Twistor inspired Higgs phenomenology

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European Physical Society HEP07,

Manchester, July 2007

Introduction

Witten's proposal to relate perturbative gauge theory amplitudes to topological string theory in twistor space has inspired new ways of calculating amplitudes in massless gauge theories:

- ✓ MHV rules

  Cachazo, Svrcek and Witten

  ⇒ NEW analytic results for some QCD tree amplitudes with any number of legs

- ✓ BCF on-shell recursion relations — Britto, Cachazo and Feng (and Witten)

  ⇒ NEW compact results for some multileg QCD tree amplitudes

- ✓ Unitarity and cut-constructibility

  Bern, Dixon, Dunbar, Kosower; Britto, Cachazo and Feng; . . .

  ⇒ NEW analytic one-loop amplitudes in massless supersymmetric theories

- ✓ Recursive derivation of rational terms

  Bern, Dixon, Kosower + Berger, Forde; Xiao,Yang, Zhu

  ⇒ NEW analytic one-loop amplitudes for multigluon amplitudes

Outline of Talk

Interesting to explore the strengths and weaknesses of the new methods for other Standard Model processes of phenomenological relevance

✓ Processes involving Higgs

- The Higgs model in the large top-mass limit
- Tree-level Higgs plus multi-parton amplitudes

Dixon, EWNG, Khoze; Badger, EWNG, Khoze

- One-loop Higgs plus multi-parton amplitudes

Badger, EWNG; Berger, Del Duca, Dixon; Badger, EWNG, Risager

The Higgs Model

✓ In the large top mass limit, we have the effective interaction

$$\mathcal{L}_H^{\rm int} = \frac{C}{2}\, H\, \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} \,, \qquad C = \frac{\alpha_s}{6\pi v}\left(1 + \mathcal{O}(\alpha_s)\right)$$

Wilczek; Shifman, Vainshtein, Zakharov

✓ Previously known amplitudes (in large m_t limit)

H + n partons	no-loops	one-loop	two-loop
2	✓	✓	✓
3	✓	✓	✓
4	✓	✓	
5	✓		
6			

✓ Higgs cross section, Higgs transverse momentum, background to weak boson scattering,

The Higgs Model

- ✓ Introduce a complex field $\phi = \frac{1}{2}(H + iA)$ and divide $\mathcal{L}_H^{\rm int}$ into two terms, containing purely selfdual (SD) and purely anti-selfdual (ASD) gluon field strengths

$$\begin{aligned}
\mathcal{L}_{H,A}^{\rm int} &= \frac{1}{2}\Big[H\,{\rm Tr}\,G_{\mu\nu}G^{\mu\nu} + iA\,{\rm Tr}\,G_{\mu\nu}\,{}^*G^{\mu\nu}\Big] \\
&= \phi\,{\rm Tr}\,G_{SD\,\mu\nu}G^{\mu\nu}_{SD} + \phi^\dagger\,{\rm Tr}\,G_{ASD\,\mu\nu}G^{\mu\nu}_{ASD}
\end{aligned}$$

Dixon, EWNG and Khoze

- ✓ Natural link with QCD when momentum of Higgs $\longrightarrow 0$
- ✓ Higgs amplitudes obtained by adding ϕ and $\phi^\dagger$ amplitudes

The Higgs Model

- ✓ The key point is that the amplitudes for ϕ plus n gluons, and those for $\phi^\dagger$ plus n gluons, separately have a simpler structure than the amplitudes for H.

