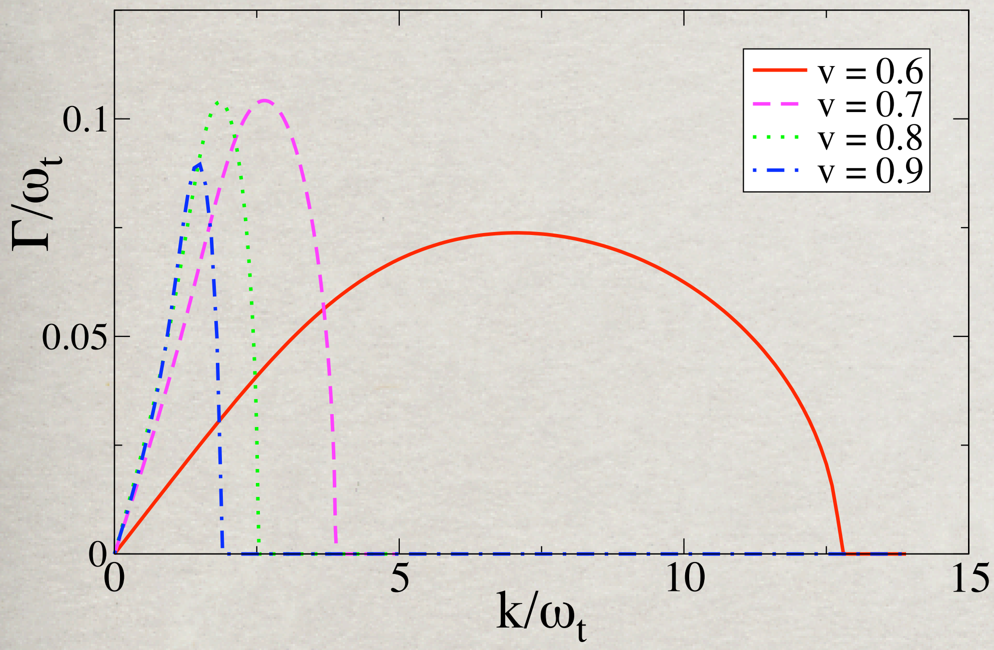
Unstable modes

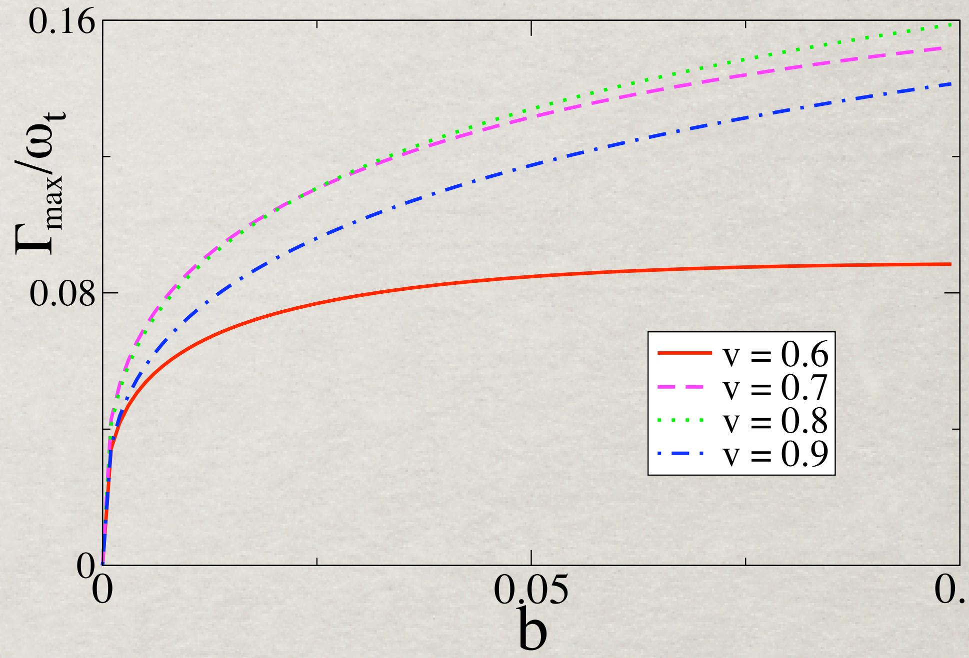
Main features:

- Instabilities only for velocity of the jet larger than the speed of sound of the plasma (as for the Mach-cone Casalderrey-Solana et. al hep-ph/0411315)
- Momentum of the collective mode **k** smaller than a threshold value
- With increasing values of b the unstable modes grows faster
- Non trivial dependence on θ

$\mathbf{k}$ parallel to $\mathbf{v}$

b=0.02





✦ Unstable for $c_s < v < 1$

✦ Maximum for velocities v ~ 0.7-0.8

✦ For v=1, the instability disappears(dimensional contraction which occurs in the eikonal limit, see e.g. Jackiw hep-th/9112020)

