Inhomogeneous Inflation as a probe of the shape of the inflationary potential

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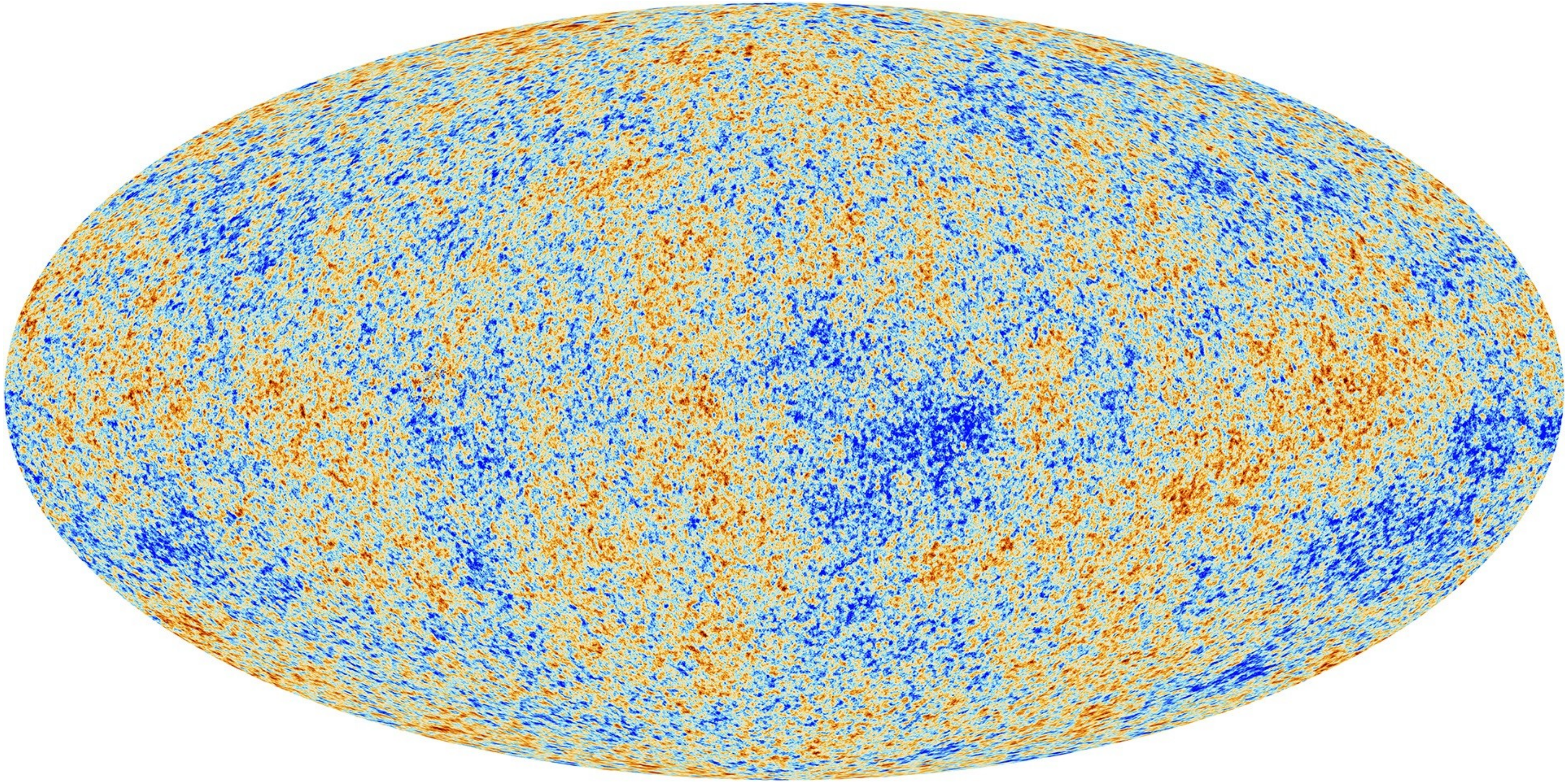
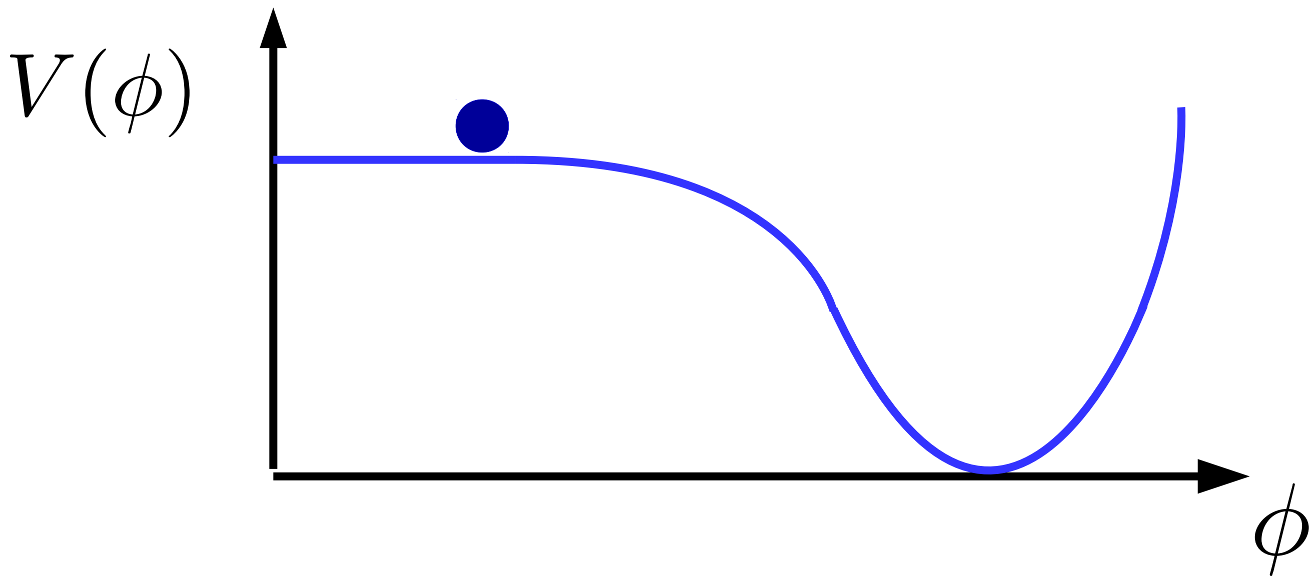
GR CHOMBO

Homogeneity Problem



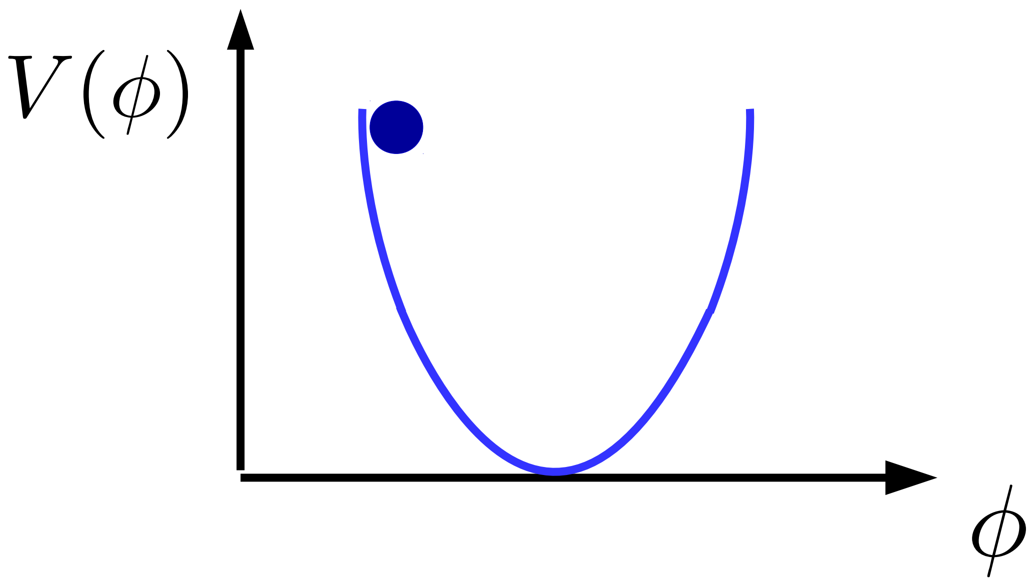
Inflation



- Homogeneous
- Slow-roll

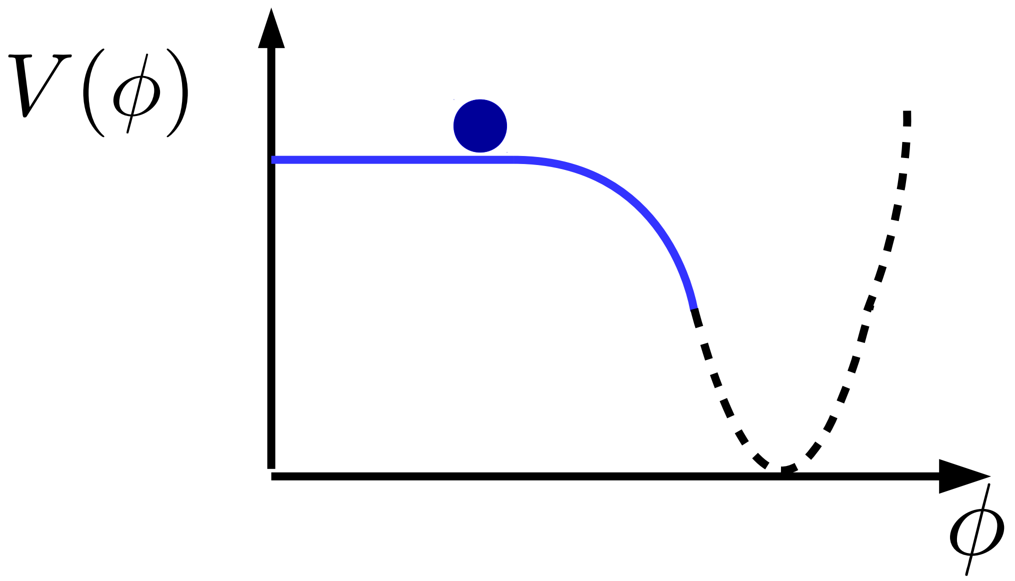
$$H^2 = \frac{8\pi}{3M_{\rm Pl}^2}\left(\cancel{\frac{1}{2}\dot{\phi}^2} + \frac{1}{2}\cancel{(\nabla\phi)^2} + V(\phi)\right)$$

Inflation



Convex

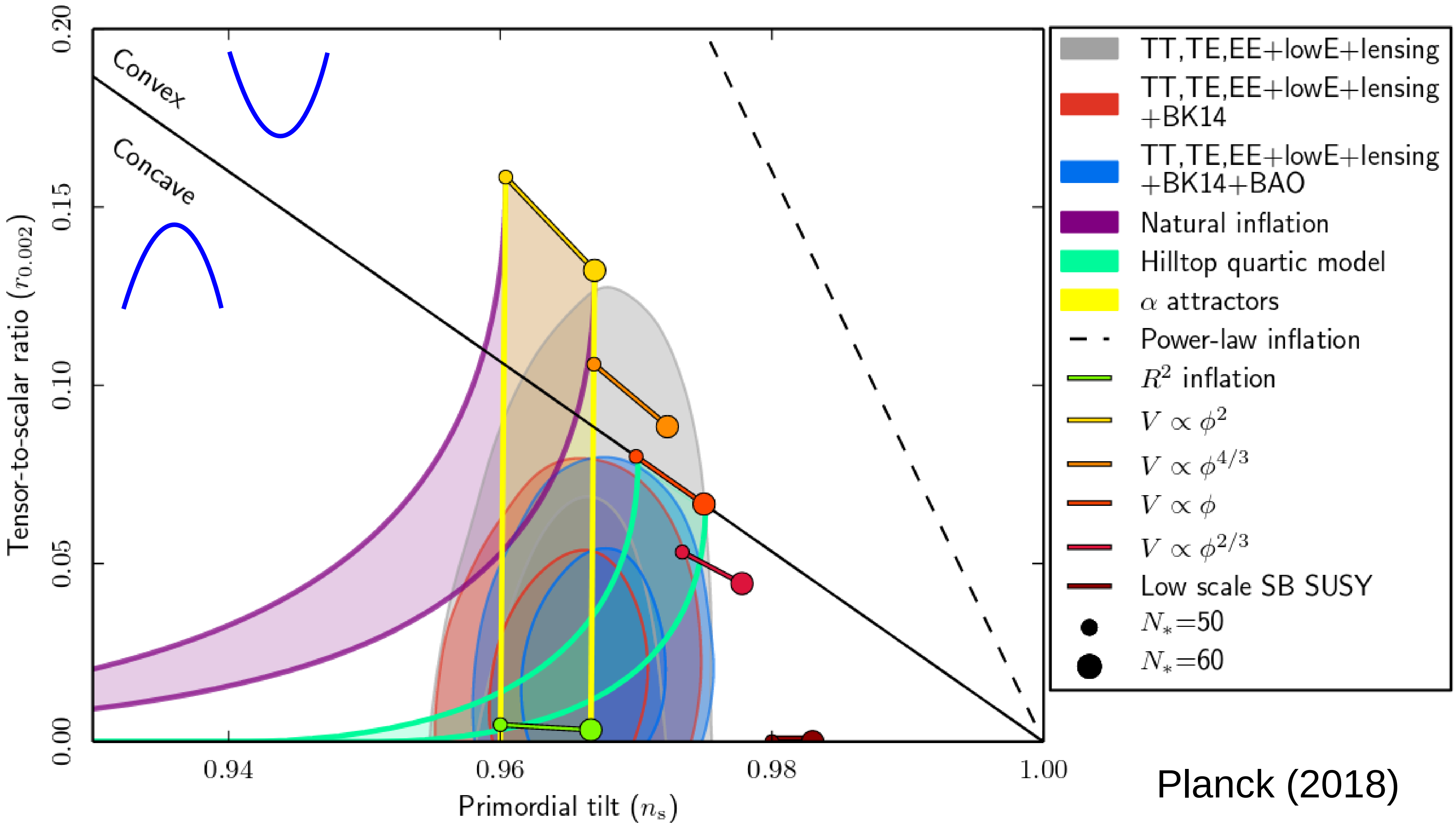
$$\frac{d^2V}{d\phi^2} > 0$$



Concave

$$\frac{d^2V}{d\phi^2} < 0$$

Inflation



Planck (2018)

Motivation

Problem to solve: homogeneity

Assumption: Pre-inflationary homogeneity

Can inflation still inflate for inhomogeneous initial conditions?

Inhomogeneous Inflation

Numerical Relativity in Early Universe Cosmo.