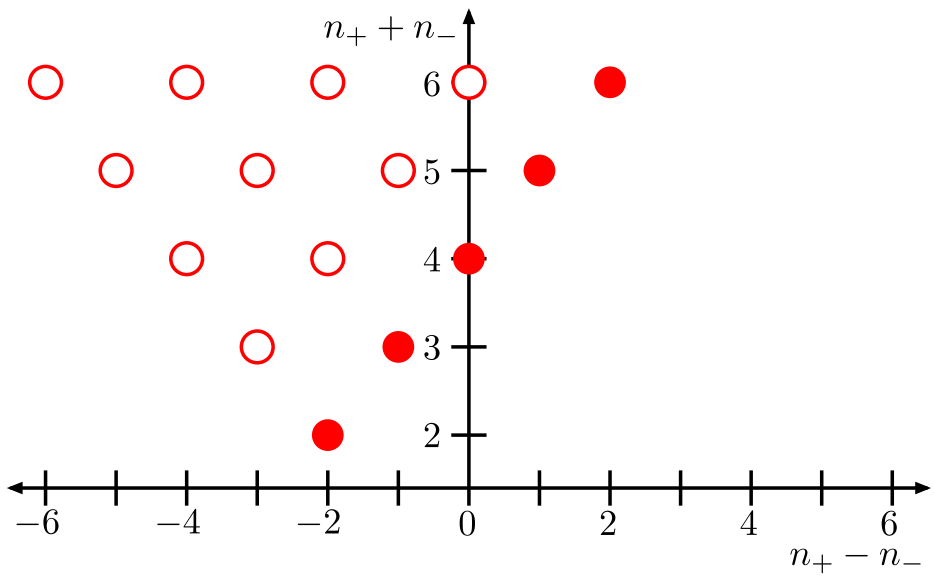
- ✓ It can be shown that (Berends-Giele currents/SUSY WI) the colour ordered subamplitudes are
  - ✓ $A_n(\phi, 1^\pm, 2^+, 3^+, \ldots, n^+) = 0$
  - ✓ $A_n(\phi^\dagger, 1^\pm, 2^+, 3^+, \ldots, n^+) \neq 0$

- ✓ The ϕ-MHV amplitudes, with precisely two negative helicities, are the first non-vanishing ϕ amplitudes.

ϕ plus multi-gluon tree amplitudes

- ✓ Furthermore, the 'ϕ-MHV' amplitudes have precisely the same form as the QCD case — except for the implicit momentum carried out of the process by the Higgs boson.

$$A_n(\phi, 1^+, \ldots, p^-, \ldots, q^-, \ldots, n^+) = \frac{\langle p\, q\rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle n-1,\, n\rangle \langle n\, 1\rangle}$$

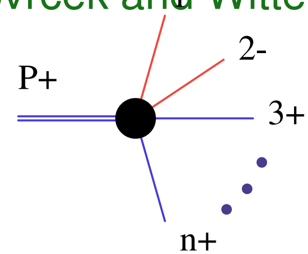


- ✓ Solid red dots represent fundamental ϕ-MHV vertices.
- ✓ Open circles are composite ϕ amplitudes, which are built from the ϕ-MHV vertices plus pure-gauge-theory MHV vertices.

MHV rules

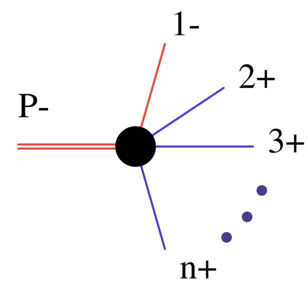
Start from on-shell MHV amplitude and define off-shell vertices

Cachazo, Svrcek and Witten



$$V(1^-,2^-,3^+,\ldots,n^+,P^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\cdots\langle n-1n\rangle\langle nP\rangle\langle P1\rangle}$$

and



$$V(1^-,2^+,3^+,\ldots,n^+,P^-) = \frac{\langle 1P\rangle^4}{\langle 12\rangle\cdots\langle n-1n\rangle\langle nP\rangle\langle P1\rangle}$$

connected by scalar propagators
Crucial step is off-shell continuation $P^2 \neq 0$:

$$\langle iP\rangle = \frac{\langle i^-|\not{P}|\eta^-]}{[P\eta]} = \sum_j \frac{\langle i^-|\not{j}|\eta^-]}{[P\eta]}$$

where $P = \sum_j j$ and η is lightlike auxiliary vector

MHV rules for Higgs+gluon amplitudes

The MHV rules for computing Higgs plus n-gluon scattering amplitudes can be summarized as follows:

- ✓ For the ϕ couplings, everything is exactly like the MHV rules (except for the momentum carried by ϕ).
- ✓ For $\phi^\dagger$, we just apply parity. That is, we compute with ϕ, and reverse the helicities of every gluon. Then we let $\langle i\, j\rangle \leftrightarrow [j\, i]$ to get the desired $\phi^\dagger$ amplitude.
- ✓ For H, we add the ϕ and $\phi^\dagger$ amplitudes.

