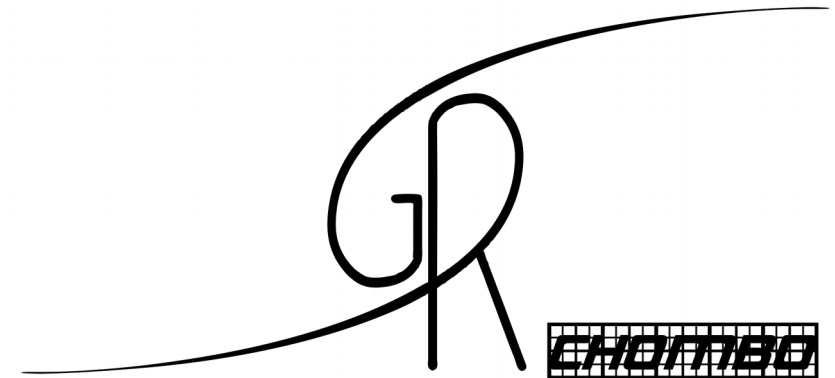


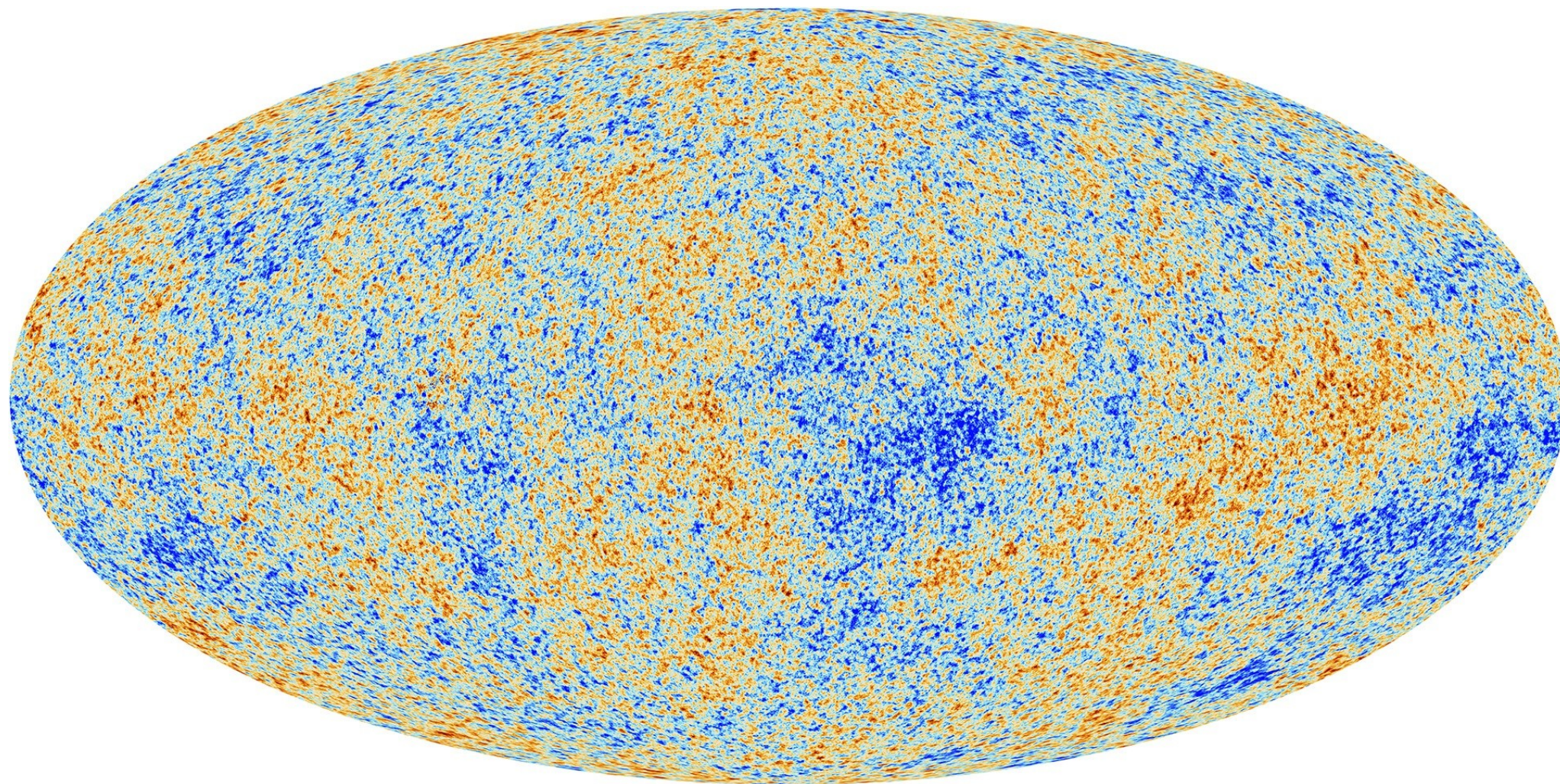
Inhomogeneous Inflation as a probe of the shape of the inflationary potential

Josu C. Aurrekoetxea

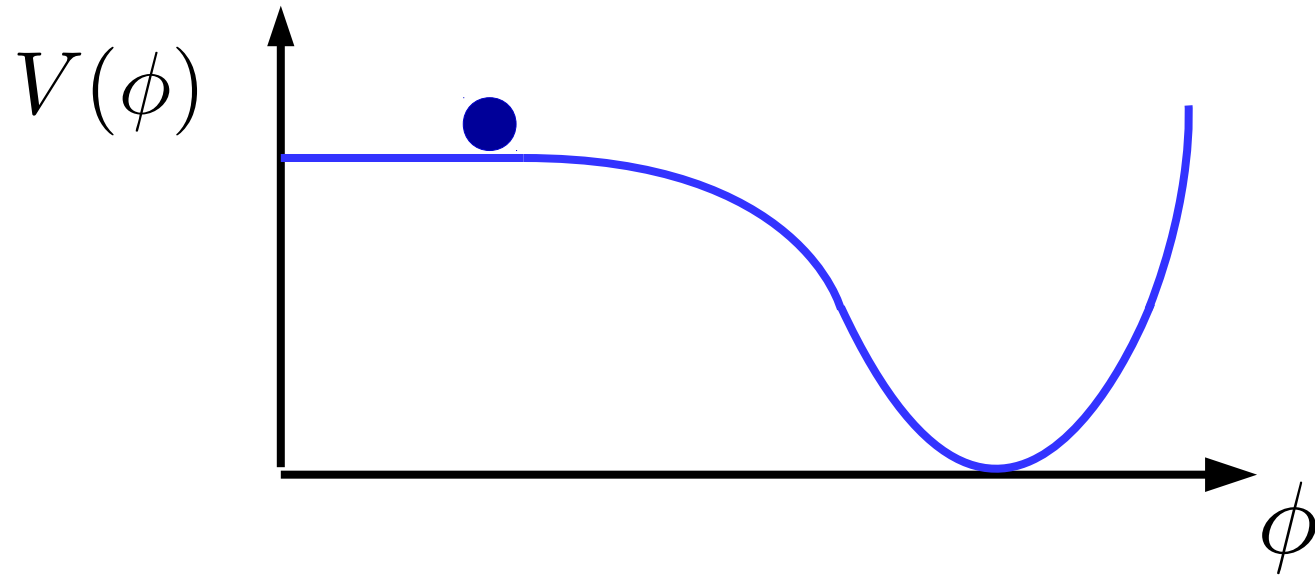
with Katy Clough, Raphael Flauger, Eugene Lim



Homogeneity Problem



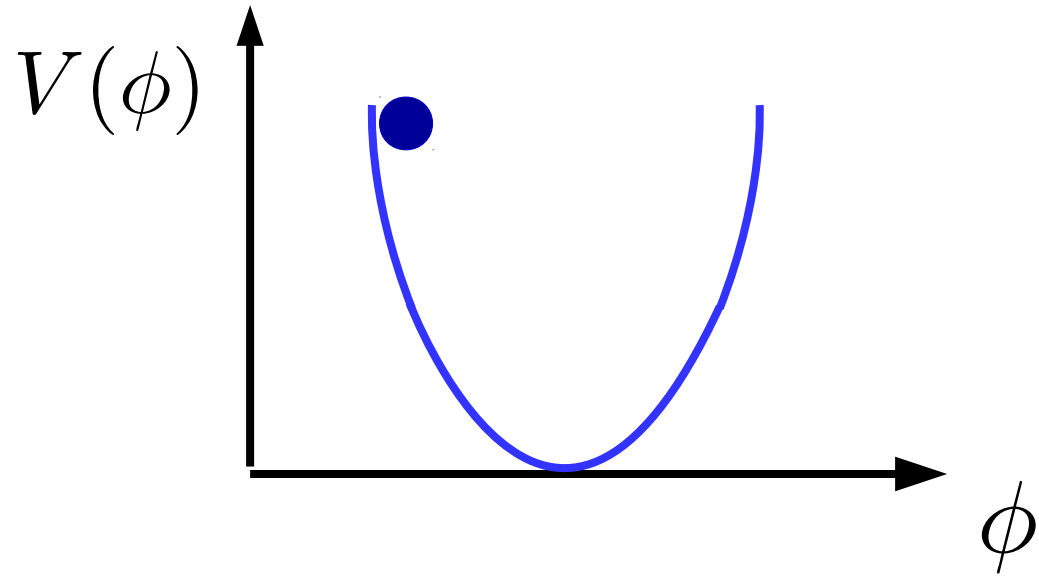
Inflation



- Homogeneous
- Slow-roll

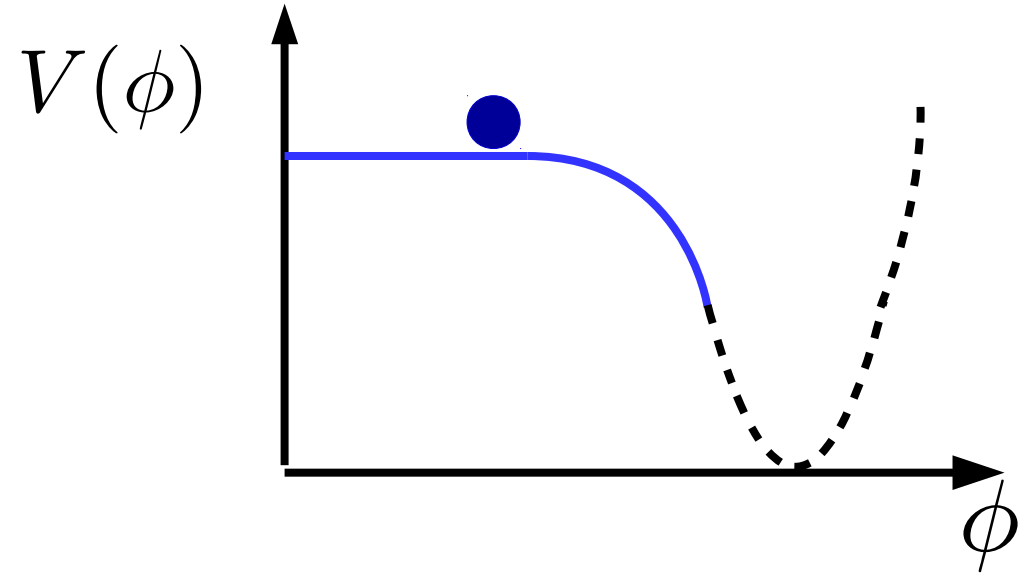
$$H^2 = \frac{8\pi}{3M_{\text{Pl}}^2} \left(\cancel{\frac{1}{2} \dot{\phi}^2} + \frac{1}{2} \cancel{(\nabla\phi)^2} + V(\phi) \right)$$