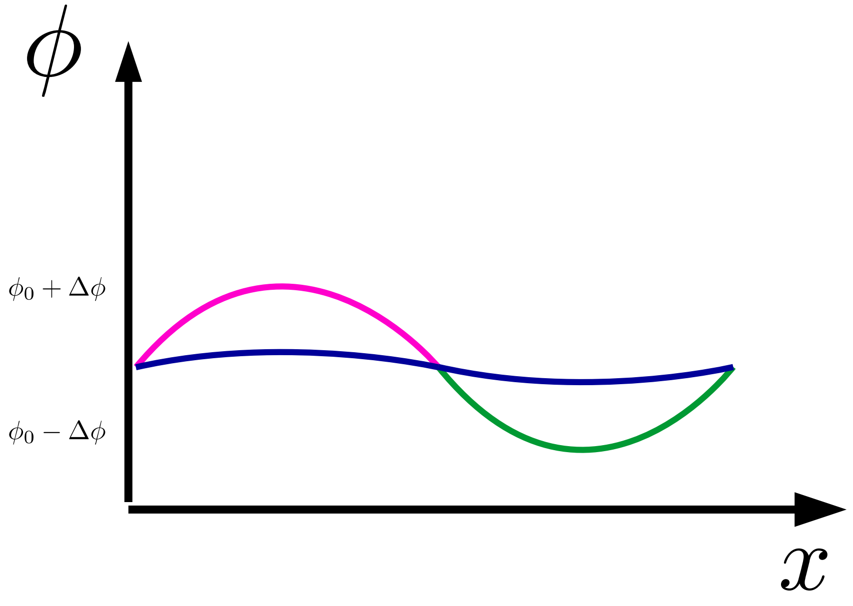
Some past/recent work:

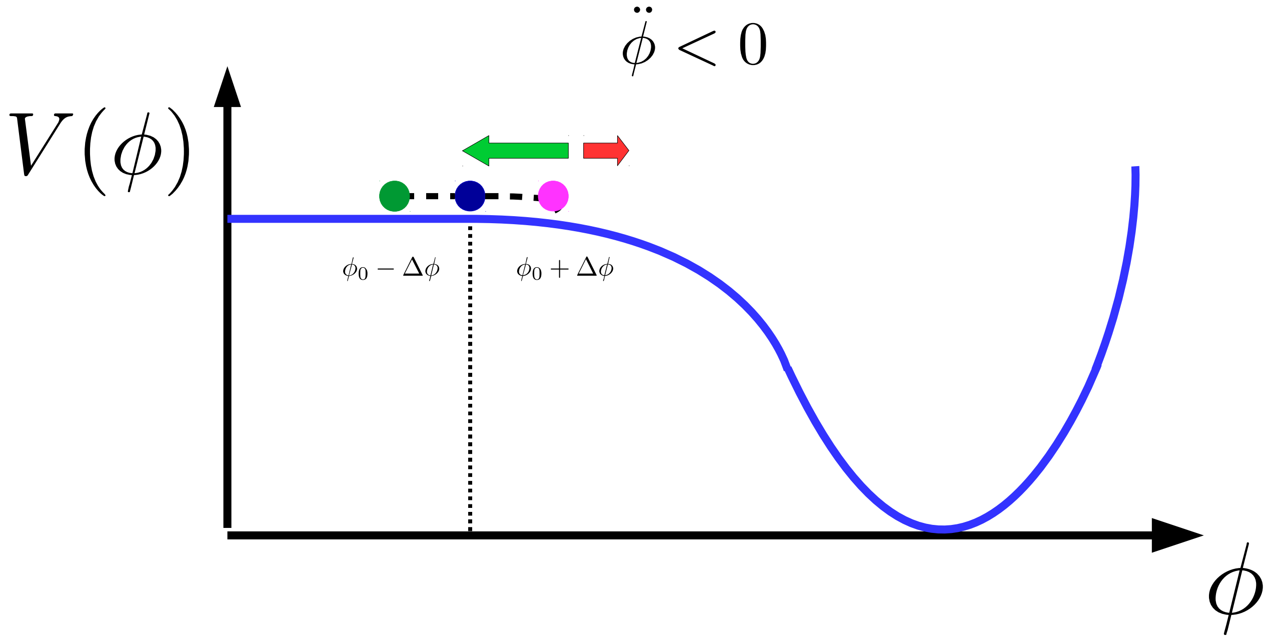
- Goldwirth, Piran (1989)
- Laguna, Kurki-Suonio, Matzner (1991)
- Garfinkle, Chet Lim, Pretorius, Steinhardt (2008)
- Xue, Garfinkle, Pretorius, Steinhardt (2013)
- East, Kleban, Linde, Senatore (2016)
- Clough, Lim, DiNunno, Fischler, Flauger, Paban (2016)
- Clough, Flauger, Lim (2017)
- . . .

Inhomogeneous Inflation

Mechanism of failure:

Pull-back

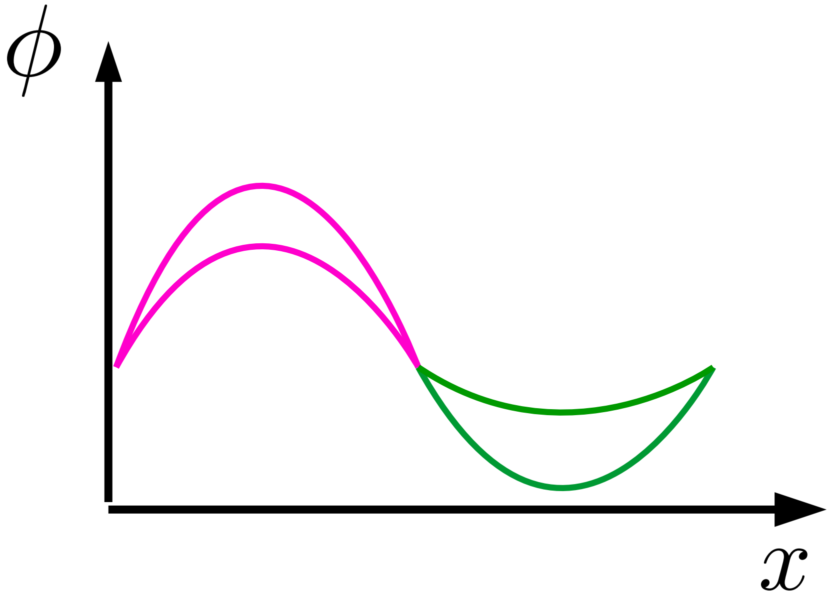


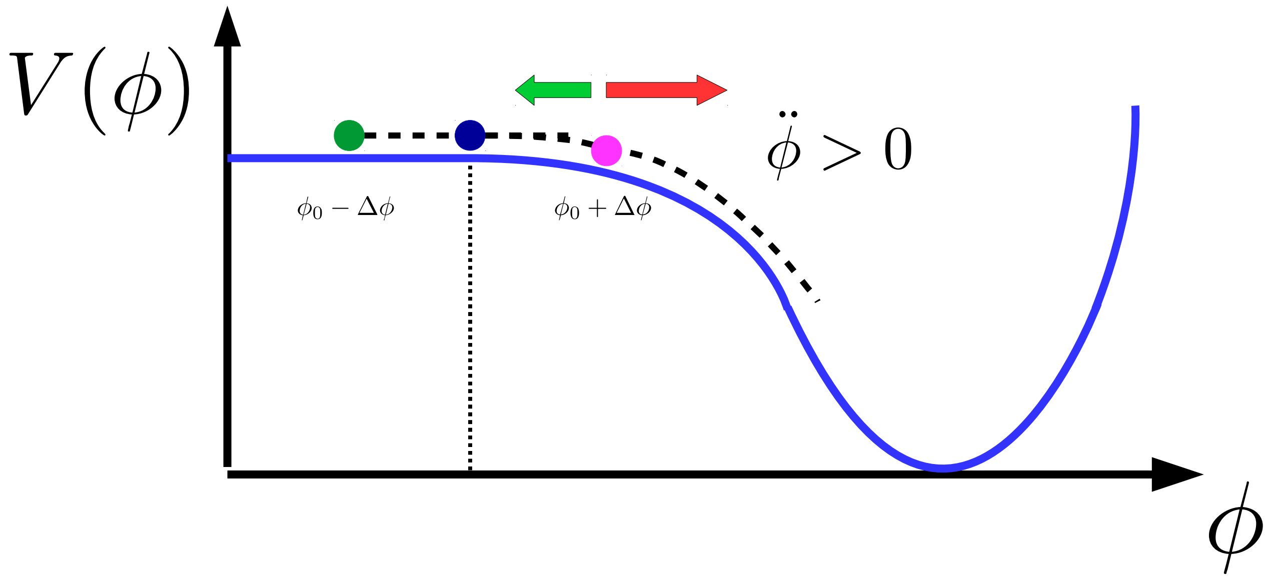


Inhomogeneous Inflation

Mechanism of failure:

Roll-down

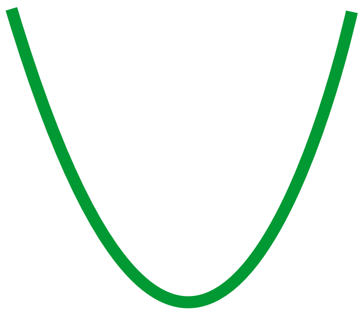




Result

Convex models CAN'T fail

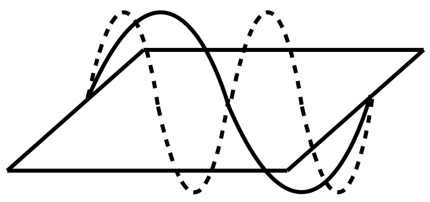
Concave models CAN fail



better than

Proof

$$k = 2\pi n H$$



Ansatz: $$\phi(\mathbf{x}, t=0) = \phi_0 + \Delta\phi\ e^{i\vec{k}\cdot\vec{x}}$$

KG: $$\ddot{\phi} + 3H\dot{\phi} + k^2\Delta\phi + \frac{dV}{d\phi} = 0$$

$$\ddot{\phi} = -k^2\Delta\phi - \frac{dV(\phi_{\max})}{d\phi}$$

Proof

$$f(\phi_{\max}) = k^2 \Delta\phi + \frac{dV(\phi_{\max})}{d\phi}$$

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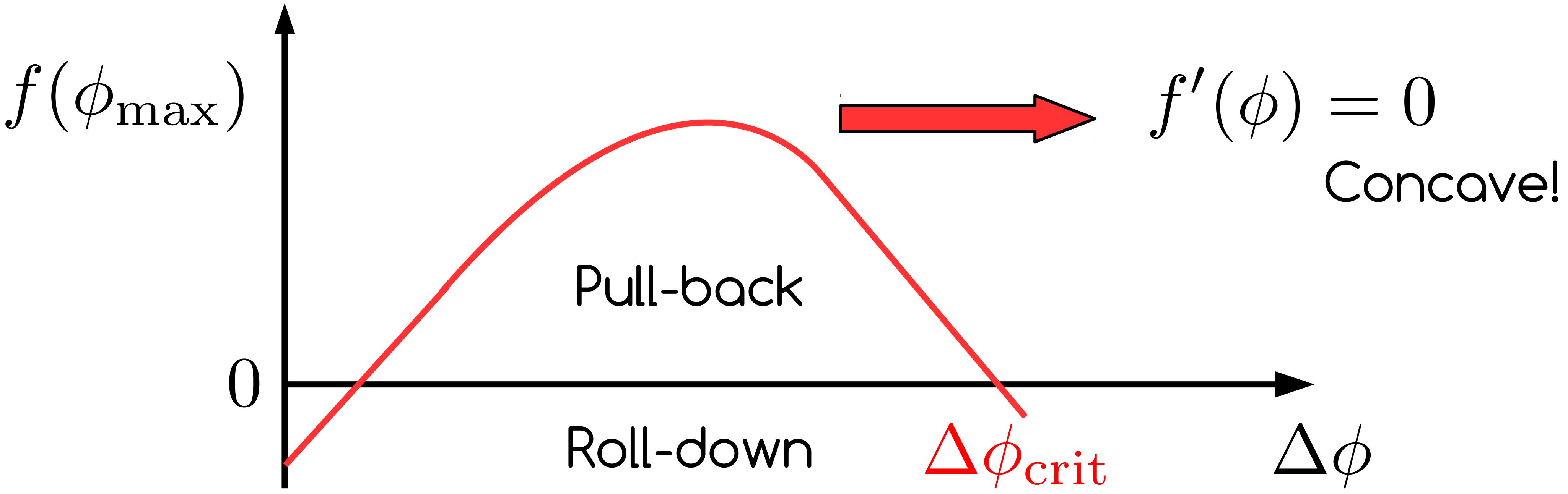
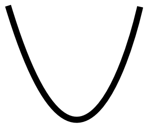


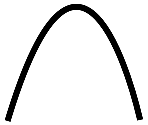
Proof

$$f'(\phi_{\max}) = k^2 + \frac{d^2V}{d\phi^2}$$

> 0 > 0

< 0

Convex $\frac{d^2V}{d\phi^2} > 0$ 

Concave $\frac{d^2V}{d\phi^2} < 0$ 

$$k^2 + \frac{d^2V}{d\phi^2} \neq 0$$ Robust!

$$k^2 + \frac{d^2V}{d\phi^2} = 0$$ Can fail!