These rules can easily be used to reproduce all of the available analytic formulae for tree-level Higgs $+\, n$-gluon scattering $(n \leq 5)$ at tree level and derive new expressions for $n \geq 6$.

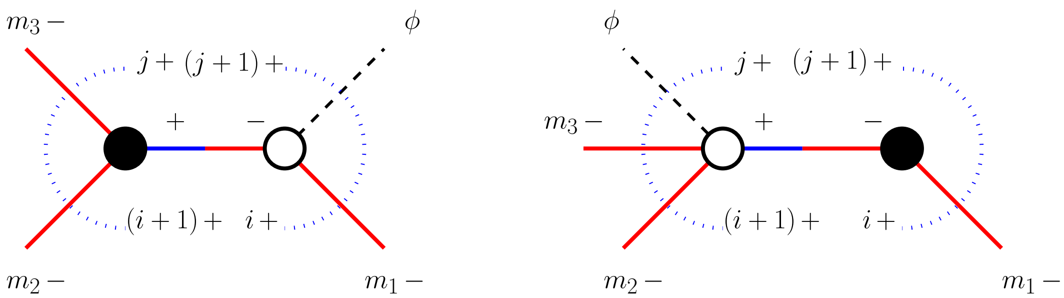
Dixon, EWNG and Khoze

- ✓ Easily extended to include massless quarks

Badger, EWNG and Khoze

NMHV $A_n(\phi, m_1^-, m_2^-, m_3^-, \ldots)$ amplitudes

$$A_n(\phi, m_1^-, m_2^-, m_3^-) = \frac{1}{\prod_{l=1}^{n} \langle l,\, l+1\rangle} \sum_{i=1}^{2} \sum_{C(m_1,m_2,m_3)} A_n^{(i)}(m_1, m_2, m_3)$$



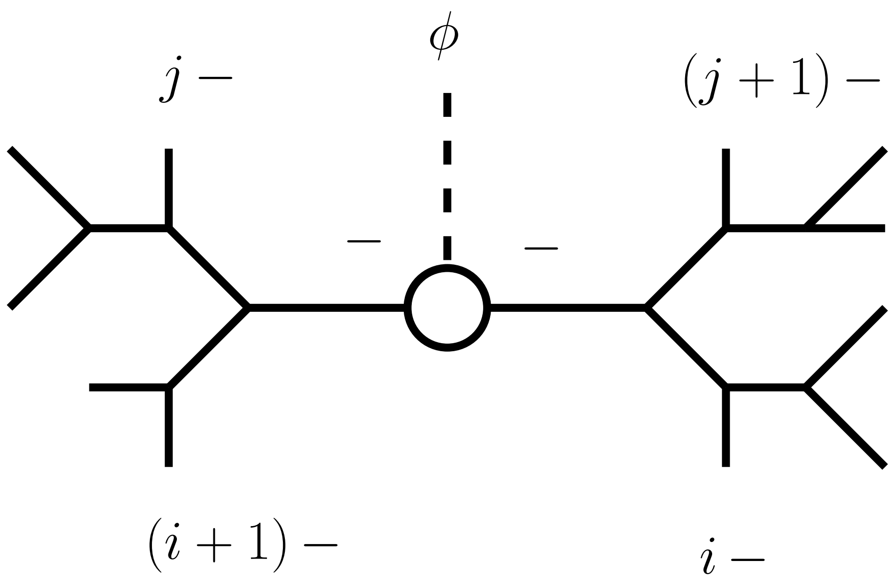
$$A_n^{(1)}(m_1, m_2, m_3) = \sum_{i=m_1}^{m_2-1} \sum_{j=m_3}^{m_1-1} \frac{\langle m_2\, m_3\rangle^4 \langle m_1|\not{q}_{i+1,j}|\xi]^4}{D(i, j, q_{i+1,j})},$$