Inflation



Convex

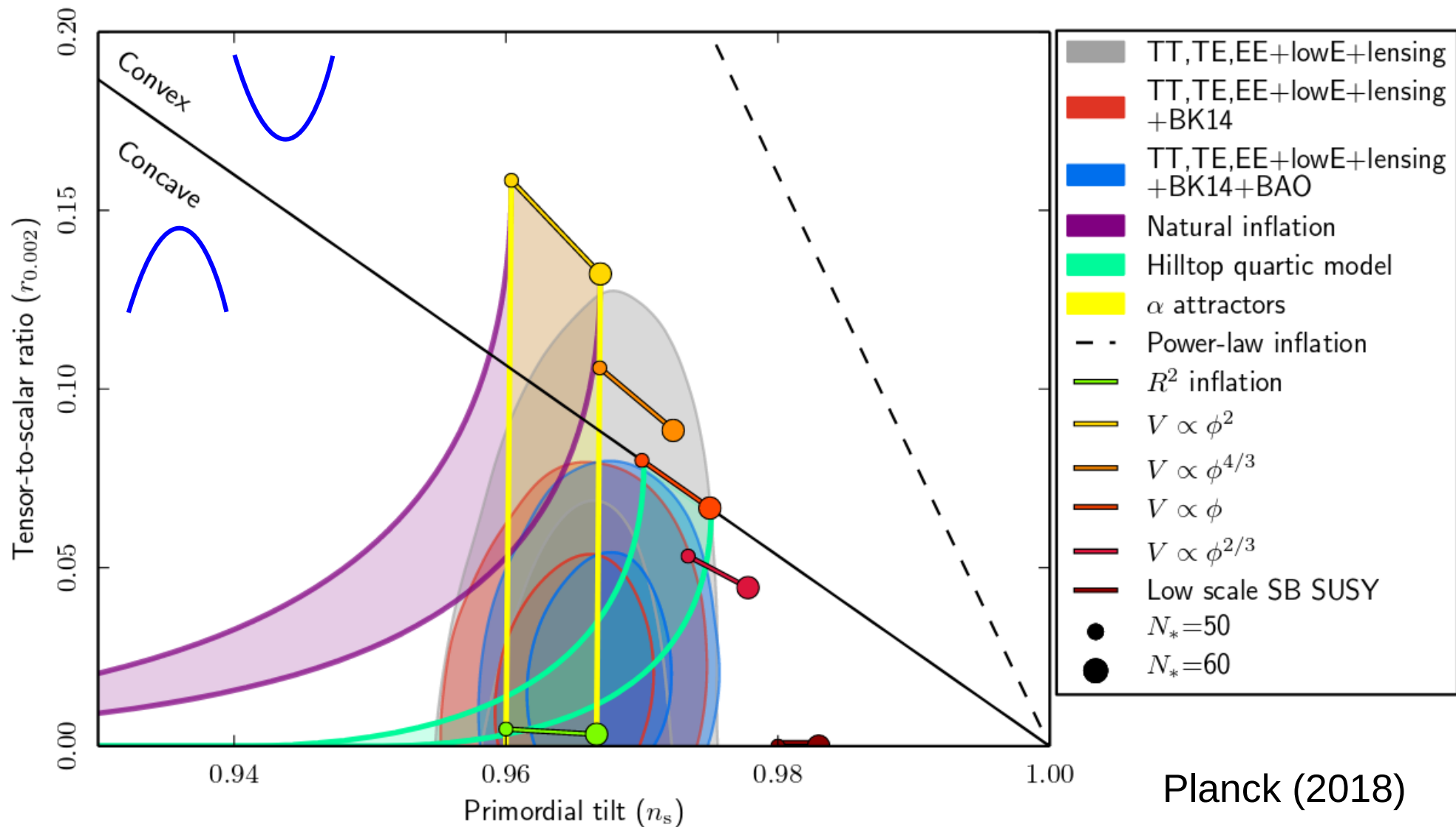
$$\frac{d^2 V}{d\phi^2} > 0$$



Concave

$$\frac{d^2 V}{d\phi^2} < 0$$

Inflation



Motivation

Problem to solve:

homogeneity

Assumption:

Pre-inflationary homogeneity

Can inflation still inflate for
inhomogeneous initial conditions?

Inhomogeneous Inflation

Numerical Relativity in Early Universe Cosmo.

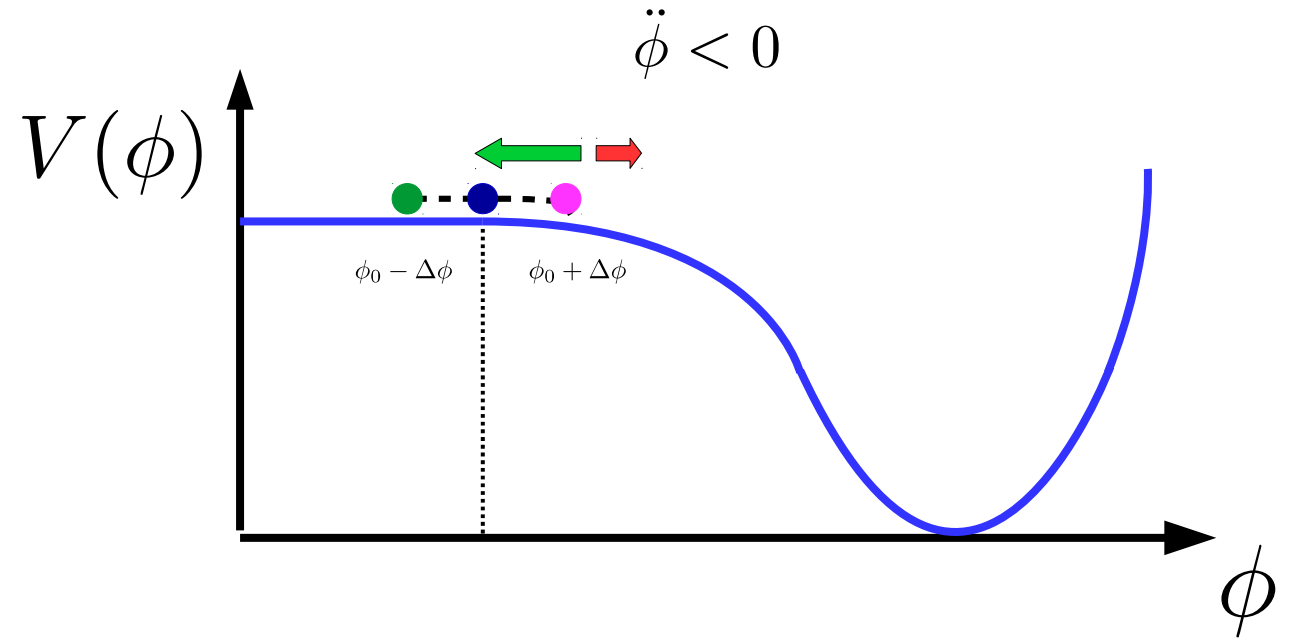
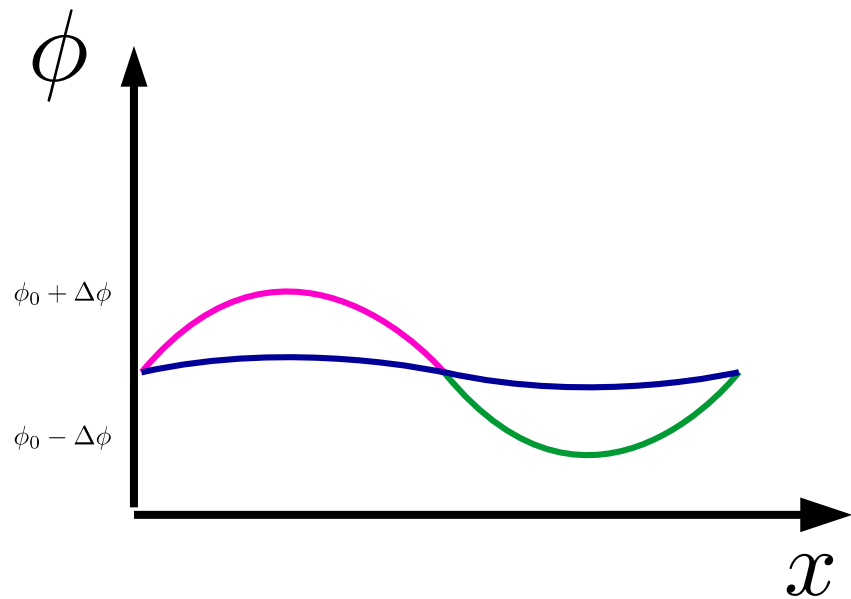
Some past/recent work:

- Goldwirth, Piran (1989)
- Laguna, Kurki-Suonio, Matzner (1991)
- Garfinkle, Chet Lim, Pretorius, Steinhardt (2008)
- Xue, Garfinkle, Pretorius, Steinhardt (2013)
- East, Kleban, Linde, Senatore (2016)
- Clough, Lim, DiNunno, Fischler, Flauger, Paban (2016)
- Clough, Flauger, Lim (2017)
- ...

Inhomogeneous Inflation

Mechanism of failure:

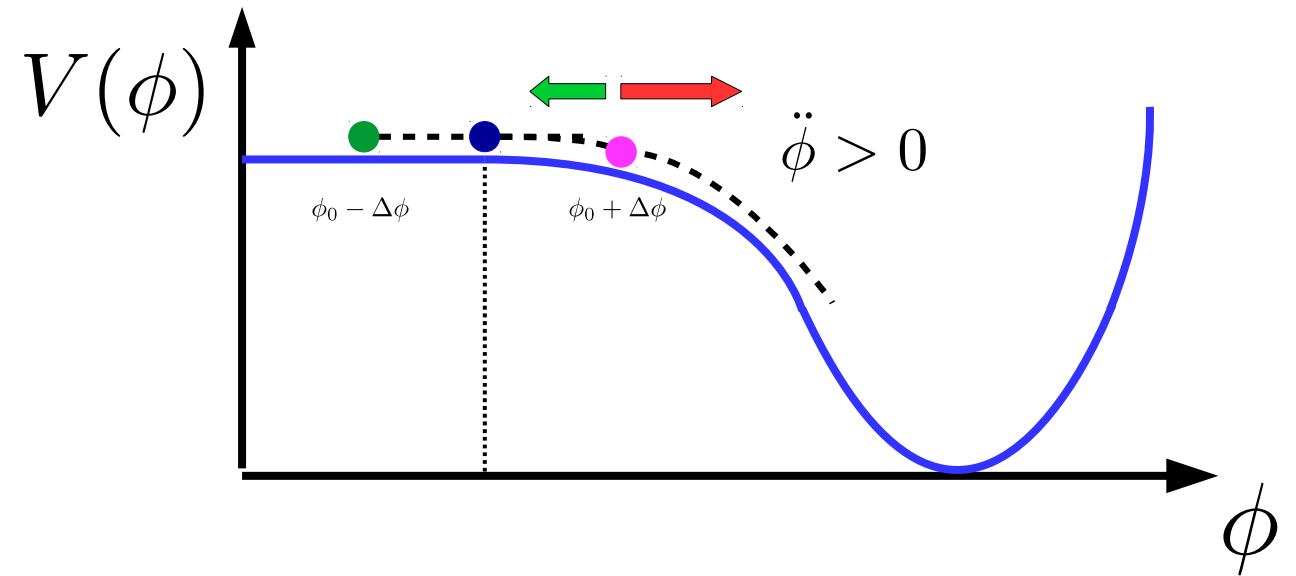
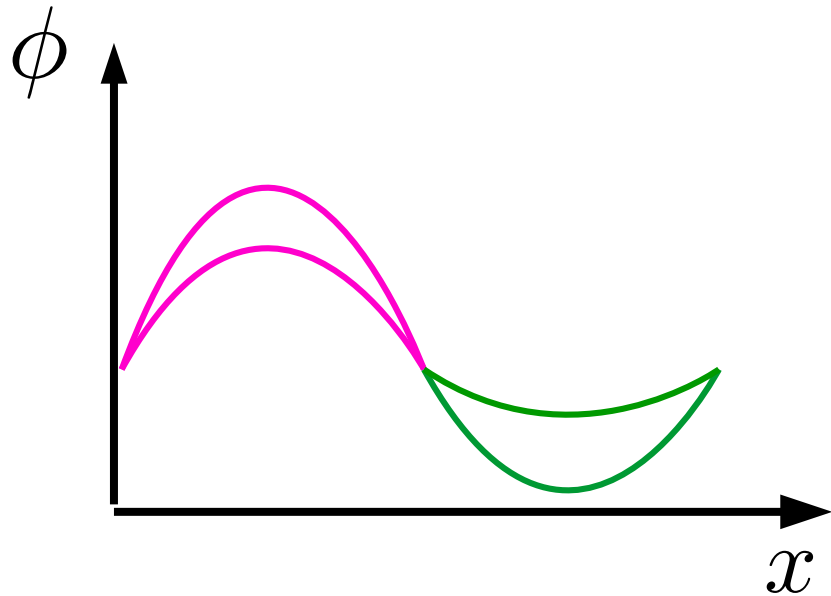
Pull-back



Inhomogeneous Inflation

Mechanism of failure:

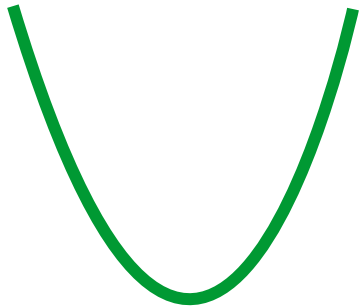
Roll-down



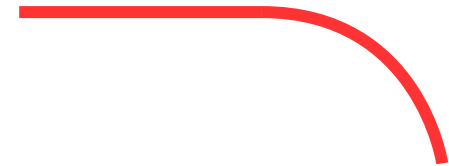
Result

Convex models CAN'T fail

Concave models CAN fail



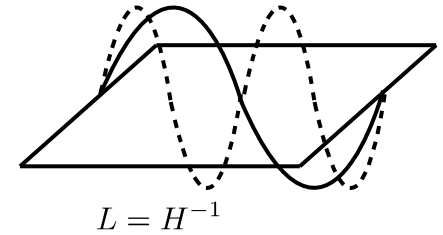
better than



Proof

Ansatz: $\phi(\mathbf{x}, t = 0) = \phi_0 + \Delta\phi e^{i\vec{k}\cdot\vec{x}}$

$$k = 2\pi nH$$



KG: $\ddot{\phi} + 3H\dot{\phi} + k^2\Delta\phi + \frac{dV}{d\phi} = 0$

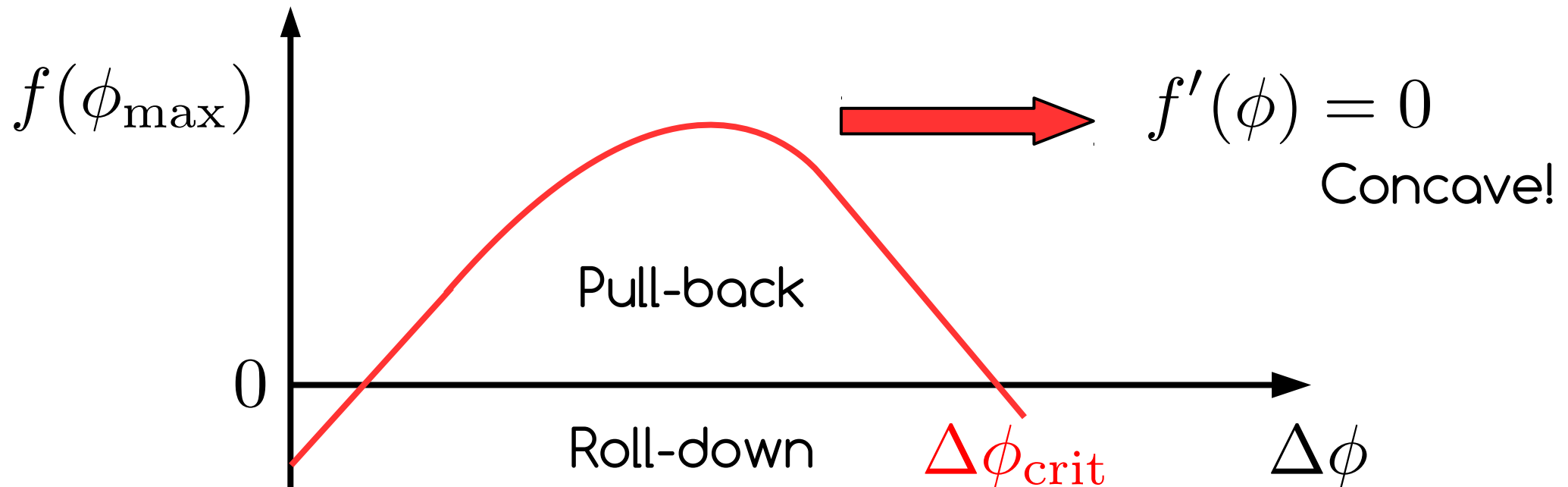
$$\ddot{\phi} = -k^2\Delta\phi - \frac{dV(\phi_{\max})}{d\phi}$$