Results

Results

Need to use FULL Numerical Relativity:

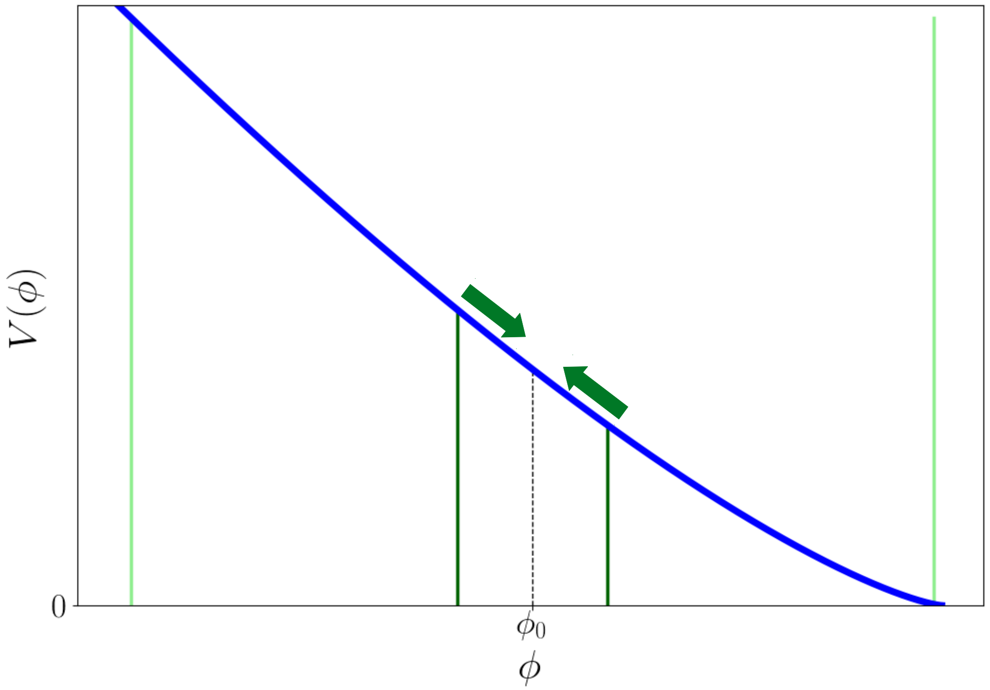
Solve non-linear Einstein eq + Scalar Field:

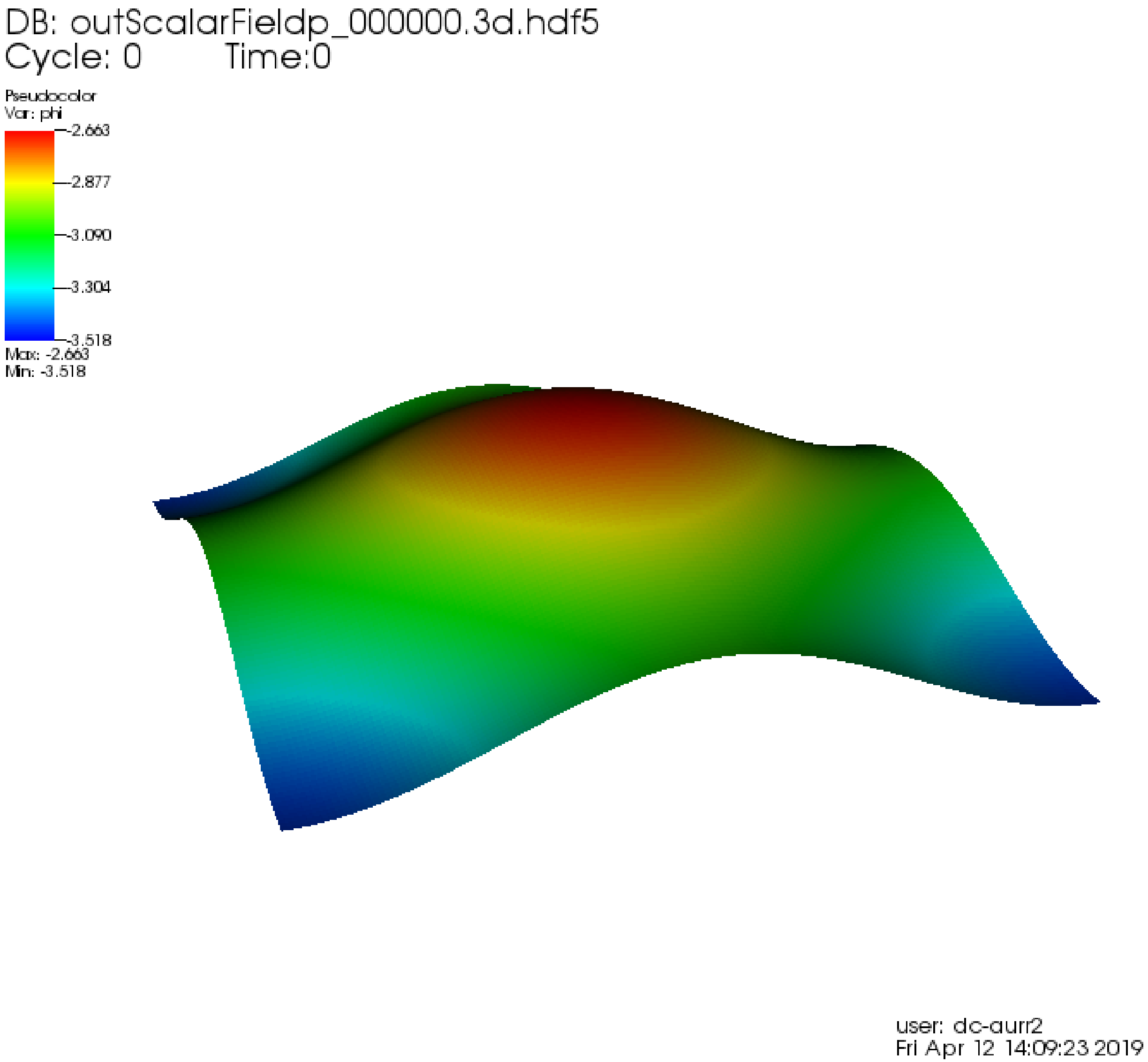
We use GRChombo:

3+1 Numerical Relativity + Adaptive Mesh

Convex

$$V(\phi) = \lambda M_{\rm Pl}^{8/3}\,(-\phi)^{4/3}$$





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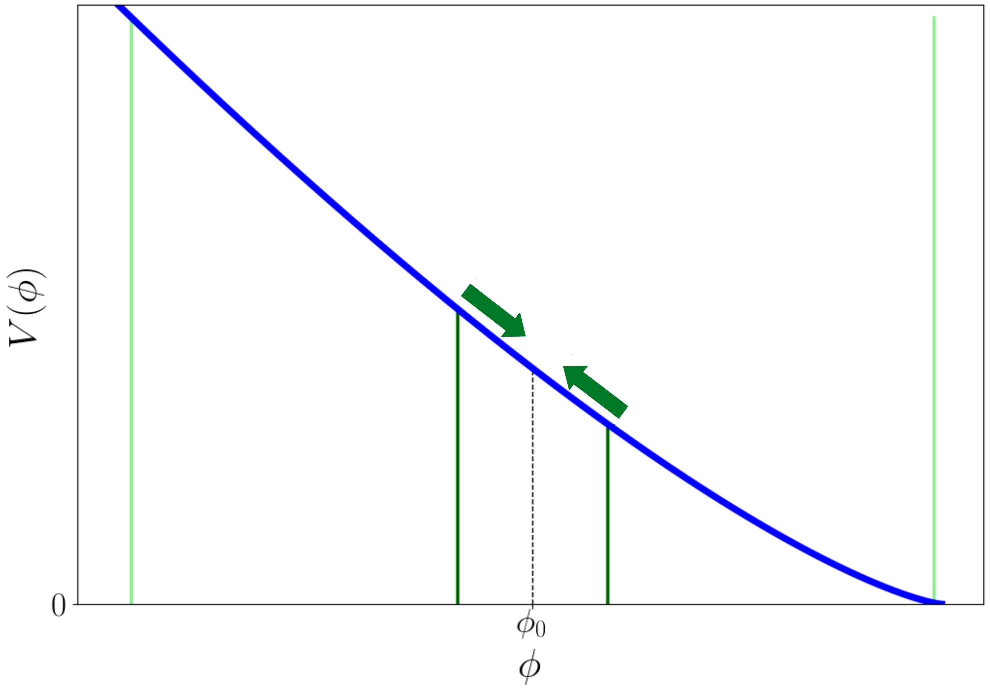

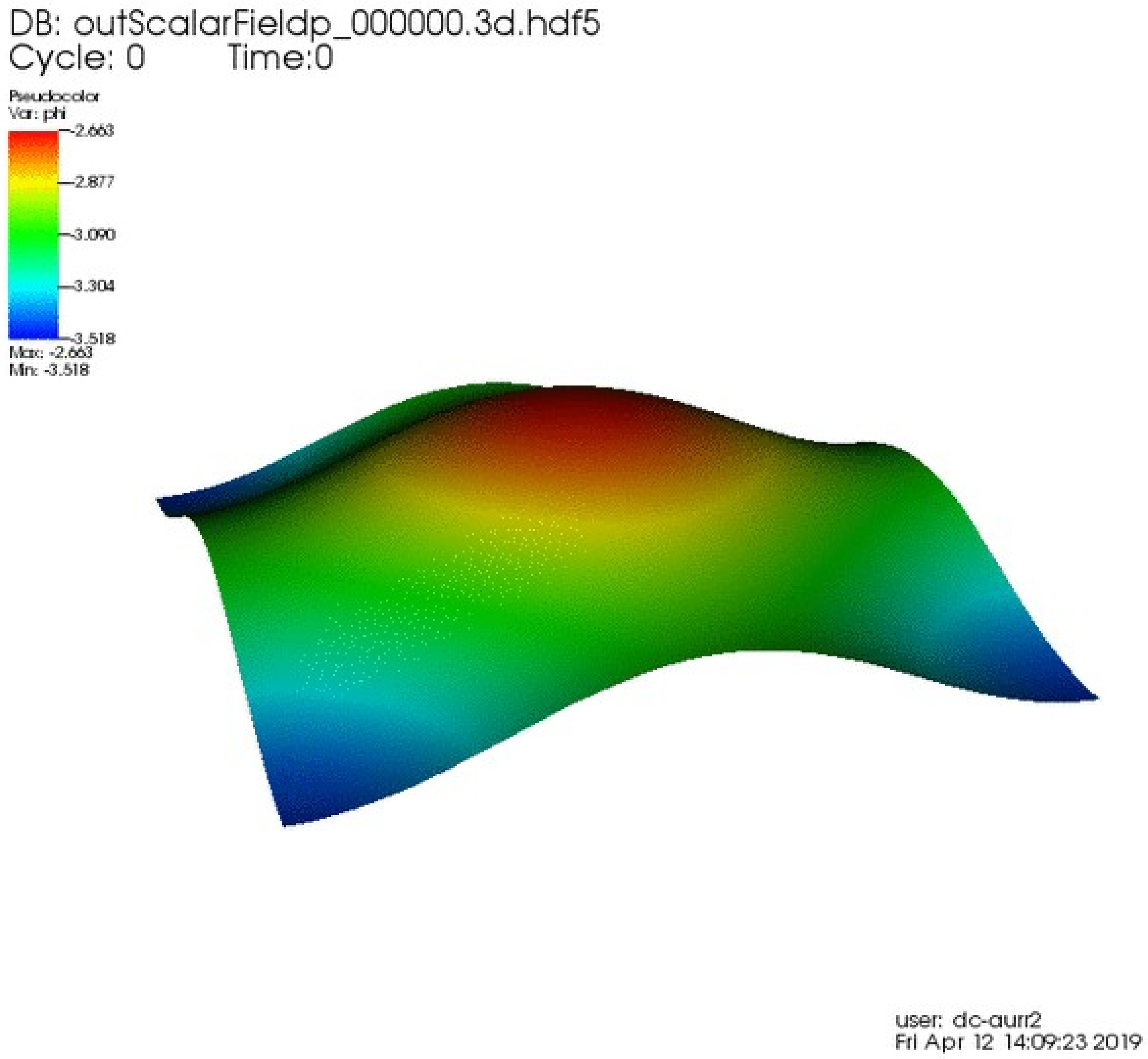

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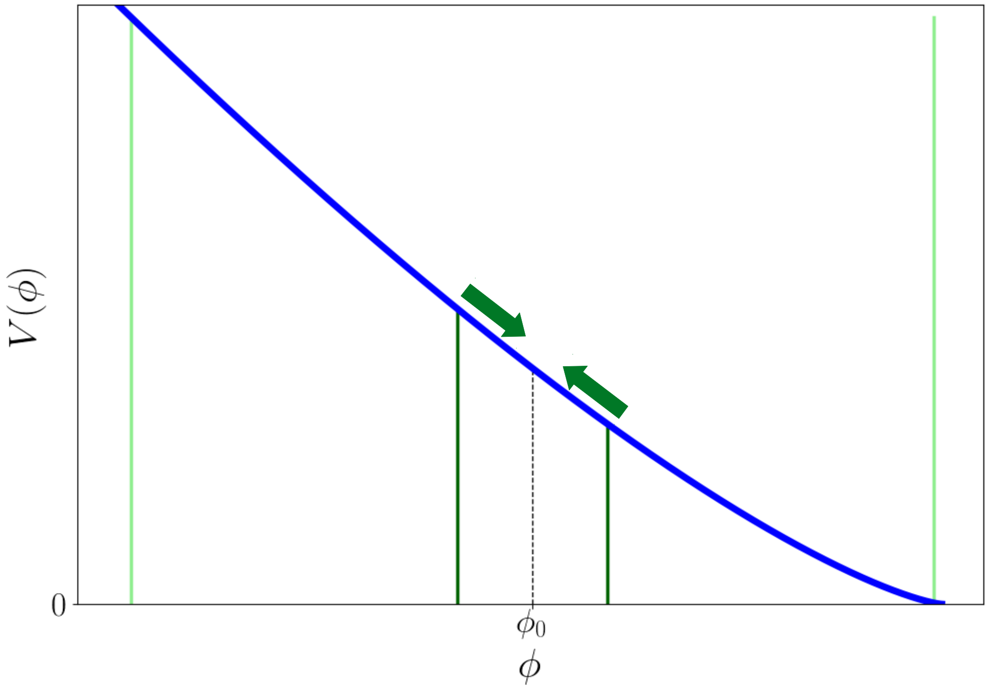

