INFLATION WITH STRONGLY NON-GEODESIC MOTION, NON-GAUSSIANITIES AND SWAMPLAND CONJECTURES

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J.F., S. Garcia-Saenz, L.Pinol, S. Renaux-Petel, J. Ronayne

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Top-Down

↳ Negative curvature in field space

e.g. α-ATTRACTORS

UV EMBEDDINGS & INFLATION

Swampland Conjectures

REQUIRE MULTI-FIELD INFLATION WITH LARGE BENDING

A. Achucarro & G. Palma '18

η-Problem

REQUIRE INFLATING ON STEEP POTENTIALS

Top-Down

↳ Negative curvature in field space

e.g. α-ATTRACTORS

$\Longrightarrow$ NEW ATTRACTORS

SIDETRACKED, HYPERINFLATION, ANGULAR INFLATION .....

S. Garcia Saenz, S. Renaux-Petel & J. Ronayne '18, A. Brown '17, D. Marsh & T. Bjorkmo '19. P. Christodoulidis, D. Roest, E. Sfakianakis ......

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REQUIRE INFLATING ON STEEP POTENTIALS

PLAN

- INFLATION IN NON-GEODESIC MOTION AND NEW ATTRACTORS
- PERTURBATIONS $\longrightarrow$ EFFECTIVE FIELD THEORY WITH IMAGINARY SPEED OF SOUND
- MATCHING THE EFT AND GOING BEYOND THE BISPECTRUM
- MODEL INDEPENDENT BOUNDS
- EXAMPLE: HYPERINFLATION

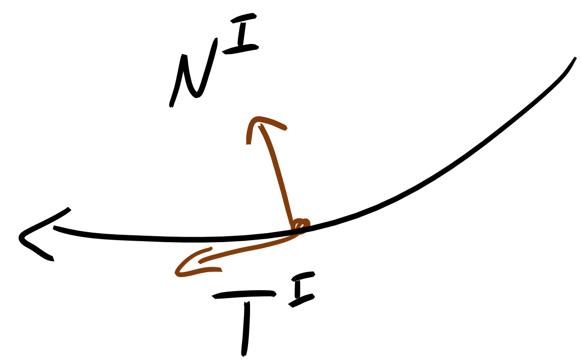
INFLATION IN NON-GEODESIC MOTION

$$\mathcal{L} = -\frac{1}{2} G_{IJ}\, \partial\phi^I \partial\phi^J - V(\phi) + M_{Pl}^2 \frac{R}{2}$$

$$\text{EOM} \equiv D_t \dot{\phi}^I + 3H\dot{\phi}^I + G^{IJ} V_J = 0$$

$$\downarrow$$

$$\ddot{\phi}^I + \Gamma^I_{JK}\, \dot{\phi}^J \dot{\phi}^K$$



Def. $\dot{\sigma}^2 = G_{IJ}\dot{\phi}^I\dot{\phi}^J$, $T^I = \dfrac{\dot{\phi}^I}{\dot{\sigma}^I}$, $N^I \perp T^I$

INFLATION IN NON-GEODESIC MOTION

N^I (EOM) $\longrightarrow$ $D_t T^I = \eta_\perp H N^I$ $\qquad \eta_\perp = -\dfrac{V_N}{H\dot{\sigma}}$

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$\eta_\perp = 0 \Longrightarrow$ GEODESIC MOTION , $D_t T^I = 0$

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SWAMPLAND REMARK   A. Achucarro & G. Palma '18

$$\varepsilon = \varepsilon_V \Big/ \left(1 + \frac{\eta_\perp^2}{9}\right) , \qquad \varepsilon_V = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \sim O(1)$$

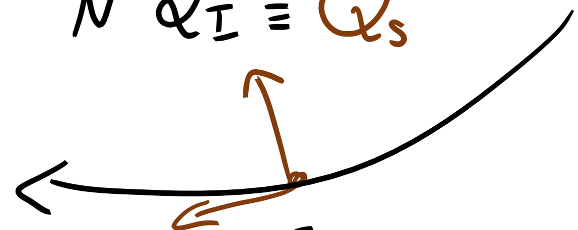
$$\not\Longrightarrow \quad \varepsilon = -\frac{\dot{H}}{H^2} \sim O(1)$$

MULTI-FIELD PERTURBATIONS

- Covariant Perturbations (flat gauge)

$$\phi^I - \phi_0^I = \delta\phi^I = Q^I - \Gamma^I_{JK} Q^J Q^K$$

$$N^I Q_I \equiv Q_s$$



$$T^I Q_I = Q_\sigma$$

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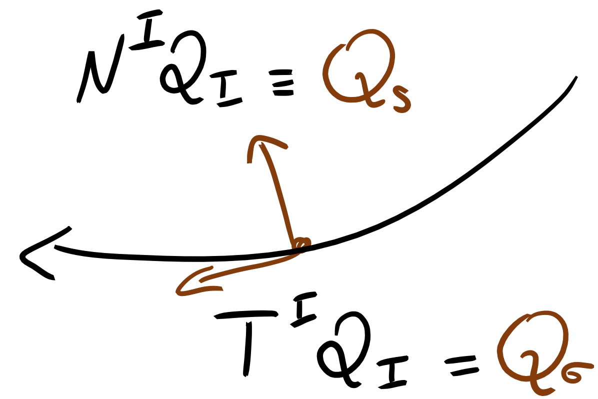

