Is Higgsplosion possible and would it be observed?

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Outline

1. Introduction: Factorial growth of amplitudes
2. Semiclassical calculation of cross-sections
3. Conjectures and interpretations
4. Conclusions

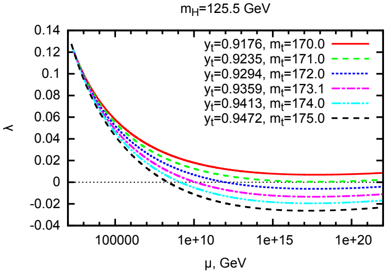
Higgs boson is discovered

But how well do we know it?

Higgs boson mass

$$m_H = 125.10 \pm 0.14\ \mathrm{GeV}$$

- Perturbative up to scale significantly above Planck scale



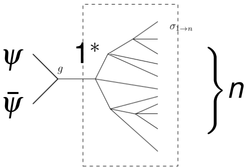
- Does it mean that there are no new scales required by SM (barring vacuum metastability)?

Factorial behaviour of large multiplicity amplitudes

$$S = \int d^4x \left(\frac{(\partial\phi)^2}{2} - \frac{\lambda}{4}\phi^4 + \frac{\mu^2}{2}\phi^2 \right)$$

Large amplitude for $n \gg \lambda^{-1}$?

$$A_n^{\text{tree}}(\text{threshold}) = n! \left(\frac{\lambda}{2m^2} \right)^{\frac{n-1}{2}}$$



- Initial state is relatively irrelevant $A(\bar{\psi}\psi \to n)$, or $A(1^* \to n)$
- Particles are massive – important (on the threshold at least)
- Spontanneous breaking – will be important

Cornwall'90, Goldberg'90, Voloshin'92, Brown'92, Argyres Kleiss Papadopoulos'93

Cross-section and the large n limit

$$\sigma(E,n) = \sum_f |\langle 0|\hat{\phi}\hat{P}_E\hat{P}_m|f\rangle|^2$$

- Ordinary perturbation theory limit is
$$\lambda \to 0, \quad n = \text{fixed constant}$$
- Semiclassical limit: large n with fixed energy per particle
$$\lambda \to 0, \quad n \to \infty$$

$$\lambda n = \text{fixed constant}$$
$$\varepsilon \equiv \frac{E - nm}{nm} = \text{fixed constant}$$

Exponentiation

$$\sigma(\varepsilon, n) \propto \exp\left(\frac{1}{\lambda} F(\lambda n, \varepsilon)\right)$$

Libanov Rubakov Son Troitskii'94,95

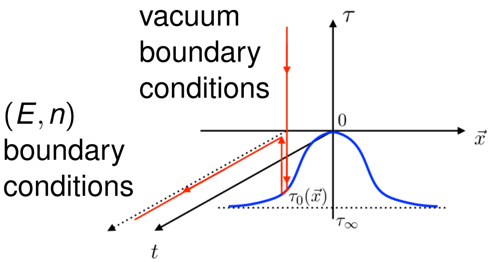
Semiclassical approach

$$\sigma(E,n) = \sum_f |\langle 0|\hat{\phi}\hat{P}_E\hat{P}_m|f\rangle|^2$$

Calculated as a saddle point

$$\sigma(\varepsilon,n) \simeq \exp\left(-2\mathrm{Im}S[\phi] + \text{boundary terms}\right)$$

with ϕ being solution in complex time singular at $\tau_0(x)$



- Conjecture: exponent does not depend on initial state

Son'95

Known results for $\sigma(\varepsilon, n) \propto \exp\left(\frac{1}{\lambda} F(\lambda n, \varepsilon)\right)$

Both explicit pertubative and semiclassic

$$F(\lambda n, \varepsilon) =$$

$$\lambda n \left(\ln \frac{\lambda n}{4} - 1 \right)$$ tree level threshold amplitude

$$+\lambda n \left(\frac{3}{2} \left(\ln \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right)$$ tree level form-factor

$$+2B\lambda^2 n^2$$ 1-loop threshold amplitude

$$+O(\lambda^3 n^3) + O(\lambda^2 n^2 \cdot \varepsilon) + O(\lambda n \cdot \varepsilon^2)$$

- Semiclassic calculation cross-checked by explicit diagram calculation
- Valid only for $\lambda n \ll 1$, $\varepsilon \ll 1$
- $F < 0$ in its region of validity – no exponential growth

Known results for $\sigma(\varepsilon, n) \propto \exp\left(\frac{1}{\lambda} F(\lambda n, \varepsilon)\right)$

Semiclassic

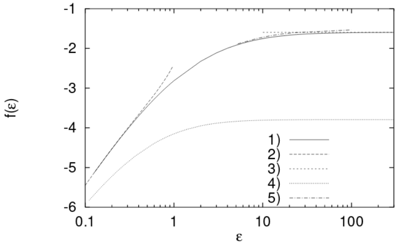
$$F(\lambda n, \varepsilon) =$$

$$\lambda n \left(\ln \frac{\lambda n}{4} - 1 \right)$$ tree level threshold amplitude

$$+\lambda n f(\varepsilon)$$ tree level form-factor

$$+\cdots$$

- Tree level at arbitrary energy
- Valid only for $\lambda n \ll 1$, any ε
- $F < 0$ in its region of validity



Khlebnikov'92, Bezrukov'98

Large $\lambda n \gg 1$ – Higgsplosion!