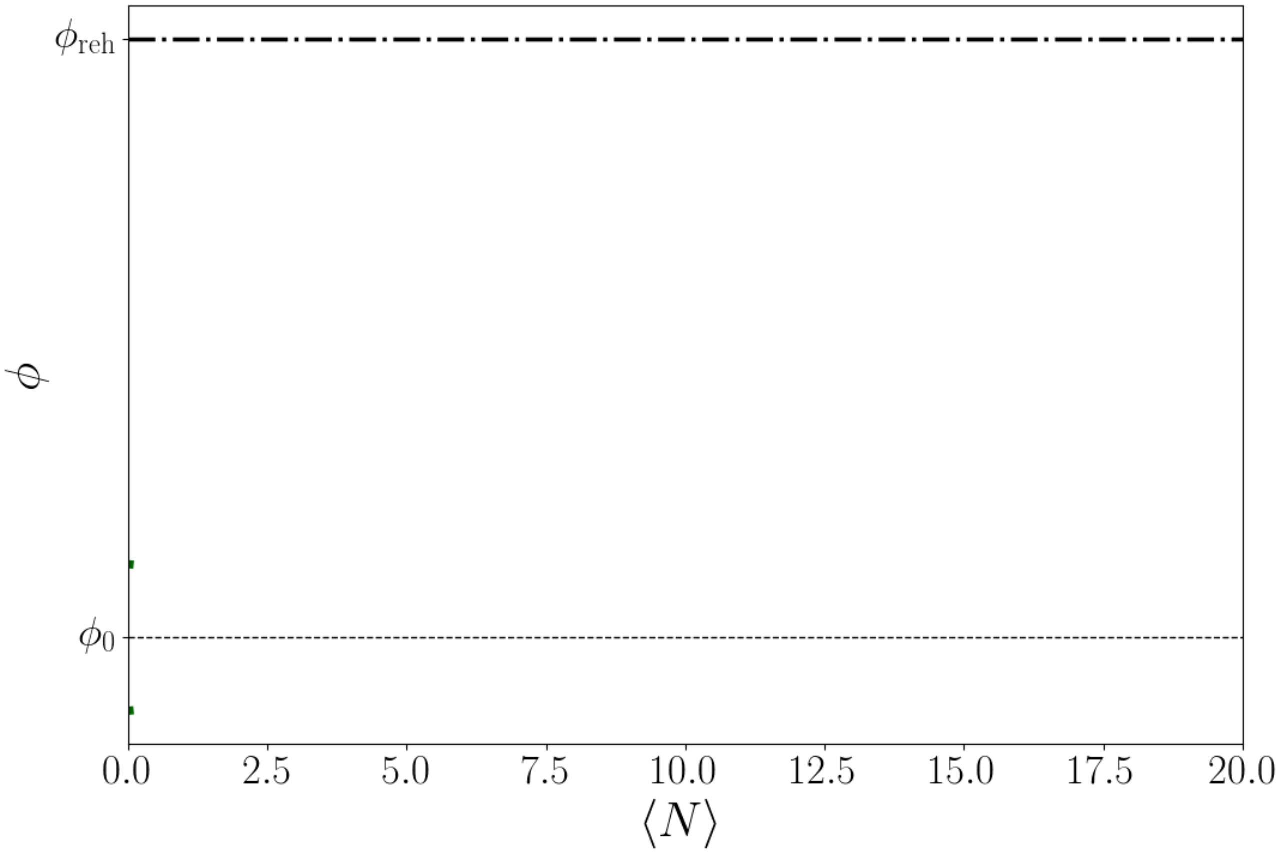

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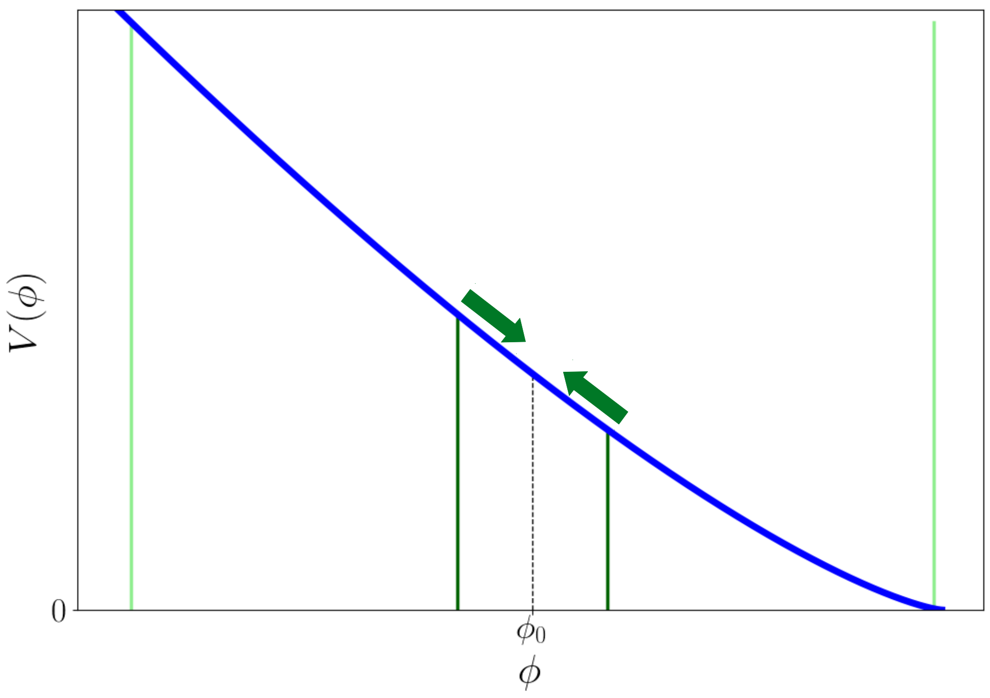

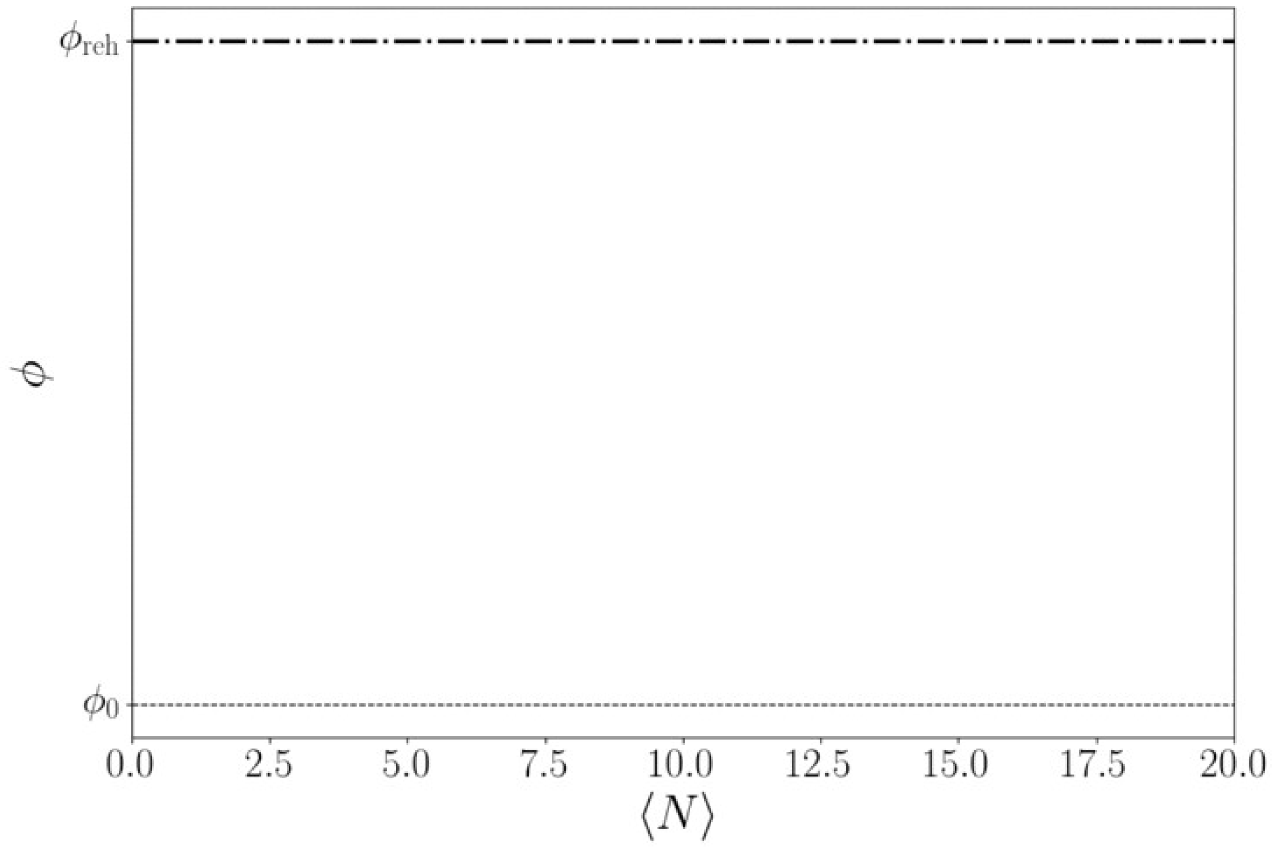

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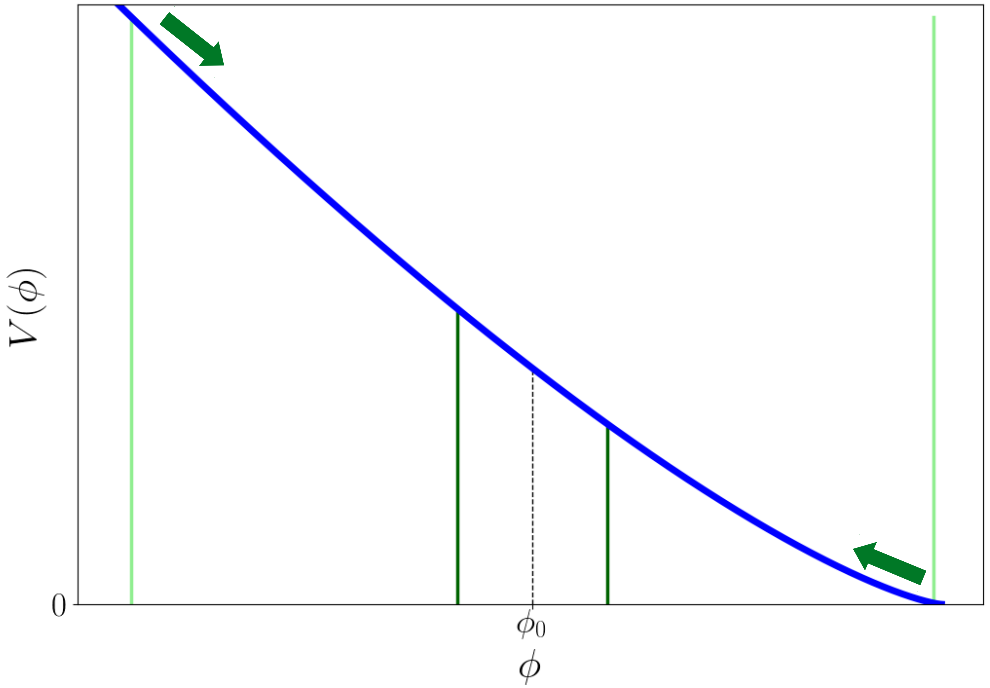