- EOM entropic fluctuation

$$\ddot{Q}_s + 3H\dot{Q}_s + \left(\frac{k^2}{a^2} + m_s^2\right) Q_s = -\alpha\, \eta_\perp\, \dot{\zeta}$$

$$V_{;ss} + \varepsilon R_{sf} H^2 M_{Pl}^2 - H^2 \eta_\perp^2$$

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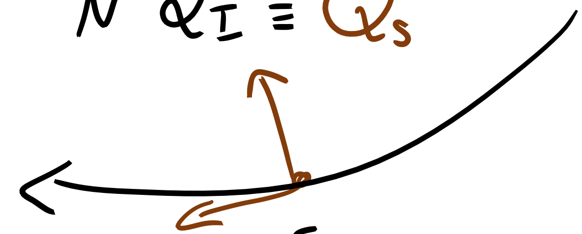

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- EOM ENTROPIC FLUCTUATION

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- N.B. $\eta_\perp^2 \gg 1$ " $\Longrightarrow$ " $m_s^2 < 0$; $|m_s| \gg H$

MULTI-FIELD PERTURBATIONS

$$m_s^2 < 0 \quad \not\Longrightarrow \quad \text{UNSTABLE BKG}$$

MULTI-FIELD PERTURBATIONS

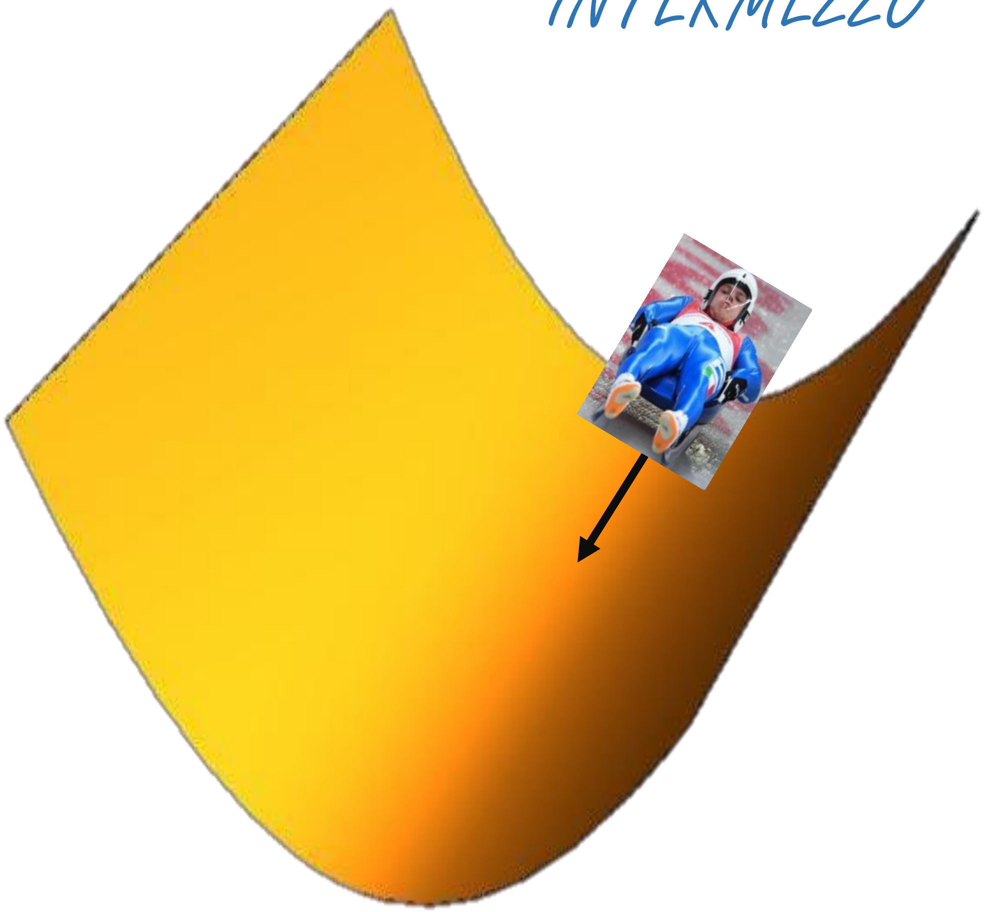
$$m_s^2 < 0 \quad \not\Rightarrow \quad \text{UNSTABLE BKG}$$

- SUPER-HUBBLE $\frac{k}{a} \ll H$; $\dot{\zeta} = \frac{2H^2 \eta_\perp}{\dot{\sigma}} Q_s \longrightarrow \text{EoM}[Q_s]$

$$\Rightarrow \ddot{Q}_s + 3H\dot{Q}_s + m_{s,\text{EFF}}^2 Q_s = 0$$

$$m_{s,\text{EFF}}^2 = \underbrace{V_{ss} + \varepsilon R_{sf} H^2 \pi_{Pl}^2 - H^2 \eta_\perp^2}_{m_s^2 < 0} + 4H^2\eta_\perp^2 > 0$$