✦ The threshold value of the momentum **decreases** with **increasing velocity**

k othogonal to **v**

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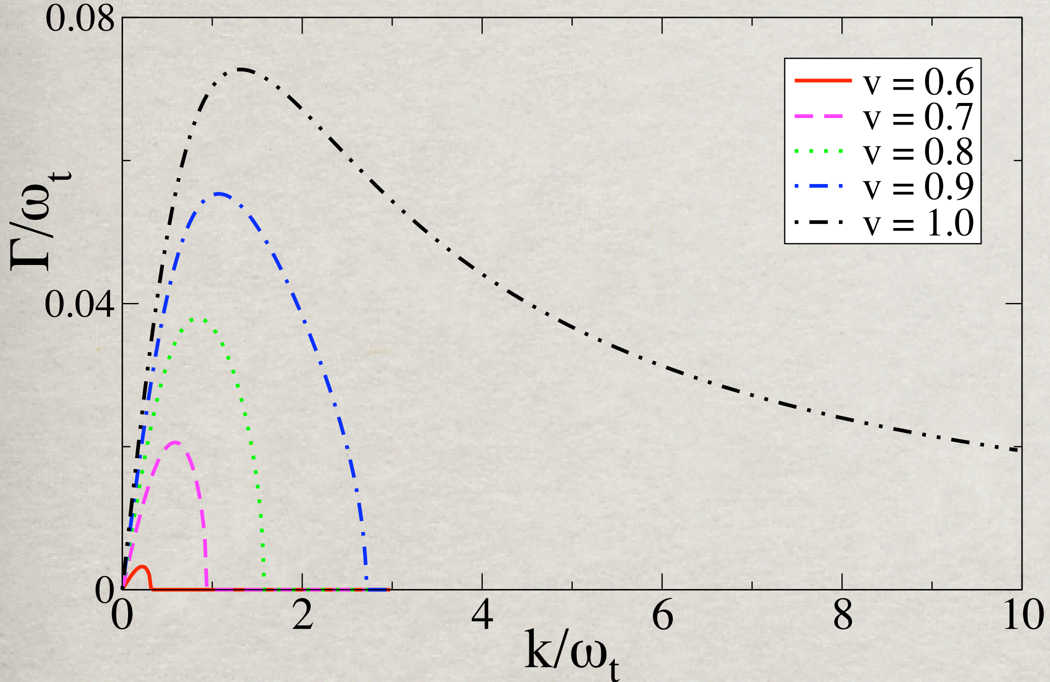
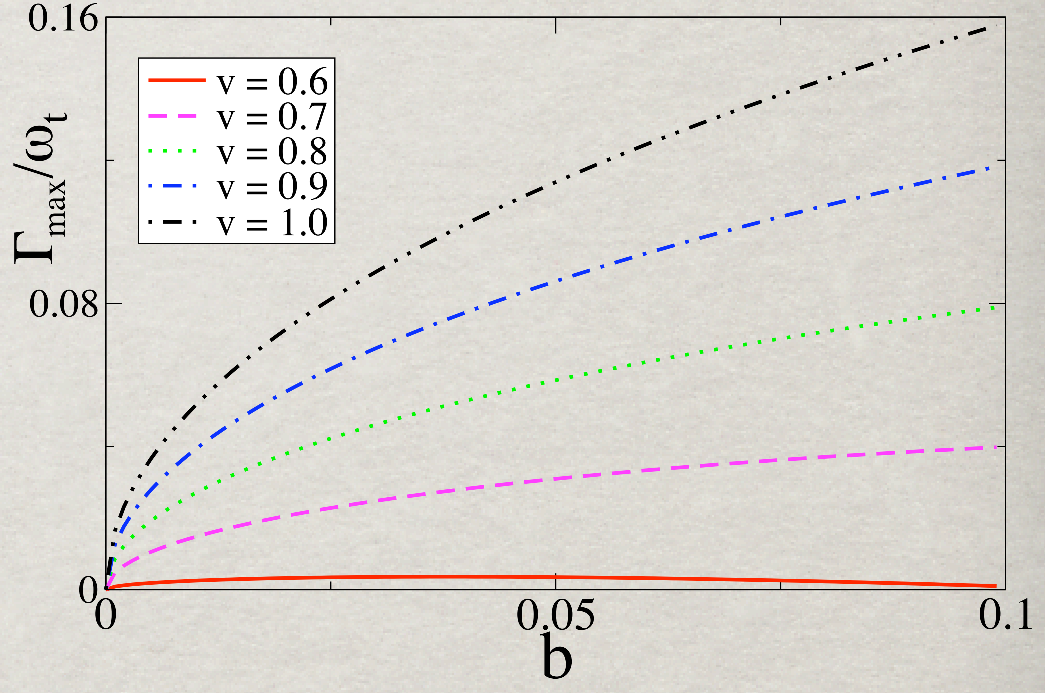




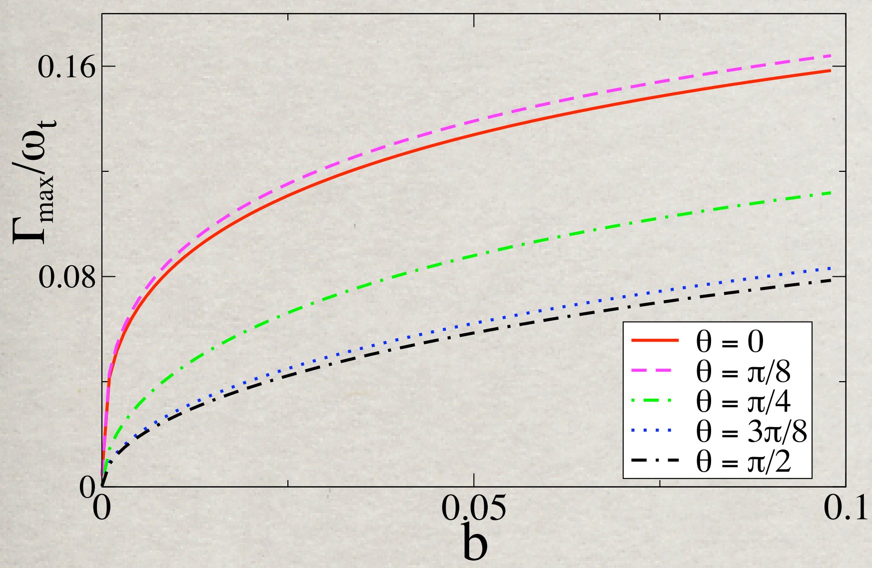
✦ Unstable for $c_s < v$

✦ Has a maximum for $v = 1$

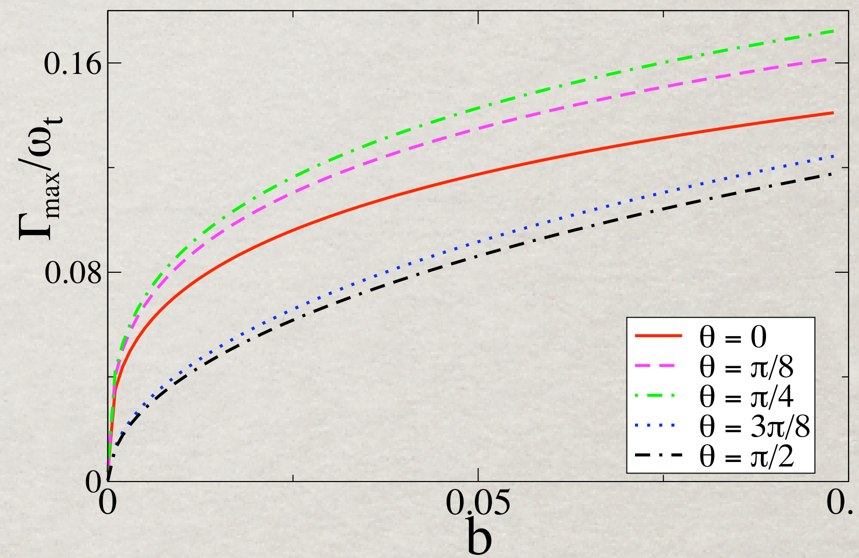
✦ The threshold value of the **momentum increases** with **increasing velocity**

Oblique case

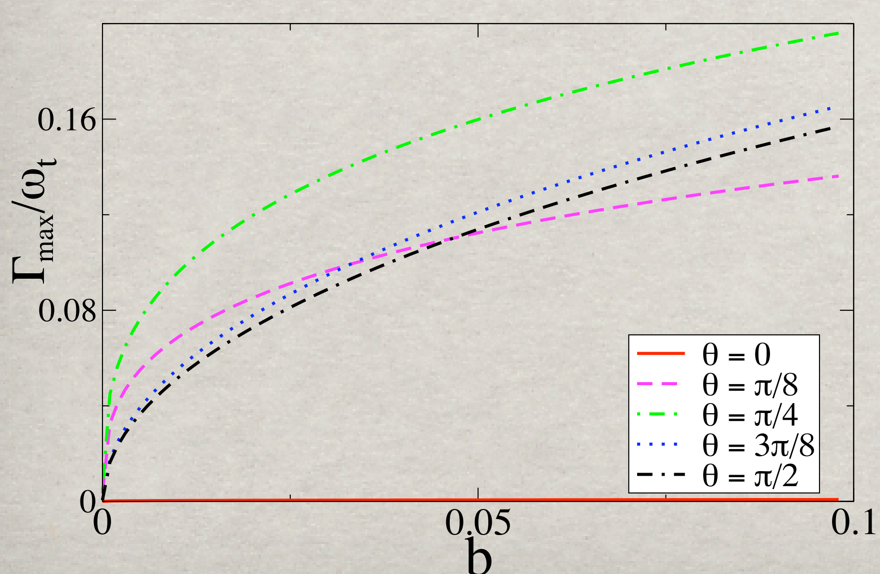
v=0.8



v=0.9



v=1.0



✦For $v \sim c_s$ modes collinear with v are dominant.

