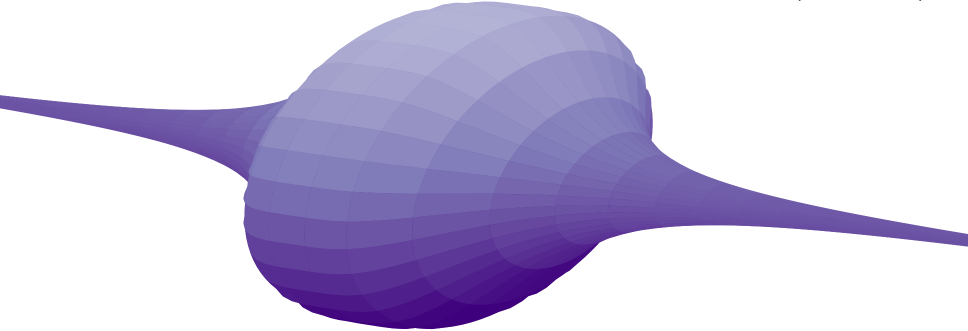
The University of Manchester

The Geometry of Inflation

arXiv:1806.02431 (PRD 2018)
arXiv:1812.07095 (PRD 2019)



Kieran Finn
with Apostolos Pilaftsis and Sotirios Karamitsos

Monday 1st July 2019

"God always geometrizes" - Plato

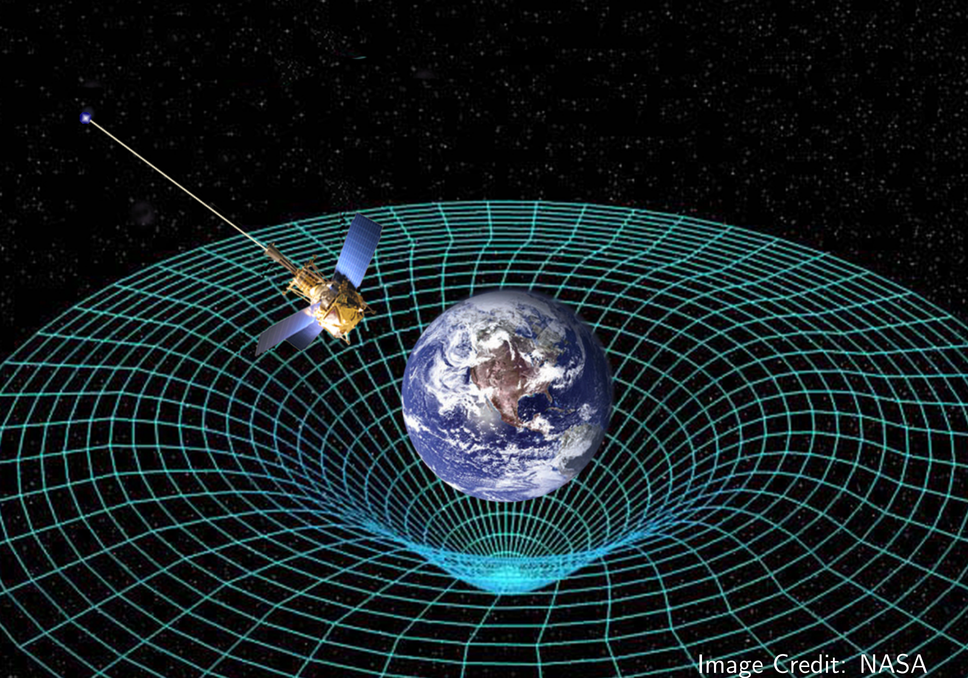


Image Credit: NASA

Geometric Interpretation of Homogeneous Field Theories

Lagrangian

- $L = \frac{1}{2} k_{ab}(\boldsymbol{\varphi}) \dot{\varphi}^a \dot{\varphi}^b$

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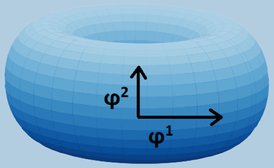
Field Space



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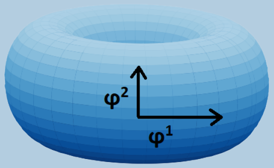

Equations of Motion

- $\ddot{\varphi}^a + \Gamma^a_{bc}\dot{\varphi}^b\dot{\varphi}^c = 0$
- $\Gamma^a_{bc} \equiv \frac{1}{2} k^{ad}\left(k_{bd,c} + k_{cd,b} - k_{bc,d}\right)$

Noether Symmetries

- $\varphi^a \rightarrow \varphi^a + \xi^a(\varphi)$
- $\nabla_b\xi_c + \nabla_c\xi_b = 0$

Geometric Interpretation of Homogeneous Field Theories

Lagrangian

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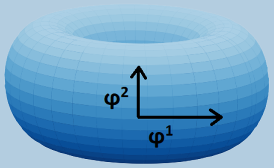