INTERMEZZO



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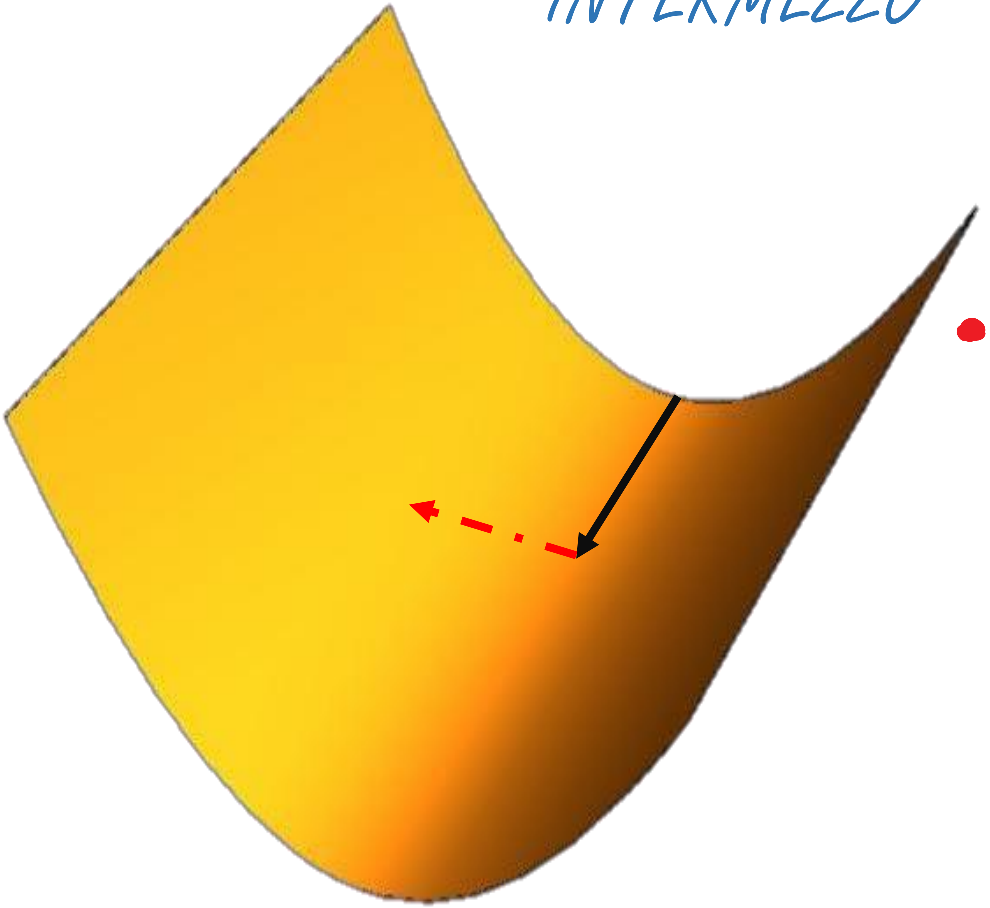

- GEOMETRICAL DESTABILIZATION
  S. Renaux-Petel & K. Turzinsky '15

$$m^2_{s,\mathrm{EFF}} = V_{ss} + \epsilon R_{sf} H^2 \eta_{\perp}^2 < 0$$

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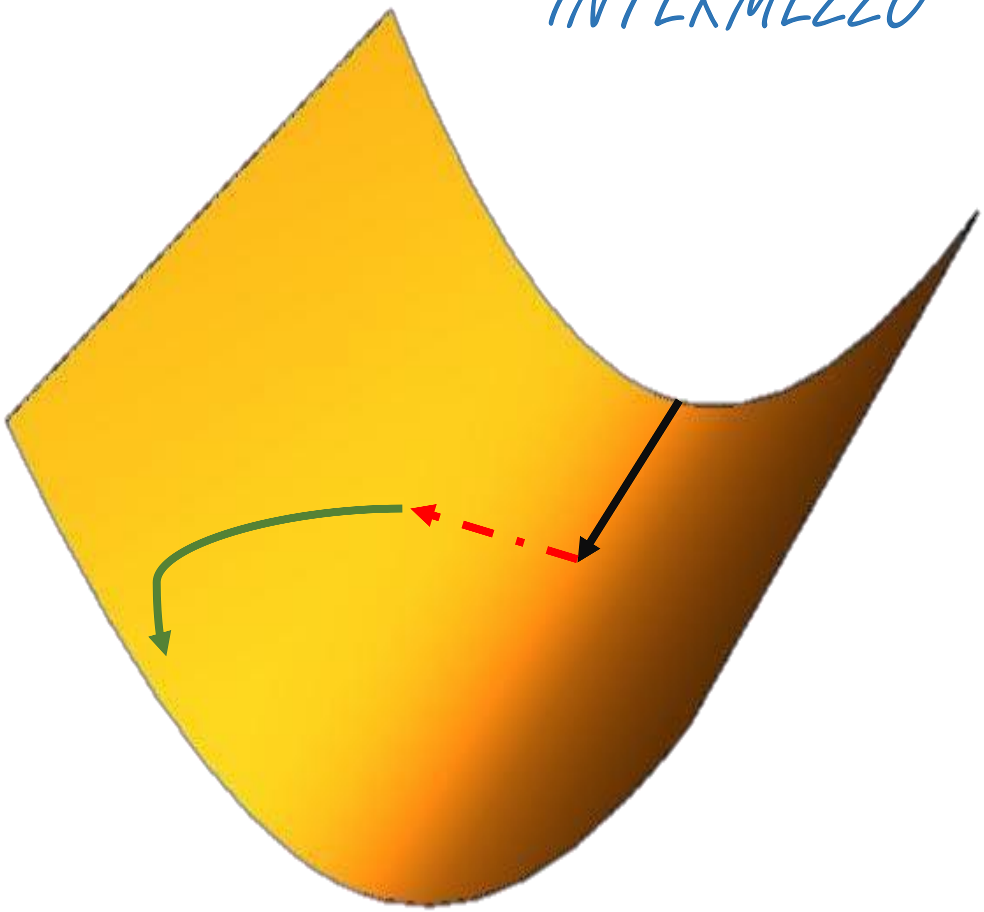

