Stability in open strings with broken supersymmetry

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July 1, 2019

Based on work in collaboration with

S. Abel, E. Dudas and D. Lewis [arXiv:1812.09714].

25th International Symposium on Particles, Strings and Cosmology,
PASCOS 2019, Manchester

Introduction

■ Supersymmetry guaranties stability of any Minkowski background under quantum corrections.

■ For Phenomonology and Cosmology, susy must be broken :

- **If "explicit" breaking,** the effective potential is

$$\mathcal{V}_{\text{quantum}} \sim M_{\text{s}}^d$$

- **If susy spontaneously broken at tree level, in flat space**

e.g. by a **stringy Scherk-Schwarz mechanism,** [Kounnas, Porrati,'88] [Antoniadis, Dudas, Sagnotti, '98]

$$M = \frac{M_{\text{s}}}{2R} \qquad \Longrightarrow \qquad \mathcal{V}_{\text{quantum}} \sim M^d$$

$\Longrightarrow$ **1) We want to find Non-Generic Models that lower this order of magnitude and 2) study the moduli stability.**

■ **At weak coupling, in open strings compactified on a torus.**

■ **Assume the lightest mass scale in the background is M**
$\Longrightarrow$ the 1-loop effective potential $\mathcal{V}$ is dominated by the light Kaluza-Klein states,

$$\mathcal{V} = \big(n_{\rm F} - n_{\rm B}\big)\, \xi\, M^d + \mathcal{O}\Big(\big(M_0 M\big)^{\frac{d}{2}} e^{-M_0/M}\Big)\,, \qquad \xi > 0$$

- **$n_{\rm F}$, $n_{\rm B}$ are the numbers of massless fermionic and bosonic degrees of freedom.**

- M_0 is the string scale, or an Higgs-like scale.

■ Let us switch on small mass scales i.e. moduli : Because we compactify on a torus ($\mathcal{N} = 4$ in 4D), **they are Wilson lines (WL)**

$$\mathcal{V} = \mathcal{V}\big|_{a=0} + M^d \sum_{\substack{\text{massless} \\ \text{spectrum}}} \sum_{r,I} Q_r a_r^I + \cdots$$

- a_r^I is the WL along the internal circle I of the r-th Cartan $U(1)$.
- Q_r is the charge of the massless spectrum running in the loop.
- combining states Q_r and $-Q_r \implies 0$: **No Tadpole.**

■ At quadratic order [Kounnas, H.P, '16] [Coudarchet, H.P., '18]

$$\mathcal{V} = \big(n_{\rm F} - n_{\rm B}\big)\, \xi\, M^d + M^d \Bigg(\sum_{\substack{\text{massless}\\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless}\\ \text{fermions}}} Q_r^2 \Bigg) \big(a_r^I\big)^2 + \cdots$$

$\Longrightarrow$ **The higher $\mathcal{V}$ is, the more unstable it is.**

■ We show that **tachyon free models with $\mathcal{V} \geq 0$ do exist at the quantum level, when $d \leq 5$.**

In 9 dimensions

■ Type I compactified on $S^1(R_9)$ with **Sherk-Schwarz** susy breaking

$$\mathcal{W} = \mathrm{diag}\big(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \ldots, e^{2i\pi a_{16}}, e^{-2i\pi a_{16}}\big)$$

$$\text{momentum} \quad \frac{m_9}{R_9} \longrightarrow \frac{m_9 + \frac{F}{2} + a_r - a_s}{R_9}$$

