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# Smallness of the cosmological constant and the multiple point principle

Roman Nevzorov

Glasgow University

*in collaboration with C.D.Froggatt and H.B.Nielsen*

# Outline

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- Introduction
- No-scale supergravity
- MPP inspired SUGRA model
- Cosmological constant in the MPP inspired SUGRA models
- Conclusions

Based on:

C. D. Froggatt, R. Nevzorov and H. B. Nielsen, Nucl. Phys. B 743 (2006) 133;

C. D. Froggatt, L. V. Laperashvili, R. Nevzorov and H. B. Nielsen, Phys. Atom. Nucl. 67 (2004) 582 [arXiv:hep-ph/0310127].

# Introduction

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- Astrophysical and cosmological observations indicate that there is a tiny energy density spread all over the Universe

$$\rho_{\Lambda} \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4 \sim (10^{-3} \text{ eV})^4 .$$

- At the same time the presence of a gluon condensate in the vacuum is expected to contribute an energy density

$$\rho_{QCD} \sim \Lambda_{QCD}^4 \simeq 10^{-74} M_{Pl}^4 .$$

- In the SM a much larger contribution must come from the EW symmetry breaking

$$\rho_{EW} \sim v^4 \simeq 10^{-62} M_{Pl}^4 .$$

- But the contribution of zero-modes is expected to push  $\rho_{\Lambda}$  up to  $M_{Pl}^4$ .

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- Because of the enormous cancellation between the contributions of different condensates to  $\rho_\Lambda$  the smallness of the cosmological constant should be regarded as a **fine-tuning problem**.
  - An exact global supersymmetry ensures zero value for the vacuum energy density.
  - But supersymmetry must be broken.
  - The breakdown of SUSY induces a huge and positive contribution to  $\rho_\Lambda$

$$\rho_\Lambda \sim \Lambda_{SUSY}^4,$$

where  $\Lambda_{SUSY}$  is a SUSY breaking scale.

- The non-observation of squarks and sleptons implies that  $\Lambda_{SUSY} \gg 100 \text{ GeV}$ .

# No-scale supergravity

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- The scalar potential in ( $N = 1$ ) SUGRA models is specified in terms of the Kähler function

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln |W(\phi_M)|^2 .$$

- The SUGRA scalar potential is given by

$$V(\phi_M, \phi_M^*) = \sum_{M, \bar{N}} e^G \left( G_M G^{M\bar{N}} G_{\bar{N}} - 3 \right) + \frac{1}{2} \sum_a (D^a)^2 ,$$

$$G_M \equiv \partial G / \partial \phi_M , \quad G_{\bar{M}} \equiv \partial G / \partial \phi_M^* , \quad G^{M\bar{N}} = G_{\bar{N}M}^{-1} ,$$

$$D^a = g_a \sum_{i,j} (G_i T_{ij}^a \phi_j) .$$

- SUGRA models include singlet fields which form hidden sector that gives rise to the breaking of local SUSY and induces non-zero gravitino mass

$$m_{3/2} = \langle e^{G/2} \rangle$$

- In SUGRA models  $\rho_\Lambda \sim \langle e^{G/2} \rangle \sim -m_{3/2}^2 M_{Pl}^2$ .

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- The Lagrangian of the simplest no–scale SUGRA model is invariant under imaginary translations

$$T \rightarrow T + i\beta, \quad \varphi_\sigma \rightarrow \varphi_\sigma$$

and dilatations

$$T \rightarrow \alpha^2 T, \quad \varphi_\sigma \rightarrow \alpha \varphi_\sigma.$$

- The invariance under imaginary translations and dilatations constrain Kähler function

$$K = -3 \ln \left[ T + \bar{T} - \sum_\sigma \zeta_\sigma |\varphi_\sigma|^2 \right], \quad W = \sum_{\sigma, \lambda, \gamma} \frac{1}{6} Y_{\sigma\lambda\gamma} \varphi_\sigma \varphi_\lambda \varphi_\gamma.$$

- Global symmetries ensure the vanishing of vacuum energy density in the no–scale SUGRA models.
- These symmetries also preserve supersymmetry in all vacua.

# MPP inspired SUGRA model

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- In order to achieve the appropriate breakdown of local supersymmetry dilatation invariance must be broken.
- Let us consider SUGRA model with two hidden sector fields that transform differently under the dilatations

$$T \rightarrow \alpha^2 T, \quad z \rightarrow \alpha z$$

and imaginary translations

$$T \rightarrow T + i\beta, \quad z \rightarrow z.$$

- We allow the breakdown of dilatation invariance in the superpotential of the hidden sector

$$W(z, \varphi_\alpha) = \kappa \left( z^3 + \mu_0 z^2 + \sum_{n=4}^{\infty} c_n z^n \right) + \sum_{\sigma, \lambda, \gamma} \frac{1}{6} Y_{\sigma\lambda\gamma} \varphi_\sigma \varphi_\lambda \varphi_\gamma,$$

where  $\mu_0$  and  $c_n \sim 1$ .