$$A_n^{(2)}(m_1, m_2, m_3) = \sum_{i=m_1}^{m_2-1} \sum_{j=m_3}^{m_1-1} \frac{\langle m_2\, m_3\rangle^4 \langle m_1|\not{q}_{j+1,i}|\xi]^4}{D(i, j, q_{j+1,i})}$$

$$D(i, j, q) = \langle i|\not{q}|\xi]\langle (j+1)|\not{q}|\xi]\langle (i+1)|\not{q}|\xi]\langle j|\not{q}|\xi] \frac{q^2}{\langle i,\, i+1\rangle \langle j,\, j+1\rangle}.$$

The all-minus tree amplitude

- ✓ The n-point all-minus tree amplitudes are constructed by joining $n-2$ three point vertices.
- ✓ All orders result proved by coupling off-shell Berends-Giele currents.



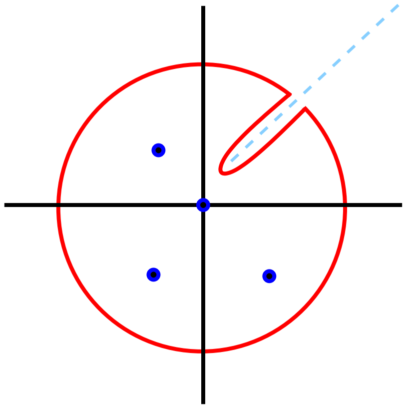
$$A_n(\phi, 1^-, \ldots, n^-) = \frac{(-1)^n M_H^4}{[1\,2]\,[2\,3]\cdots[n\,1]}$$

One-loop amplitudes

- ✓ In general, loop amplitudes contain both poles and cuts

$$A_n^1 \sim \text{(poly)logs} + \text{rational}$$

e.g. $\log(x)$ has cut for negative x



- ✓ logarithmic terms can be constructed from cuts using unitarity - double cuts, or generalised cuts
- ✓ rational parts only have simple poles and can be constructed using BCF type recursion and knowledge of factorisation properties

  Collectively this is the Unitarity Bootstrap

One-loop amplitudes

- ✓ Aim to use these methods to compute MHV $--++\ldots$ and all-minus (googly) $----\ldots$ one-loop Higgs amplitudes

  Badger, EWNG; Badger, EWNG, Risager

- ✓ Recall

$$A_n^{(1)}(H;\cdots) = A_n^{(1)}(\phi;\cdots) + A_n^{(1)}(\phi^\dagger;\cdots)$$

  so only compute ϕ amplitude

- ✓ Separate out cut-constructible (C) and rational (R) parts

$$A_n^{(1)}(\phi;\cdots) = A_n^{(1),C}(\phi;\cdots) + A_n^{(1),R}(\phi;\cdots)$$

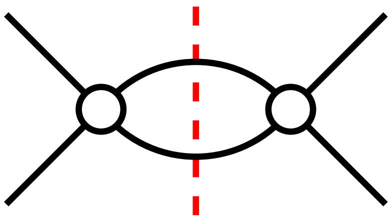
- ✓ Work in non-supersymmetric theory
- ✓ The finite ϕ anplitudes - all plus, and one-minus now available

  Berger, Del Duca, Dixon

Cut constructible parts - unitarity

At least three different methods - all based on connecting on-shell 4-dimensional vertices

Bern, Dixon, Dunbar, Kosower;Britto, Cachazo, Feng; Brandhuber, Spence, Travaglini



✓ Reconstruct coefficients of basis set of integrals - boxes, triangles, bubbles

1. Classic double cut + collinear factorisation (triple cut)

   Bern, Dixon, Dunbar, Kosower (94)