Proof

$$f(\phi_{\max}) = k^2 \Delta\phi + \frac{dV(\phi_{\max})}{d\phi}$$

Proof

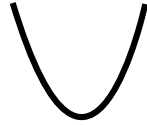
$$f(\phi_{\max}) = k^2 \Delta\phi + \frac{dV(\phi_{\max})}{d\phi}$$




Proof

$$f'(\phi_{\max}) = k^2 + \frac{d^2V}{d\phi^2}$$

> 0 > 0
 < 0

Convex $\frac{d^2V}{d\phi^2} > 0$ 

Concave $\frac{d^2V}{d\phi^2} < 0$ 

$$k^2 + \frac{d^2V}{d\phi^2} \neq 0 \quad \text{Robust!}$$

$$k^2 + \frac{d^2V}{d\phi^2} = 0 \quad \text{Can fail!}$$

Results

Results

Need to use FULL Numerical Relativity:

Solve non-linear Einstein eq + Scalar Field:

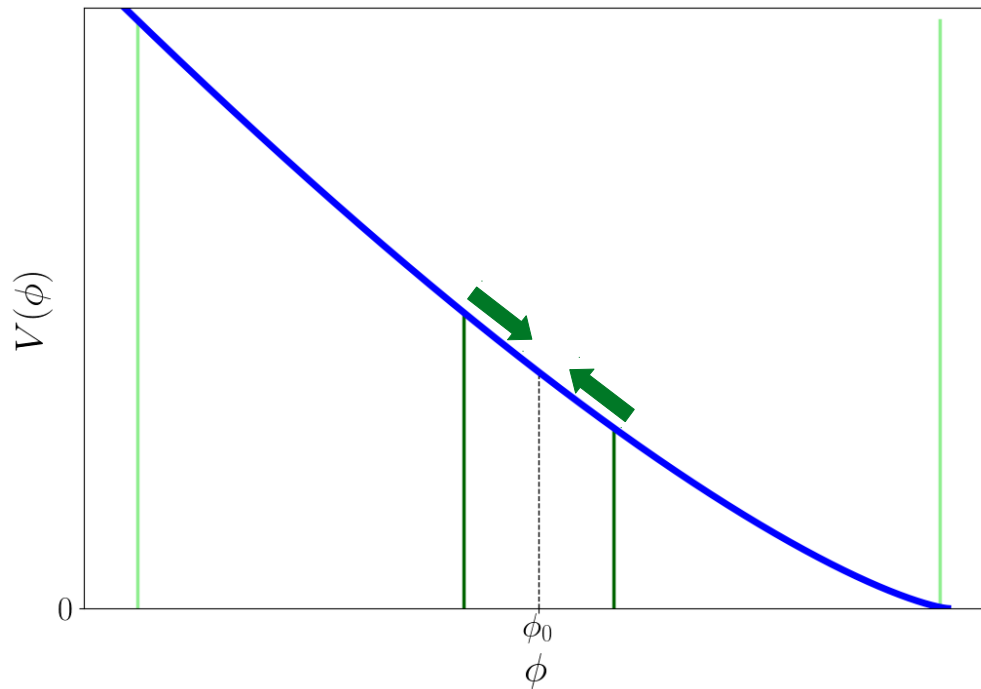
We use GRChombo:

3+1 Numerical Relativity + Adaptive Mesh



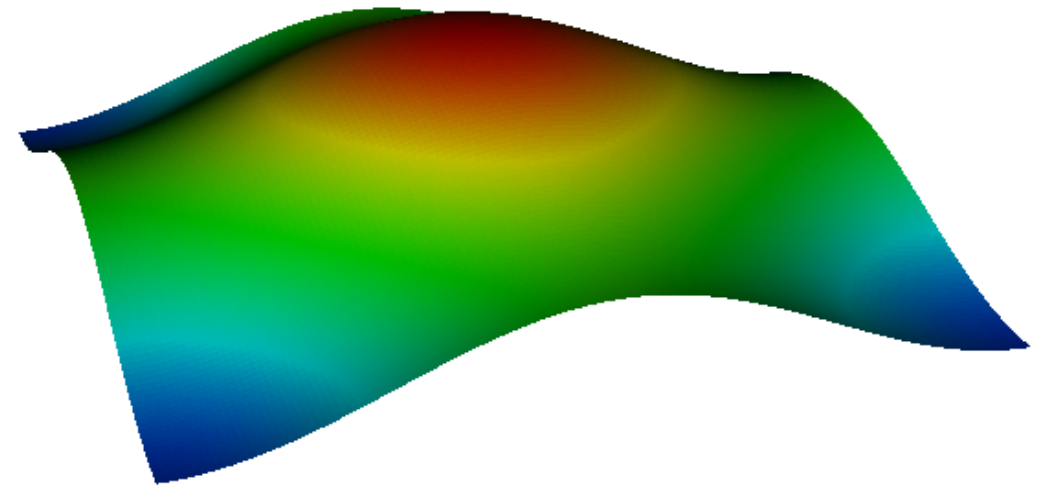
Convex

$$V(\phi) = \lambda M_{\text{Pl}}^{8/3} (-\phi)^{4/3}$$



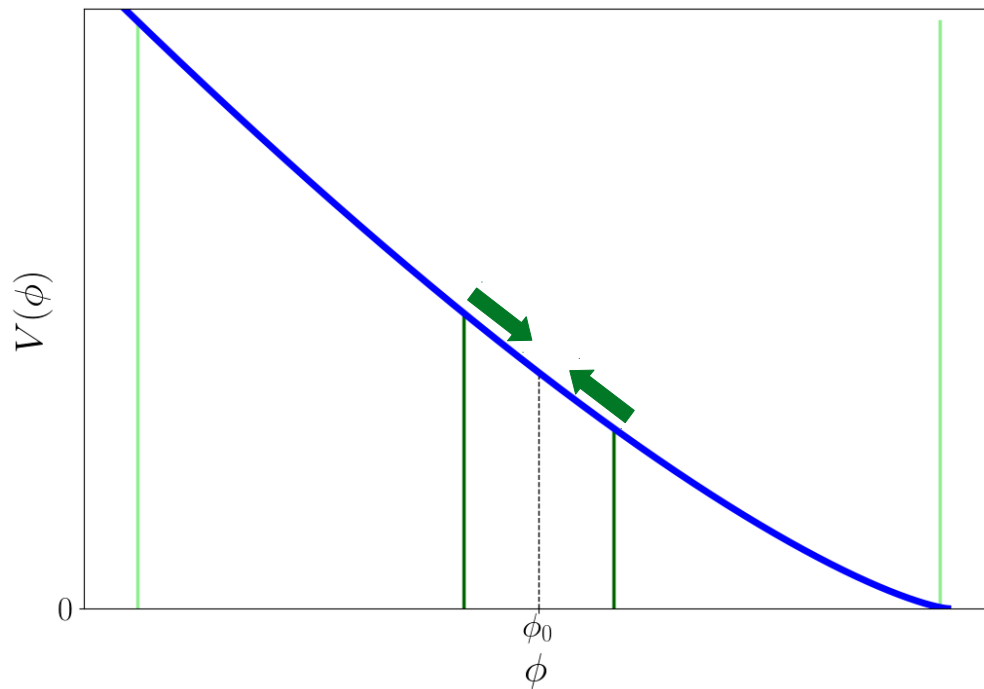
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time:0