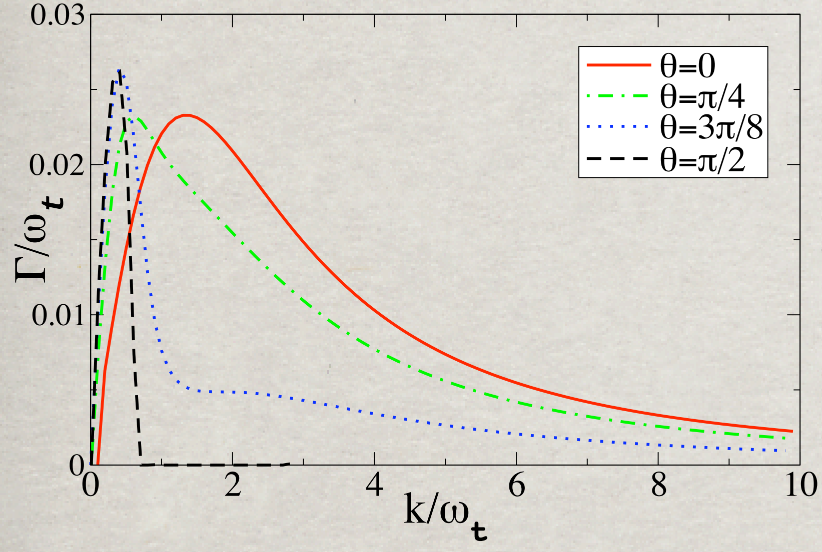
✦For ultra-relativistic velocities collinear modes are suppressed and dominant modes have

$$\theta \sim \pi/4$$

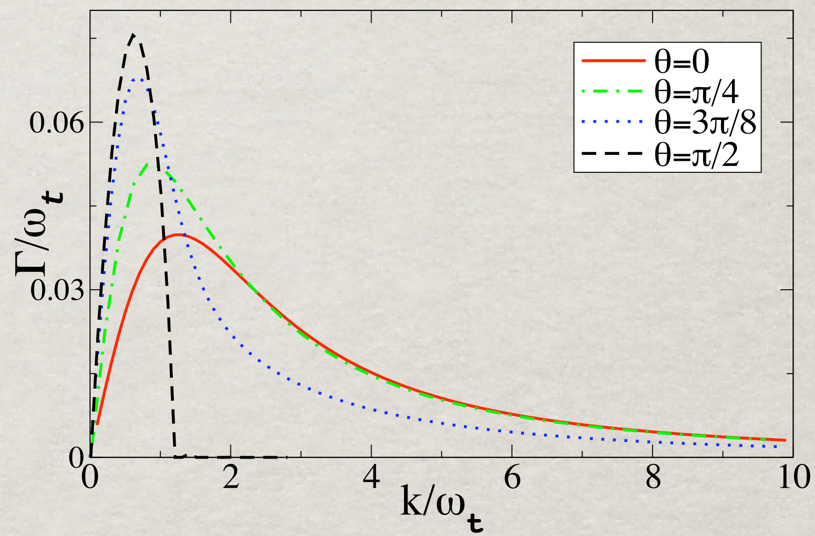
Time scale for the instability ~1-2fm/c

Preliminary from kinetic theory analysis

v=0.4



v=0.6



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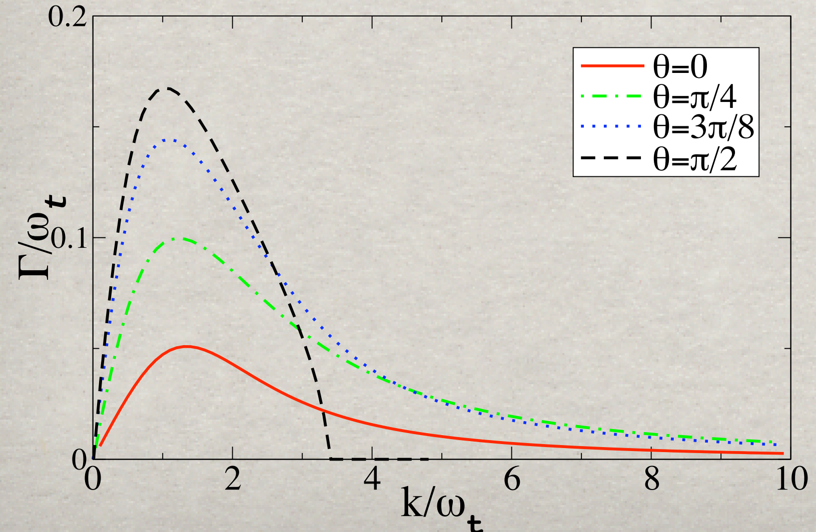
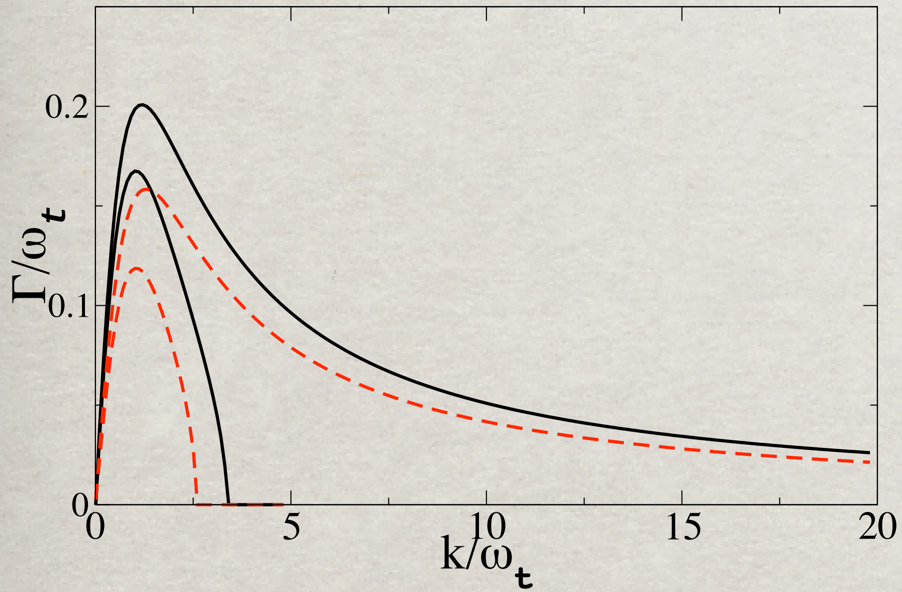