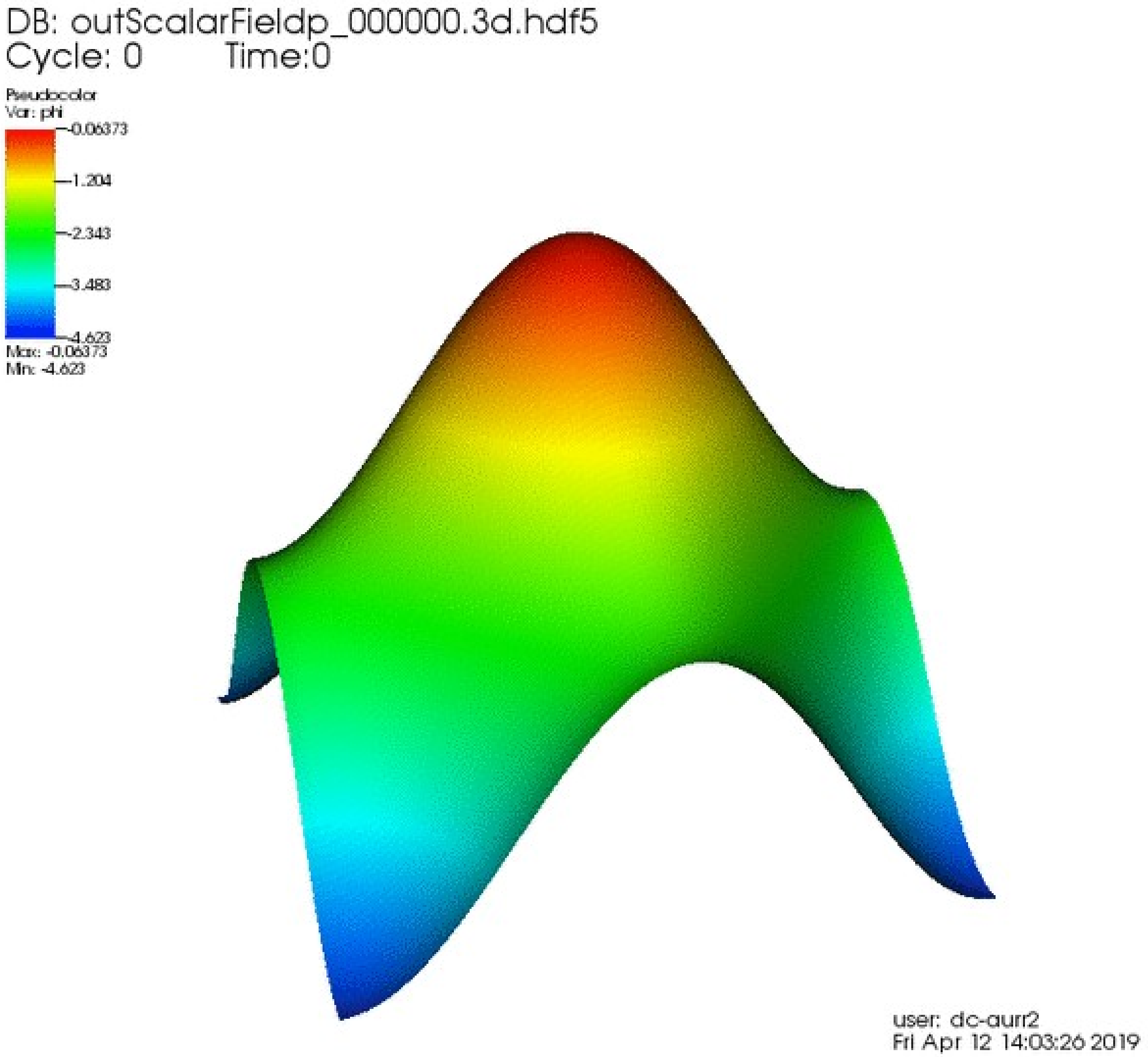

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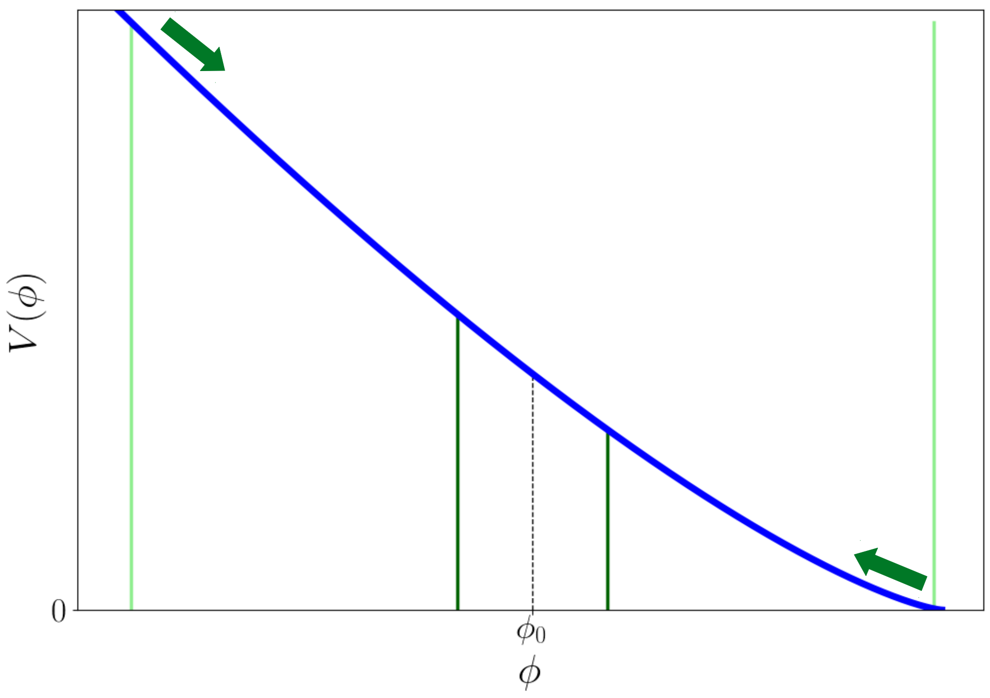
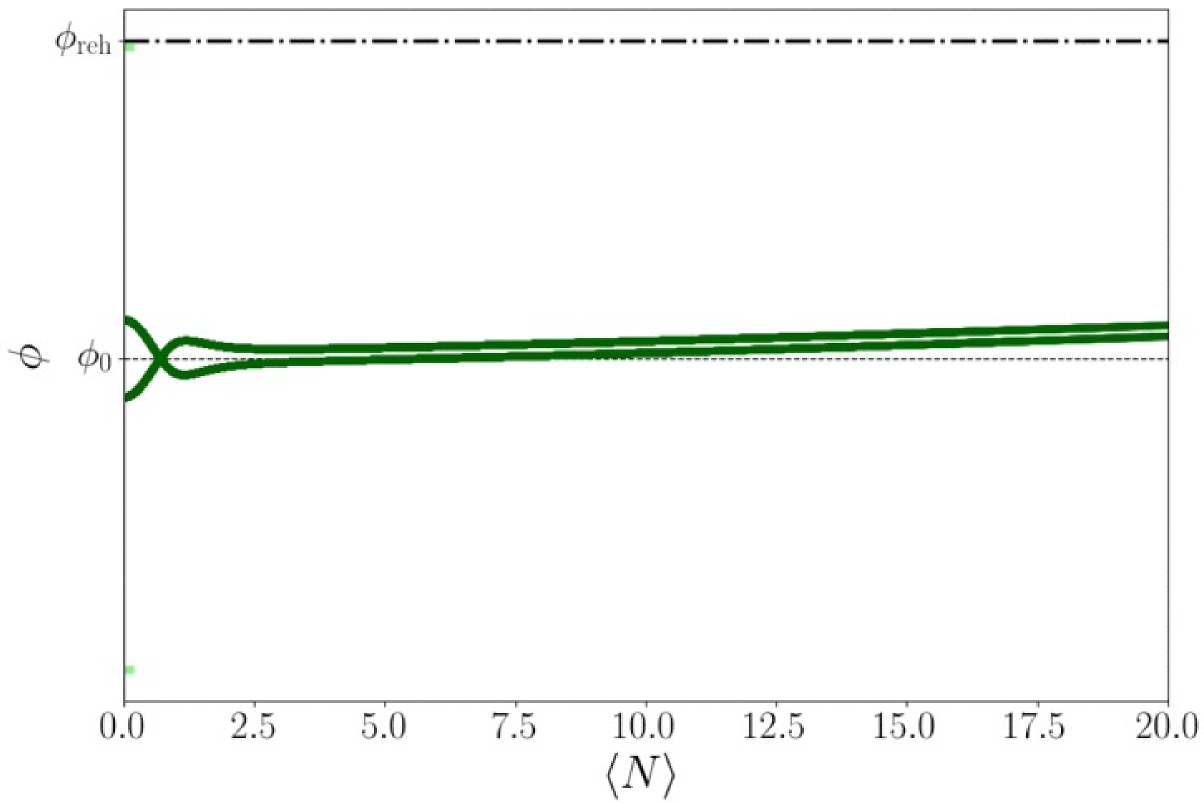
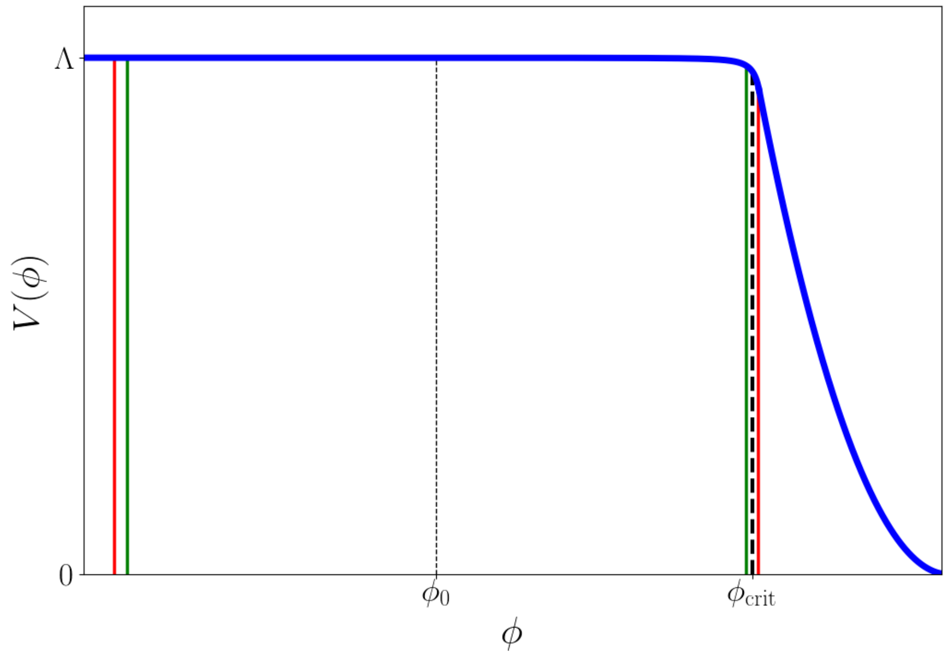




Concave (DBrane)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{\mu}{\phi}\right)^2\right)$$



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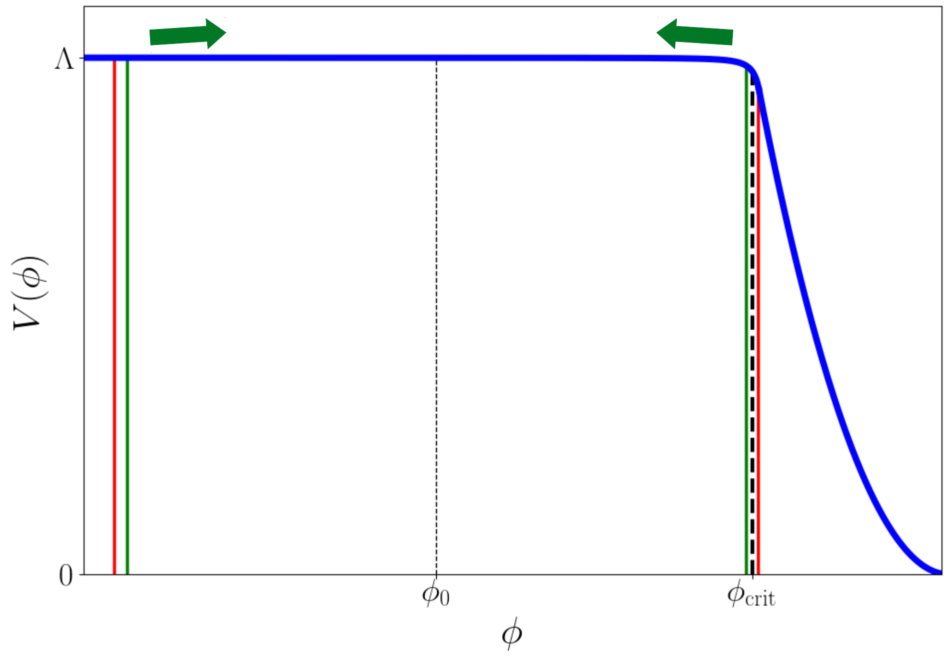

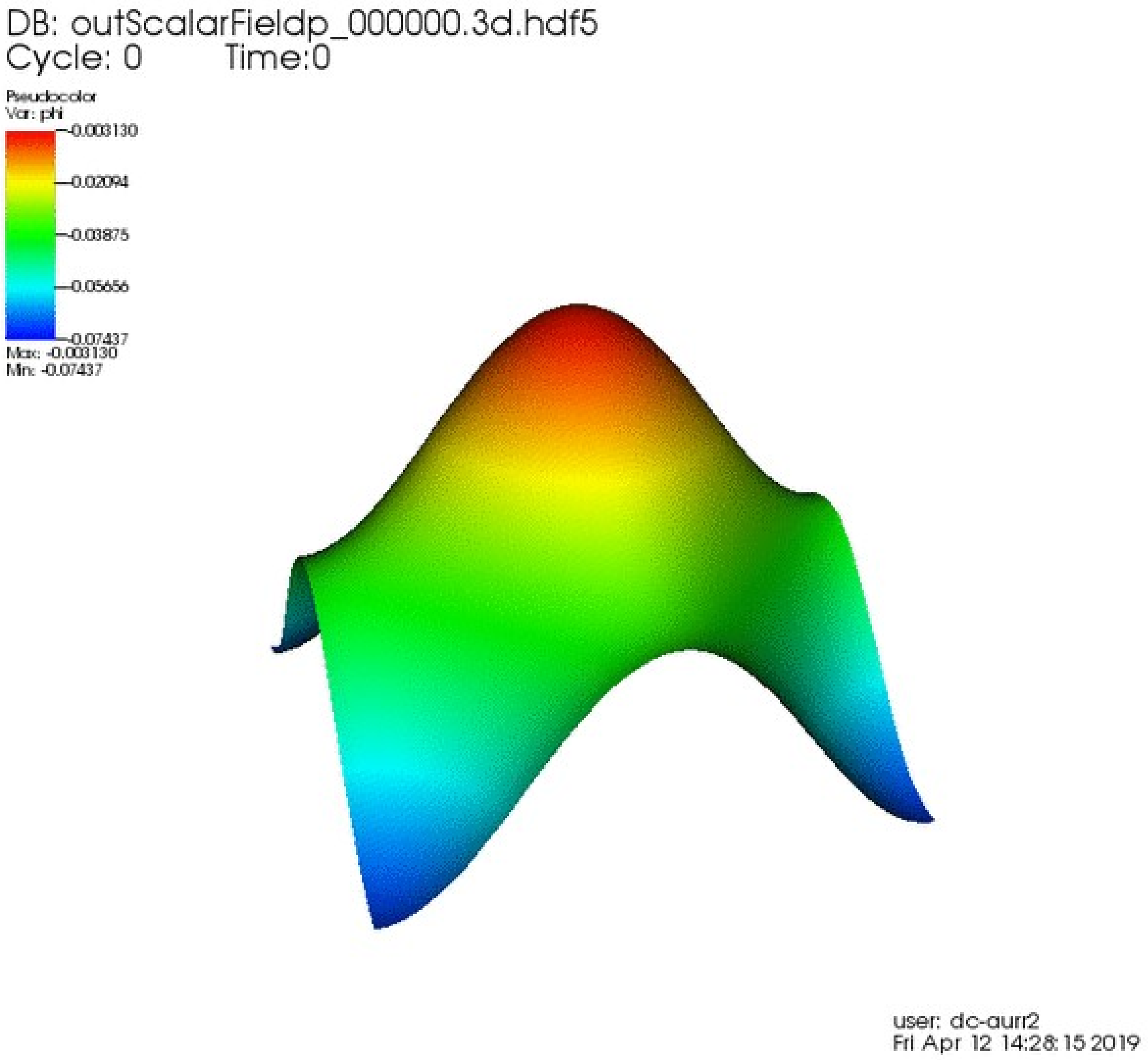

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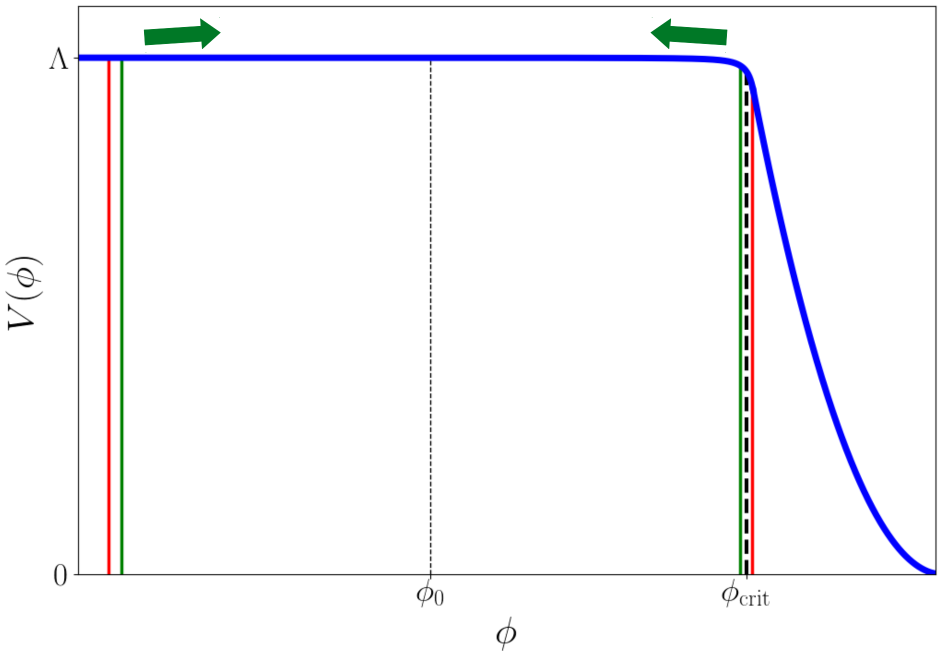