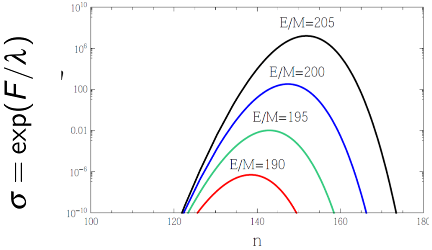
New! Semiclassic

New thin wall singular bubbles semiclassic solution

$$F(\lambda n, \varepsilon) = \lambda n \left(\log \frac{\lambda n}{4} + \boxed{0.85\sqrt{\lambda n}} + \frac{1}{2} + \frac{3}{2} \log \frac{\varepsilon}{3\pi} - \frac{25}{12}\varepsilon \right)$$

- Only in spontaneously broken theory
- Only in $d = 4$
- Calculated only for $\varepsilon = 0$
    - Conjecture – result can be extended to non-zero energies

Conjecture:

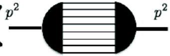


Khoze Spannowsky'17 (c.f. Voloshin Gorsky'93)

Higgsplosion and Higgspersion

(slide from Valya's and Michael's talks)

The optical theorem now relates the 1* -> nh amplitudes with the imaginary part of the self-energy (valid to all orders)

$$-\,\mathrm{Im}\,\Sigma_R(p^2) \;=\; m\,\Gamma(p^2)$$ ↔ - Im()

where $\Gamma(s) = \sum_{n=2}^{\infty} \Gamma_n(s)$ and $\Gamma_n(s) = \frac{1}{2m}\int \frac{d\Phi_n}{n!}\,|\mathcal{M}(1\to n)|^2$

and thus

$$\Delta_R(p) \;=\; \frac{i}{p^2 - m^2 - \mathrm{Re}\,\Sigma_R(p^2) + im\Gamma(p^2) + i\epsilon}$$

No information as perturbation theory breaks down for many loops, but not possible to cancel imaginary part

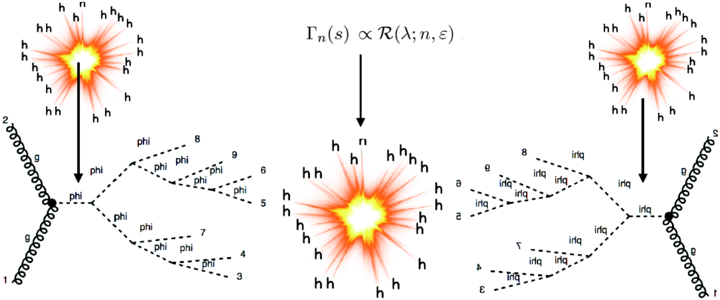
Higgsplodes

Higgsplosion as a solution of hierarchy problem

(slide from Valya’s and Michael’s talks)

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - m_h^2 - Re\tilde{\Sigma}(p^2) + im_h\Gamma(p^2)} \times \mathcal{M}_{h^*\to n\times h}$$

Include self-energy



$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 \frac{m_t^2}{m_h} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{(s - Re\Sigma(s))^2 + m_h^2\Gamma^2(s)} \times \Gamma_n(s)$$

VVK & Spannowsky 1704.0344

- Unitarity restored!

Problems with resummation

- Resummation would work only for convergent series

$$\Delta_F(p^2) = \frac{i}{p^2 - m_0^2} \sum_{n=1}^{\infty} \left(-i\Sigma(p^2) \frac{i}{p^2 - m_0^2} \right)$$

- If $\Sigma(p^2) \sim \rho(p^2) \sim p^{2N} + \cdots$ must do N subtractions

$$\Delta_F(p^2) = \Delta_F(0) + p^2 \Delta_F^{(1)}(0) + (p^2)^2 \Delta_F^{(2)}(0) + \cdots$$
$$+ (p^2)^N \int \mathrm{d}p'^2 \, \rho(p'^2) \frac{-i}{(p'^2)^N (p'^2 - p^2)}$$

- Can not predict N-order polynomial.
- No predictive power at all if $\rho(p^2) \propto \exp(+\mathrm{const} \cdot p^2)$
  - Higgsplosion (if it is there) does not cure the theory
- However – to make "calculation" relies on "reasonable" QFT, without exponential growth

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Belyaev FB Ross Shepherd'18

Other arguments

Weinberg's theorem

- $\Delta(p^2) < 1/p^2$ at large momenta
- with locality and unitarity (Källén-Lehmann representation) leads to contradiction
- Theory is not-local?
- However – if cross-sections are exponential – Källén-Lehmann representation is not directly applicable – infinite number of subtractions required.

Monin'18

Devil in the interpretation

Voloshin Gorsky'93

At threshold

$$A_{1\to n} \propto \exp(+F/\lambda)$$

- Intermediate "bubble" B with $A_{1\to B} \sim \exp(-F/\lambda)$

$$\sigma_{1\to n} = |A_{1\to B}|^2 \underbrace{|A_{B\to n}|^2 G(\varepsilon)}_{\sigma_{B\to n}}$$

- $\sigma_{B\to n} \sim O(1)$ Kobzarev'76
- $G \sim \exp(-2F/\lambda)$
- Therefore

$$\sigma_{1\to n} \sim \exp(-F/\lambda)$$

Khoze Spannowsky'17

At threshold

$$A_{1\to n} \propto \exp(+F/\lambda)$$

- Probably, the same happens away from threshold
- $$\sigma_{1\to n} \sim \exp(F/\lambda)$$
- What to do with this theory?

Do we know the answer for sure?

- Multiparticle Cross-sections in the higgs-like theories are notoriously hard to calculate at $n \gtrsim \lambda^{-1}$.
- Arguments exist both in favour and against unusual growth of these cross-sections (“Higgsplosion”)
  - Analogously, (“Higgspersion”) translates to cut-off in the propagator.
  - This would mean an additional scale predicted in SM!
- The problem is still not yet solved!