2. Generalised (quadruple cut) unitarity and holomorphic anomaly

   Britto, Cachazo, Feng

✓ Reconstruct full amplitude by doing phase space and dispersion integrals

Brandhuber, Spence, Travaglini

Cut constructible parts - unitarity

Choose to use *MHV-rules* method

Brandhuber, Spence and Travaglini

- Connect on-shell 4-dimensional vertices with off-shell propagators

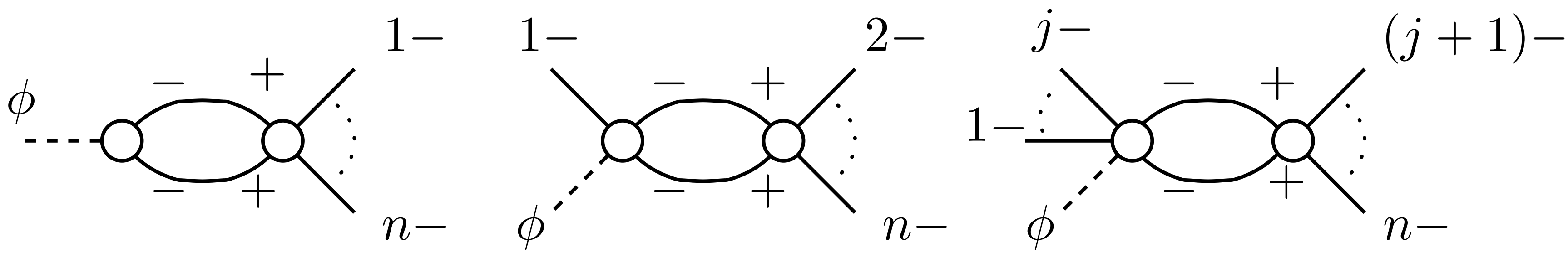
$$\int \frac{d^4L_1}{L_1^2}\frac{d^4L_2}{L_2^2}\delta^4(L_1 + P_{i+1,j} - L_2)\mathcal{A}_L(\ell_2, \ldots, -\ell_1)\mathcal{A}_R(\ell_1, \ldots, -\ell_2)$$

- Each vertex is an on-shell tree amplitude with light-like internal momenta ℓ_i
- Each propagator is continued off-shell $L_i = \ell_i + z_i\eta$
- Rewrite integrals as phase space plus dispersion integrals

$$\int \frac{dz}{z}\int dLIPS^{(4)}(\ell_1, \ell_2, P)\mathcal{A}_L(\ell_2, \ldots, -\ell_1)\mathcal{A}_R(\ell_1, \ldots, -\ell_2)$$

Cut constructible parts - unitarity

- ✓ For the all-minus amplitude, three types of contribution - all involving MHV QCD vertices with the tree-level all-minus amplitude



- ✓ The integrand is written down by inspection e.g.

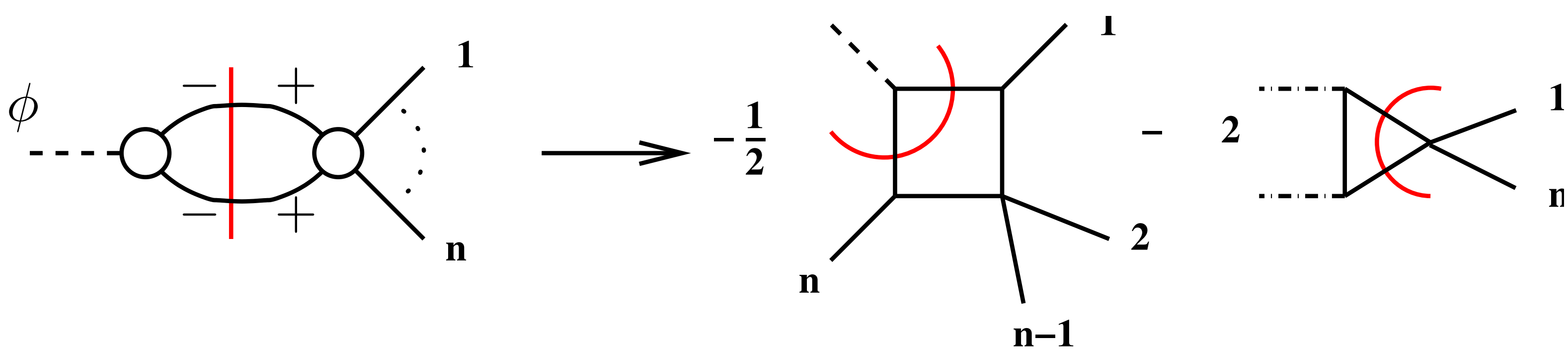
$$\mathcal{A}_L\mathcal{A}_R = \frac{m_H^4}{[l_1 l_2][l_2 l_1]} \frac{[l_1 l_2]^3}{[l_2 1][n l_1]\Pi_{\alpha=1}^{n-1}[\alpha\alpha+1]} = \mathcal{A}^{(0)}(\phi;1^-,\ldots,n^-)\frac{[l_1 l_2][1n]}{[l_2 1][n l_1]}$$