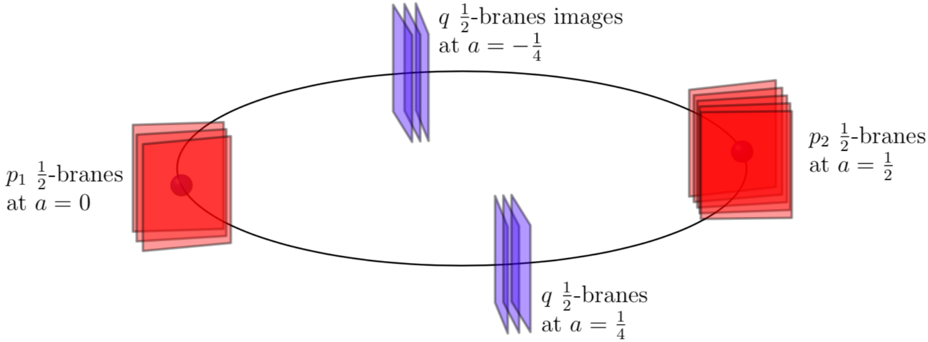
■ **T-duality $R_9 \to \tilde{R}_9 = \frac{1}{R_9}$ yields a geometric picture in Type I', where WLs become positions along $S^1(\tilde{R}_9)$:**

• There are 2 O8-orientifold planes at $\tilde{X}^9 = 0$ and $\tilde{X}^9 = \pi \tilde{R}_9$.

• The D9-branes become 32 D8 "half"-branes :
16 at $\tilde{X}^9 = 2\pi a_r \tilde{R}_9$ and 16 mirror $\frac{1}{2}$-branes at $\tilde{X}^9 = -2\pi a_r \tilde{R}_9$.

• $\frac{1}{2}$-branes and their mirrors can be coincident on an O8-plane,
$a_r = 0$ or $\frac{1}{2} \quad \Longrightarrow \quad$ ***SO*(*p*), *p* even**

• Elsewhere, a stack of q $\frac{1}{2}$-branes and the mirror stack $\Longrightarrow$ ***U*(*q*)**


q $\frac{1}{2}$-branes images
at $a = -\frac{1}{4}$
p_1 $\frac{1}{2}$-branes
at $a = 0$
p_2 $\frac{1}{2}$-branes
at $a = \frac{1}{2}$
q $\frac{1}{2}$-branes
at $a = \frac{1}{4}$

■ We look for **stable brane configurations.**

• **A sufficient condition for $\mathcal{V}$ to be extremal** with respect to the a_r is that **no mass scale exist between 0 and M.**

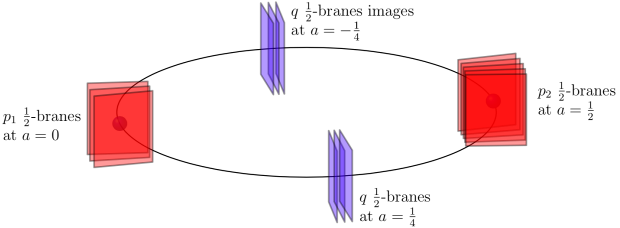
This corresponds to $\boldsymbol{a = 0}$ **or** $\boldsymbol{\frac{1}{2}}$ **only.**

• Moreover, $m_9 + \frac{1}{2} + \frac{1}{2} - 0$ can vanish :
Super-Higgs and Higgs compensate $\Longrightarrow$ massless fermions.

This is necessary to have $n_\mathrm{F} - n_\mathrm{B} \geq 0$.

■ However, $\boldsymbol{a = \pm\frac{1}{4}}$ is also special :

• $m_9 + \frac{1}{2} + \frac{1}{4} - (-\frac{1}{4})$ can vanish $\Longrightarrow$ massless fermions.



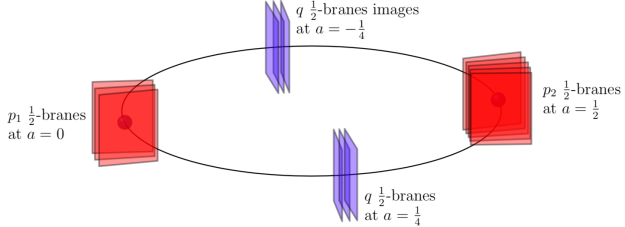
- $SO(p_1) \times SO(p_2) \times U(q) \qquad \times U(1)^2$ for $G_{\mu 9}$, RR-2-form $C_{\mu 9}$

$$n_{\rm B} = 8\left(8 + \frac{p_1(p_1-1)}{2} + \frac{p_2(p_2-1)}{2} + q^2\right)$$

$$n_{\rm F} = 8\left(p_1 p_2 + \frac{q(q-1)}{2} + \frac{q(q-1)}{2}\right)$$

- Bifundamental $\boldsymbol{(p_1, p_2)}$ and antisymmetric $\oplus$ $\overline{\text{antisymmetric}}$

- **$n_{\rm F} - n_{\rm B}$ is minimal for $p_1 = 32$, $p_2 = 0$, $q = 0$,** which **suggests that the $SO(32)$ brane configuration yields an absolute minimum, stable.**



■ We have described the moduli space where p_1, p_2 are even.