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- We also assume that the dilatation invariance is broken in the Kähler potential of the observable sector

$$K = -3 \ln \left[ T + \bar{T} - |z|^2 - \sum_{\sigma} \zeta_{\sigma} |\varphi_{\sigma}|^2 \right] + \sum_{\sigma, \lambda} \left( \frac{\eta_{\sigma\lambda}}{2} \varphi_{\sigma} \varphi_{\lambda} + h.c. \right) + \sum_{\sigma} \xi_{\sigma} |\varphi_{\sigma}|^2 .$$

- Such breakdown of global symmetry preserves a zero value of the energy density in all vacua.
- The scalar potential of the hidden sector takes a form

$$V(T, z) = \frac{1}{3(T + \bar{T} - |z|^2)^2} \left| \frac{\partial W(z)}{\partial z} \right|^2 .$$

- When  $c_n = 0$  this SUGRA scalar potential has two minima with zero vacuum energy density

$$z = 0, \quad z = -\frac{2\mu_0}{3} .$$



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- In the vacuum where  $z = -2\mu_0/3$  local supersymmetry is broken so that gravitino and all scalar particles get non-zero masses:

$$m_{3/2} = \frac{4\kappa\mu_0^3}{27\left\langle\left(T + \bar{T} - \frac{4\mu_0^2}{9}\right)^{3/2}\right\rangle}, \quad m_\sigma \sim \frac{m_{3/2}\xi_\sigma}{\zeta_\sigma}.$$

- In the vacuum with  $z = 0$  local SUSY remains intact and the low-energy limit of this theory is described by a pure SUSY model in flat Minkowski space.
  - The vanishing of  $\rho_\Lambda$  can be considered as a result of degeneracy of all possible vacua in the considered theory, one of which is supersymmetric with  $\langle W \rangle = 0$ .
  - The presence of degenerate vacua with broken and unbroken local supersymmetry leads to the natural realisation of the multiple point principle (MPP).
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- MPP postulates the existence of the maximal number of phases with the same energy density which are allowed by a given theory.
  - Being applied to supergravity MPP implies the existence of a phase with global SUSY in flat Minkowski space.
  - Such vacuum is realised only if SUGRA scalar potential has a minimum where the following conditions are satisfied

$$\left\langle W(z_i^0) \right\rangle = \left\langle \frac{\partial W(z_i)}{\partial z_j} \right\rangle_{z_i=z_i^0} = 0,$$

that requires an extra fine-tuning in general.

- In the considered no-scale SUGRA models the MPP conditions are fulfilled without any extra fine-tuning.

# Cosmological constant

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- According to MPP the physical and supersymmetric vacua have the same energy density.
- Since the vacuum energy density of supersymmetric states in flat Minkowski space is zero  $\rho_\Lambda$  in the physical vacuum vanishes in the leading approximation.
- However non-perturbative effects in the observable sector may lead to the breakdown of SUSY in the supersymmetric phase.
  - The supersymmetry breakdown can be caused by the strong interactions that give rise to a non-zero positive value for the cosmological constant
$$\rho_\Lambda \simeq \Lambda_{SQCD}^4.$$
  - In particular, large top quark Yukawa coupling may induce  $t$ -quark condensate that breaks SUSY.

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- We assume that at high energy scale the gauge and Yukawa couplings are the same in both vacua.
  - In the supersymmetric vacuum the QCD interaction becomes strong at