- ✓ Spinor algebra to simplify integrand

$$\frac{[l_1 l_2][1n]}{[l_2 1][n l_1]} \to \frac{2P\cdot nP\cdot 1 - P^2 n\cdot 1}{4\ell_1\cdot 1n\cdot\ell_2} - \frac{P\cdot 1}{2\ell_1\cdot 1} - \frac{P\cdot n}{2n\cdot\ell_2}$$

Cut constructible parts - unitarity

- ✓ Only the contributions corresponding to a cut in a particular channel are produced
  i.e. not the whole box function, but only the part that has a cut in that channel



van Neerven, NPB 268 (1986) 453

- ✓ Only one of the four hypergeometric functions in F^{2me} is produced. The other hypergeometric functions are obtained summing over the different classes of diagrams - and reconstruct entire box functions

$A_n^{(1),C}(\phi, 1^-, \ldots, n^-)$

Summing over the permutations of the three topologies,

$$
\begin{aligned}
A_n^{(1),C}(\phi,1^-,\ldots,n^-) &= A_n^{(0)}(\phi,1^-,\ldots,n^-) \\
&\Bigg[\sum_{i=1}^{n}\left(F_3^{1m}(s_{i,n+i-2}) - F_3^{1m}(s_{i,n+i-1})\right) \\
&- \frac{1}{2}\sum_{i=1}^{n}\sum_{j=i+2}^{n+i-2} F_4^{2me}(s_{i,j}, s_{i+1,j-1}; s_{i,j+1}, s_{i+1,j}) \\
&- \frac{1}{2}\sum_{i=1}^{n} F_4^{1m}(s_{i,i+2}; s_{i,i+1}, s_{i+1,i+2})\Bigg]
\end{aligned}
$$

- ✓ F_3^{1m}, F_4^{1m}, F_4^{2me} scaled loop integrals
- ✓ Satisfies known infrared pole structure
- ✓ Satisfies cut-constructible part of double collinear limit

$A_n^{(1),C}(\phi, 1^-, 2^-, 3^+ \dots, n^+)$

$$A_n^{(1),C}(\phi,1^-,2^-,3^+\dots,n^+) = A_n^{(0)}(\phi,1^-,2^-,3^+,\dots,n^+)$$

$$\times\Bigg[\sum_{i=1}^{n}\left(\mathrm{F}_3^{1\mathrm{m}}(s_{i,n+i-2}) - \mathrm{F}_3^{1\mathrm{m}}(s_{i,n+i-1})\right)$$

$$-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=i+2}^{n+i-2}\mathrm{F}_4^{2\mathrm{me}}(s_{i,j}, s_{i+1,j-1}; s_{i,j+1}, s_{i+1,j}) - \frac{1}{2}\sum_{i=1}^{n}\mathrm{F}_4^{1\mathrm{m}}(si, i+2; s_{i,i+1}, s_{i+1,i+2})$$

$$+\sum_{i=4}^{n}\Bigg(\frac{2}{3}\left(1-\frac{N_F}{N}\right)\left[\frac{\mathrm{tr}_-(1P_{i,n}(i-1)2)^3}{s_{12}^3}L_3(s_{i-1,1}, s_{i,1}) + \frac{\mathrm{tr}_-(2P_{3,i-1}i1)^3}{s_{12}^3}L_3(s_{2,i}, s_{2,i-1})\right]$$

$$-\left(1-\frac{N_F}{N}\right)\left[\frac{\mathrm{tr}_-(1P_{i,n}(i-1)2)^2}{s_{12}^2}L_2(s_{i-1,1}, s_{i,1}) + \frac{\mathrm{tr}_-(2P_{3,i-1}i1)^2}{s_{12}^2}L_2(s_{2,i}, s_{2,i-1})\right]$$