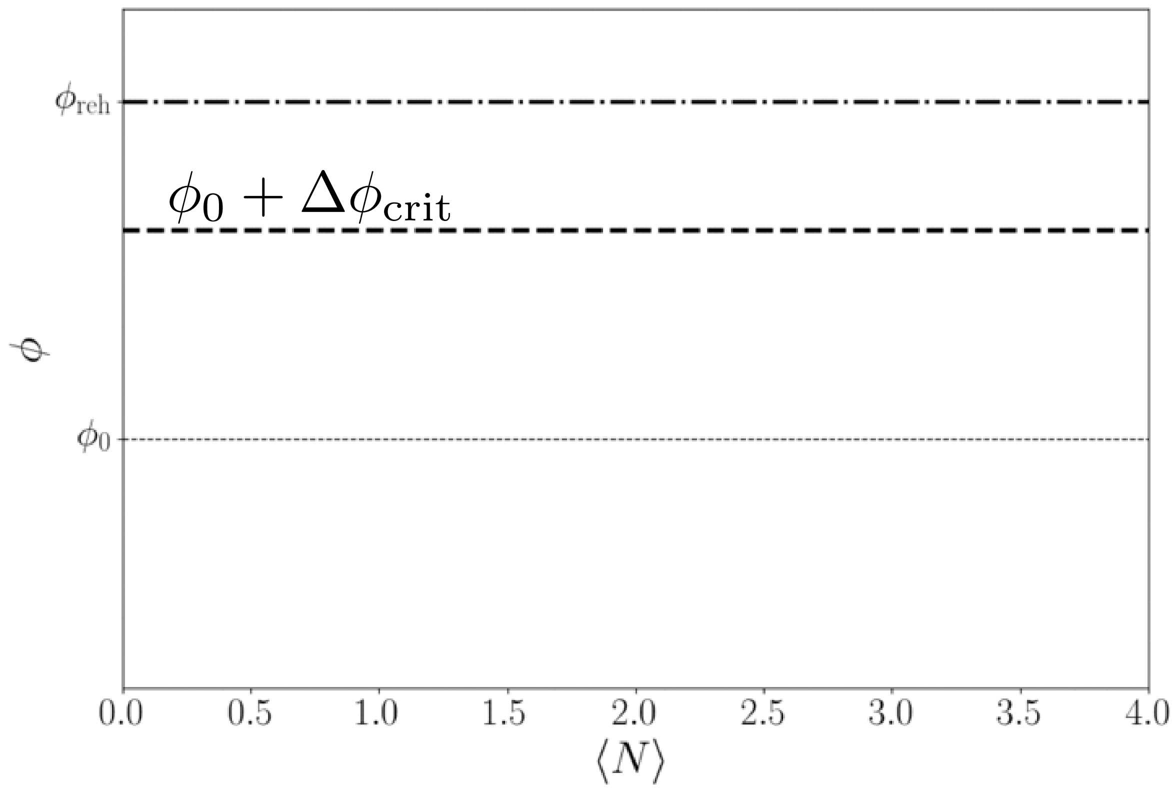

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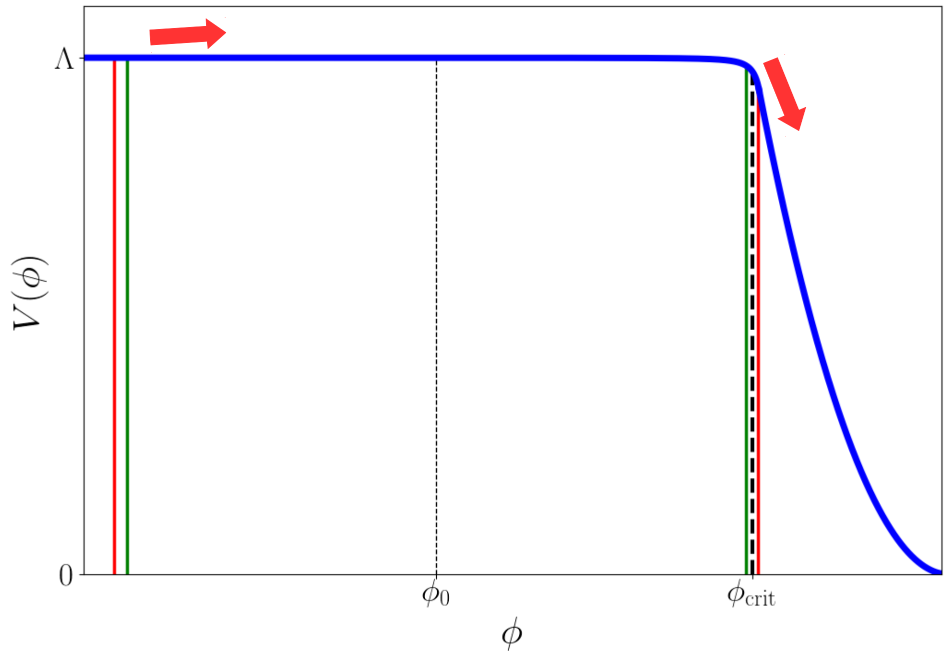

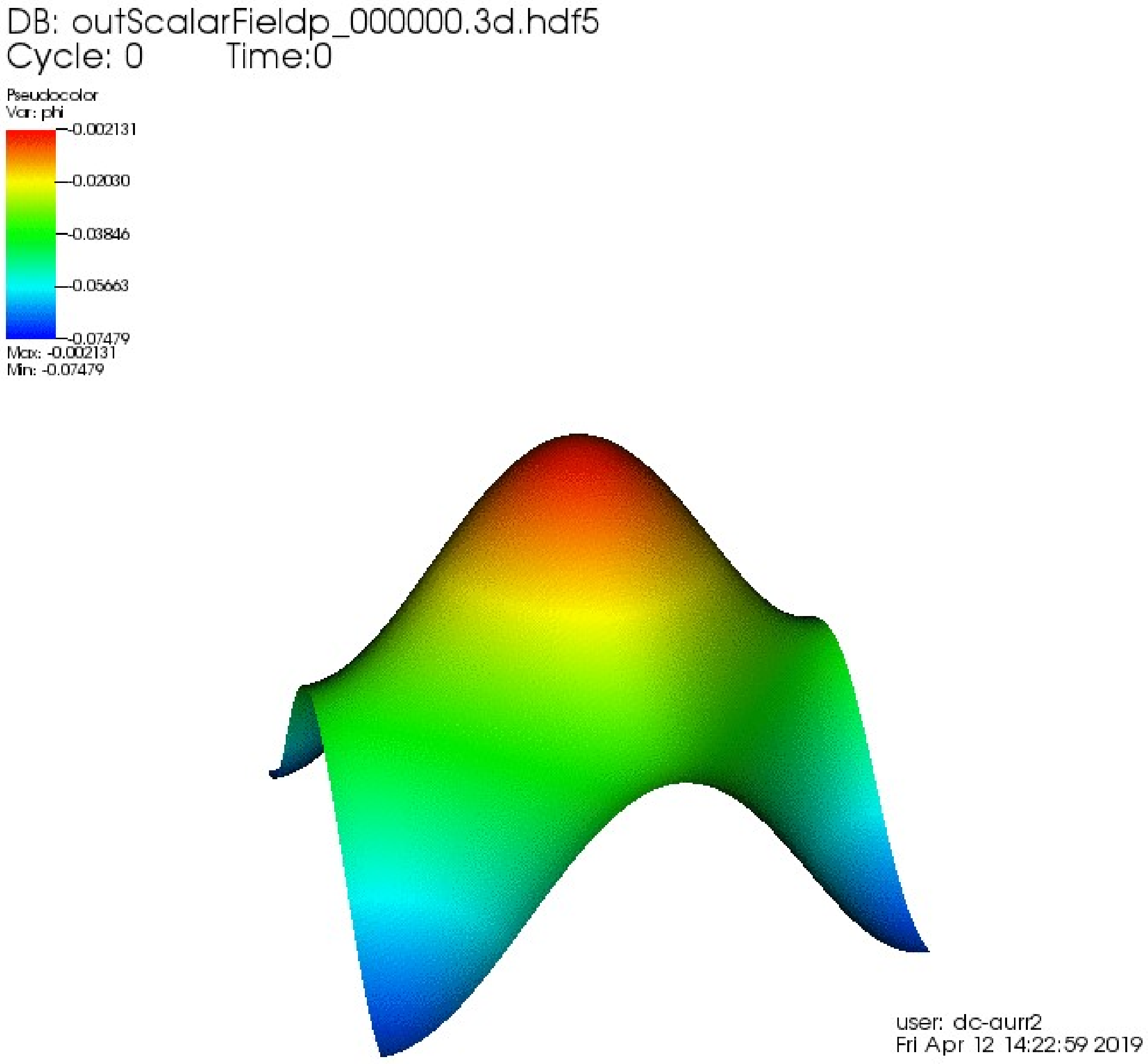

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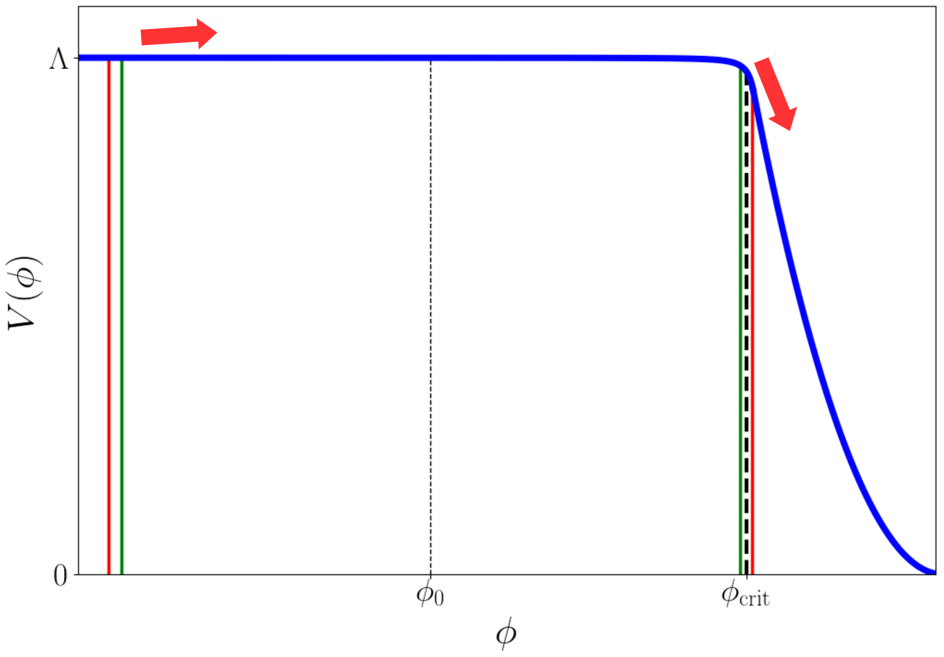

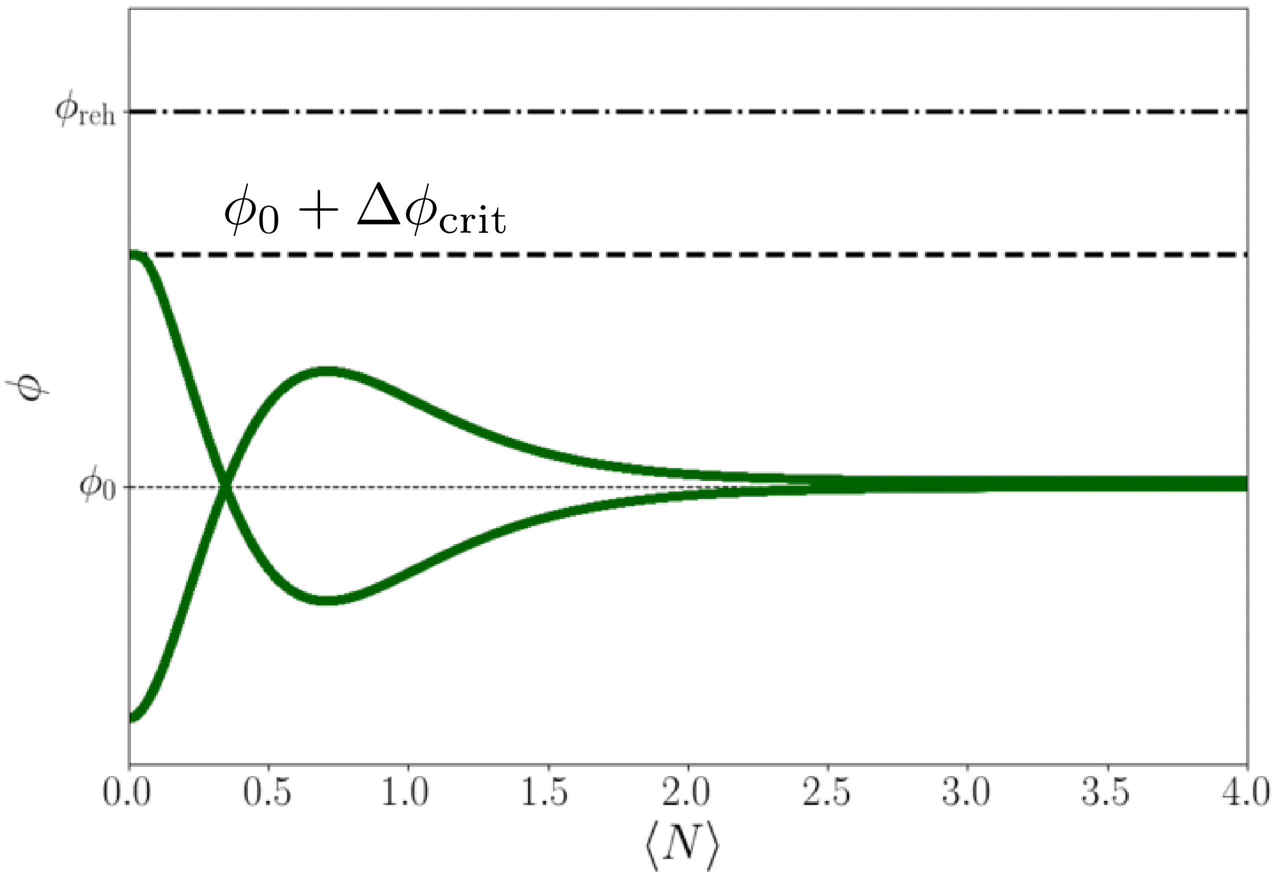

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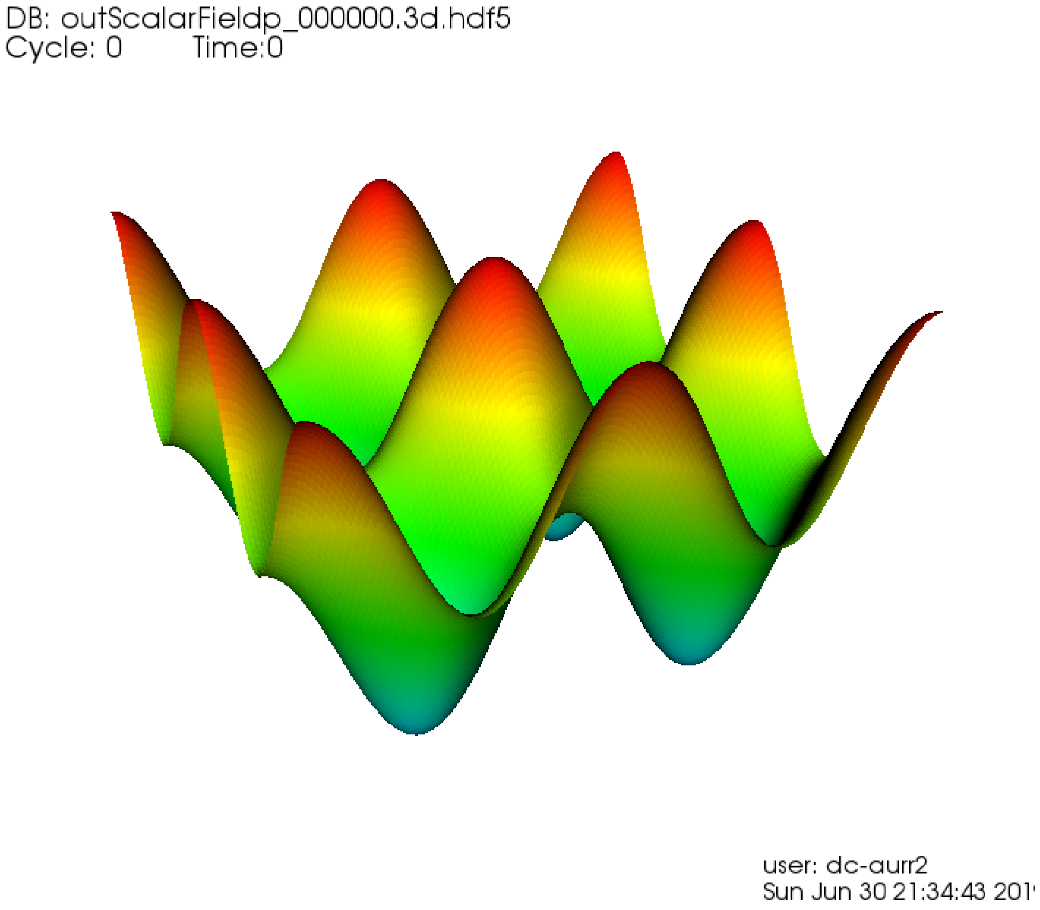