Pseudocolor
Var: phi
-2.663
-2.877
-3.090
-3.304
-3.518
Max: -2.663
Min: -3.518



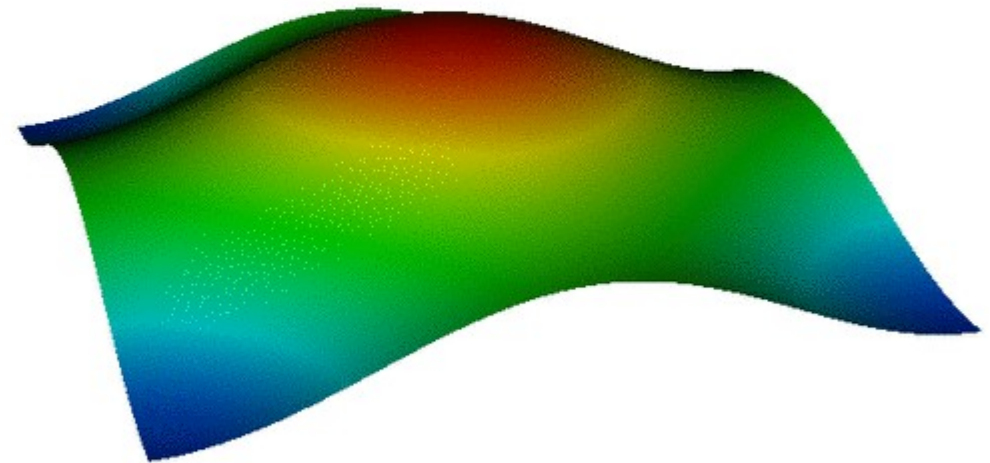
Convex

$$V(\phi) = \lambda M_{\text{Pl}}^{8/3} (-\phi)^{4/3}$$



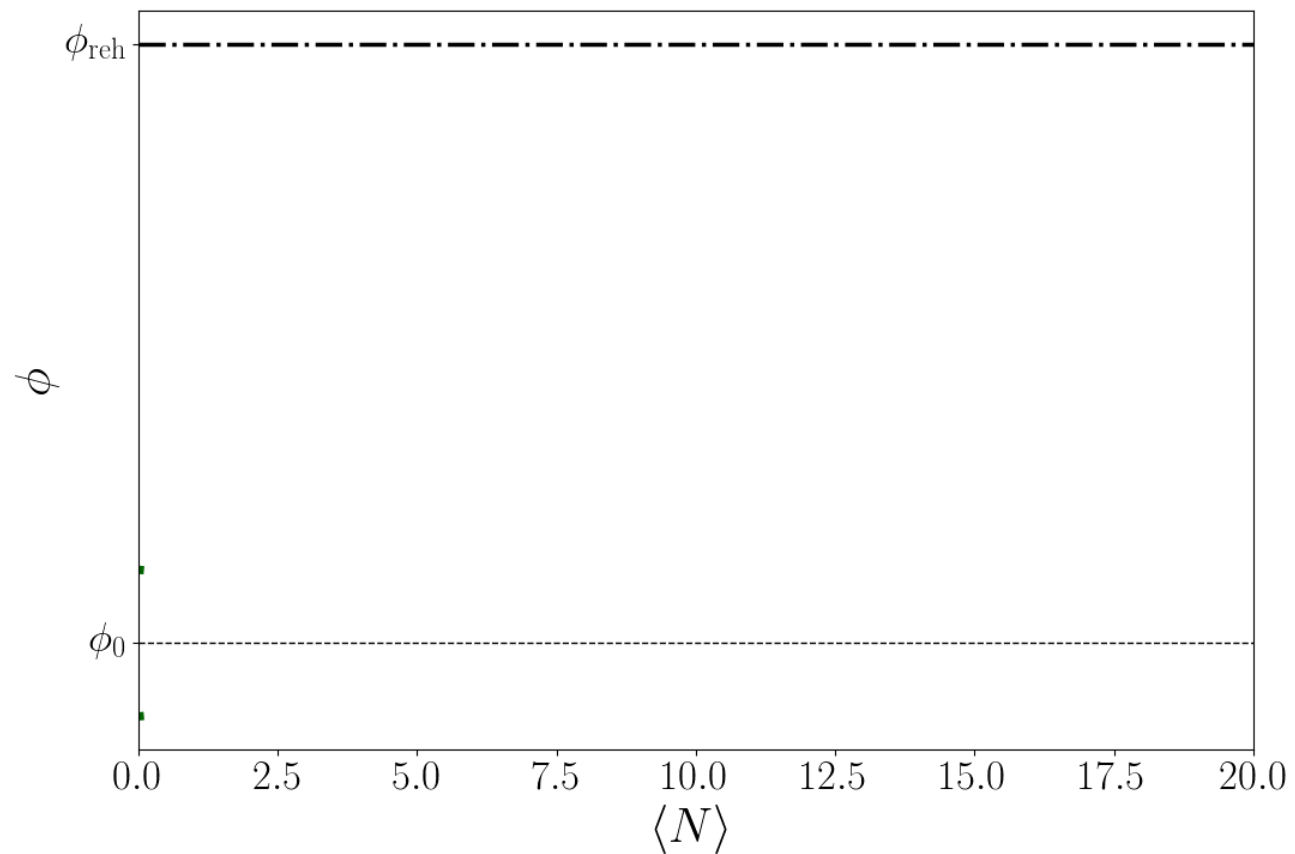
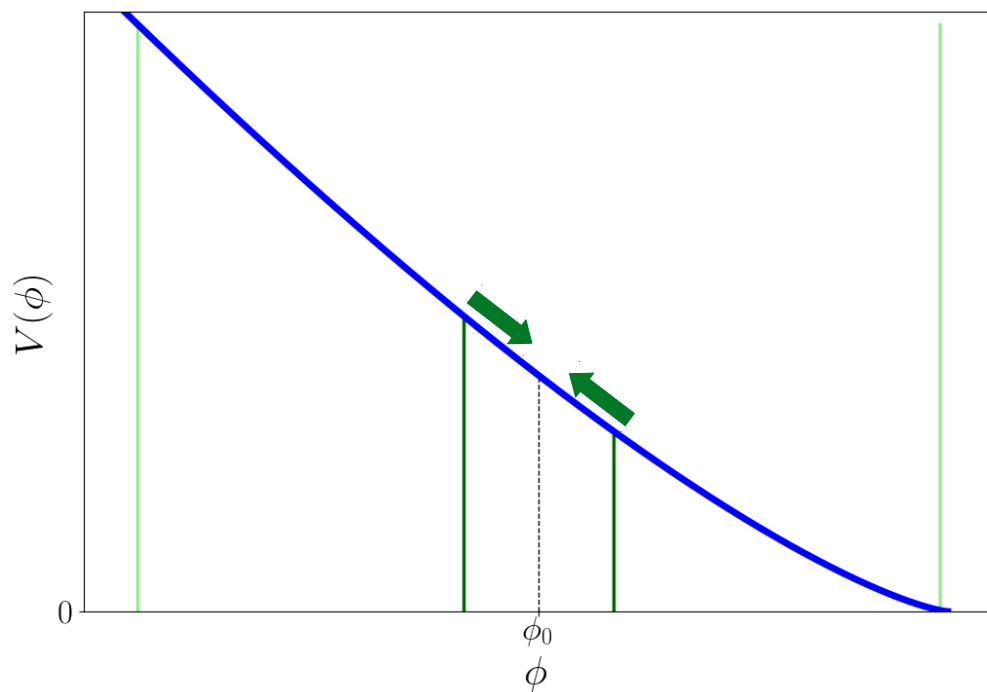
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time:0

Pseudocolor
Var: phi
-2.663
-2.877
-3.090
-3.304
-3.518
Max: -2.663
Min: -3.518



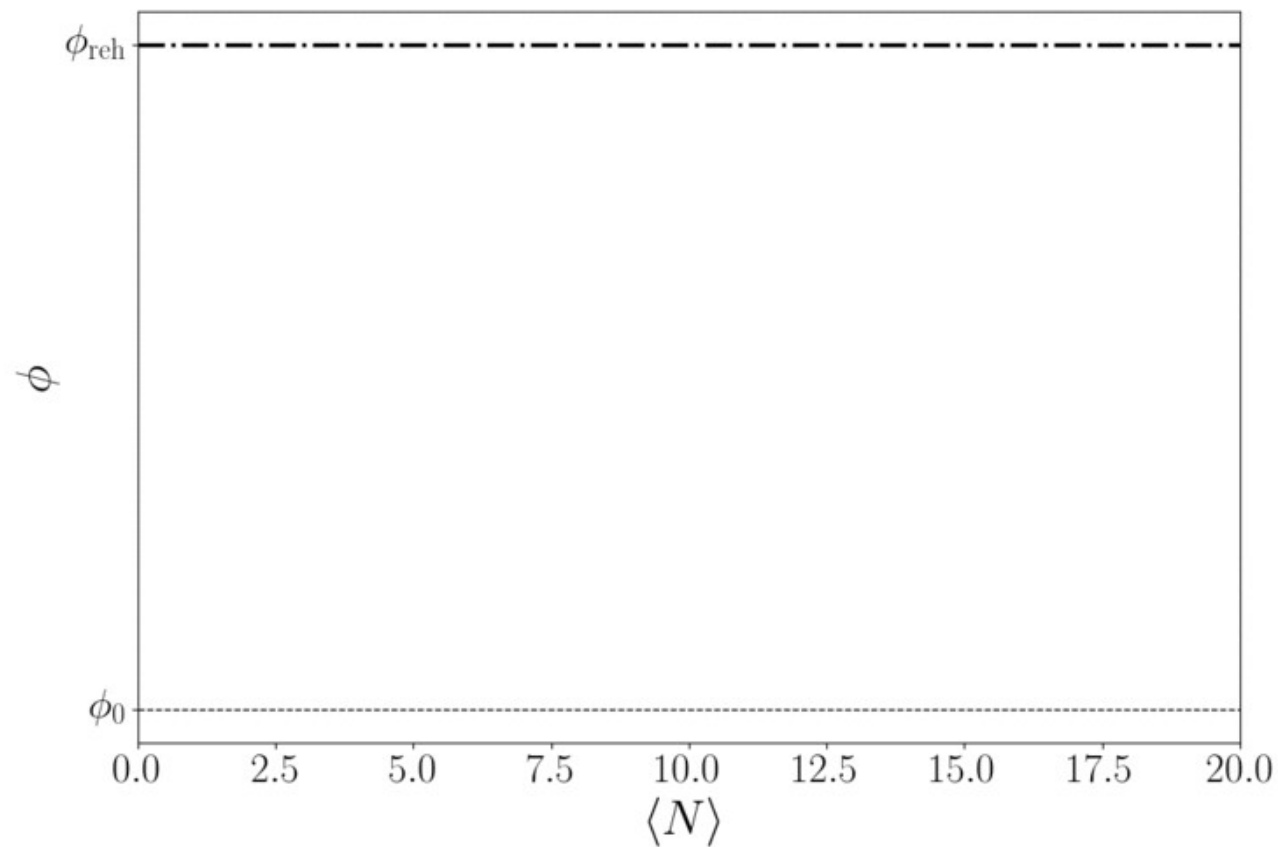
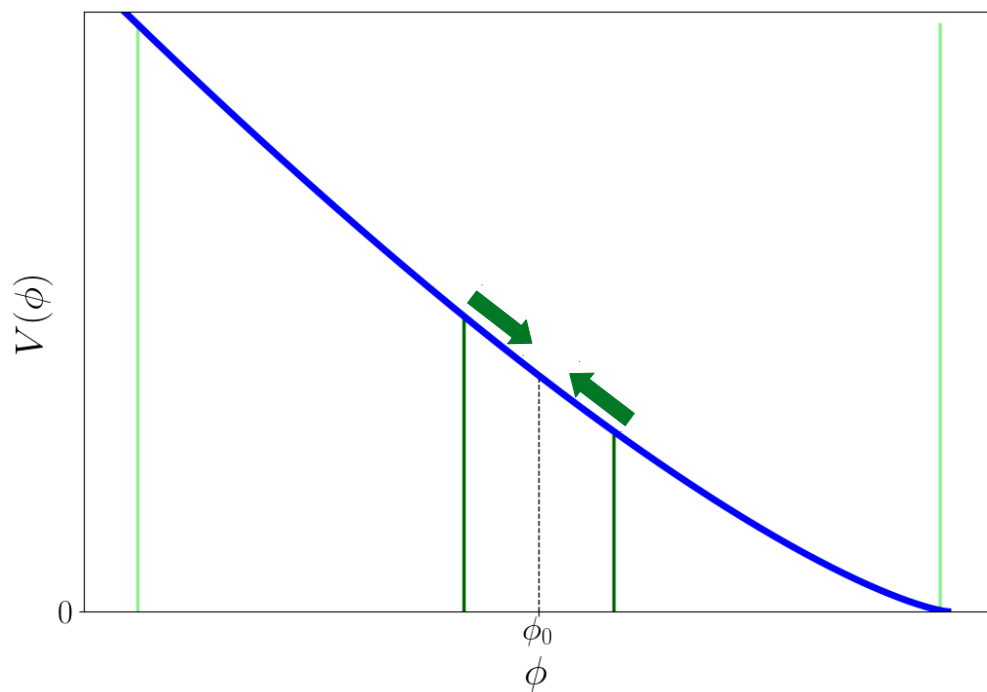
Convex