$$+4\left(1-\frac{N_F}{4N}\right)\left[\frac{\mathrm{tr}_-(1P_{i,n}(i-1)2)}{s_{12}}L_1(s_{i-1,1}, s_{i,1}) + \frac{\mathrm{tr}_-(2P_{3,i-1}i1)}{s_{12}}L_1(s_{2,i}, s_{2,i-1})\right]\Bigg)\Bigg]$$

with

$$L_k(s,t) = \frac{\log(s/t)}{(s-t)^k}$$

Unphysical poles

- ✓ Cut terms have spurious poles (coming from tensor triangle integrals)

$$\frac{\log(s_1/s_2)}{(s_1-s_2)^2}$$

- ⇒ rational terms must have (predictable) spurious poles that that do not obey factorisation properties

$$-\frac{(s_1+s_2)}{2s_1s_2(s_1-s_2)}$$

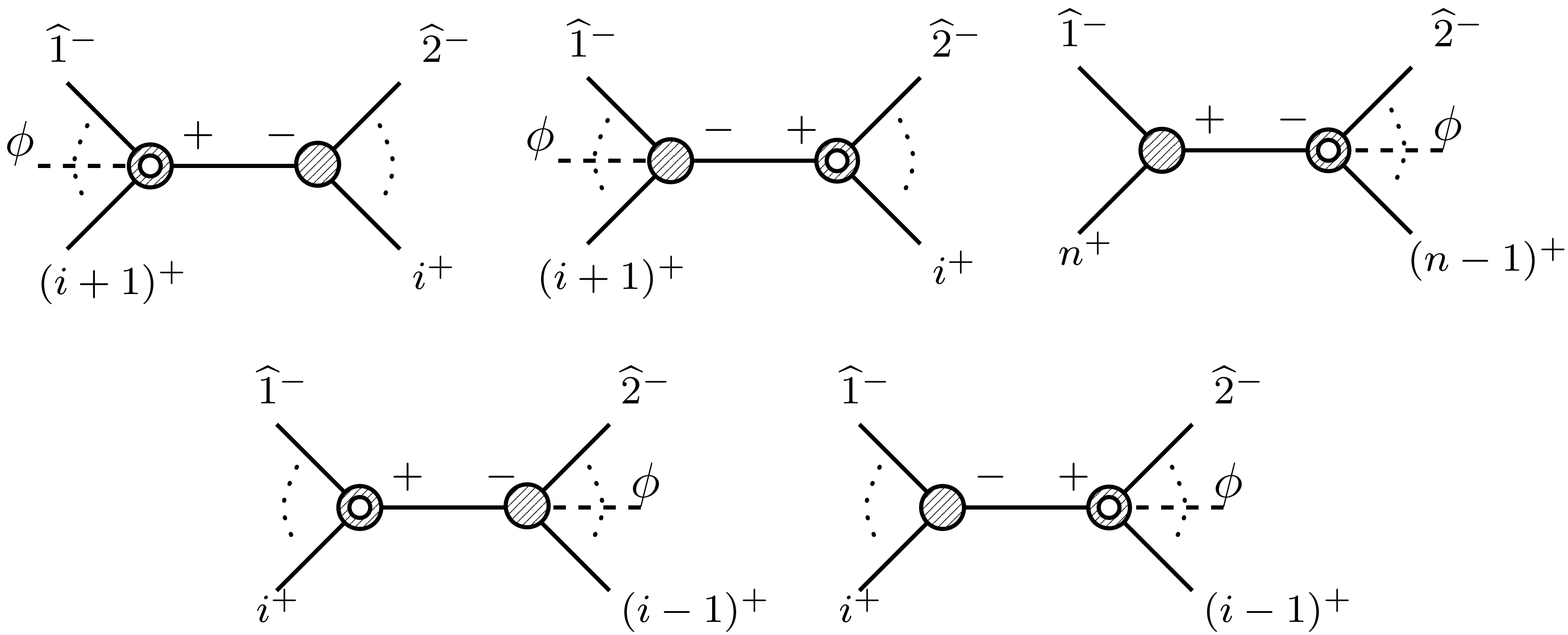
- ✓ Define completed cut term

$$\frac{\log(s_1/s_2)}{(s_1-s_2)^2} - \frac{(s_1+s_2)}{2s_1s_2(s_1-s_2)}$$

- ✓ They must be there - and are generated by tensor triangle reduction

On-shell recursion

- ✓ Once cut-completion terms have been defined (and their residue on physical poles removed) can use factorisation properties to establish a recursion relation.
- ✓ on-shell recursion for ϕ-MHV amplitudes



- ✗ Still not fully understood for all helicity configurations

One-loop Higgs amplitudes

$$A_n^{(1),C}(H;\cdots) = A_n^{(1),C}(\phi;\cdots) + A_n^{(1),C}(\phi^\dagger;\cdots)$$

$$A_4^{(1),R}(H;1^-,2^-,3^-,4^-)$$

$$= \frac{N_p}{96\pi^2}\left[-\frac{s_{13}\langle 4|1+3|2]^2}{s_{123}\,[1\,2]^2\,[2\,3]^2} + \frac{\langle 3\,4\rangle^2}{[1\,2]^2} + 2\frac{\langle 3\,4\rangle\,\langle 4\,1\rangle}{[1\,2]\,[2\,3]} + \frac{s_{12}s_{34}+s_{123}s_{234}-s_{12}^2}{2\,[1\,2]\,[2\,3]\,[3\,4]\,[4\,1]}\right]$$

$+ 3$ cyclic perms

$$\mathcal{A}^{(1),R}(H;1^-,2^-,3^+,4^+)$$

$$= \frac{N_p}{96\pi^2}\left[\frac{(s_{12}+s_{23})\langle 2|1+3|4]^2}{s_{123}\langle 23\rangle^2[12]^2} - \frac{s_{234}\langle 12\rangle[41]}{\langle 23\rangle\langle 34\rangle[12]^2} - \frac{\langle 2|1+3|4][34]}{\langle 23\rangle[12]^2}\right.$$
