Parallel Session

High-Energy Electroweak Physics

Unitarity Cuts and Reduction of Master Integrals for One-Loop Amplitudes

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Constructing Loop Amplitudes From Tree Amplitudes

LHC might see the emergence of new physics

- At least the Higgs boson will be found
- New heavy particles give complex final states in terms of leptons and jets
- Standard Model physics gives significant background
- Precise understanding of the background is beneficial both for searching for the signal and after the discovery of new physics
- New technical challenge: calculate differentical cross-section of 5,6,… leg processes in NLO accuracy in QCD pertrubation theory

At LO very useful software packages
MADGRAPH, ALPGEN, HELAC, CompHEP, …
The ultimate goal is to get similar software packages with NLO QCD and perhaps EWK corrections. DREAMS?

Wishes vs. Realities at NLO

Status of the theoretical calculations

All $2 \to 1$, $2 \to 2$, $2 \to 3$ processes are calculated or feasible to calculate

"Traditonal Feynman diagram technique"

pp → ttj ***dittmaier,uwer, weinzierl***

"Fully automized numerical Feynman diagram technique":

pp→ ZZZ ***Lazopoulos, Melnikov, Petriello***

"Semi-numerical Feynman diagram technique" for virtual corrections

Ellis, Giele, Zanderighi

Other approaches:

Grace, Nagy, Soper, Binoth et.al. …

automated generation of Feynman diagrams, numerical evaluation of tensor integrals, subtraction method, sector decomposition

Wish list of experimentalists

1. p+p→V+V+jet
2. p+p → V+V+V
3. p+p→ 4jets
4. p+p→ V+ 3jets
5. p+p → V+V+2jets
6. p+p → V+V+b+b
7. p+p → t+t+2jets
8. p+p → t+t +b+b
9. p+p → t+t+V +V

Unitarity cut methods for one-loop calculations

One loop amplitudes in terms of tree amplitudes of physical states

- Four dimensional unitarity cut method + structure of the collinear limit

  bern dixon kosower: pp $\rightarrow$ W, Z + 2 jets (1998)

- D=4-2ε dimensional unitarity cut method

  van Nerveen, Bern, Morgan, Bern, Dixon, Dunbar, Kosower

  four dimensoinal unitarity cut – dispersion integrals are convergent,
  subtraction terms are related to ultraviolet behaviour

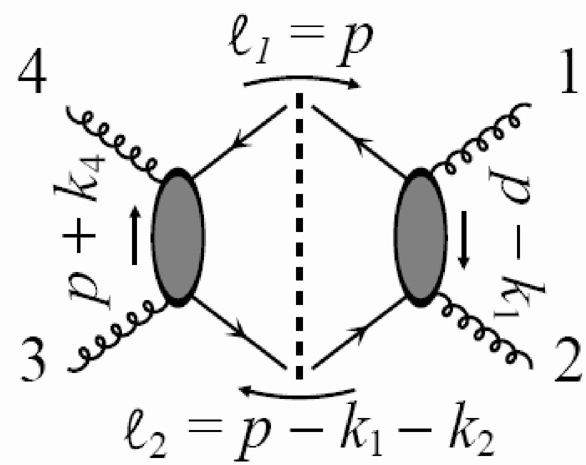
  Cut-constructible parts and rational parts

  D-dimenson integrals are convergent but one has to use D-dimensional states, "tree level input" is more complicated

- Use all information provided by perturbation theory

Basic setup

$$T^\dagger - T = -2iT^\dagger T$$



$$-i\,\mathrm{Disc}\; A_4(1,2,3,4)\Big|_{s-\mathrm{cut}} = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\delta^{(+)}(\ell_1^2 - m^2)\, 2\pi\delta^{(+)}(\ell_2^2 - m^2) \times A_4^{\mathrm{tree}}(-\ell_1, 1, 2, \ell_2)\, A_4^{\mathrm{tree}}(-\ell_2, 3, 4, \ell_1)\,,$$

Bern, Dixon, Dunbar, Kosower; Bern,Morgan;

Unitarity cut method: recent developments

i) Twistors and use of complex kinematics

Witten; Caczazo, Witten

ii) On-shell recursion relations for tree amplitudes

Britto, Feng, Caczaco; Britto, Feng, Caczazo, Witten

iii) Generalized unitarity:

more than two internal particles are on-shell

Britto, Caczazo, Feng; Brandhuber, Spencer, Travagliani

iv) Spinorial integration

Caczazo, Witten, Britto, Feng, Mastrolia, Svreck (D=4)

Anastasiou, Britto, Feng, Kunszt (D=4-2ε)