$$V(\phi) = \lambda M_{\text{Pl}}^{8/3} (-\phi)^{4/3}$$



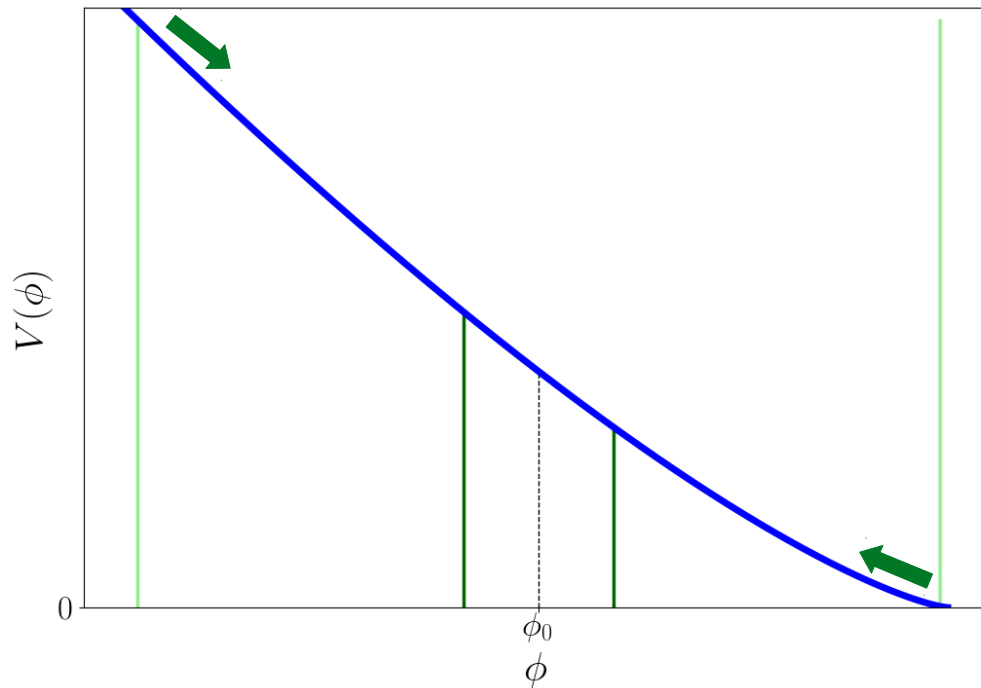
Convex

$$V(\phi) = \lambda M_{\text{Pl}}^{8/3} (-\phi)^{4/3}$$



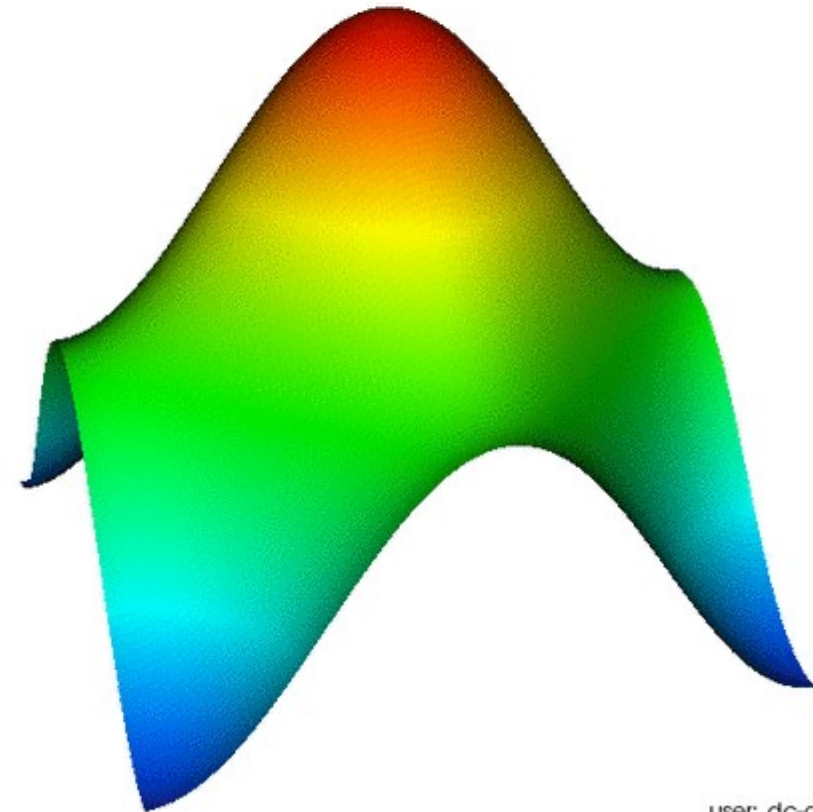
Convex

$$V(\phi) = \lambda M_{\text{Pl}}^{8/3} (-\phi)^{4/3}$$



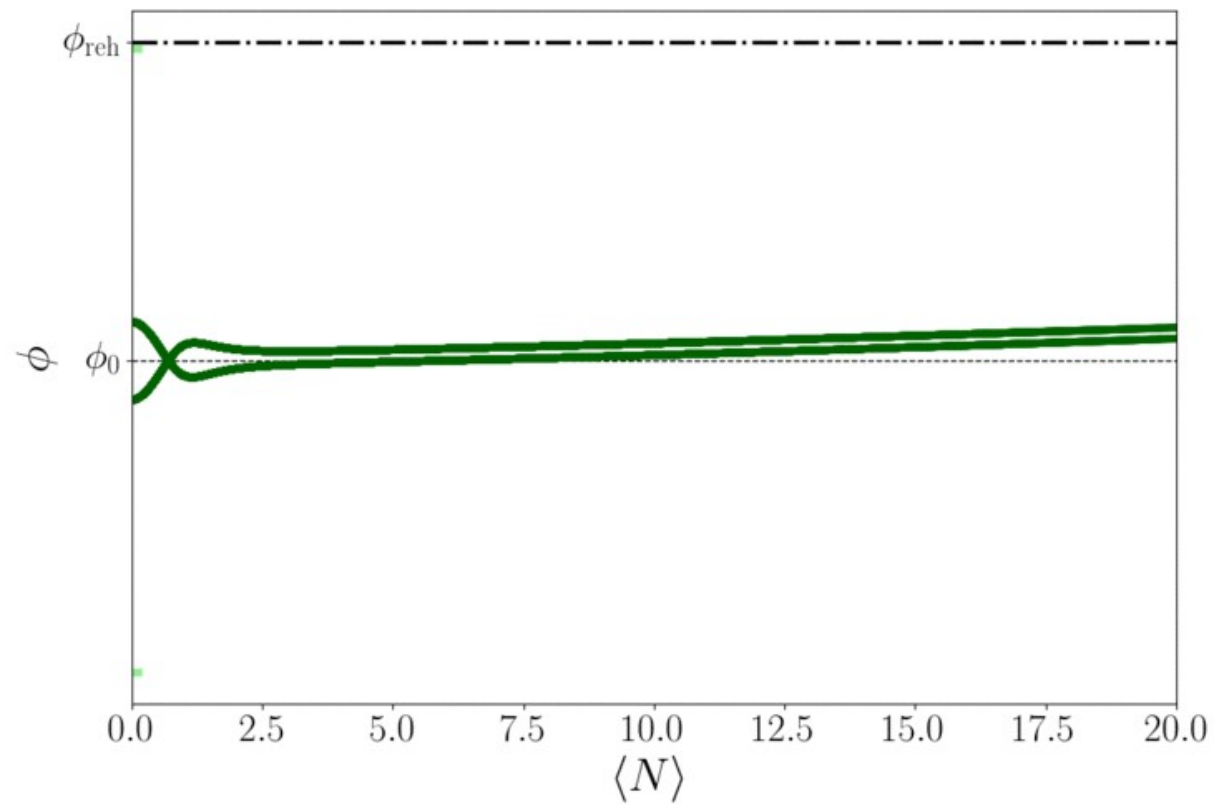
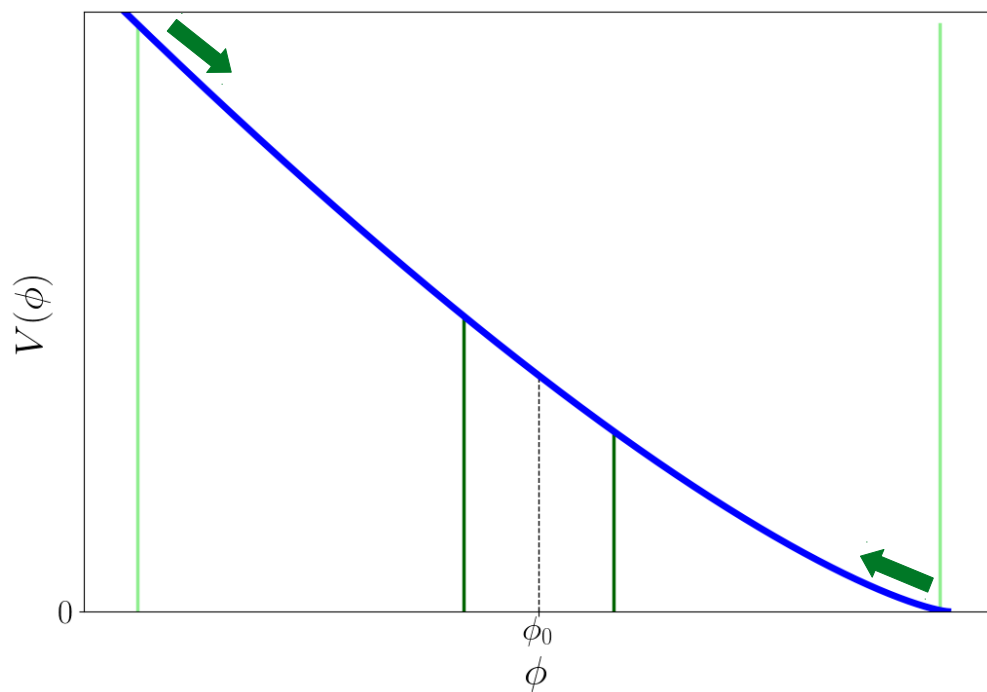
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time:0

Pseudocolor
Var: phi
-0.06373
-1.204
-2.343
-3.483
-4.623
Max: -0.06373
Min: -4.623



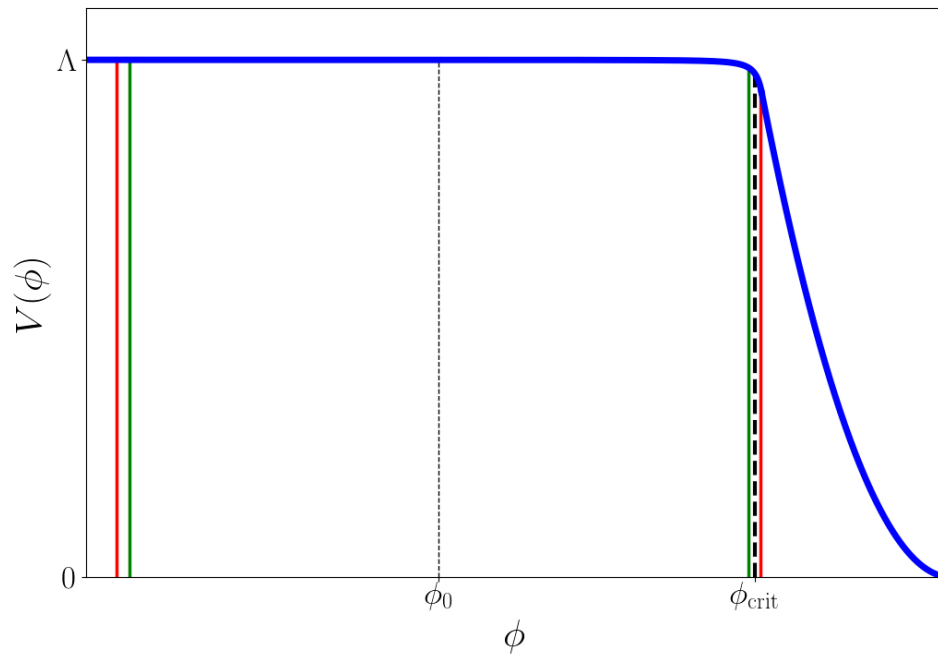
Convex

$$V(\phi) = \lambda M_{\text{Pl}}^{8/3} (-\phi)^{4/3}$$



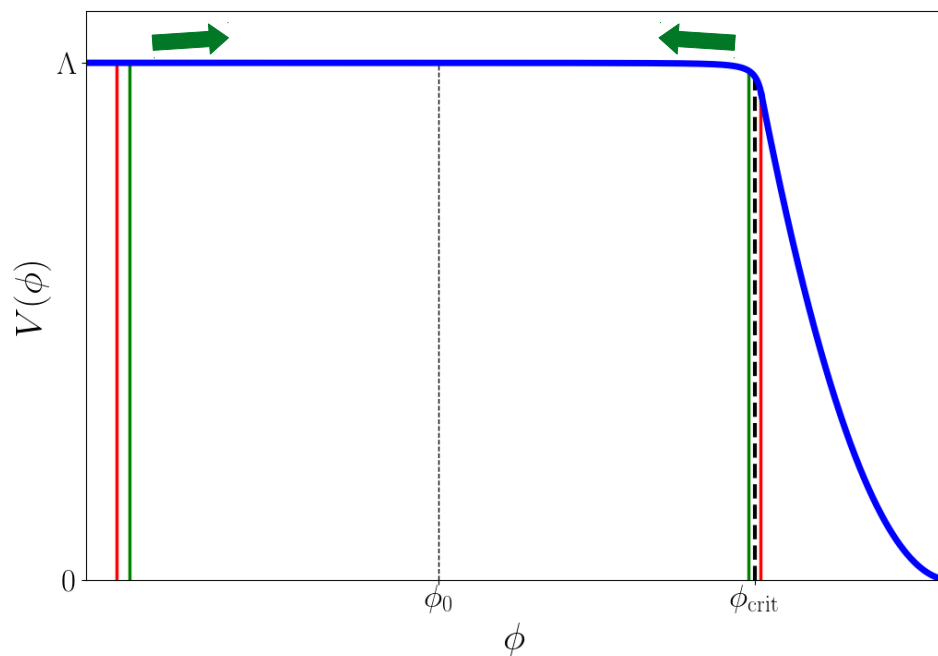
Concave (DBrane)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{\mu}{\phi} \right)^2 \right)$$



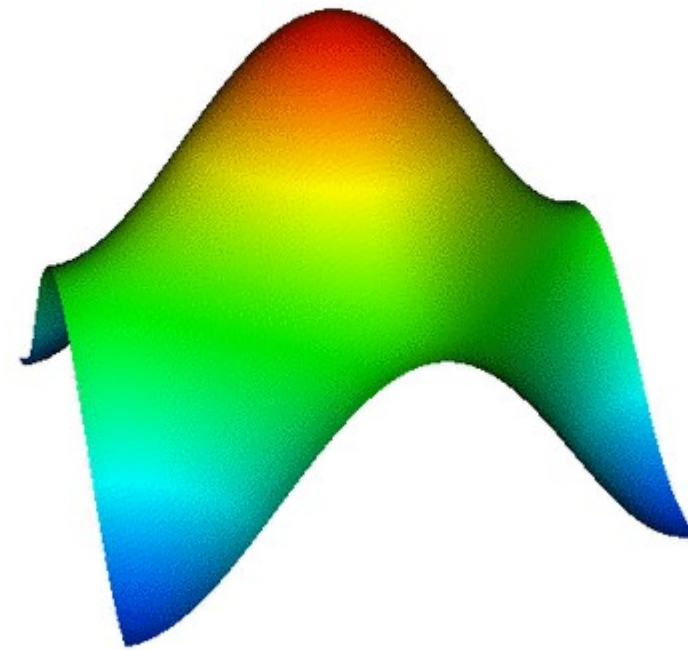
Concave (DBrane)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{\mu}{\phi} \right)^2 \right)$$