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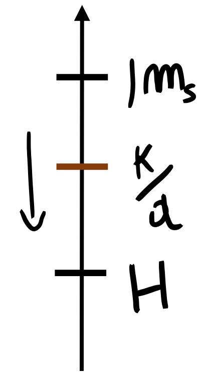
SINGLE FIELD EFFECTIVE FIELD THEORY

- INTEGRATING OUT THE ENTROPIC MODE $|m_s^2| \gg \frac{k^2}{a^2}$



$$Q_\zeta^{EFT} = -\frac{2\dot{\sigma}\eta_\perp}{m_s^2}\dot{\zeta} \longrightarrow S_2^{EFT}[\zeta] = \int d\tau\, d^3x\, \varepsilon a^2 \left[\frac{(\zeta')^2}{c_s^2} - (\partial_i\zeta)^2\right]$$

- $$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}$$

A. Achucarro et al. '11

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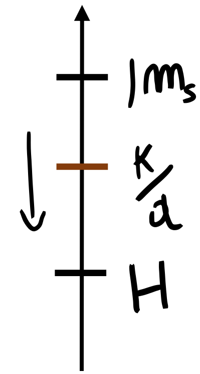

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- $$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2} = \frac{m_{s,Eff}^2}{m_s^2}$$

$m_{s,Eff}^2 > 0 \Longleftarrow$ Stable Background

$m_s^2 < 0 \quad$ " $\Longleftarrow$ " $\quad \eta_\perp^2 \gg 1$

A. Achucarro et al. '11

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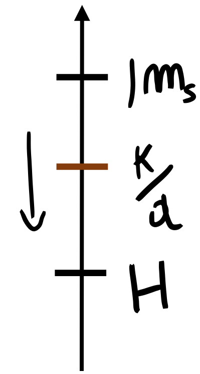

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$>0 \Longleftarrow$ Stable Background

<0 " $\Longleftarrow$ " $\eta_\perp^2 \gg 1$

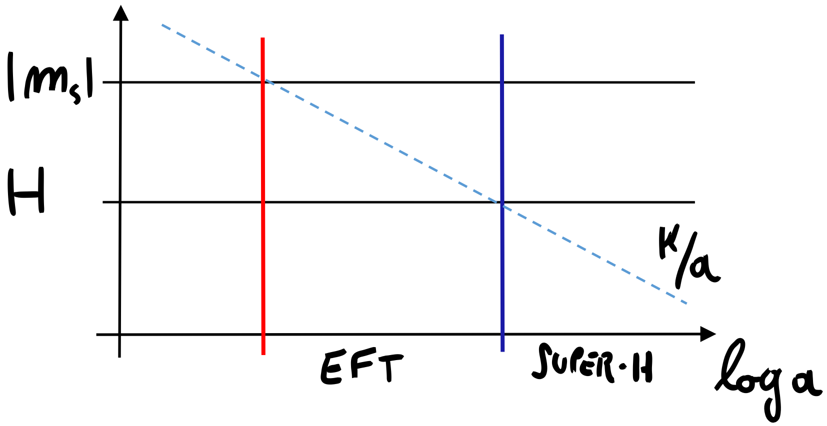
A. Achucarro et al. '11

$\Longrightarrow$ $\boxed{c_s^2 < 0 \quad \text{EFT WITH IMAGINARY SPEED OF SOUND}}$

S. Garcia Saenz, S. Renaux-Petel '18
J.F., S. Garcia Saenz, L. Pinol, ,
S. Renaux-Petel, J. Ronayne '19

SINGLE FIELD EFFECTIVE FIELD THEORY

TRANSIENT INSTABILITY



SINGLE FIELD EFFECTIVE FIELD THEORY

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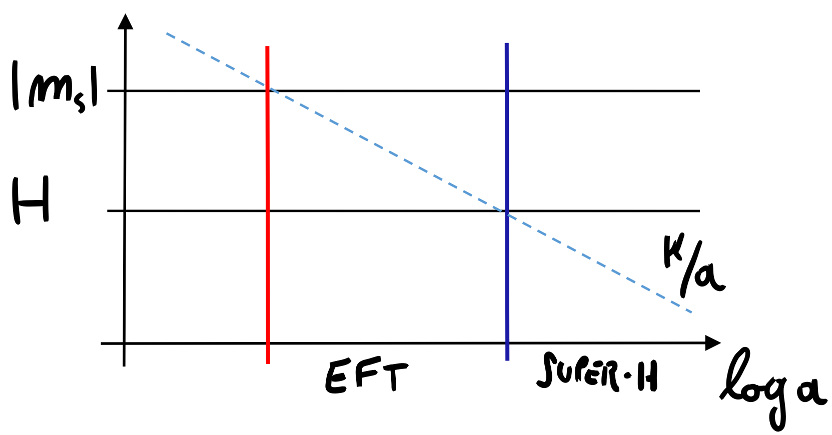

Validity of the EFT

$$\frac{|c_s|\,k}{a} = x H\,, \qquad x \lesssim \frac{|m_s||c_s|}{H}$$

$$N_{S.H} - N_{EFT} = \ln x$$

- ***MODE FUNCTION SOLUTION***

$$\zeta(\tau) = \left(\frac{2\pi}{k^3}\right)^{\frac{1}{2}} \alpha \cdot \left[e^{k|c_s|\tau + x}\left(k|c_s|\tau - 1\right) - \rho\, e^{i\psi} e^{-(k|c_s|\tau + x)}\left(k|c_s|\tau + 1\right)\right]$$