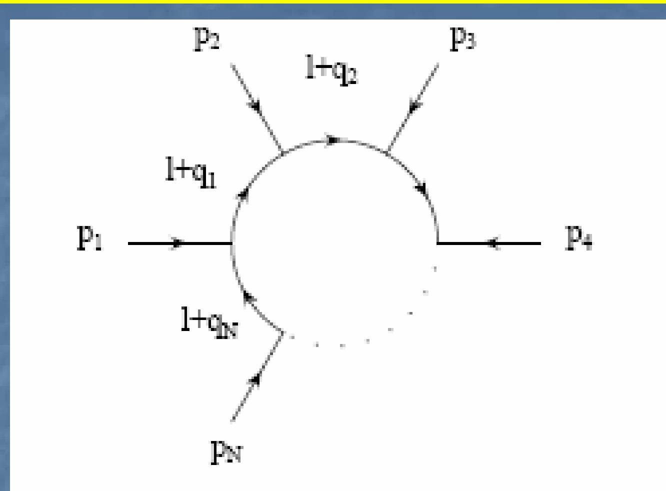
v) On shell recursion relation for loop amplitudes

Bern, Dixon, Kosower

vi) Algebraic tensor reduction

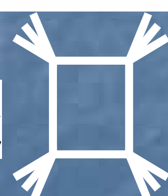
Ossala, Pittau, Papadopoulos

1. Representation of one-loop N-point amplitude in terms of 9 master integras



$$\mathcal{A}_N(p_1, p_2, \ldots, p_N) = \int d\,l \, \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N}$$

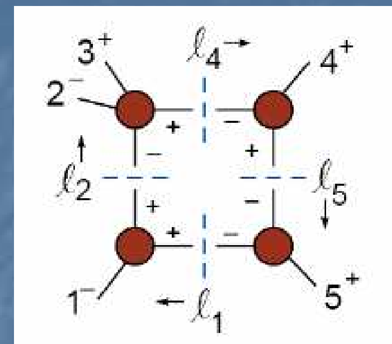
$$d_i = (l + q_i)^2 - m_i^2 = (l + \sum_{j=1}^{i} p_i)^2$$

$A_N(\{p_i\}) = \sum d_i$  $+ \sum c_i$  $+ \sum b_i$ 

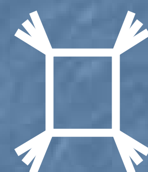
Representation of integrand in tems of four, three, two poles

2. Generalized unitarity to read out coefficients

$$T^\dagger - T = -2iT^\dagger T$$



Britto, Caczo, Feng

$A(\{p_i\}) = \sum_{ijk} d_{ijk}$  $+ \sum_{ij} c_{ij}$  $+ \sum b_i$ 

$$\frac{i}{(l+P_i)^2} \to (2\pi)\delta((l+P_i)^2)\,, \qquad d_0 = d_i = d_j = d_k = 0$$

3. Factorized expression for the cut diagrams

$$d_{ijk} = \frac{1}{2}\sum_{a=1}^{2} A_1(l_{ijk;a})A_2(l_{ijk;a})A_3(l_{ijk;a})A_4(l_{ijk;a})$$

Twistors, complex momenta

Short hand notation for Weyl-spinors, related to particle i with momentum p_i:

$$\lambda_i^\alpha = \overline{u}_-(p_i)_\alpha = \langle i-| \,, \qquad \overline{\lambda}_{i,\dot\alpha} = \overline{u}_+(p_i)_{\dot\alpha} = \langle i-|$$

$$\lambda_{i,\alpha} = u_+(p_i)_\alpha = |i+\rangle \,, \qquad \overline{\lambda}_i^{\dot\alpha} = u_-(p_i)^{\dot\alpha} = |i-\rangle$$

We can reconstruct the momenta from the spinors as

$$(\sigma^\mu p_{i,\mu})_{\alpha\dot\alpha} = u_+(p_i)\overline{u}_+(p_i)_{\dot\alpha} = \lambda_\alpha \overline{\lambda}_{\dot\alpha}$$

$$\langle jl\rangle = \lambda_j^\alpha \lambda_{l,\alpha} \,, \qquad [jl] = \overline{\lambda}_{j,\dot\alpha}\overline{\lambda}_l^{\dot\alpha}$$

For real momenta $\tilde\lambda$ is complex conjugate of λ and

$$\langle j\, l\rangle = \sqrt{s_{jl}}\, e^{i\phi_{jl}} \,, \qquad [j\, l] = \pm\sqrt{s_{jl}}\, e^{-i\phi_{jl}}$$