• **The moduli space admits a second, disconnected part, where p_1, p_2 are odd $\Longrightarrow$ One $\frac{1}{2}$-brane is frozen at $a = 0$, and one frozen at $a = \frac{1}{2}$** [Schwarz,'99]

$$\mathcal{W} = \mathrm{diag}\left(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \ldots, e^{2i\pi a_{15}}, e^{-2i\pi a_{15}}, 1, -1\right)$$

• **$n_{\mathrm{F}} - n_{\mathrm{B}}$ is minimal for $p_1 = 31$, $p_2 = 1$, $q = 0$,** which **suggest that the $SO(31) \times SO(1)$ brane configuration is an absolute minimum, stable.** ($SO(1)$ is to remind the frozen brane ie $(p_1, 1)$ bifundamental fermion)

■ **To demonstrate these expectations, we compute the 1-loop potential**

$$\mathcal{V} = \frac{\Gamma(5)}{\pi^{14}}\, M^9 \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\mathcal{W})}{(2n_9+1)^{10}} + \mathcal{O}\Big((M_s M)^{\frac{9}{2}} e^{-\pi \frac{M_s}{M}}\Big)$$

It involves the torus + Klein bottle + annulus + Möbius amplitudes :

$$\begin{aligned}\mathcal{N}_{2n_9+1}(\mathcal{W}) &= 4\Big(-16 - 0 - (\mathrm{tr}\, \mathcal{W}^{2n_9+1})^2 + \mathrm{tr}\,(\mathcal{W}^{2(2n_9+1)})\Big)\\ &= -16\left(\sum_{\substack{r,s=1\\ r\neq s}}^{N} \cos\big(2\pi(2n_9+1)a_r\big)\cos\big(2\pi(2n_9+1)a_s\big) + N - 4\right)\end{aligned}$$

(where $N = 16$ or 15)

■ **For $a_r = 0, \frac{1}{2}, \pm\frac{1}{4}$**

$$\mathcal{N}_{2n_9+1}(\mathcal{W}) = n_{\mathrm{F}} - n_{\mathrm{B}} \quad \Longrightarrow \quad \mathcal{V} = \big(n_{\mathrm{F}} - n_{\mathrm{B}}\big)\, \xi\, M^d + \mathcal{O}\Big((M_s M)^{\frac{9}{2}} e^{-\pi \frac{M_s}{M}}\Big)$$

- **The $\boldsymbol{U(q)}$ groups always have unstable WLs**

$\Longrightarrow$ **All $\frac{1}{2}$-branes must sit on the O8-planes.**

- For $p_1 \geq 2$, the WLs of $\boldsymbol{SO(p_1)}$ have $(\text{masse})^2 \propto p_1 - 2 - p_2$.

  For $p_2 \geq 2$, those of $\boldsymbol{SO(p_2)}$ have $(\text{masse})^2 \propto p_2 - 2 - p_1$.

  Both cannot be ≥ 0, $\Longrightarrow$ **$\boldsymbol{p_2}$ must be 0 or 1.**

■ **Conclusion in 9 dimensions :**

$\boldsymbol{SO(32)}$ and $\boldsymbol{SO(31)\times SO(1)}$ are stable brane configurations

with **$\boldsymbol{M}$ running away**

NB : $0 - n_{\rm B} = -4032$ and $n_{\rm F} - n_{\rm B} = -3536$, which is higher because

- the dimension of $SO(31)$ is lower
- the frozen $\frac{1}{2}$-brane at $a = \frac{1}{2}$ induces a fermionic bifundam $(p_1, 1)$.