- ✦For v<<1 all modes seem to be equally unstable
- ✦For ultra-relativistic velocities collinear modes are suppressed and dominant modes have large angles

Comparison kinetic-fluid

Transverse modes comparison

$$c_s^a = 1/\sqrt{3}$$



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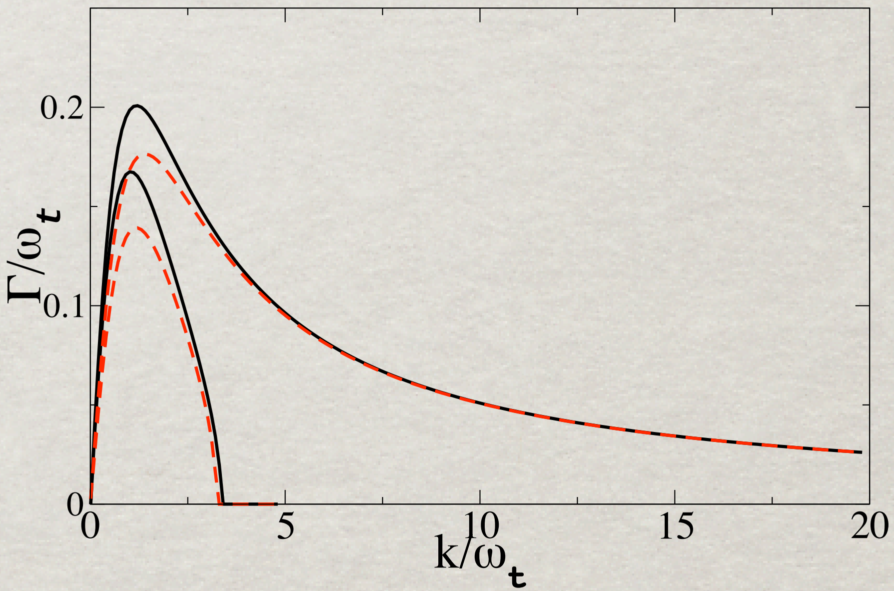


Rather good agreement.

The fluid approach underestimates the growth rate of the instability.

Summary

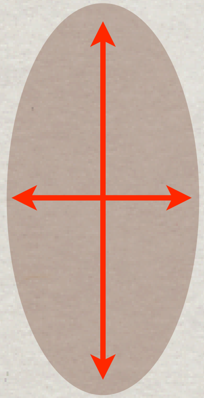
- Heavy-ion collisions give a unique setting for understanding QCD
- Plasma instabilities may play a role in the jet-quenching
- While traveling across the plasma jet degrade losing energy and momentum exciting the collective modes of the system
- Similar results with kinetic theory (HTL) valid for $g \ll 1$
- Outlook: calculation of energy loss, more complex configurations

Backup slides

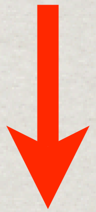
ELLIPTIC FLOW AND HYDRO

Elliptic flow: anisotropy in the **momentum** distribution of the hadrons

Overlapping region in peripheral collisions



space anisotropy



momentum anisotropy

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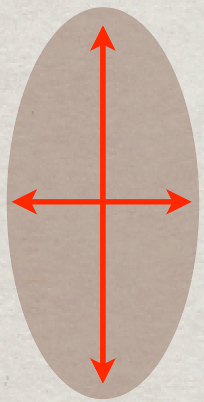

space anisotropy

↓

pressure anisotropy

↓

momentum anisotropy

Hydrodynamics !

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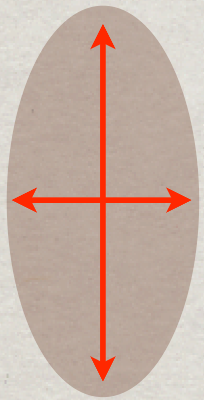

space anisotropy

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pressure anisotropy

↓

momentum anisotropy

Hydrodynamics !

- Pressure anisotropy converted into velocity anisotropy:

$$\frac{dv}{dt} = \frac{1}{\rho}\nabla P$$

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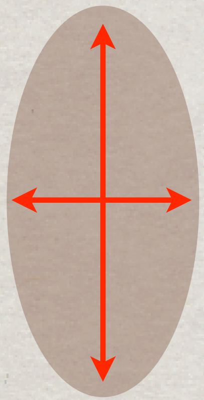

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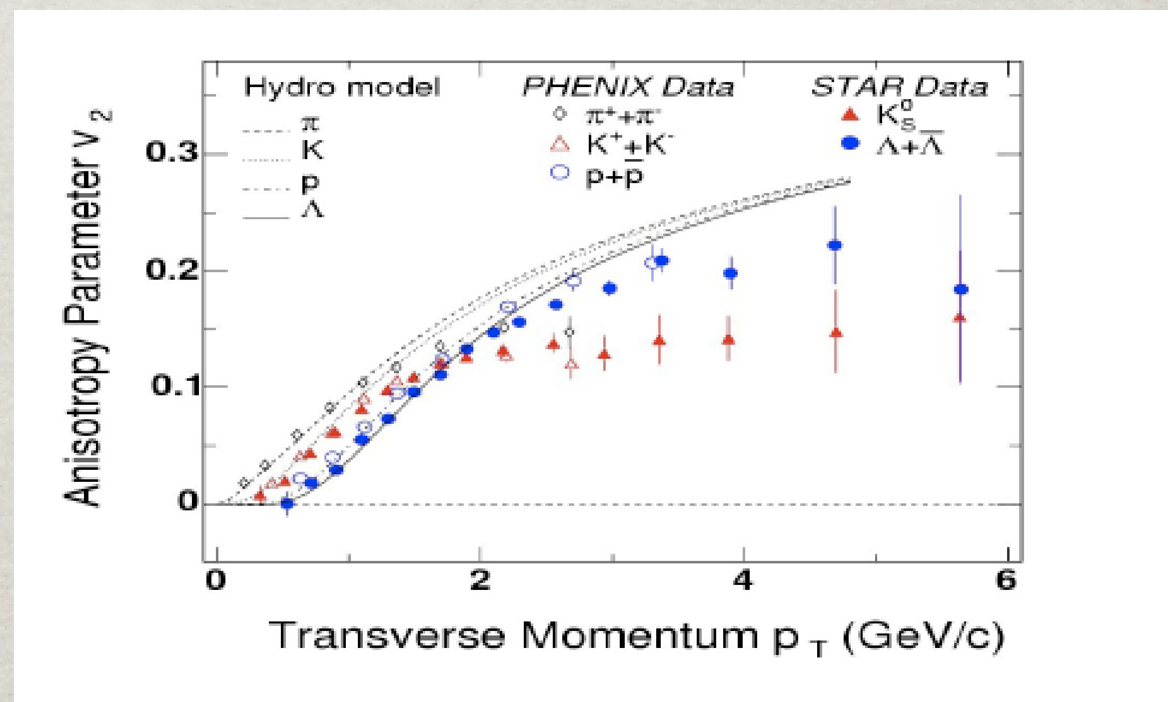