Equations of Motion

- $\ddot{\varphi}^a + \Gamma^a_{bc}\dot{\varphi}^b \dot{\varphi}^c = -k^{ab} V_{,b}$
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Noether Symmetries

- $\varphi^a \rightarrow \varphi^a + \xi^a(\varphi)$
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- $V_{,a}\xi^a = 0$

The Eisenhart Lift for Field Theories

L.P. Eisenhart: 1928

KF, S. Karamitsos and A. Pilaftsis: 2018 (arXiv:1806.02431)

Original Theory

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Lifted Theory

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Extended Field Space

- $\phi^A = \{\varphi^a, \chi\}$
- $L = \frac{1}{2} G_{AB} \dot{\phi}^A \dot{\phi}^B$

$$G_{AB} = \begin{pmatrix} k_{ab} & 0 \\ 0 & \dfrac{1}{V(\varphi)} \end{pmatrix}$$

Extension to Non-Homogeneous Field theories

Original Theory

- $\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}k_{ab}(\boldsymbol{\varphi})\,\partial_\mu\varphi^a\partial_\nu\varphi^b - V(\boldsymbol{\varphi})\right)$

- $\nabla^\mu\nabla_\mu\varphi^a + \Gamma^a_{bc}\partial_\mu\varphi^b\partial^\mu\varphi^c = -k^{ab}V_{,b}$

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Extended Theory

- $\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}k_{ab}(\varphi)\,\partial_\mu\varphi^a\partial_\nu\varphi^b + \frac{1}{2}\frac{1}{V(\varphi)}\nabla_\mu B^\mu\nabla_\nu B^\nu\right)$
- $\nabla^\mu\nabla_\mu\varphi^a + \Gamma^a_{bc}\partial_\mu\varphi^b\partial^\mu\varphi^c = -\frac{1}{2}\left(\frac{\nabla_\nu B^\nu}{V(\varphi)}\right)^2 k^{ab}V_{,b}$
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The Eisenhart Lift for Inflation

- $S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} R + \frac{1}{2} (\partial^\mu \varphi)(\partial_\mu \varphi) - V(\varphi) \right)$
- $g_{\mu\nu} = \mathrm{diag}\left(1, -a^2, -a^2, -a^2\right), \qquad \varphi = \varphi(t)$

Homogeneous Inflation

- $L = -3a\dot{a}^2 + \frac{1}{2} a^3 \dot{\varphi}^2 - a^3 V(\varphi)$

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Field Space Metric for Inflation

- $G_{AB} = \begin{pmatrix} -6\,a & 0 & 0 \\ 0 & a^3 & 0 \\ 0 & 0 & \dfrac{1}{a^3 V(\varphi)} \end{pmatrix}$

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- $\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{2}\left(\frac{\dot{\chi}}{a^3V}\right)^2 V'(\varphi) = 0$
- $H^2 + 2\frac{\ddot{a}}{a} = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\left(\frac{\dot{\chi}}{a^3V}\right)^2 V(\varphi)$
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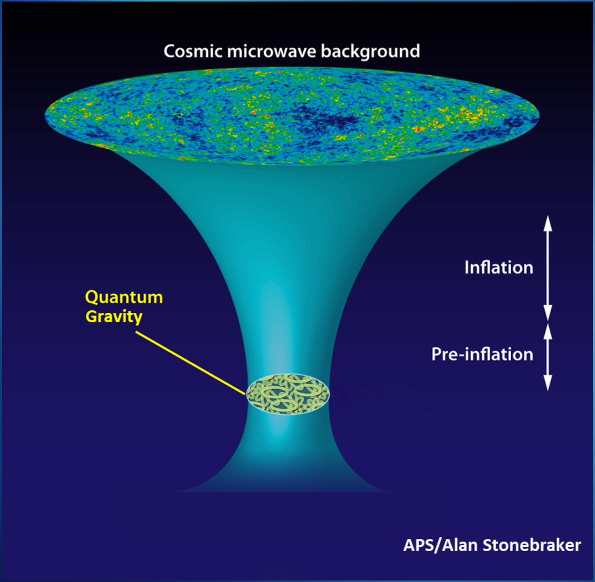
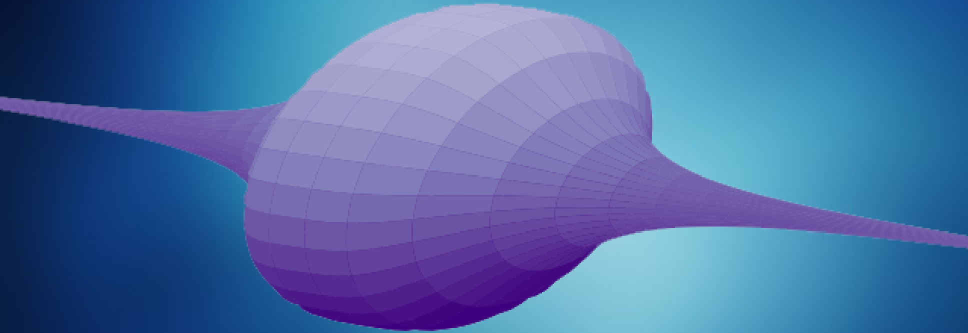
Initial Conditions for Inflation