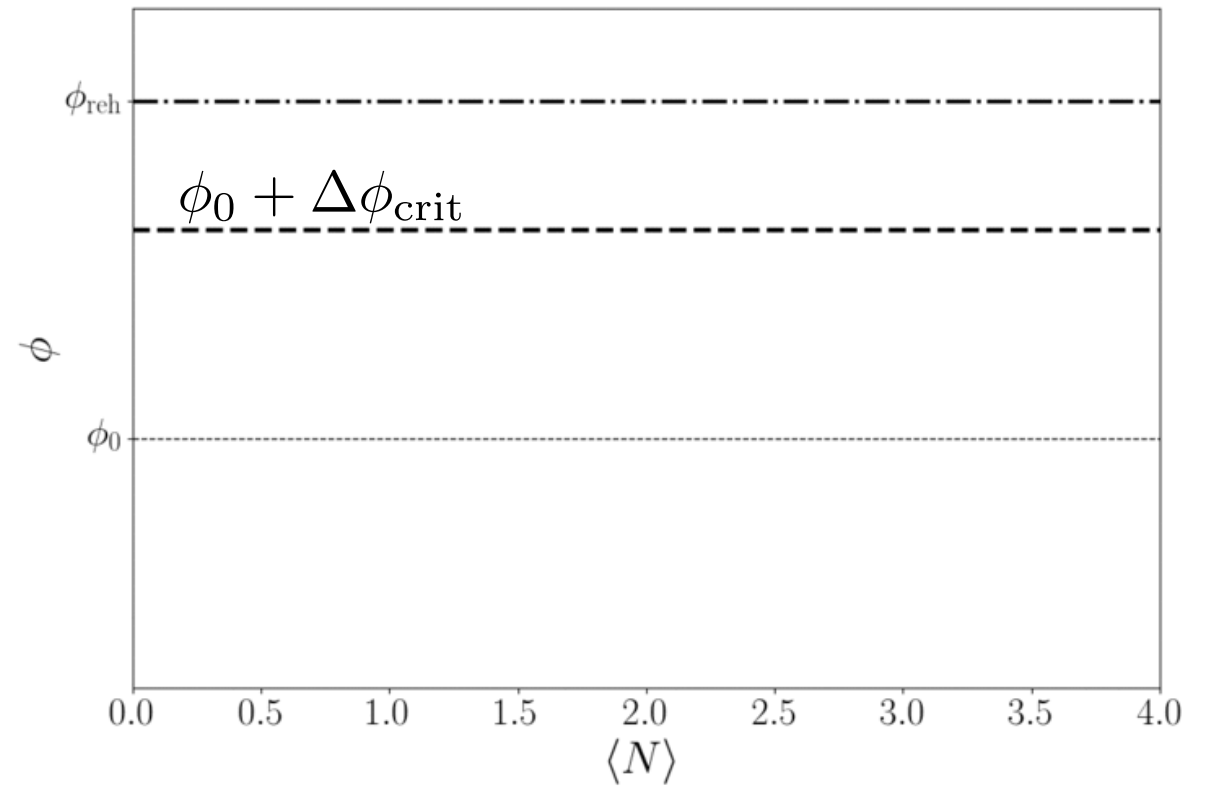
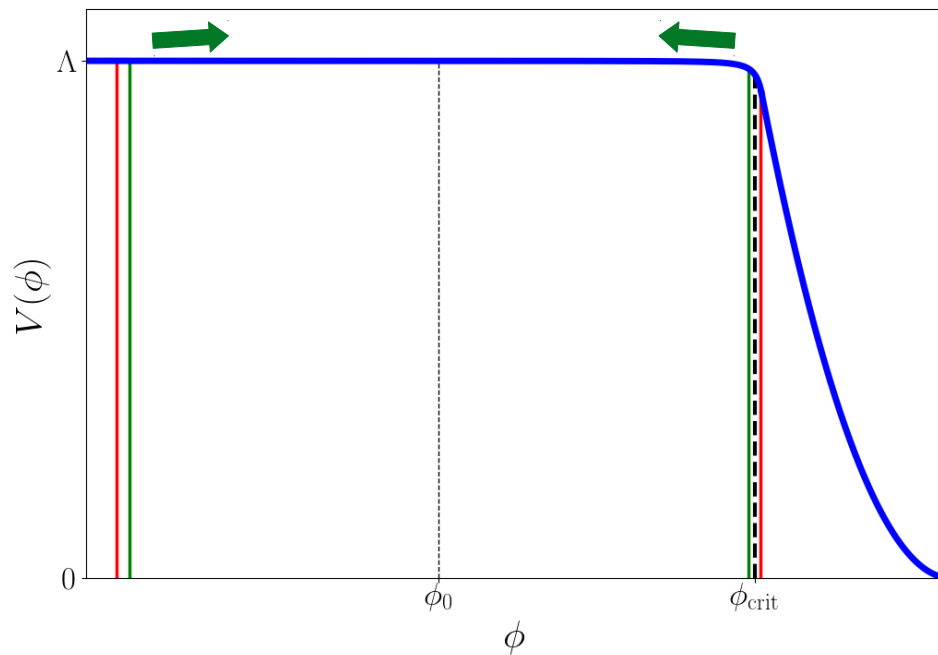
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time:0

Pseudocolor
Var: phi
-0.003130
-0.02094
-0.03875
-0.05656
-0.07437
Max: -0.003130
Min: -0.07437



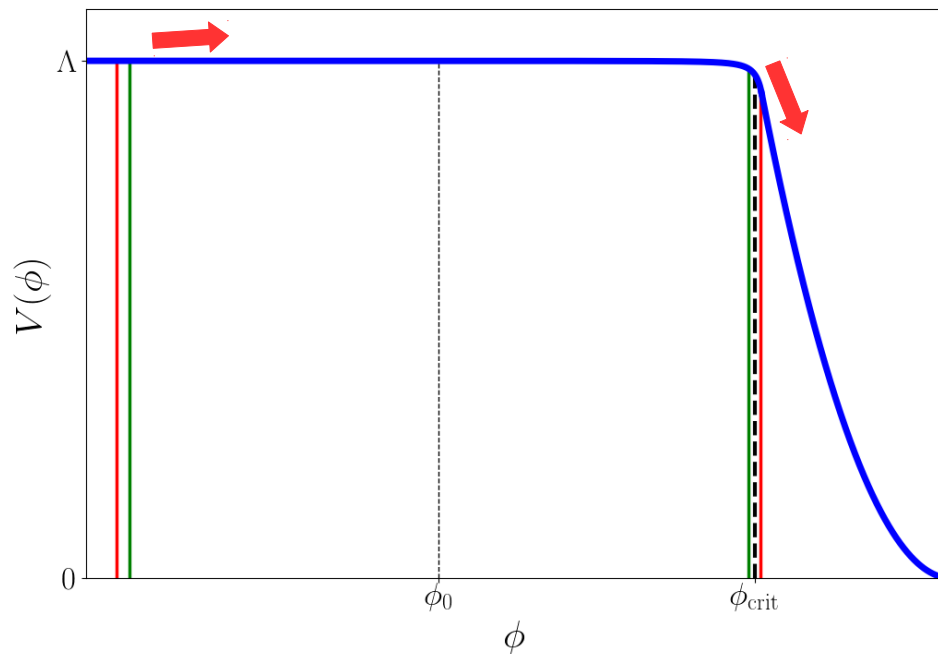
Concave (D)Brane

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{\mu}{\phi} \right)^2 \right)$$



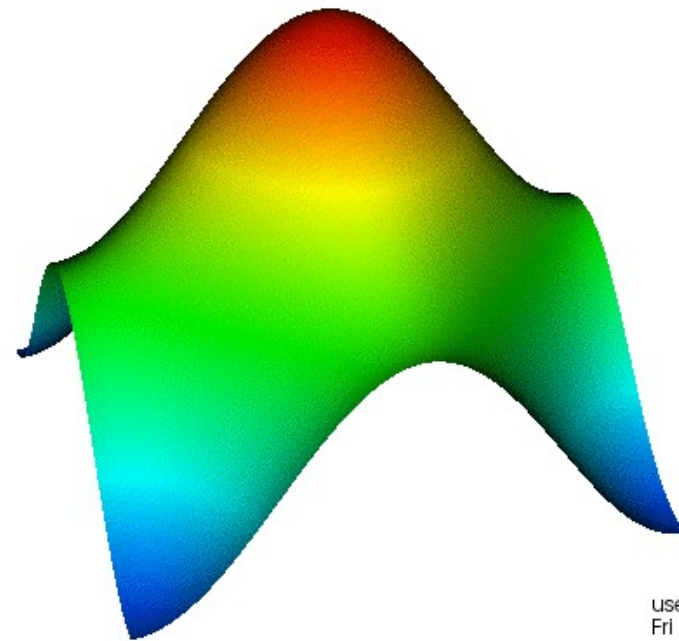
Concave (DBrane)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{\mu}{\phi} \right)^2 \right)$$



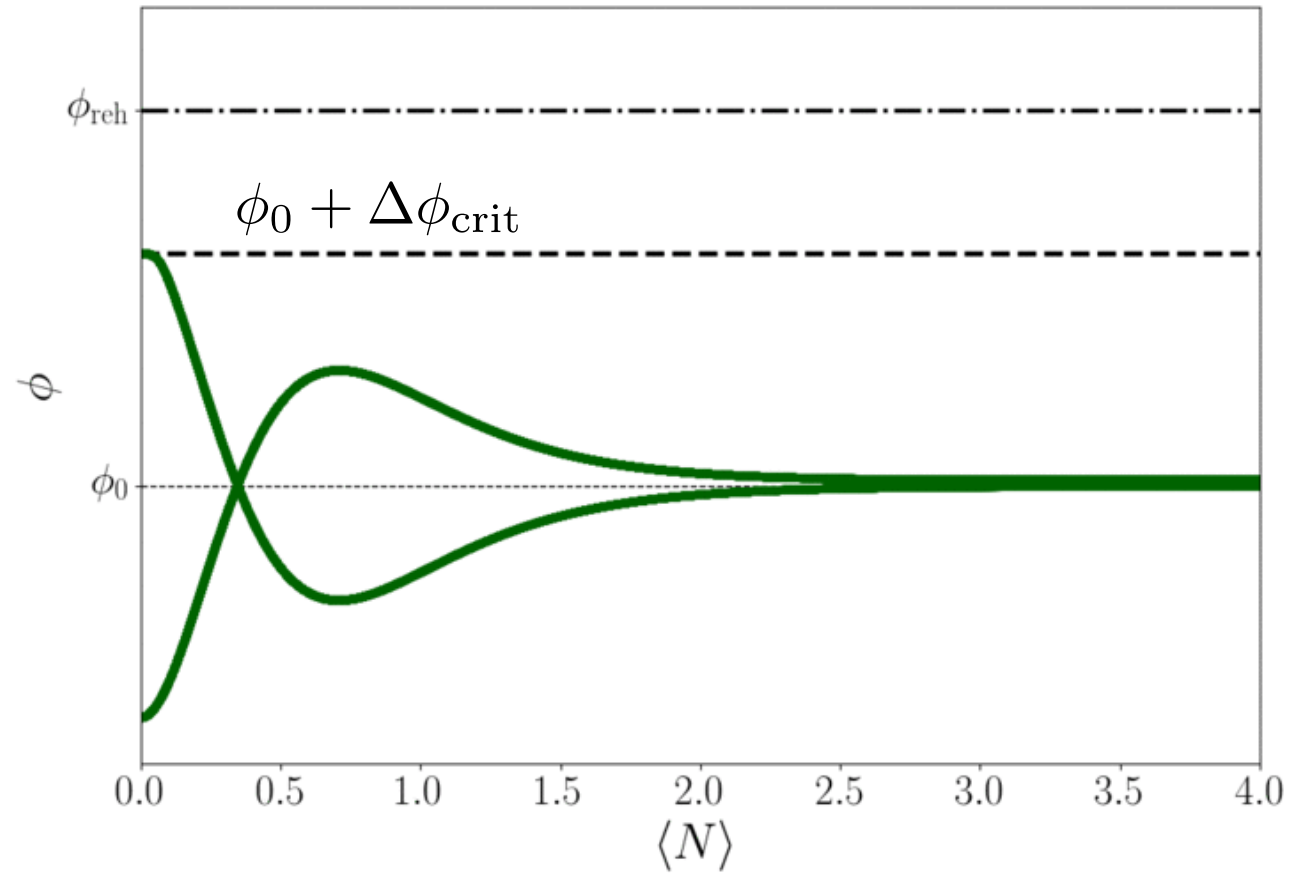
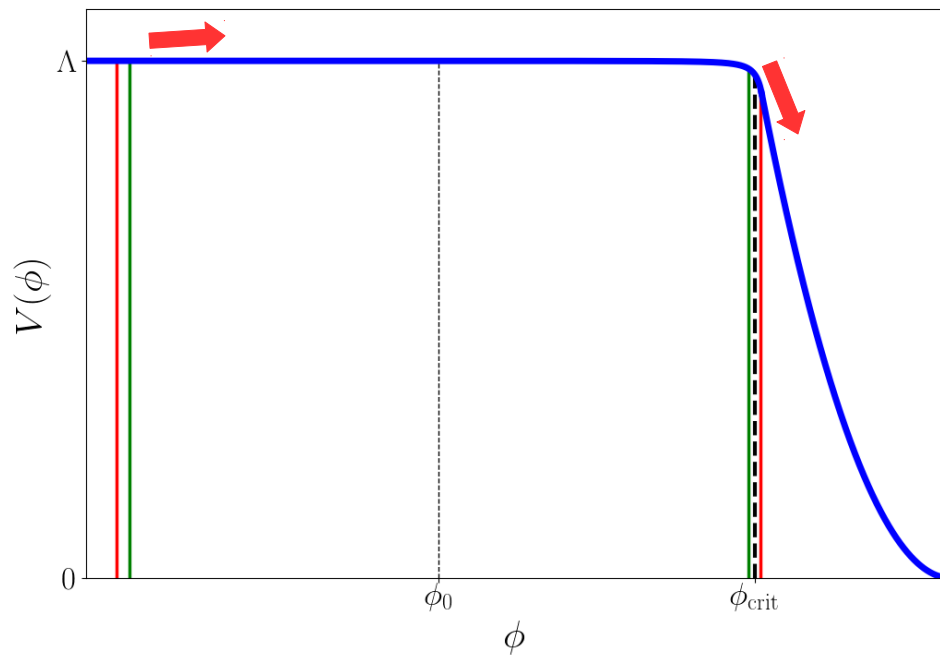
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time:0

Pseudocolor
Var: phi
-0.002131
-0.02030
-0.03846
-0.05663
-0.07479
Max: -0.002131
Min: -0.07479