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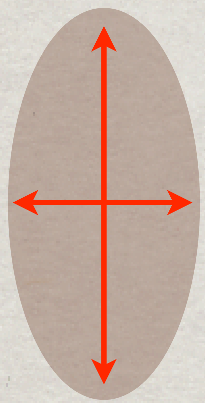

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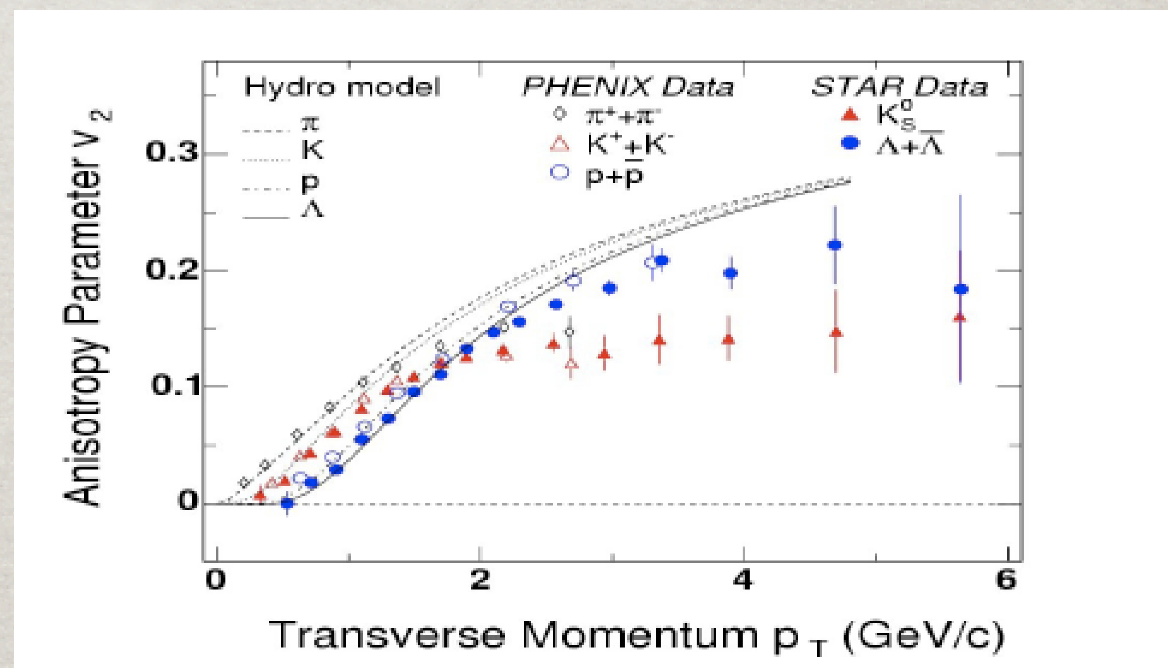


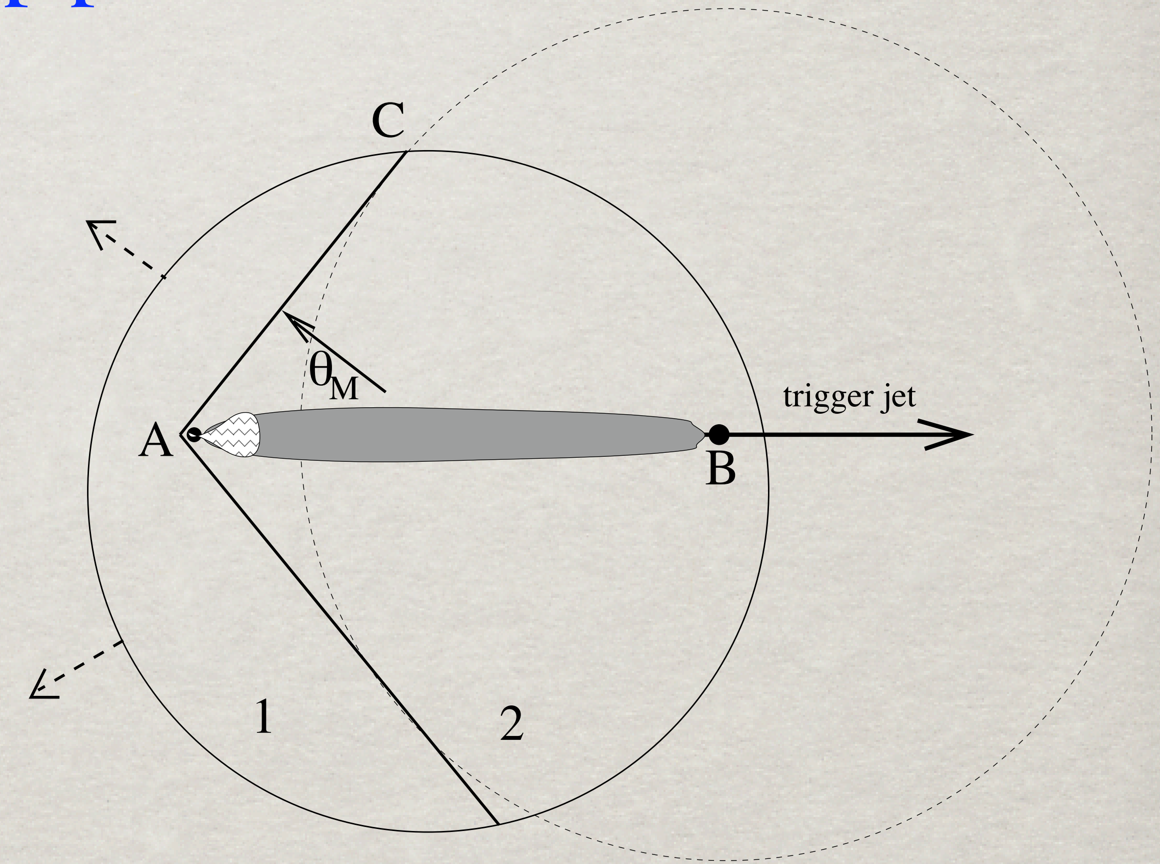
Open problem: hydro requires very short equilibration times ~0.6 fm/c how does the system equilibrate so quickly?