Overall amplitude   *Growing mode*   *Decaying mode*

SINGLE FIELD EFFECTIVE FIELD THEORY

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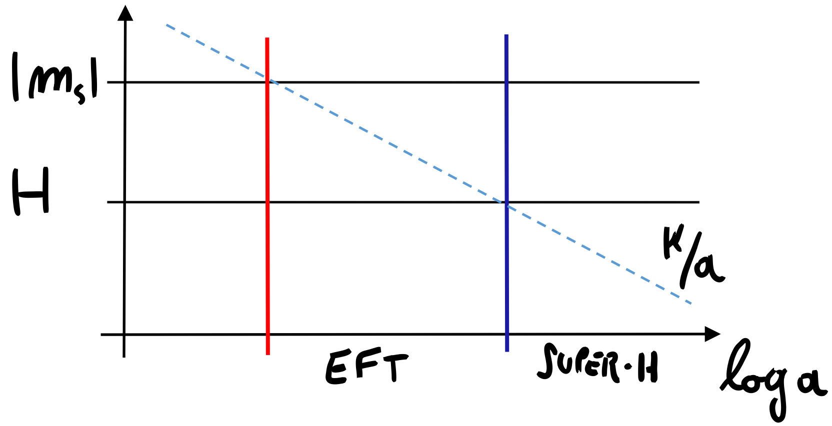

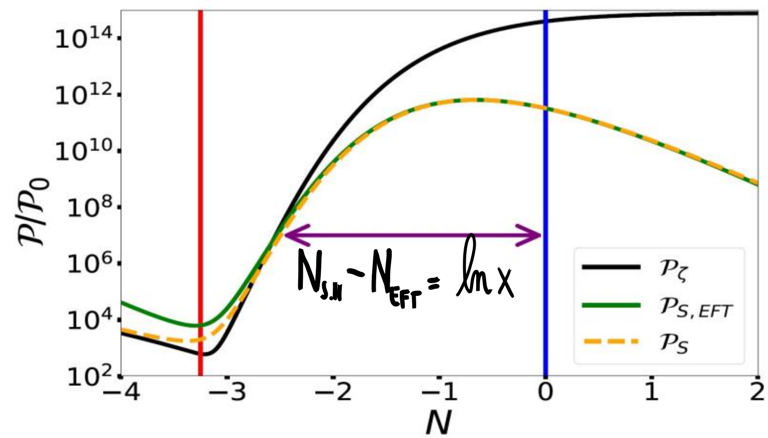

- MODE FUNCTION SOLUTION $\longrightarrow$ $P_\zeta = \alpha^2 e^{2x}$

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Overall amplitude   Growing mode   Decaying mode

ROADMAP: USEFULNESS OF THE EFT

EFT $\longrightarrow \dfrac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \sim f_{NL}(A, x) \xleftrightarrow{\text{MATCHING}}$ **NUMERICS**

PyTransport 2.0

D. Mulryne and J. Ronayne

$$S^3_{\rm EFF} = \int d\tau\, d^3x\, \frac{a\,\varepsilon\, M_{\rm Pl}^2}{H} \left(\frac{1}{c_s^2} - 1 \right) \left[\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right]$$

WHY THE EFFECTIVE FIELD THEORY?

↳ *Understanding enhancement of certain shapes (e.g. flattened NG)*

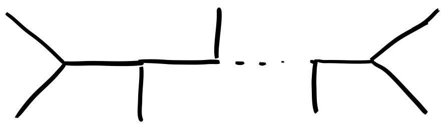
↳ ***More importantly, beyond 3pt functions:***

Estimate higher order correlators $\langle \zeta^n \rangle$ ***and set perturbative bounds***

THEORETICAL CONSTISTENCY BOUNDS

J.F., S. Garcia Saenz, L. Pinol, , S. Renaux-Petel, J. Ronayne '19

$$\langle \zeta^n \rangle = \langle 0 | \bar{T}\left(e^{i\int H dt}\right) \zeta_I \, T\left(e^{-i\int H dt}\right) | 0 \rangle$$

$\supset$  $\sim \left(H^{(3)}\right)^{n-2}$

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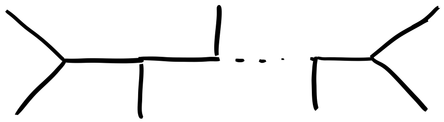
$\supset$  $\sim \left(H^{(3)}\right)^{n-2}$

ENHANCMENT OF THE POWER SPECTRUM

$$\Longrightarrow \quad f_{NL}^{n-2} \sim \frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} = \left[\frac{P_\zeta}{P_0} \times \left(1 + \frac{1}{|c_s|^2}\right)\right]^{n-2}$$

PERTURBATIVITY VIOLATION Shandera et al. '08 '10 Bartolo et al '10

IF

$$f_{NL}^{n-2} \times P_\zeta^{\frac{n-2}{2}} < 1 \quad \Longrightarrow \quad \boxed{\frac{P_\zeta}{P_0}\left(1 + \frac{1}{|c_s|^2}\right) \gtrsim 10^4}$$

MODEL INDEPENDENT THEORETICAL BOUND

HYPERINFLATION $\sim 10^{15}$

THEORETICAL CONSTISTENCY BOUNDS

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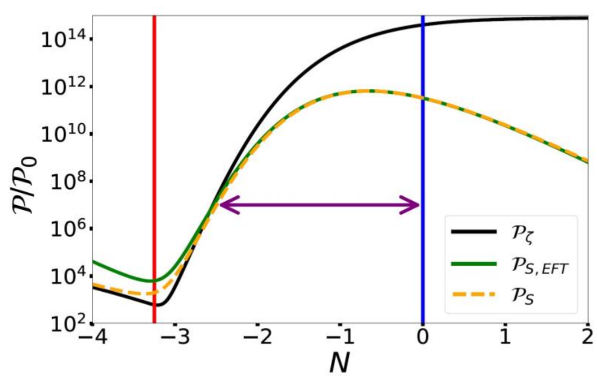