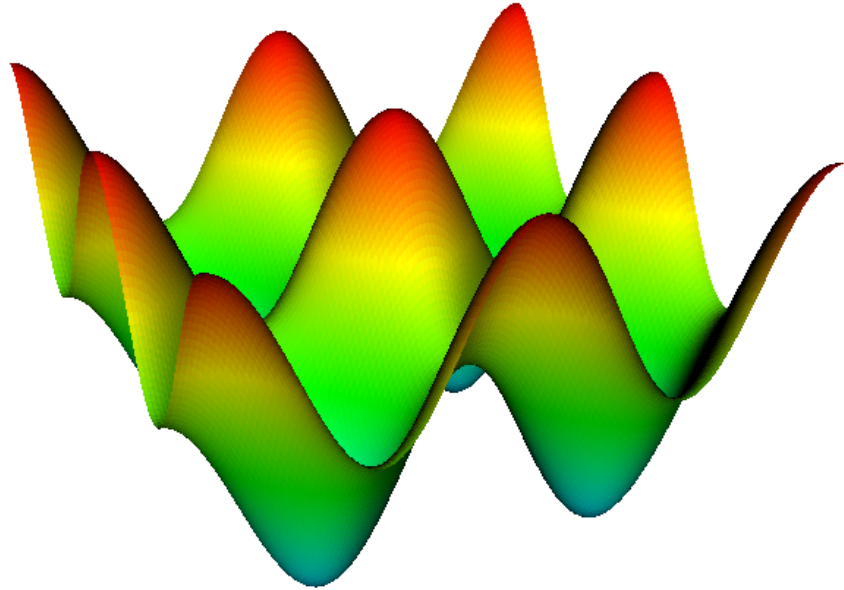
Concave (DBrane)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{\mu}{\phi} \right)^2 \right)$$



Concave (DBrane)

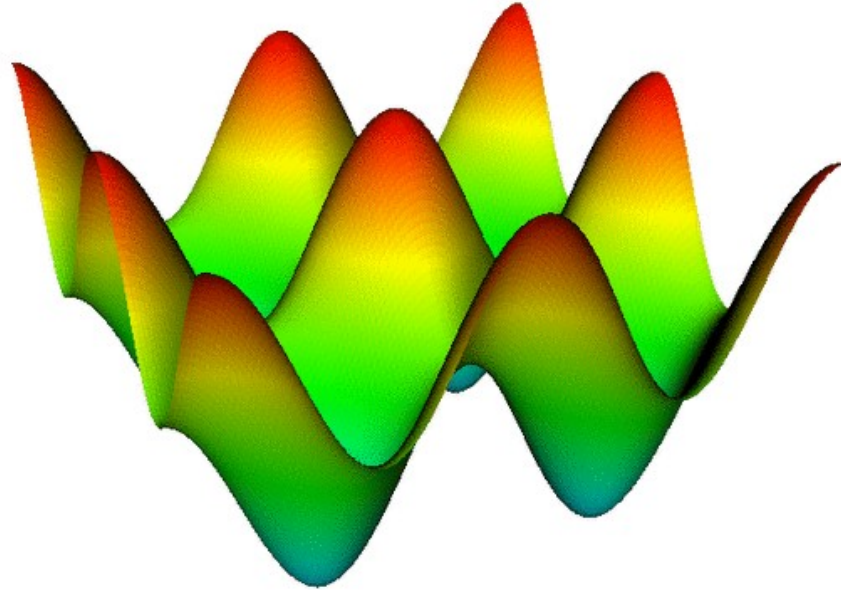
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time: 0



user: dc-aur2
Sun Jun 30 21:34:43 201

Concave (DBrane)

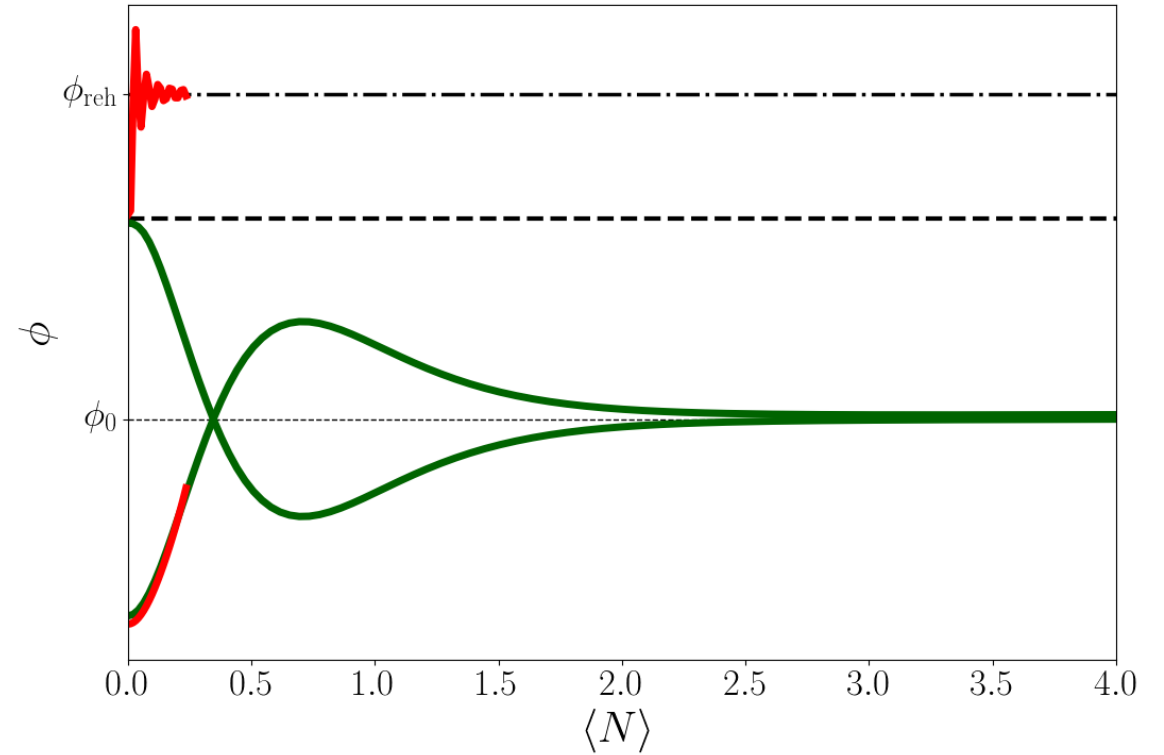
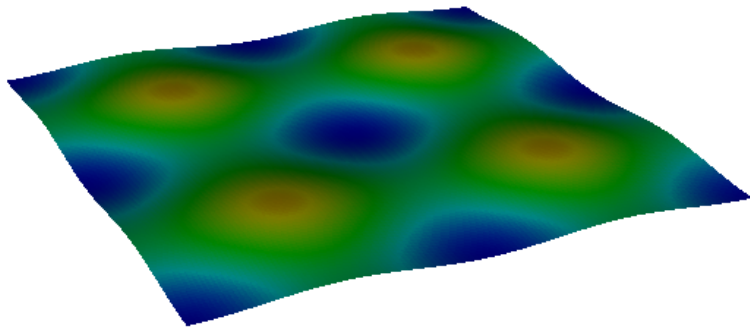
DB: outScalarFieldp_000000.3d.hdf5
Cycle: 0 Time: 0



user: dc-aur2
Sun Jun 30 21:34:43 201

Concave (DBrane)

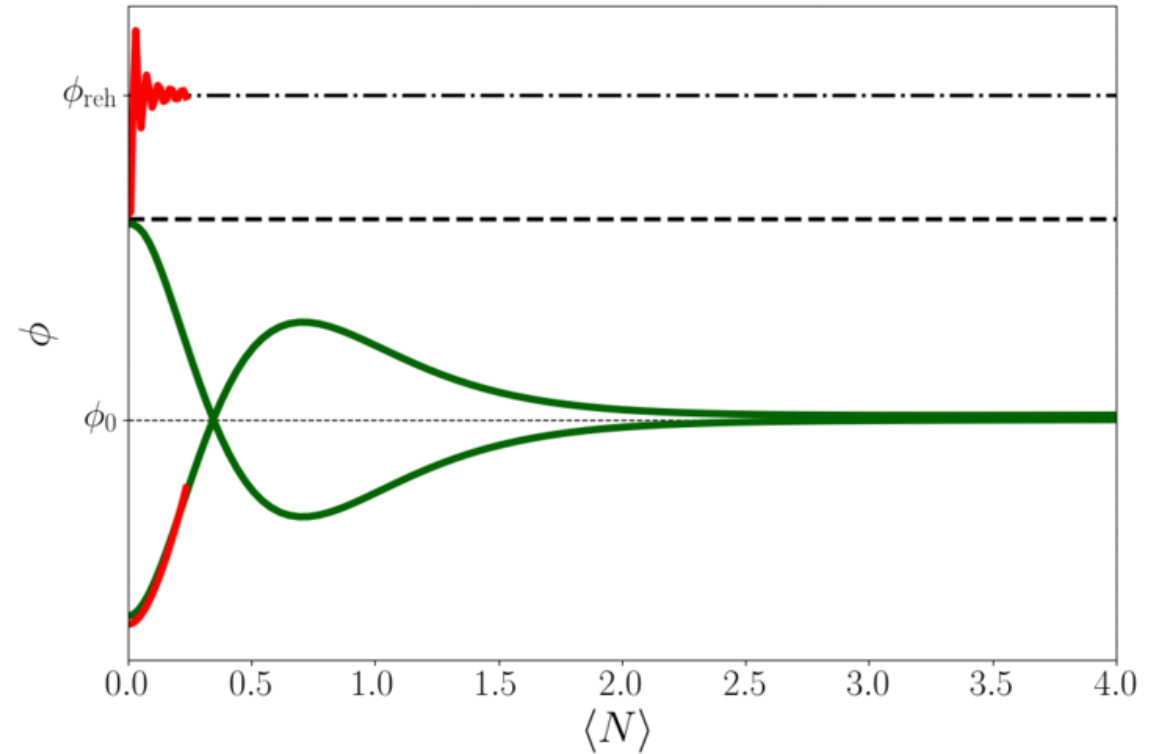
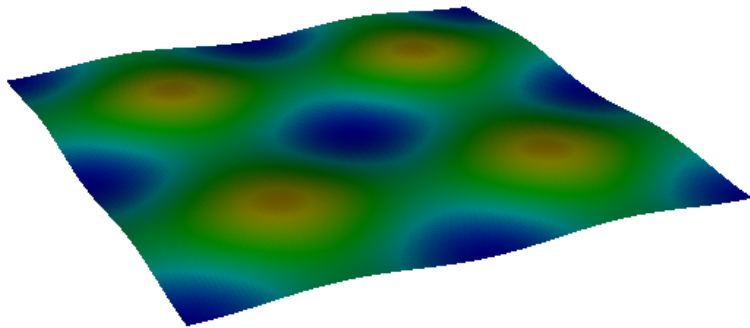
DB: outScalarFieldp_001495.3d.hdf5
Cycle: 1495 Time:93.4375



user: dc-aurr2
Sun Jun 30 21:41:51 2011

Concave (DBrane)