Conical flow

Casalderrey-Solana et. al hep-ph/0411315

- An **ultrasonic jet** produces a Mach shock-wave
- The angle of emission of partons is simply given by

$$\cos\theta_M \simeq c_s/c$$



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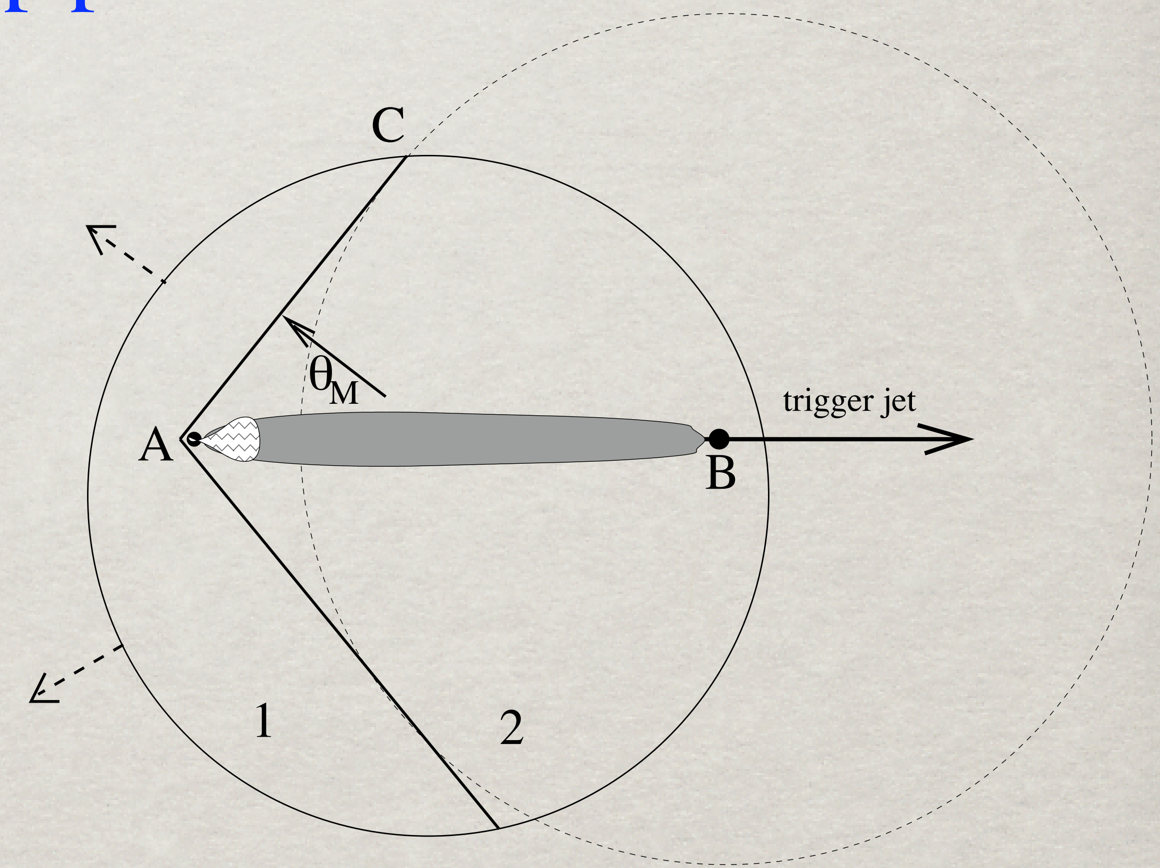
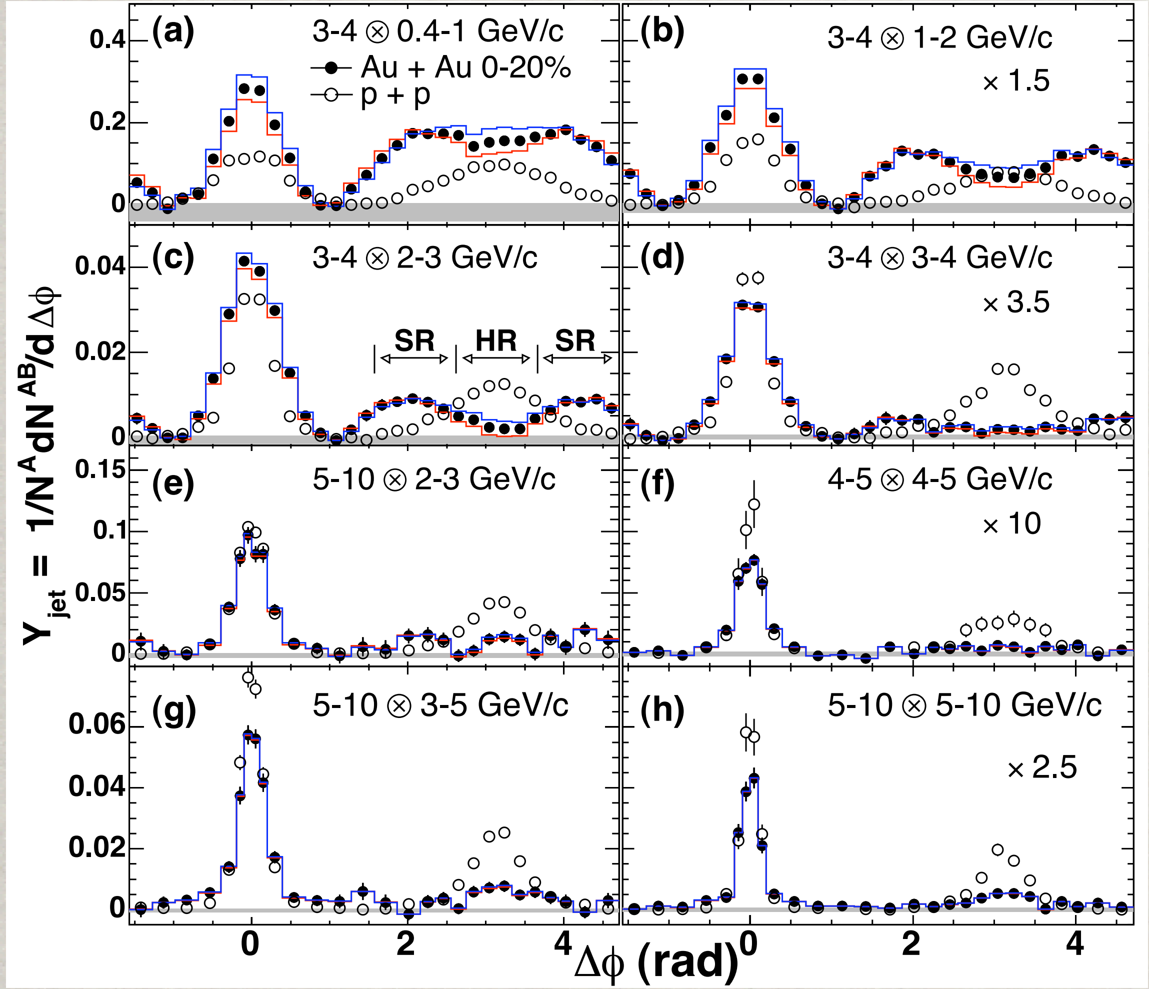


They find $\theta_M \simeq 1.2$ in good agreement with experimental data.