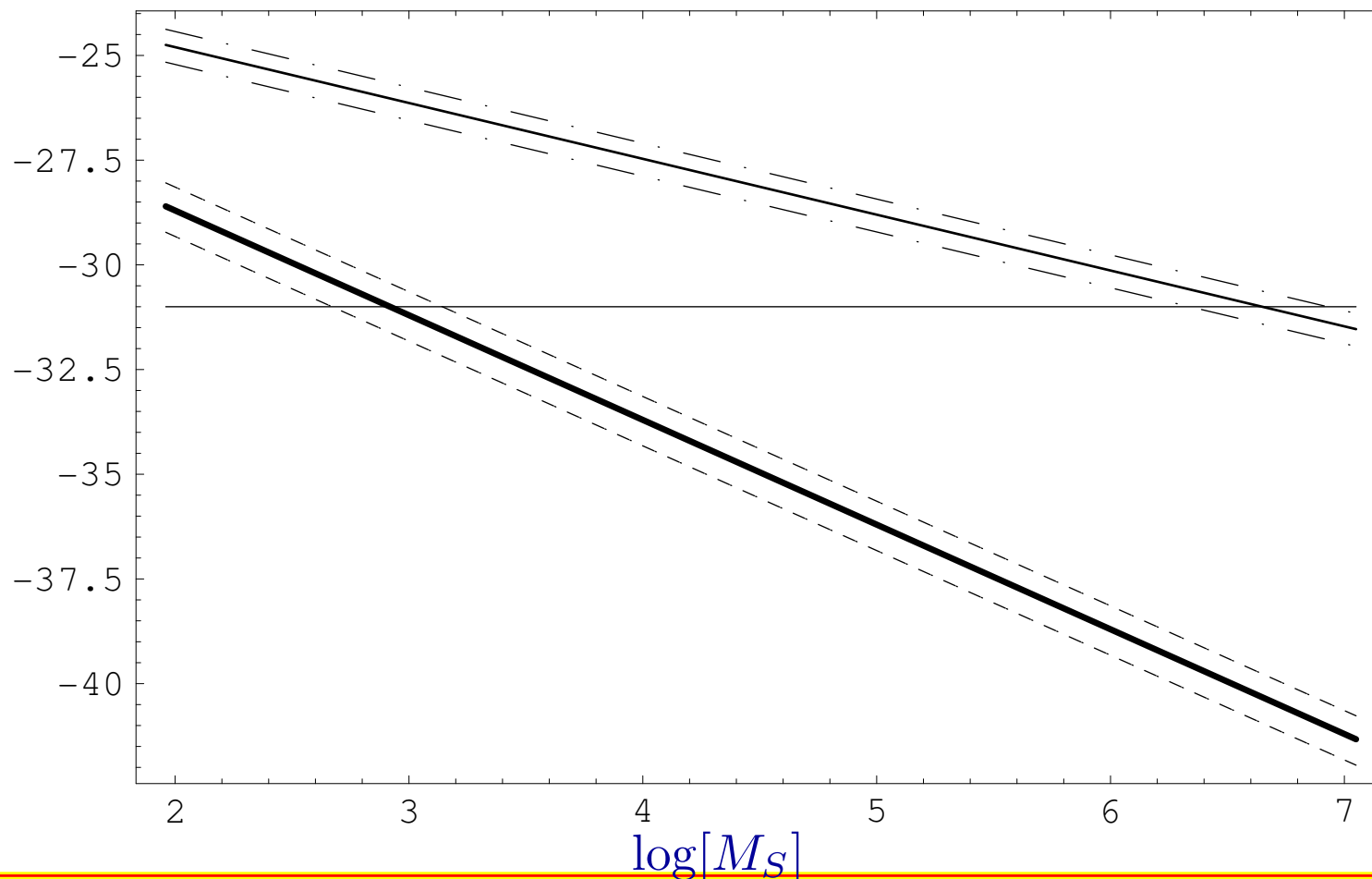
$$\Lambda_{SQCD} = M_S \exp \left[ \frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)} \right], \quad \frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2},$$

where  $M_S$  is a SUSY breaking scale and  $\tilde{b}_3 = -7$ .

- In the MSSM ( $b_3 = -3$ ) the measured value of  $\rho_\Lambda$  is reproduced for  $M_S = 10^3 - 10^4$  TeV.
- If the MSSM particle content is supplemented by a pair of  $5 + \bar{5}$  multiplets ( $b_3 = -2$ ) then the observed value of  $\rho_\Lambda$  can be obtained even for  $M_S \simeq 1$  TeV.

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- In the physical vacuum extra particles gain masses  $\sim M_S$  due to the presence of  $\eta(5 \cdot \bar{5})$  term in  $K(\phi_M, \phi_M^*)$ .

$\log[\Lambda_{SQCD}/M_{Pl}]$



# Conclusions

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- In no-scale supergravity global symmetries protect local supersymmetry and a zero value for the cosmological constant.
- The breakdown of these symmetries that ensures the vanishing of  $\rho_\Lambda$  near the physical vacuum leads to the natural realization of the multiple point principle (MPP).
  - MPP requires the degeneracy of all global vacua.
  - MPP also predicts the existence of a supersymmetric phase in flat Minkowski space that results in the vanishing of  $\rho_\Lambda$  to first approximation.
- Non-perturbative effects can give rise to the breakdown of SUSY in the supersymmetric vacuum inducing tiny and positive value of  $\rho_\Lambda$ , i.e.

$$\rho_\Lambda \ll 10^{-100} M_{Pl}^4.$$