NB : In lower dim, we have more O-planes on which we can freeze more $\frac{1}{2}$-branes $\Longrightarrow n_{\rm F} - n_{\rm B} \geq 0$.

In d dimensions

■ **Type I on $\boldsymbol{T^{10-d}}$** with metric G_{IJ} and **Scherk-Schwarz along $\boldsymbol{X^9}$**

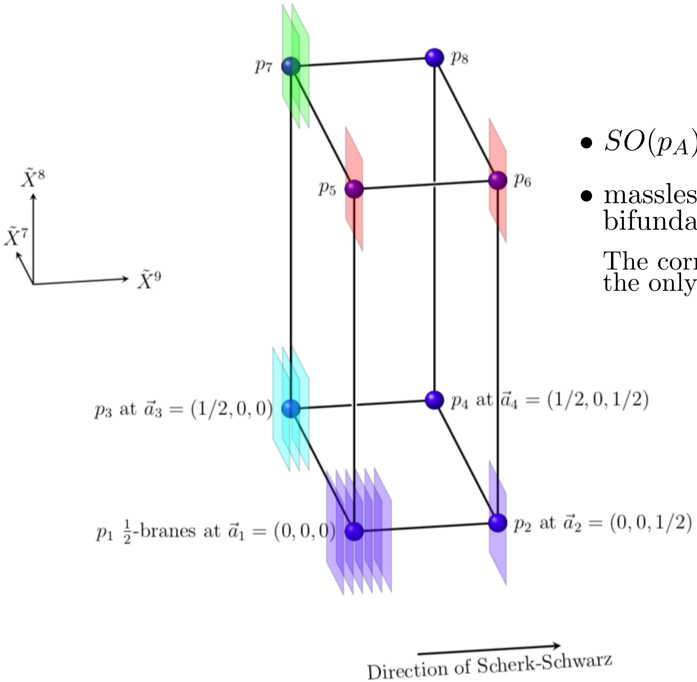
$$M = \frac{\sqrt{G^{99}}}{2}\, M_s$$

■ **Type I'** picture obtained by **T-dualizing $\boldsymbol{T^{10-d}}$:**

- **$\boldsymbol{2^{10-d}}$ O$(\boldsymbol{d-1})$-planes located at the corners of a $(\boldsymbol{10-d})$-dimensional box.**

- **32 "half" $(\boldsymbol{d-1})$-branes.**

■ **$\boldsymbol{\mathcal{V}}$ is extremal when the 32 $\boldsymbol{\frac{1}{2}}$-branes are located on the O-planes.**



- $SO(p_A)$ at corner A
- massless fermionic bifundamental (p_{2A-1}, p_{2A})

  The corners $2A-1$, $2A$ are the only ones close

■ **The WLs masses can be found from the potential, or**

$$\text{mass}^2 \propto \Big(\sum_{\substack{\text{massless}\\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless}\\ \text{fermions}}} Q_r^2 \Big) \propto p_{2A-1} - 2 - p_{2A} \qquad \text{as in 9D}$$

■ Stability implies

$$\boldsymbol{SO(p_{2A-1})} \textbf{ with 0 or 1 frozen } \boldsymbol{\tfrac{1}{2}}\textbf{-brane at corner } \boldsymbol{2A}$$

■ $\boldsymbol{n_{\mathrm{F}} - n_{\mathrm{B}}}$ **can be positive or negative.**

- 23 models have $n_{\mathrm{F}} - n_{\mathrm{B}} = 0$, e.g. in $d \leq 5$:

$$SO(4)\times\big[SO(1)\times SO(1)\big]^{14} \;\text{ or }\; \big[SO(5)\times SO(1)\big]\times\big[SO(1)\times SO(1)\big]^{13}, \;\ldots$$

- All can be realized with WL matrices in $SO(32)$ (rather than $O(32)$ $\Longrightarrow$ they are **consistent non-perturbatively and should admit heterotic duals.**

■ **The potential depends on G_{IJ} and**

$$a^I_\alpha = \langle a^I_\alpha \rangle + \varepsilon^I_\alpha, \qquad \langle a^I_\alpha \rangle \in \left\{0, \frac{1}{2}\right\}, \qquad \alpha = 1, \dots, 32, \quad I = d, \dots, 9$$

- **The Ramond-Ramond moduli C_{IJ}** have no mass term :

Because they are also WLs, but there are no perturbative states charged under the associated $U(1)$'s, $C_{\mu I}$.