Mach Cone and hydro

Phenix collaboration 0705.3238 [nucl-ex]



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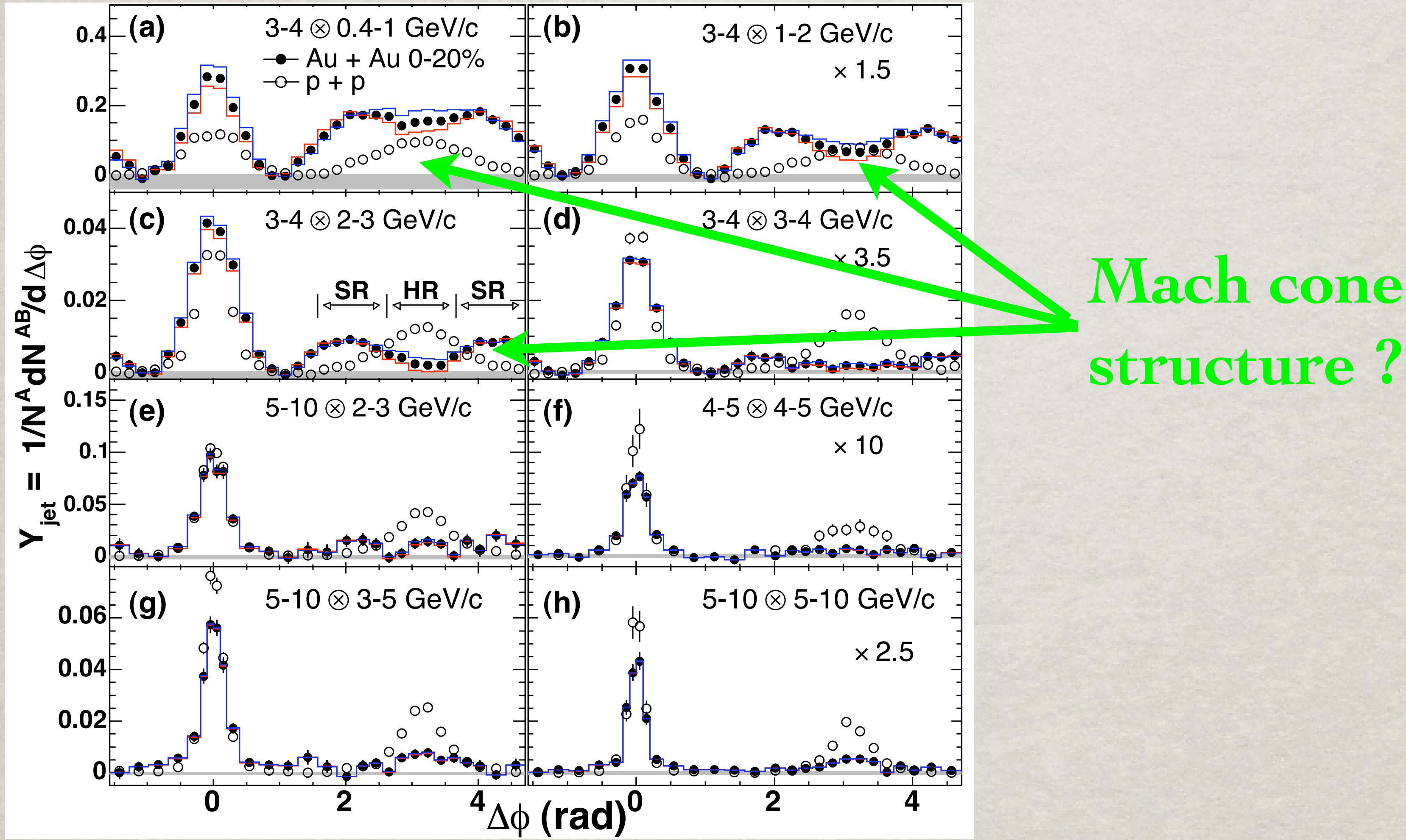

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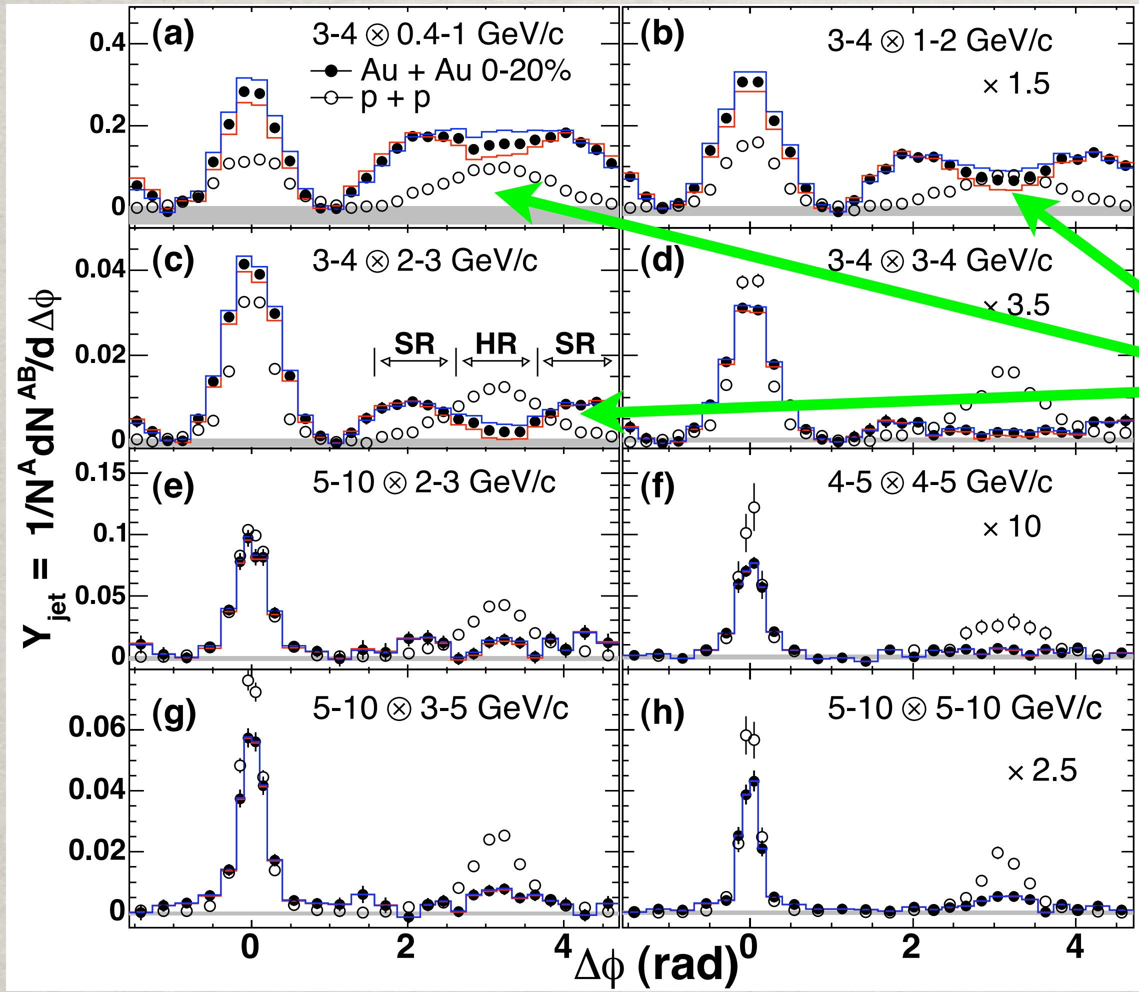

Mach cone structure ?