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CONCLUSIONS

- Multi-field inflation with internal geometry naturally arises in UV complete theories.
- Large bending may be required to satisfy the refined de Sitter Swampland conjecture.
- Perturbative analysis reveals a transient tachyonic instability EFT with $c_s^2 < 0$.
  - Exponential enhancement of P_ζ
  - Large flattened bispectrum f_{NL}^{flat} Match with numerics
  - Stringent bound on the enhancement of P_ζ
  - E.g. Hyperinfaltion is ruled out

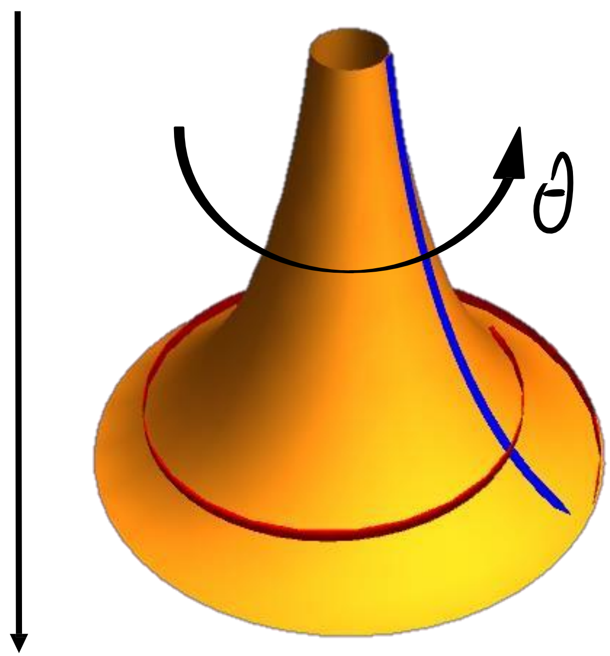
Example: HYPERINFLATION A. Brown '17

Inflating on a hyperbolic geometry $R_{\mathrm{sf}} = -2/M^2$

$$G_{IJ}\, d\phi^I d\phi^J = d\phi^2 + M^2 \sinh\left(\frac{\phi}{M}\right) d\theta^2 \quad ; \quad V = V(\phi)$$

Conserved quantity

$J = a^3 \cdot$ (ANGULAR MOMENTUM)



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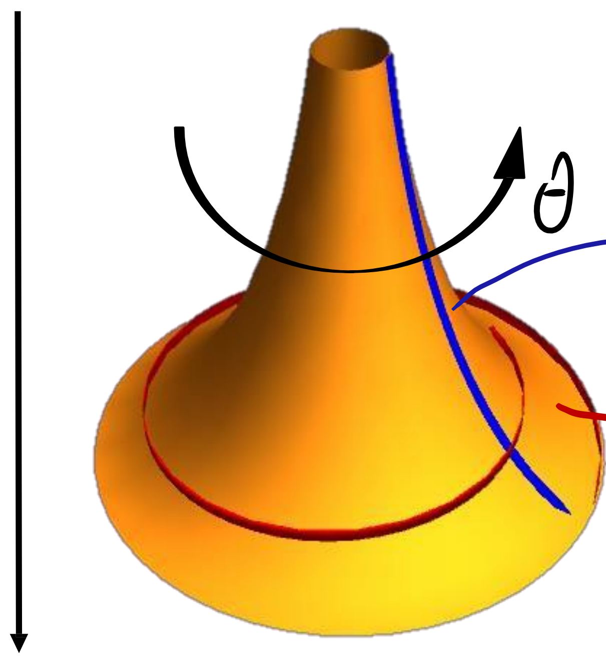


Conserved quantity

$$J = a^3 \cdot \underbrace{(\text{ANGULAR MOMENTUM})}_{L}$$

- Radial geodesic $J = 0$

  unstable for steep potential

- New attractor: spiraling $J \neq 0$

$$\frac{3M}{M_{Pl}} < M_{Pl}\frac{V'}{V} \ll \frac{M_{Pl}}{M}\,, \quad M\frac{|V''|}{V'} \ll 1$$

$\longrightarrow$ Inflate without slow-rolling & large bending

e.g. $V = \frac{m^2\phi^2}{2}$, $M = 10^{-2} M_{Pl}$ $\Longrightarrow$ $\eta_\perp^2 \sim 100$, $m_s^2/H^2 \sim -200$ $\longrightarrow$ $c_s^2 \approx -1$

MATCHING THE EFT

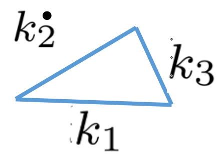
NUMERICS

PyTransport 2.0

D. Mulryne and J. Ronayne '16

Full Bispectrum $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$

MATCHING



EFT

$c_s^2 \approx -1$

$$\frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \sim f_{NL}(A, x)$$

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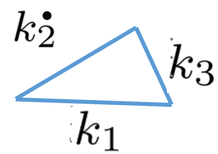