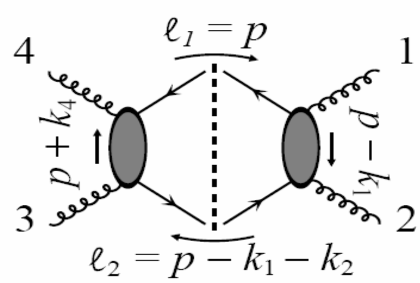
For complex momenta we can choose λ_1, λ_2, λ_3 proportional while their complex conjugates are not proportional and $p_1 + p_2 + p_3 = 0$, the three point amplitudes

$$A_3^{\mathrm{tree}}(1^-, 2^-, 3^+) = i\frac{\langle 1\, 2\rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \langle 3\, 1\rangle}$$

may not be wanishing

Witten

Reduction at the integrand level I



Tensor cut loop integral requires reduction

Ossola, Papadopoulos, Pittau

Partial fractioning of the integrand and reading out residua

Numerical Unitarity Cut Method, W. Giele's talk at Les Houches

$$
\begin{aligned}
\mathcal{A}_N(l) &= \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N} \\
&= \sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} \frac{\overline{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1 \le i_1 < i_2 < i_3 \le N} \frac{\overline{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} \\
&+ \sum_{1 \le i_1 < i_2 \le N} \frac{\overline{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1 \le i_1 \le N} \frac{\overline{a}_{i_1}(l)}{d_{i_1}}
\end{aligned}
\qquad (1)
$$

Reduction at the integrand level II

The residue is taken at special loop momentum defined by the generalized unitarity condition

The first terms on the RHS are given by factors of tree amplitudes at special cut-mom. configurations

$$\begin{aligned}
\bar{d}_{ijk}(l) &= \mathrm{Res}_{d_i=d_j=d_k=d_0=0}\left[\, A_N(l) d_i d_j d_k d_0 \,\right] \\
\bar{c}_{ij}(l) &= \mathrm{Res}_{d_i=d_j=d_0=0}\left(A_N(l) d_i d_j d_0 - \sum_{k\neq ij} \frac{\bar{d}_{ijk}(l)}{d_k(l)} \right) \\
\bar{b}_i(l) &= \mathrm{Res}_{d_i=d_0=0}\left(A_N(l) d_i d_0 - \sum_{k\neq i} \frac{\bar{c}_{ij}(l)}{d_j(l)} - \sum_{k\neq i} \frac{\bar{d}_{ijk}(l)}{d_j(l) d_k(l)} \right)
\end{aligned} \tag{1}$$

Unitarity conditions are trivially fullfilled if we use van Neerven Vermaseren basis to parametize ***l***

vNV basis for tensor reduction

Instead of $g_{\mu\nu}$, p_i use $w^{\mu\nu}$ and v_i for expandig vectors and tensors

box

$$l^\mu = V_4^\mu + \alpha\, n^\mu$$

$$V_4^\mu = -\frac{1}{2} p_1^2\, v_1^\mu - \frac{1}{2}(p_{12}^2 - p_1^2)\, v_2^\mu - \frac{1}{2}(p_{123}^2 - p_{12}^2)\, v_3^\mu$$

$$v_1^\mu = \frac{\delta^{\mu p_2 p_3}_{p_1 p_2 p_3}}{\Delta(p_1,p_2,p_3)};\quad v_2^\mu = \frac{\delta^{p_1 \mu p_3}_{p_1 p_2 p_3}}{\Delta(p_1,p_2,p_3)};\quad v_3^\mu = \frac{\delta^{p_1 p_2 \mu}_{p_1 p_2 p_3}}{\Delta(p_1,p_2,p_3)};\quad n_1^\mu = \varepsilon^{\mu p_1 p_2 p_3}$$

$$w_\nu^\mu = \frac{n^\mu n_\nu}{\Delta(p_1,p_2,p_3)}$$

$$l_\pm^\mu = V_4^\mu \pm i\sqrt{V_4^2} \times n^\mu$$

triangle

$$l^\mu = V_3^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu$$

$$V_2^\mu = -\frac{1}{2} p_1^2\, v_1^\mu(p_1,p_2) - \frac{1}{2}(p_{12}^2 - p_1^2)\, v_2^\mu - \frac{1}{2}(p_{123}^2 - p_{12}^2)\, v_3^\mu$$

Forde's (OPP) basis for triple cut

With three delta-functions containing l, only single free parameter remain for the loop integral denoted by t.

$$l^\mu = a_0^\mu t + \frac{1}{t} a_1^\mu + a_2^\mu$$

$$Disc_3(A) = \int dt J_t A_1 A_2 A_3 = \int J_t \sum_{i=0}^{m} f_i t^i + \sum_{\text{boxes}} d_l D_0^{\text{cut}}$$

$$c_{ij} = -[\text{Inf}_t A_1 A_2 A_3](t)_{t=0}$$

triangle

Direct numerical implementation ?