- We take $G^{99} \ll |G_{ij}| \ll G_{99}$, $i, j = d, \dots, 8$, to not have mass scales $< M$

$$\mathcal{V} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\varepsilon, G)}{|2n_9+1|^{d+1}} + \mathcal{O}\left((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\right)$$

$$\mathcal{N}_{2n_9+1}(\varepsilon,G)=4\left\{-16-\sum_{(\alpha,\beta)\in L}(-1)^F \cos\left[2\pi(2n_9+1)\left(\varepsilon^9_\alpha-\varepsilon^9_\beta+\frac{G^{9i}}{G^{99}}(\varepsilon^i_\alpha-\varepsilon^i_\beta)\right)\right] \times \mathcal{H}_{\frac{d+1}{2}}\left(\pi|2n_9+1|\frac{(\varepsilon^i_\alpha-\varepsilon^i_\beta)\hat{G}^{ij}(\varepsilon^j_\alpha-\varepsilon^j_\beta)}{\sqrt{G^{99}}}\right) + \sum_\alpha \cos\left[4\pi(2n_9+1)\left(\varepsilon^9_\alpha+\frac{G^{9i}}{G^{99}}\varepsilon^i_\alpha\right)\right]\mathcal{H}_{\frac{d+1}{2}}\left(4\pi|2n_9+1|\frac{\varepsilon^i_\alpha\,\hat{G}^{ij}\,\varepsilon^j_\alpha}{\sqrt{G^{99}}}\right)\right\}$$

where $\hat{G}^{ij}=G^{ij}-\frac{G^{i9}}{G^{99}}\,G^{99}\,\frac{G^{9j}}{G^{99}}$ and $\mathcal{H}_\nu(z)=\frac{2}{\Gamma(\nu)}\,z^\nu K_\nu(2z)$

$$\mathcal{V} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\varepsilon, G)}{|2n_9+1|^{d+1}} + \mathcal{O}\Big((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\Big)$$

■ Setting the massive open string WLs at $\boldsymbol{\varepsilon^I_\alpha = 0}$**,**

$$\Longrightarrow \quad \mathcal{N}_{2n_9+1}(0, G) = n_{\rm F} - n_{\rm B}$$

$$\Longrightarrow \quad \mathcal{V} = \big(n_{\rm F} - n_{\rm B}\big)\, \xi\, M^d + \mathcal{O}\Big((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\Big)$$

$\Longrightarrow$ **all components of $\boldsymbol{G_{IJ}}$ are flat directions !**

(Except $M = M_{\rm s}\sqrt{G^{99}}/2$ unless $n_{\rm F} - n_{\rm B} = 0$)

■ **$\boldsymbol{G_{IJ}}$ and the RR-moduli $\boldsymbol{C_{IJ}}$ should be stabilized in the heterotic dual**

$$(G+C)_{IJ}\big|_{\text{Type I}} = (G+B)_{IJ}\big|_{\text{heterotic}}$$

at enhanced gauge symmetry points, where there are additional massless states with non-trivial Q_r.
These states have winding numbers $\Rightarrow$ they are **D-strings in Type I.**

Conclusion

■ **In open string theory compactified on a torus,** we have found **at the quantum level but weak coupling,** backgrounds

- where **the open string moduli are stabilized.**

- **If $n_F \neq n_B$, all closed string moduli except M are flat directions at 1-loop.**
However they are expected to be stabilized at 1-loop in an heterotic framework.

- **If $n_F = n_B$, we have consistent Minkowski vacua at 1-loop** (up to exponentially suppressed terms). Even if non-trivial, it is modest, since higher loop constraints have to be enforced for maintaining flatness [Abel, Stewart, '17], up to a residual higher order cosmological constant in g_s.

- One has to see if the dilaton and M may be stabilized in perturbation theory. (Contributions of different loops may be of same order of magnitude when $n_F = n_B$.)