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~ 10.5

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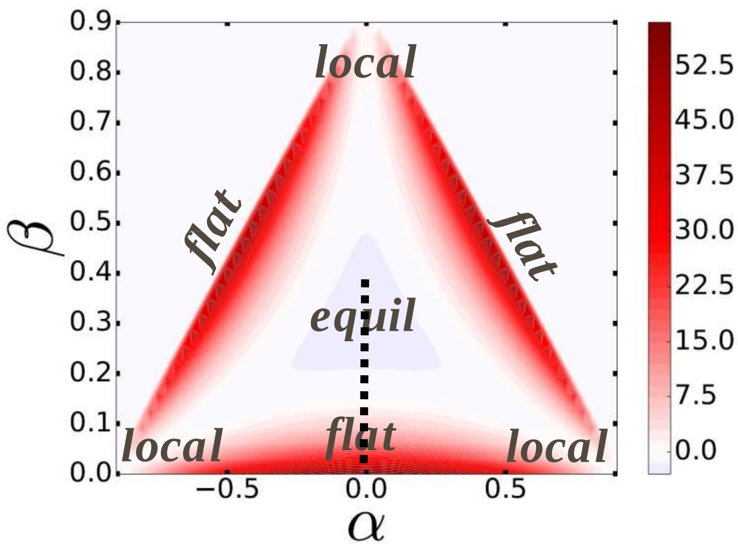
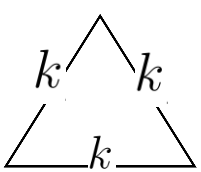
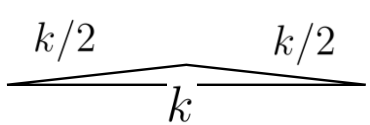


$$f_{NL}^{\rm eq} = -2.0\,, \qquad f_{NL}^{\rm flat} = 53.8$$





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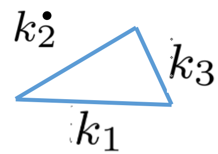

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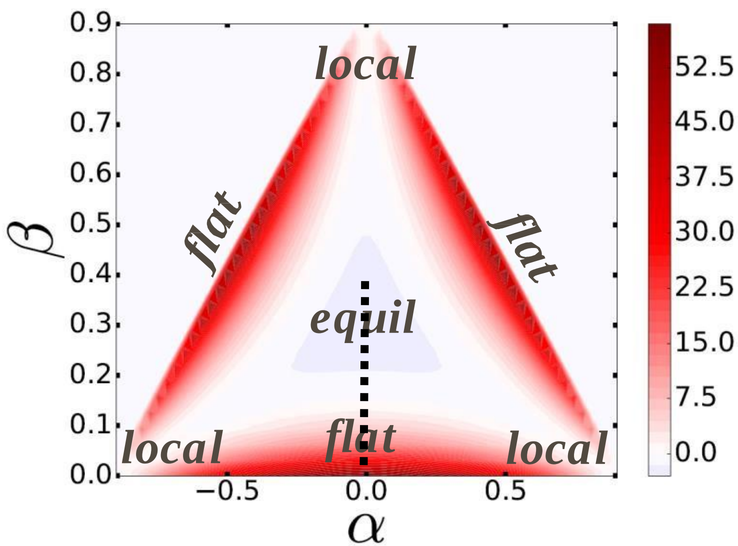
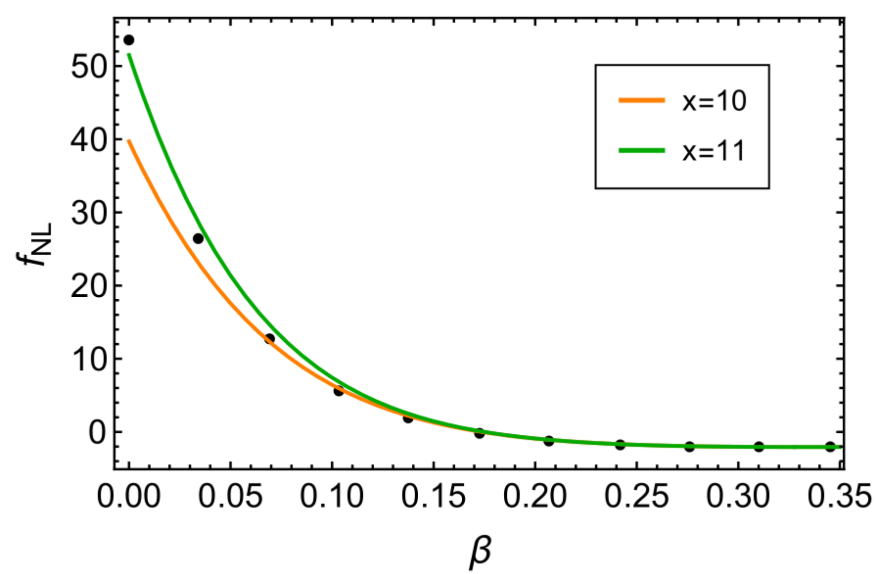
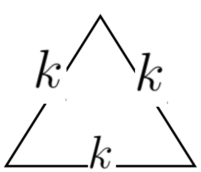
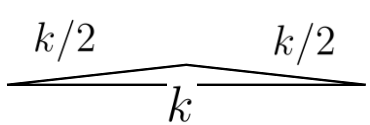
-0.3