KF and S. Karamitsos: 2019 (arXiv:1812.07095)



Phase Space Manifold



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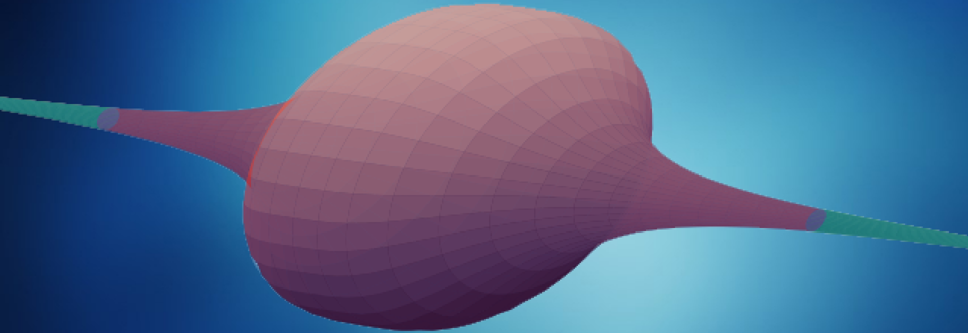
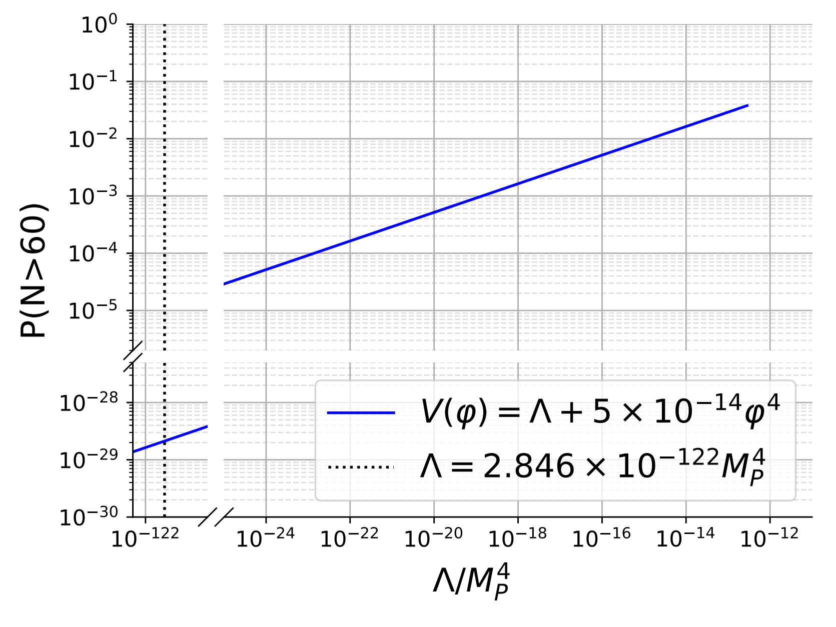


Fraction of Phase Space that allows Inflation

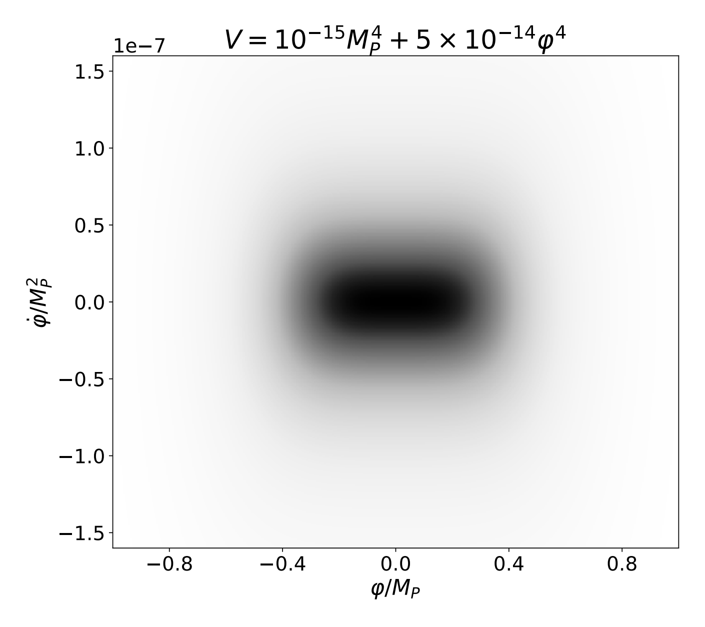


Summary

- The Eisenhart lift allows us to describe any scalar field theory geometrically
- For homogeneous theories we must add an additional scalar field
- For non homogeneous theories we must add a vector field
- We can use this to construct a geometric description of inflation
- We can use this to study the initial conditions problem

Backup Slides

Distribution of the Inflaton Field



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