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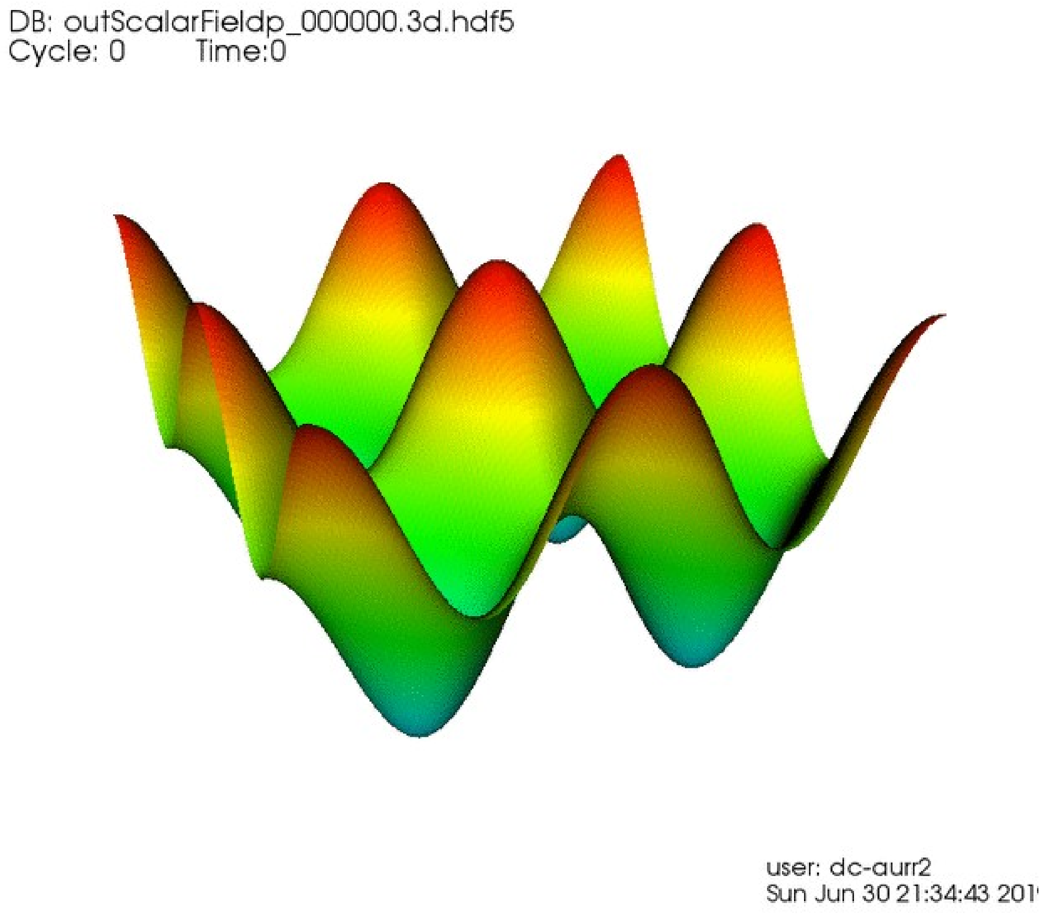

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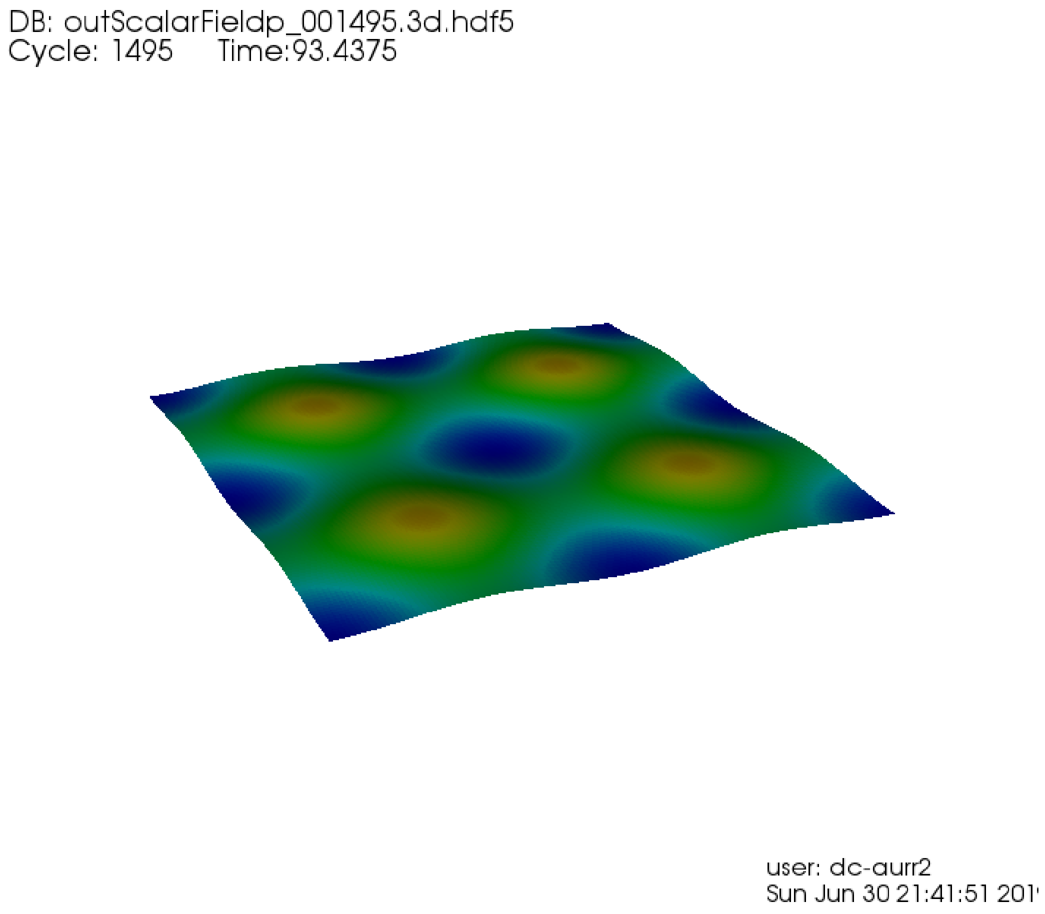

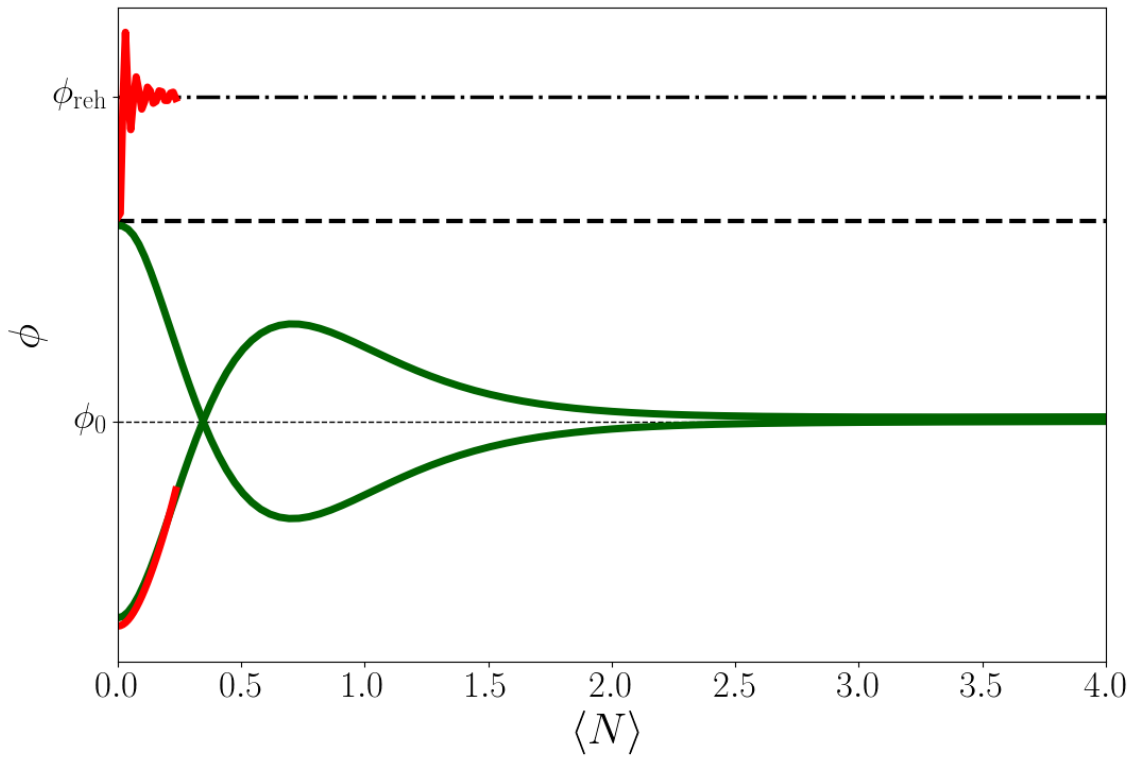

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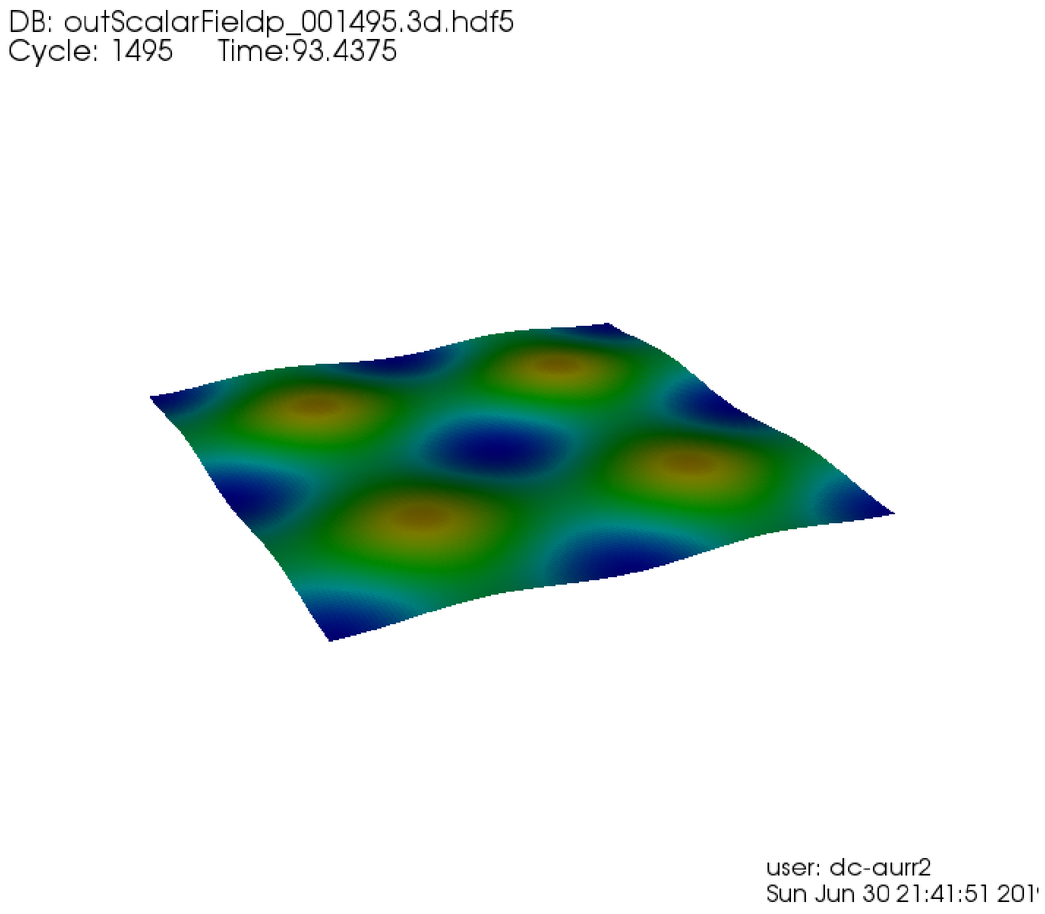