$$\left. + \frac{1}{4}\left(\frac{\langle 12\rangle}{\langle 34\rangle} - \frac{[34]}{[12]}\right)^2\right]$$

$+ 3$ perms $\qquad\qquad \rightarrow$ implemented in MCFM

Summary

By rearranging the effective Lagrangian, Higgs plus multi-parton amplitudes amenable to new on-shell methods

- Tree-level
  - ✓ Known results for up to five partons checked
  - ✓ New analytic results for any number of partons in particular helicity configurations
- Cut constructible parts of one-loop amplitudes can be rather simple
  - ✓ New analytic results for one-loop ϕ amplitude with any number of negative helicity gluons
  - ✓ New analytic results for one-loop ϕ-MHV amplitude with any number of gluons
- Rational parts of one-loop amplitudes slightly more work
  - ✓ New analytic results for one-loop ϕ amplitude with any number of positive gluons and up to two (adjacent) negative gluons

    Berger, Del Duca, Dixon; Badger, EWNG, Risager
  - ✓ New analytic results for one-loop $H \to - - --$ and $H \to - - ++$

Summary - II

- ✓ New methods already competitive with traditional methods for loop amplitudes with massless particles - gluons, quarks
- ✓ Six gluon one-loop amplitudes are new results (confirmed numerically)
- ✓ Will definitely see all six parton one-loop amplitudes in next few months
- ✗ Not necessarily the most interesting phenomenologically
- ? Will new methods be useful for amplitudes with heavy particles - top quarks, susy particles, Higgs bosons, vector bosons
- ✓ In principle heavy particles not a problem - but certainly a complication.
- ✓ yes for one Higgs plus multiparton as discussed here
- ✓ yes for one vector boson plus multiparton e.g. V + multijet
- ✓ probable for two vector boson plus multiparton e.g. VV + multijet
- ? much more difficult for $pp \to t\bar{t}b\bar{b}$

✓ ✓ Expect much more progress in 2007