Reduction at the integrand level III

Spurious terms: residual l-dependence

Finite number of spurious terms:
1 (box), 6 (triangle) 8 (box)

$$\bar{d}_{ijk}(l) = d_{ijk} + \tilde{d}_{ijk}\alpha$$

2 unknowns

$$\bar{c}_{ij}(l) = c_{ij} + \tilde{c}^{(1)}_{ijk}\alpha_1 + \tilde{c}^{(2)}_{ijk}\alpha_2 + \ldots$$

7 unknowns

Factors of tree amplitudes

Suitable for numerical evaluation
for cut-constructible part

Unitarity cut in D-2ε and the rational part

Anastasiou, Britto, Feng, Mastroli, ZK

$$\mathcal{A} = \int d^{4-2\epsilon}p\,\delta\left(p^2\right)\delta\left((K-p)^2\right)\mathcal{A}_L(p)\mathcal{A}_R(p)$$

$$\mathcal{A} = \frac{\pi^{-\epsilon}}{\Gamma(-\epsilon)}\int d\mu^2(\mu^2)^{-1-\epsilon}\int d^4\tilde{l}\,\delta\left(\tilde{l}^2-\mu^2\right)\delta\left((K-\tilde{l})^2-\mu^2\right) A_L\left(\tilde{l}+\vec{\mu}\right)A_R\left(\tilde{l}+\vec{\mu}\right).$$

Massive cut lines, with D=4 and integration over the mass parameter μ
OPP-generalized unitarity cut reduction for the massive D=4 tensor integrals or reduction with spinorial integrals.

$$u = \frac{4\mu^2}{K^2}$$

$$\mathcal{A} = \int_0^1 du\,u^{-1-\epsilon}\sum_i f_i(u)\mathcal{L}_i(u)$$

i=B,C,D

$f_i(u)$ is polynomial

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Cut tensor integrals and their reduction

$$\mathrm{Bub}^{(n)} = \int_0^1 du\, u^{-1-\epsilon} u^n \sqrt{1-u}$$

$$\mathrm{Tri}^{(n)} = \int_0^1 du\, u^{-1-\epsilon} u^n \ln\left(\frac{Z+\sqrt{1-u}}{Z-\sqrt{1-u}}\right)$$

$$\mathrm{Box}^{(n)} = \int_0^1 du\, u^{-1-\epsilon} \frac{u^n}{\sqrt{B-Au}} \times$$

$$\ln\left(\frac{D-Cu-\sqrt{1-u}\sqrt{B-Au}}{D-Cu+\sqrt{1-u}\sqrt{B-Au}}\right)$$

Cut tensor integrals and their reduction

$$\mathrm{Bub}^{(n)} = F^{(n)}_{2\to 2}\mathrm{Bub}^{(0)}$$

$$\mathrm{Tri}^{(n)}(Z) = F^{(n)}_{3\to 3}(Z)\mathrm{Tri}^{(0)}(Z) + F^{(n)}_{3\to 2}(Z)\mathrm{Bub}^{(0)}$$

$$F^{(n)}_{2\to 2} = \frac{(-\epsilon)_{\frac{3}{2}}}{(n-\epsilon)_{\frac{3}{2}}}, \qquad F^{(n)}_{3\to 3} = \frac{-\epsilon}{n-\epsilon}(1-Z^2)^n$$

Unitarity cut in D-2ε and the rational part

Reduction of f_i (u) to constant generates \epsilon dependence of the coefficient of the integral function

Rational part is given by the order \epsilon term of the box coefficient function

Massive cut lines, D-dimensional intermediate states

Numerical implementation ?

Numerical Implementation of Unitarity Techniques

K.Ellis W. Giele, Z.K.

- Numerical program for 4,5,6 gluon one-loop amplitudes
- Comparisons to analytic 4,5,6 one-loop gluon amplitudes
- Time for 6 gluon: 107 secs/10,000 events

Concluding Remarks

- Remarkable progress since 2004
- Cut constructible part: fast numerical im implementations (OPP,EGK,..)

  numerical stability?
- Rational part: numerical implementation of recursive technique? (BBDFK…)

  i) A bit more than just tree amplitudes

  ii) More tranditional methods (Zhu, Binoth,Heinrich)