Simulations of Chaudhuri and Heinz, PRL 97,62301 (2006) with hydro do not give the Mach-cone structure. However they considered only colorless components!

Mach Cone and hydro

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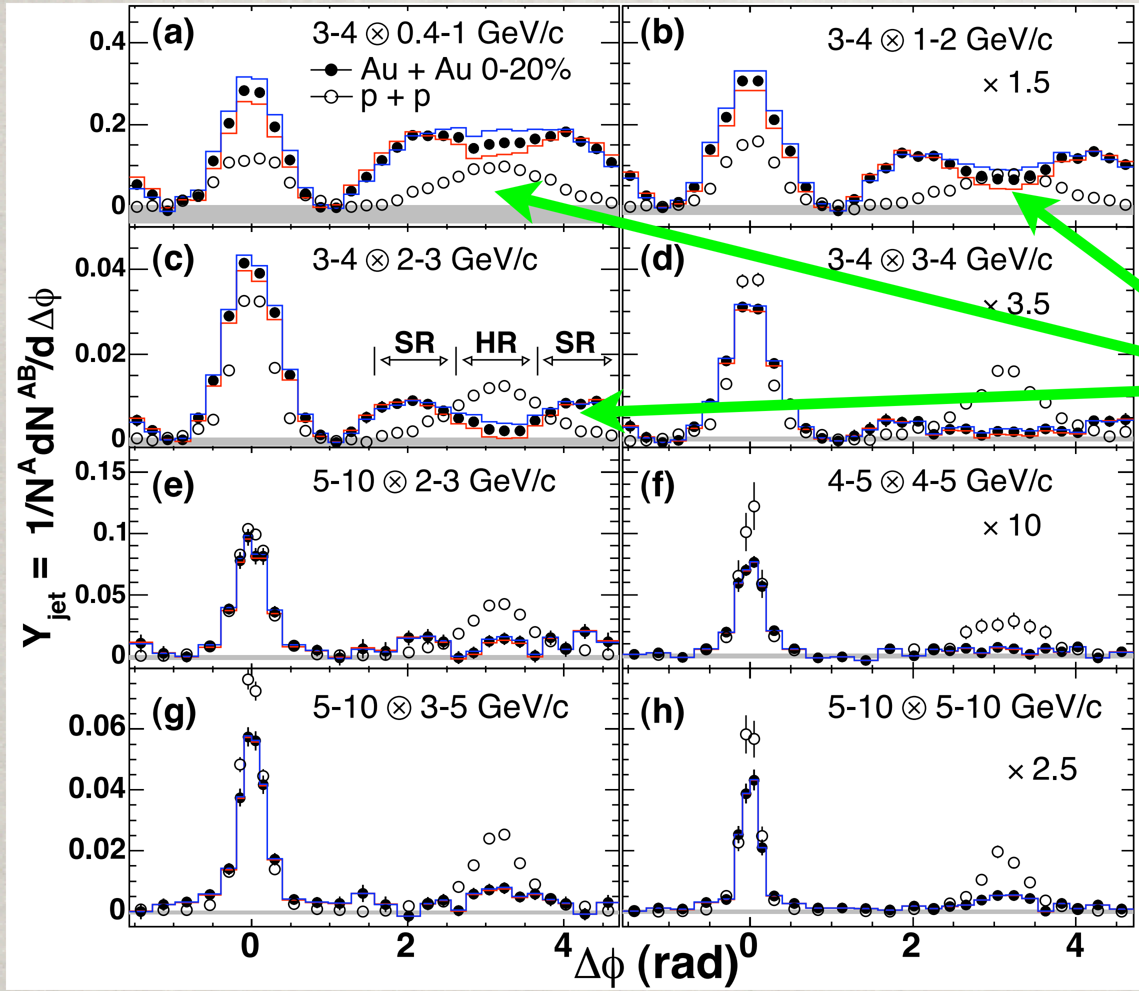

Mach cone structure ?

Simulations of Chaudhuri and Heinz, PRL 97,62301 (2006) with hydro do not give the Mach-cone structure. However they considered only colorless components!

General idea: if hydro can reproduce soft phenomena, then it might be able to reproduce some aspects of jet quenching.

Our idea: include color fluctuations

QCD Vlasov equations

Kinetic equations for quarks, antiquarks and gluons

$$p^\mu D_\mu Q(p,x) + \frac{g}{2} p^\mu \left\{F_{\mu\nu}(x), \partial_p^\nu Q(p,x)\right\} = C$$
$$p^\mu D_\mu \bar{Q}(p,x) - \frac{g}{2} p^\mu \left\{F_{\mu\nu}(x), \partial_p^\nu \bar{Q}(p,x)\right\} = \bar{C}$$
$$p^\mu \mathcal{D}_\mu G(p,x) + \frac{g}{2} p^\mu \left\{\mathcal{F}_{\mu\nu}(x), \partial_p^\nu G(p,x)\right\} = C_g$$

In Vlasov approximation one neglects the collision terms

And Maxwell equations $D_\mu F^{\mu\nu}(x) = \delta j_t^\nu(x)$

Considering small fluctuations around an equilibrium distribution

$$Q(p,x) = f_{FD}^{\text{eq.}}(p_0) + \delta Q(p,x)\ , \qquad \bar{Q}(p,x) = f_{FD}^{\text{eq.}}(p_0) + \delta \bar{Q}(p,x)\ , \qquad G(p,x) = f_{BE}^{\text{eq.}}(p_0) + \delta G(p,x)$$

We get the polarization tensor in linear response analysis

$$ip_\nu F^{\mu\nu} \simeq -\Pi^{\mu\nu} A_\nu$$

Jet quenching

High transverse momentum partons lose energy in matter prior to forming hadrons. **Suppression of the away-side yields at high p_t** .

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- A jet loses energy mainly by radiative processes
- To describe this phenomenon QCD has to be supplemented with medium-induced parton energy loss

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- A jet loses energy mainly by radiative processes
- To describe this phenomenon QCD has to be supplemented with medium-induced parton energy loss

Many models on the market, see e.g. hep-ph/0304151