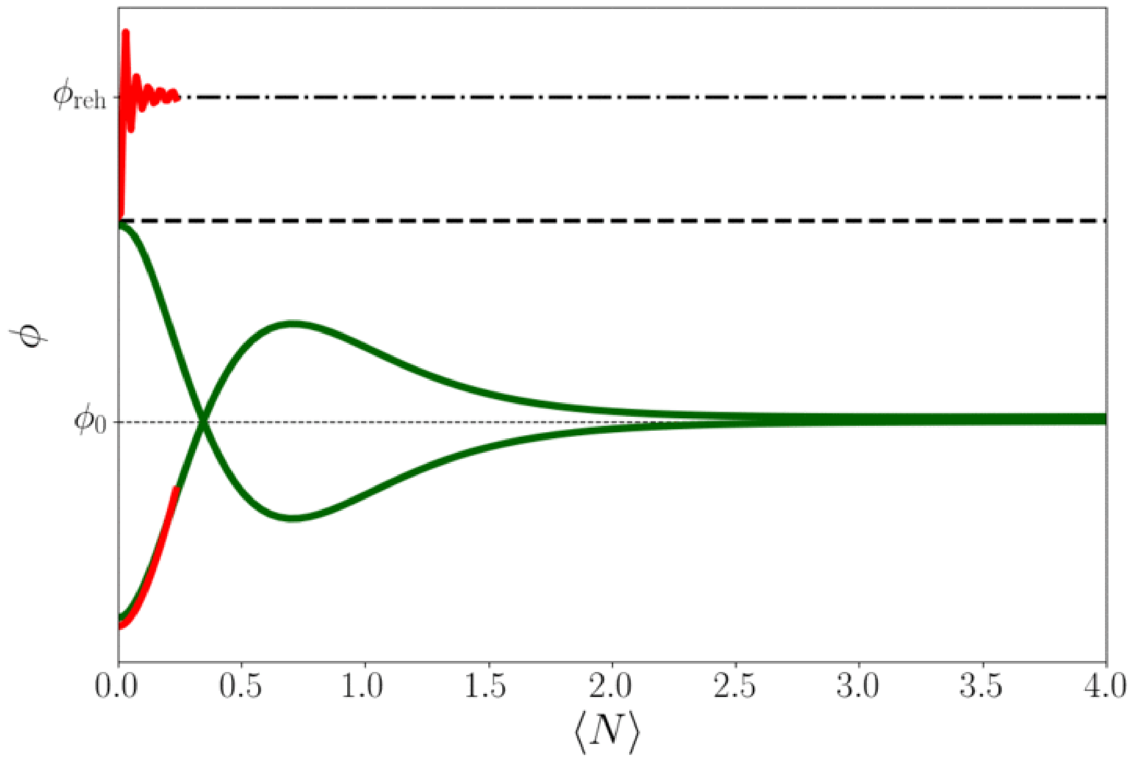

Initial Conditions

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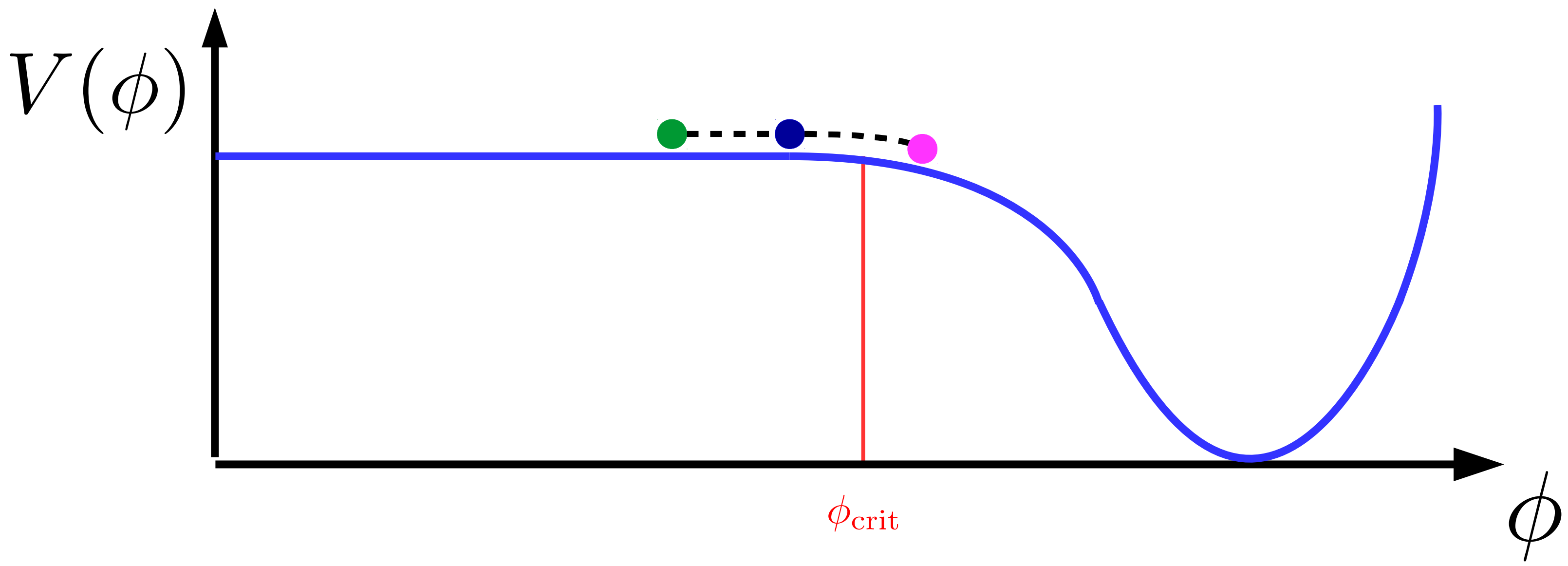
One can use this to learn about the initial conditions ϕ_0

$$f(\phi_{\max}) = f(\phi_0, \Delta\phi)$$



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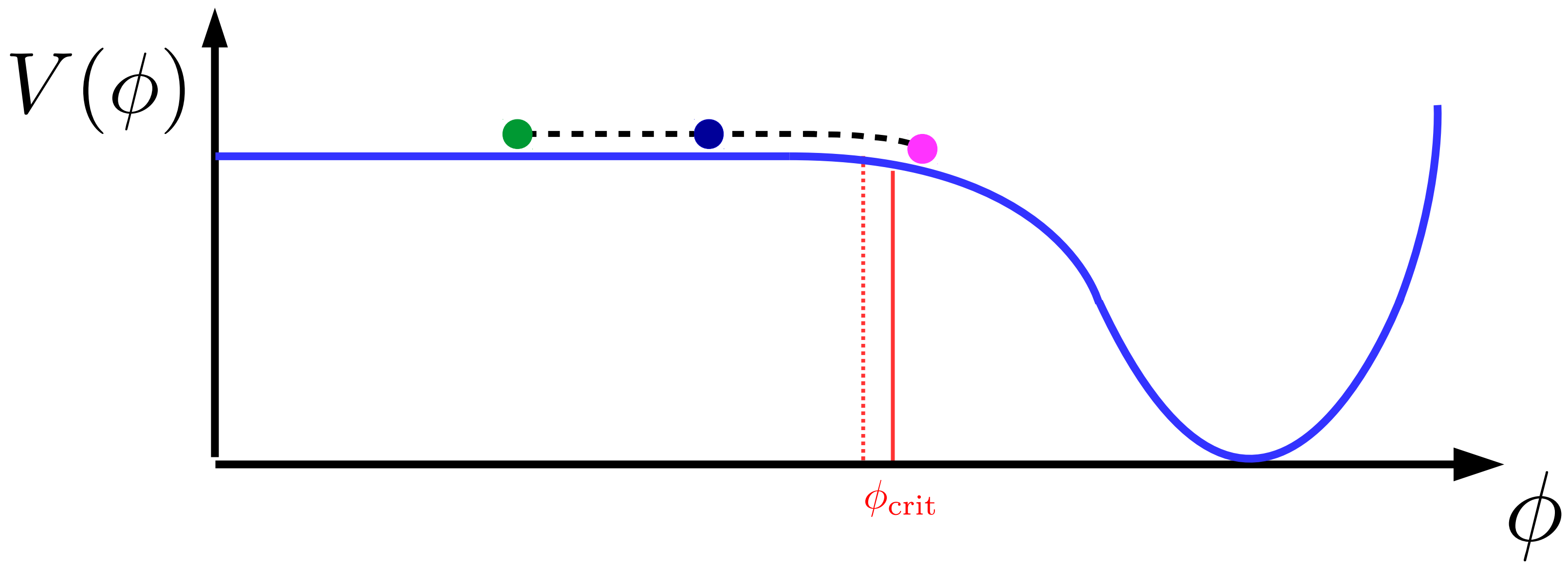

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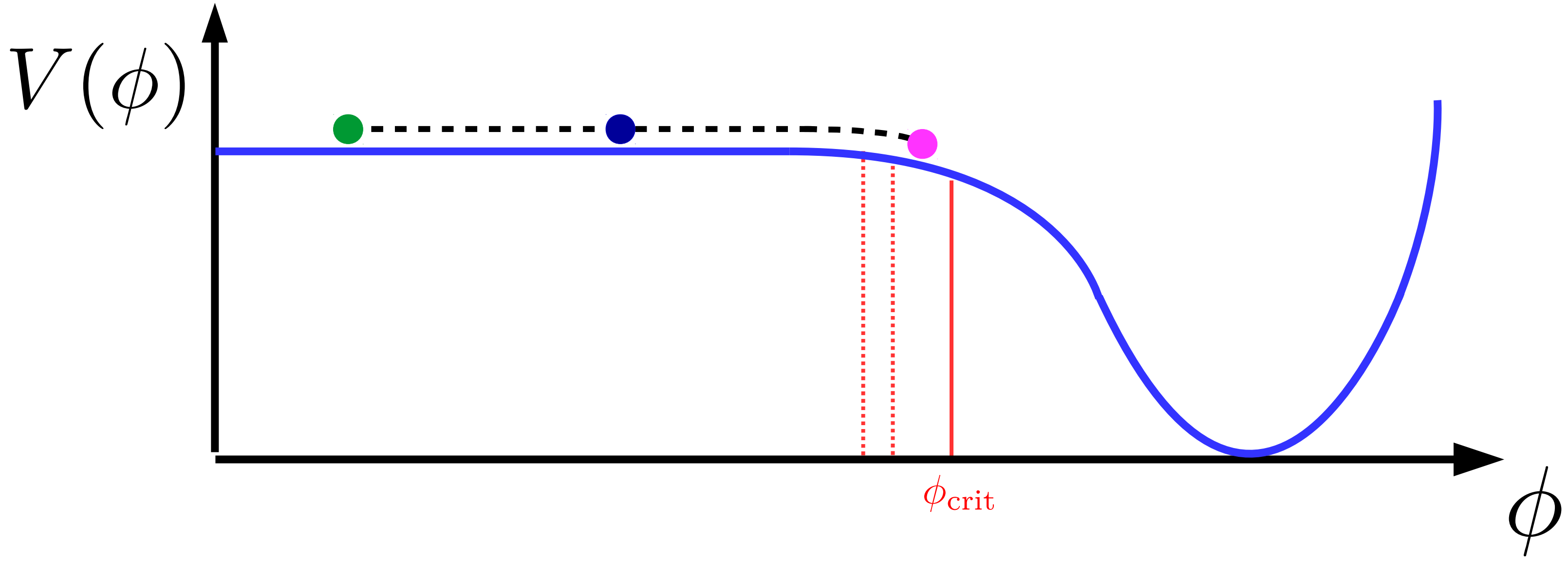

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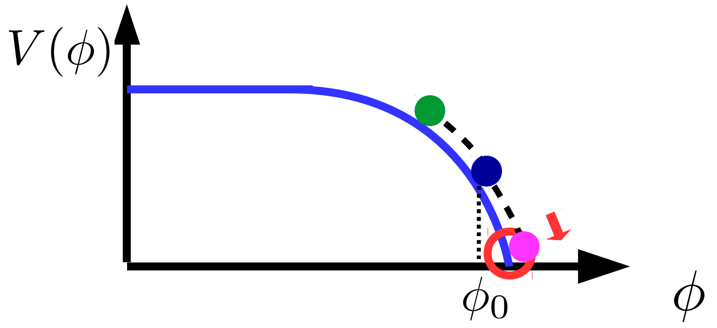


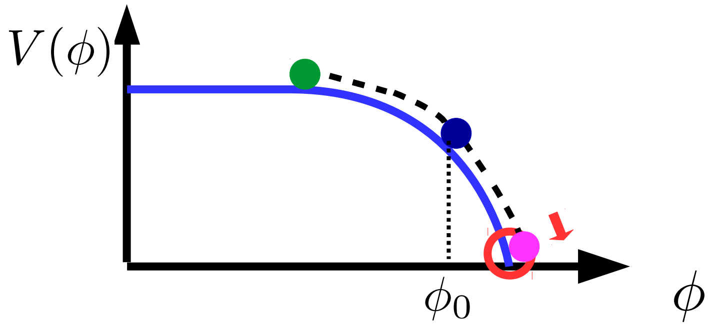
$$f(\phi_0, \Delta\phi) = k^2 \Delta\phi + \frac{dV}{d\phi}$$

Most negative $\frac{dV}{d\phi}$ at $\hat{\phi}$

If $f(\hat{\phi}) > 0$ safe

$$\Delta\phi = \hat{\phi} - \phi_0$$

Initial Conditions



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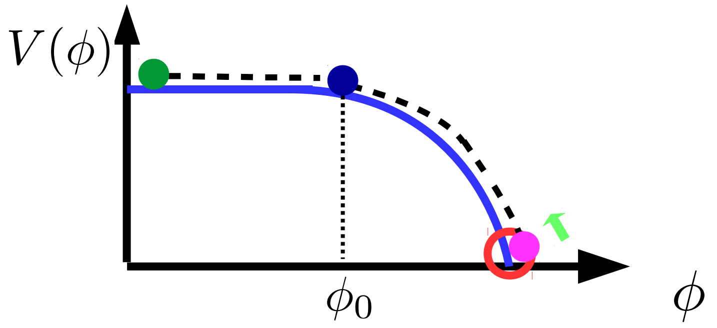

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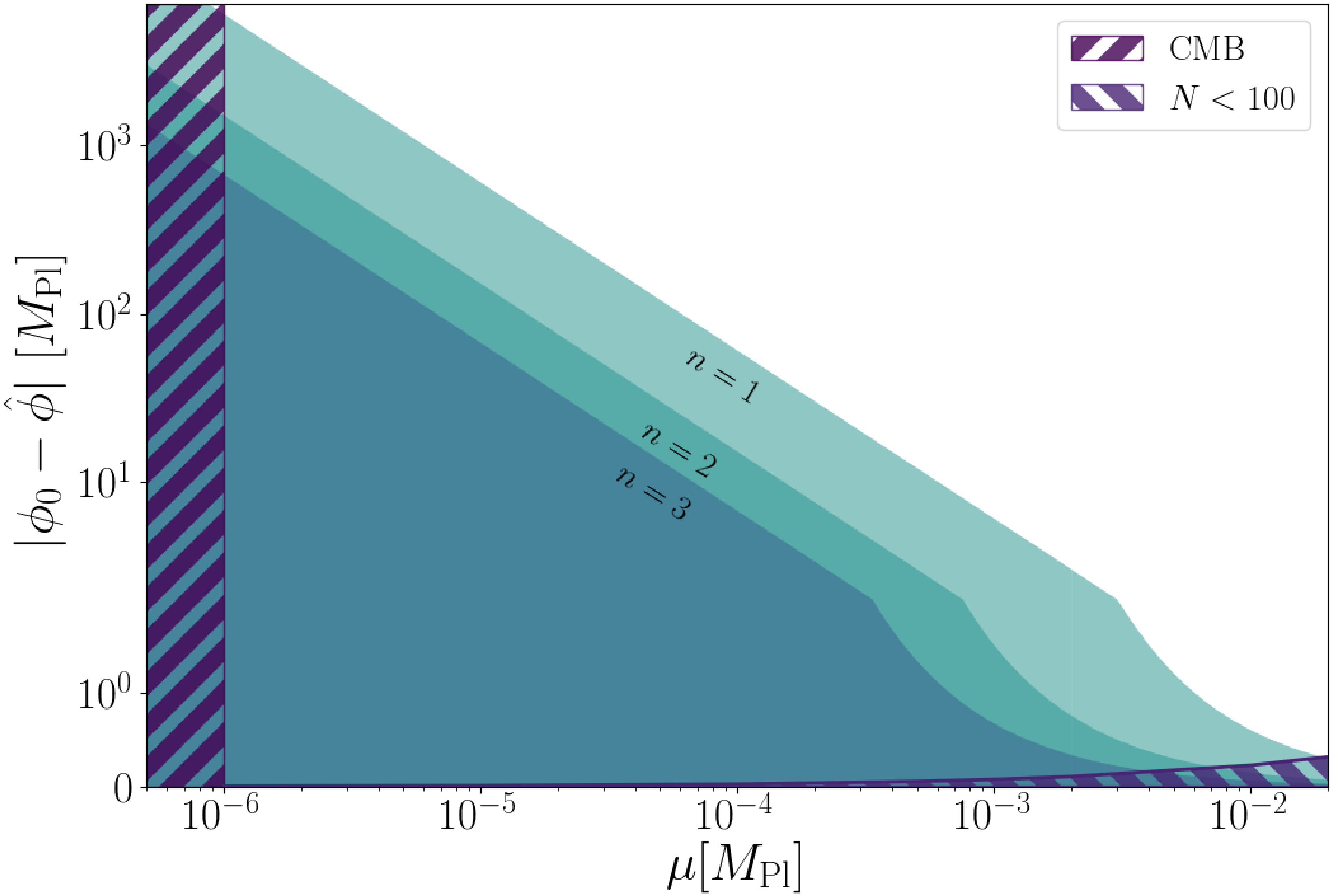
$$\Delta\phi = \hat{\phi} - \phi_0$$

$$\boxed{\phi_0 < \hat{\phi} + \frac{1}{k^2}\frac{dV(\hat{\phi})}{d\phi}}$$

Initial Conditions



Summary

- Concave models are generically less robust than Convex
- Higher modes make the model more robust
- One can use this to learn about initial conditions ϕ_0

Thank you