DB: outScalarFieldp_001495.3d.hdf5
Cycle: 1495 Time:93.4375



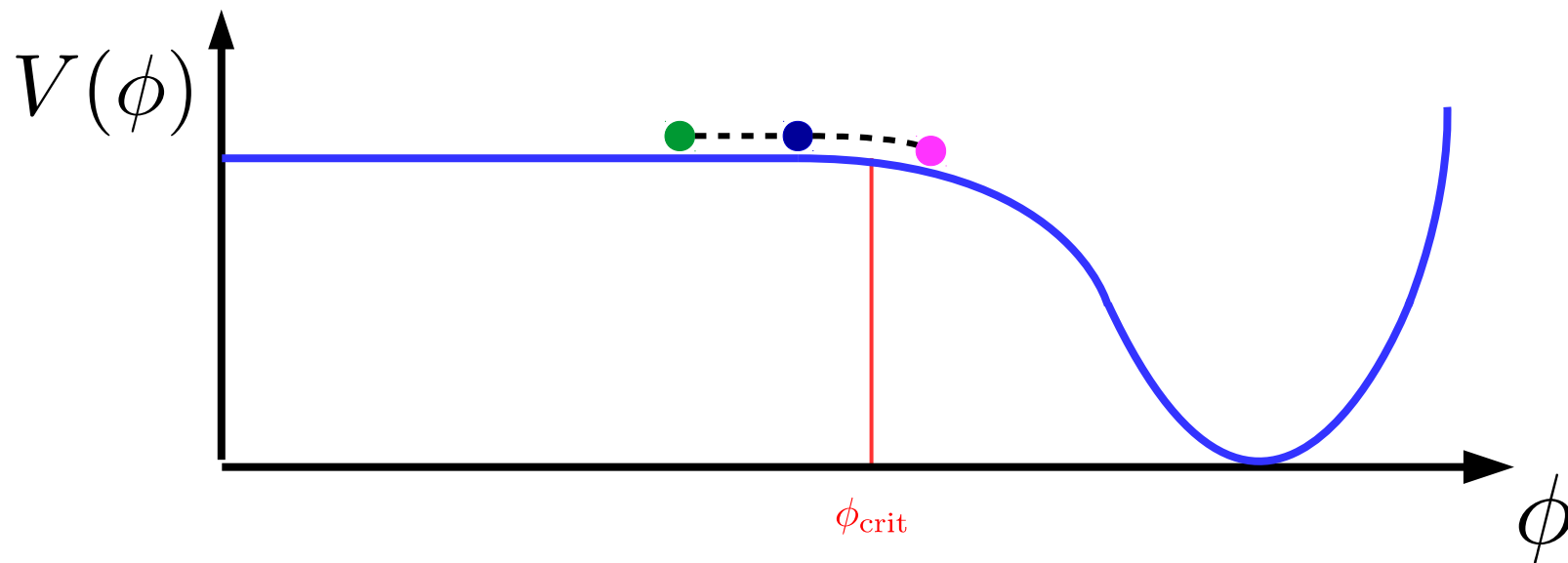
user: dc-aurr2
Sun Jun 30 21:41:51 2011

Initial Conditions

Initial Conditions

One can use this to learn about the initial conditions ϕ_0

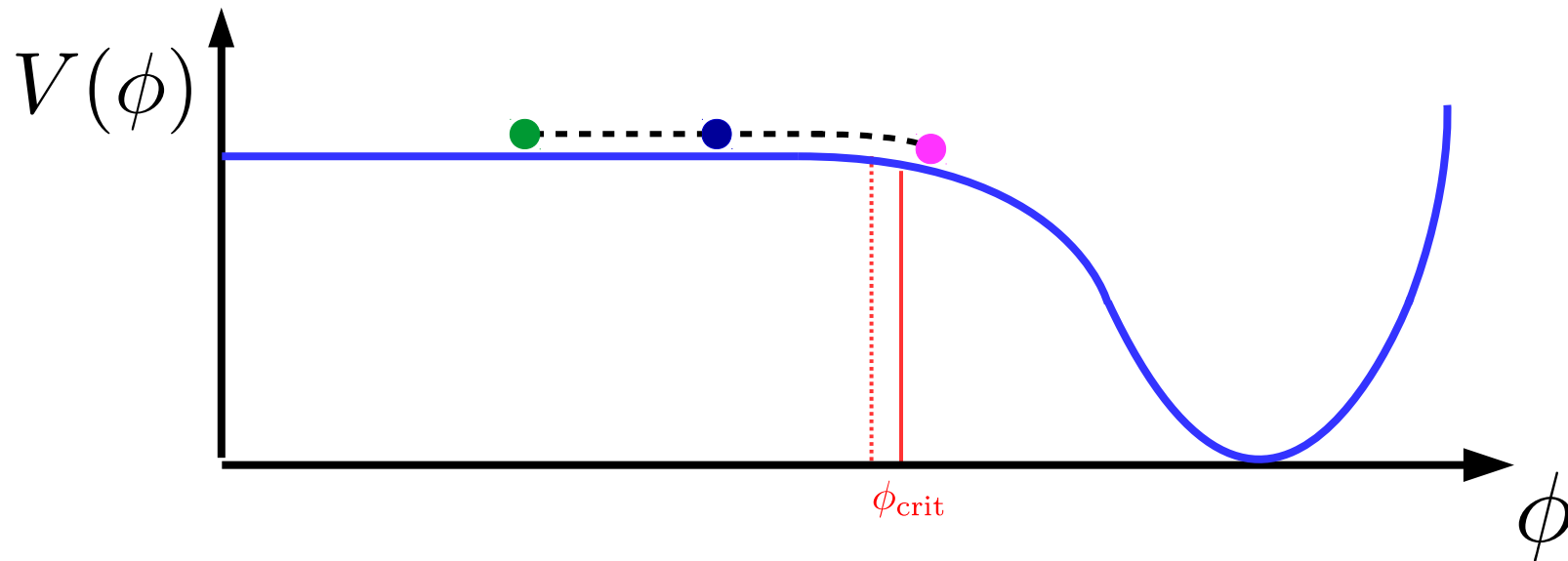
$$f(\phi_{\max}) = f(\phi_0, \Delta\phi)$$



Initial Conditions

One can use this to learn about the initial conditions ϕ_0

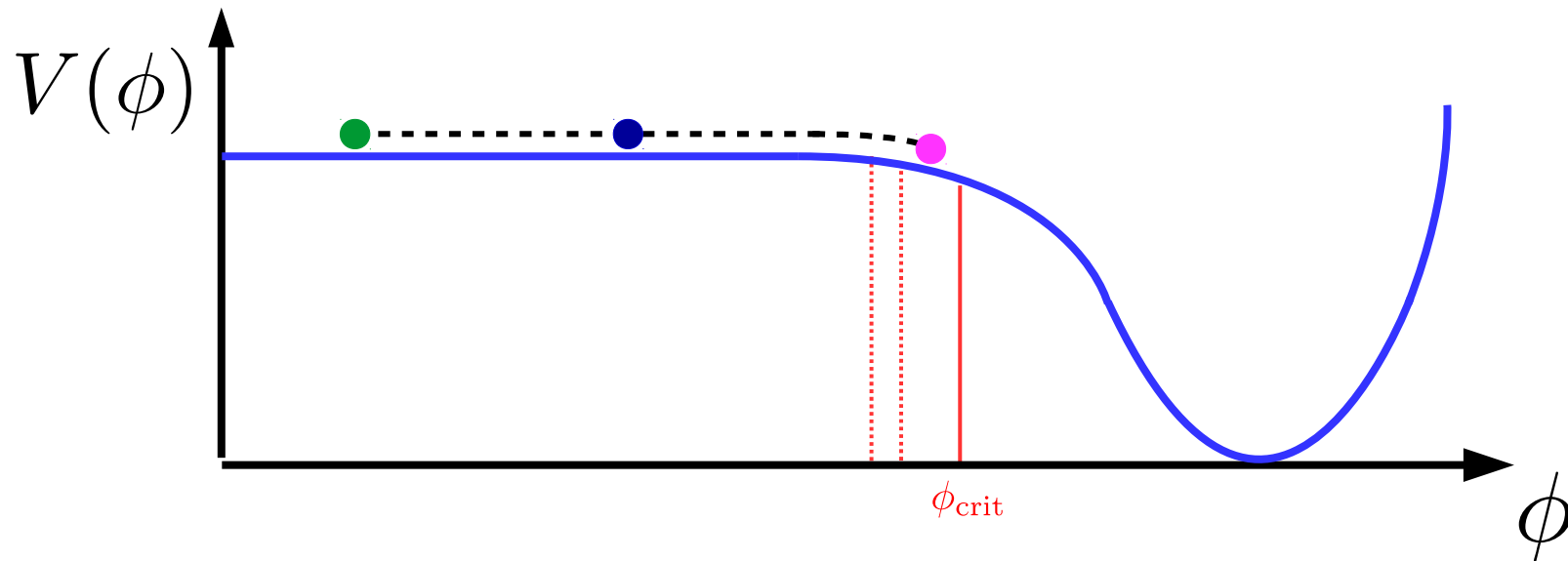
$$f(\phi_{\max}) = f(\phi_0, \Delta\phi)$$



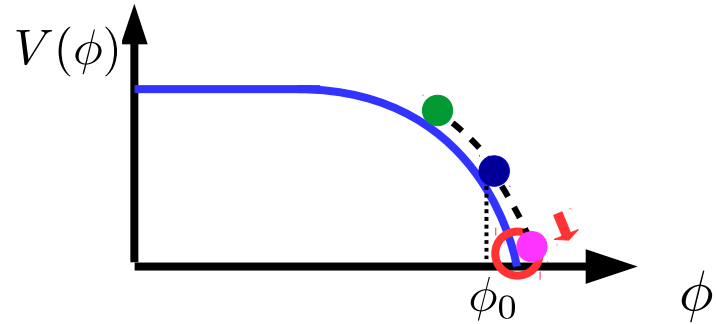
Initial Conditions

One can use this to learn about the initial conditions ϕ_0

$$f(\phi_{\max}) = f(\phi_0, \Delta\phi)$$



Initial Conditions

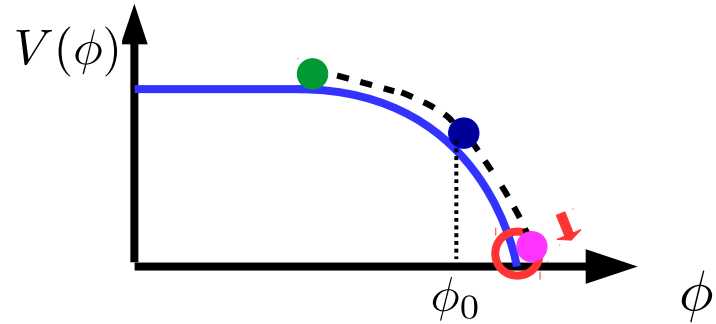


$$f(\phi_0, \Delta\phi) = k^2 \Delta\phi + \frac{dV}{d\phi}$$

Most negative $\frac{dV}{d\phi}$ at $\hat{\phi}$ If $f(\hat{\phi}) > 0$ safe

$$\Delta\phi = \hat{\phi} - \phi_0$$

Initial Conditions

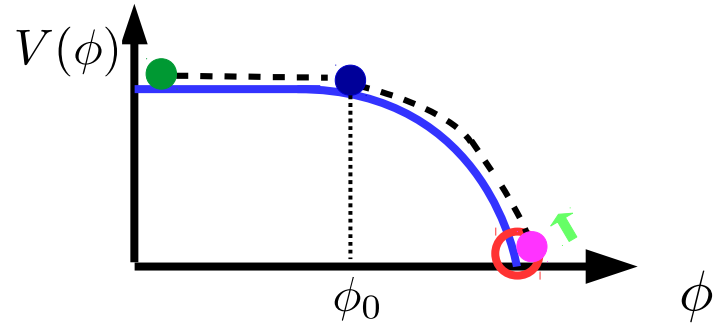


$$f(\phi_0, \Delta\phi) = k^2 \Delta\phi + \frac{dV}{d\phi}$$

Most negative $\frac{dV}{d\phi}$ at $\hat{\phi}$ If $f(\hat{\phi}) > 0$ safe

$$\Delta\phi = \hat{\phi} - \phi_0$$

Initial Conditions



$$f(\phi_0, \Delta\phi) = k^2 \Delta\phi + \frac{dV}{d\phi}$$

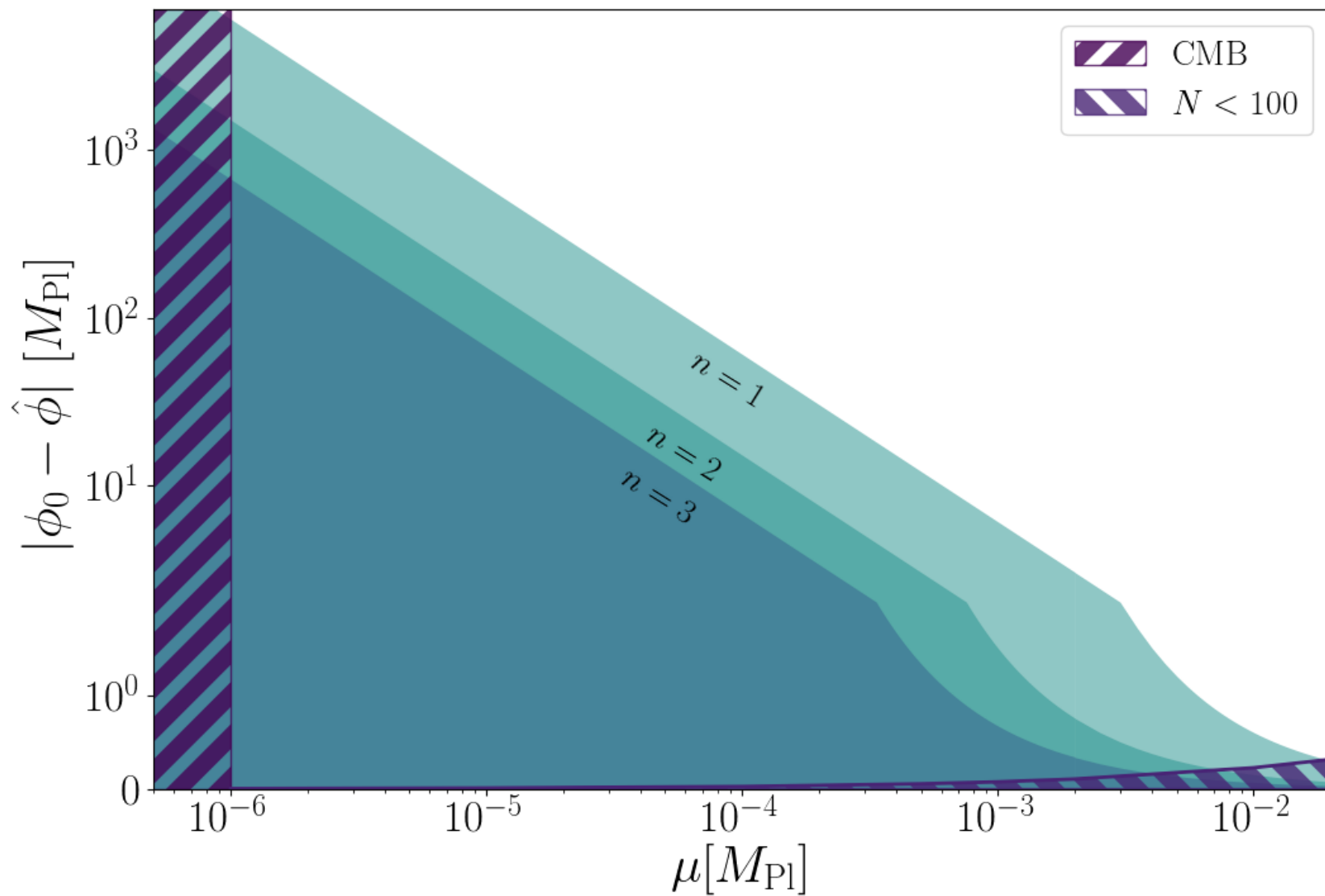
Most negative $\frac{dV}{d\phi}$ at $\hat{\phi}$

If $f(\hat{\phi}) > 0$ safe

$$\Delta\phi = \hat{\phi} - \phi_0$$

$$\phi_0 < \hat{\phi} + \frac{1}{k^2} \frac{dV(\hat{\phi})}{d\phi}$$

Initial Conditions



Summary

- Concave models are generically less robust than Convex
- Higher modes make the model more robust
- One can use this to learn about initial conditions ϕ_0

Thank you