$$f_{NL}^{\rm eq} = -2.0\,, \qquad f_{NL}^{\rm flat} = 53.8$$





MATCHING THE EFT

J.F., S. Garcia Saenz, L. Pinol, , S. Renaux-Petel, J. Ronayne '19

NUMERICS

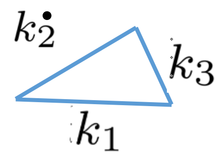
PyTransport 2.0

D. Mulryne and J. Ronayne '16

Full Bispectrum $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$

MATCHING

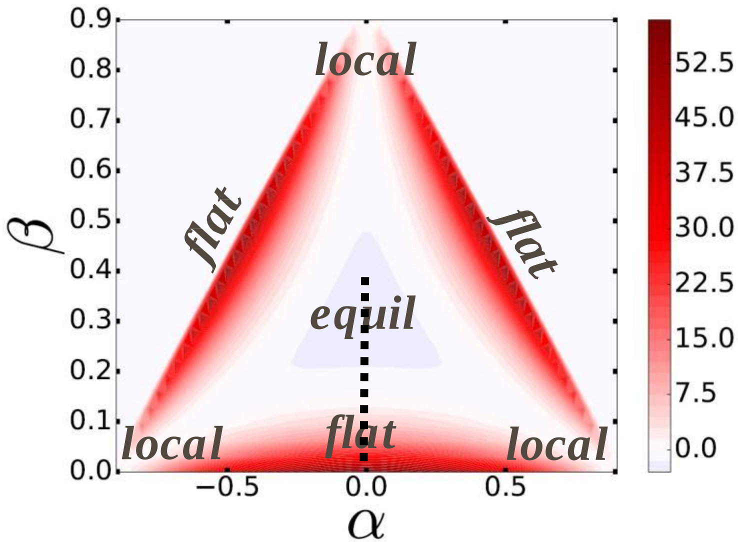
$\longleftrightarrow$

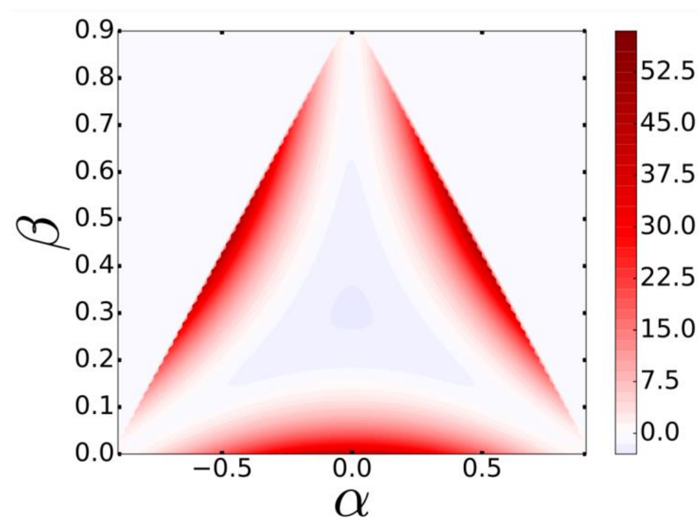


EFT

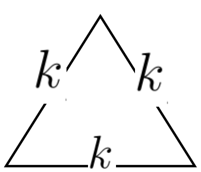
$c_s^2 \approx -1$

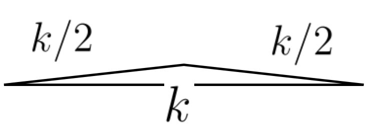
$$\frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \sim f_{NL}(A, x), \quad A \sim 10.5, \quad x \to -0.3$$





$f_{NL}^{\rm eq} = -2.0\,,$ $\qquad f_{NL}^{\rm flat} = 53.8$





THEORETICAL CONSTISTENCY BOUNDS

$$\langle \zeta^n \rangle = \langle 0 | \bar{T}\left(e^{i\int H dt}\right) \zeta_I \, T\left(e^{-i\int H dt}\right) | 0 \rangle$$

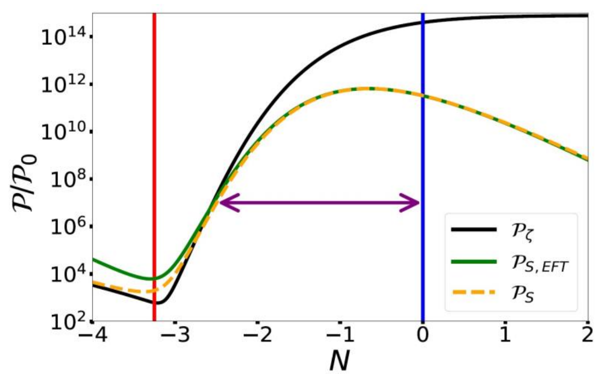
$\supset$  $\sim \left(H^{(3)}\right)^{n-2}$

ENHANCMENT OF THE POWER SPECTRUM

$$\Longrightarrow \quad f_{NL}^{\,n-2} \sim \frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} = \left[\frac{P_\zeta}{P_0} \times \left(1 + \frac{1}{|c_s|^2}\right)\right]^{n-2}$$

PERTURBATIVITY VIOLATION

Shandera et al. '08 '10 Bartolo et al '10



$$\boxed{\frac{P_\zeta}{P_0}\left(1 + \frac{1}{|c_s|^2}\right) \gtrsim 10^4 \quad \text{MODEL INDEPENDENT THEORETICAL BOUND}}$$

HYPERINFLATION $\sim 10^{15}$

CONCLUSIONS

- Multi-field inflation with internal geometry naturally arises in UV complete theories.
- Large bending may be required to satisfy the refined de Sitter Swampland conjecture.
- Perturbative analysis reveals a transient tachyonic instability EFT with $c_s^2 < 0$.
    - Exponential enhancement of P_ζ
    - Large flattened bispectrum f_{NL}^{flat} Match with numerics
    - Stringent bound on the enhancement of P_ζ
    - E.g. Hyperinfaltion is ruled out