Gravity, Scale Invariance and the Hierarchy Problem

Mikhail Shaposhnikov, Andrey Shkerin

Based on

MS, AS, Phys. Lett. B783, 253 (2018), 1803.08907
MS, AS, JHEP 1810, 024 (2018), 1804.06376
AS, Phys. Rev. D99 (2019) 115018, 1903.11317

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Motivation: drowning by numbers…

The fact is that

$$\frac{G_F \hbar^2}{G_N c^2} \sim 10^{33}$$

where G_F - Fermi constant, G_N - Newton constant

Quantum complications:

G.F. Giudice, (2008) 155, 0801.2562

Let M_X be some heavy mass scale. Then, one expects

$$\delta m^2_{H,X} \sim M_X^2$$

Even if one assumes that there are no heavy thresholds beyond the EW scale, then, naively,

$$\delta m^2_{H,grav.} \sim M_P^2$$

Framework

Conjectures:

- Scale Invariance: The idea of reducing an amount of dimensionful parameters as a way towards the fundamental theory seems fruitful. Besides, SI can protect the Higgs mass against large radiative corrections.
- No degrees of freedom beyond the EW scale: ``Minimalistic approach.'' Experimental data?
- Dynamical gravity: Since the Planck mass is involved

Ways to generate the Higgs vev

- Coleman-Weinberg mechanism…

  S. R. Coleman, E. J. Weinberg'73; A. D. Linde'76,'77; S. Weinberg'76

  …comes in tension with experiment in the SM framework.
  But it may work in extensions of the SM:

  E. Gildener, S. Weinberg'76; K. A. Meissner, H. Nicolai'07; S. Iso, N. Okada, Y. Orikasa'09 …

- Spontaneous symmetry breaking

  In fact, both the Planck and the Weak energy scales can be generated classically via spontaneous breaking of Scale symmetry. Example: the Higgs-Dilaton model.

  M. Shaposhnikov and D. Zenhausern'09; J. Garcia-Bellido, J. Rubio, M. Shaposhnikov, D. Zenhausern'13

  The hierarchy between the scales is encoded in the hierarchy between the dimensionless parameters.

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  The hierarchy between the scales is encoded in the hierarchy between the dimensionless parameters.

Are we ok with this?

Yes → …

No → **Challenge accepted!**

EFT approach and beyond

The Effective Field Theory paradigm:

Low energy description of Nature, provided by the SM, can be affected by an unknown UV physics only through a finite set of parameters.

This “**Naturalness principle**” is questioned now in light of the absence of signatures of new physics at the LHC.

G.F. Giudice, PoS EPS-HEP2013 (2013) 163, 1307.7879

What if one goes beyond the EFT approach? Many examples are known:

- Multiple point criticality principle D. L. Bennett, H. B. Nielsen’94; C. D. Froggatt, H. B. Nielsen’96
- Asymptotic safety of gravity S. Weinberg’09; M. Shaposhnikov, C. Wetterich’09
- EW vacuum decay V. Branchina, E. Messina, M. Sher’14; F. Bezrukov, M. Shaposhnikov’14

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Let’s write something like $$v \sim M_P e^{-B}$$

where *v* is the Higgs vev. Then, $B \approx 37$.

The idea

In Euclidean signature, the time-independent, spatially-homogeneous vev of the scalar field is

$$\langle \varphi \rangle \sim \int \mathscr{D}\textbf{[Measure]}\, \varphi(0)\, e^{-S_E(\varphi, g_{\mu\nu}, \ldots)}$$

Let M_P be the only classical scale in the theory. Change the variable,

$$\varphi \to M_P e^{\bar{\varphi}} \ , \quad \varphi \gtrsim M_P$$

and evaluate the Path integral via saddle points of

$$\mathscr{B} = -\, \bar{\varphi}(0) + S_E(\bar{\varphi}, g_{\mu\nu}, \ldots)$$

Apply the Saddle-Point Approximation:

$$\langle \varphi \rangle = M_P e^{-B} \times \textbf{[Fluctuation factor]}$$

For this to work, it is necessary to find

- appropriate saddle points of $\mathscr{B}$,
- semiclassical parameter that would justify the SPA,
- physical argumentation that would justify the change of the scalar field variable.

Theories we consider

are within the framework outlined and motivated by phenomenology. For example:

- Models of Higgs inflation, with the scalar-gravity sector of the form

$$\frac{\mathscr{L}_E}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + \lambda\varphi^4/4$$

F. Bezrukov, D. Gorbunov, M. Shaposhnikov, 0812.3622
J. Garcia-Bellido, D. G. Figueroa, J. Rubio, 0812.4624
F. Bezrukov, J. Rubio, M. Shaposhnikov, 1412.3811

…and with some modifications at high energy scales.

- Models of Higgs-Dilaton inflation, with the scalar-gravity sector of the form

$$\frac{\mathscr{L}_E}{\sqrt{g}} = -\frac{1}{2}(\xi_\chi\chi^2 + \xi_h\varphi^2)R + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial\varphi)^2 + \lambda\varphi^4/4$$

M. Shaposhnikov and D. Zenhausern, 0809.3395
J. Garcia-Bellido, J. Rubio, M. Shaposhnikov, D. Zenhausern, 1107.2163
F. Bezrukov, G. K. Karananas, J. Rubio, M. Shaposhnikov, 1212.4148

…and with some modifications at high energy scales.

Decomposing the Instanton

Recall that we are looking for saddles of $\mathscr{B} = -\,\bar{\varphi}(0) + S_E(\bar{\varphi}, g_{\mu\nu}, \ldots)$

The source term → The instanton is singular

Physical singularity?

It depends…

This is not the first time such solutions are encountered, see

S. W. Hawking, N. Turok, arXiv:hep-th/9802030

A. Vilenkin, arXiv:hep-th/9803084

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- The instanton probes all energy scales: $B = B_{HE} + B_{LE}$

The core, governed by UV structure of the theory

The tail, sensitive to ``low energies”

Several options are possible:

- $B \approx B_{HE}$ - no sensitivity to the SM parameters (good and bad)

1803.08907, 1804.06376

- $B_{LE} \approx B_{HE}$ - all energy scales contribute (bad and good)

1903.11317

Note: We do not argue that the models we choose to test the mechanism can indeed be embedded into the UV complete theory of gravity.

Ingredients identified

For successful implementation of the mechanism, it seems important to have

- Non-minimal coupling of the scalar field to gravity

  For instance: $\frac{\mathscr{L}_E}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \underline{\xi\varphi^2})R + \frac{1}{2}(\partial\varphi)^2 + V(\varphi)$

- Approximate conformal invariance at high energies

  In fact, $B \sim \frac{1}{\textbf{deviation from the conformal point in UV}}$

- Higher-dimensional derivative operators $\sim (\partial\varphi)^4\ ,\ (\partial\varphi)^2(\partial\chi)^2\ ,\ldots$

  They are necessary to regularize the otherwise divergent (singular) instanton.

Outlook

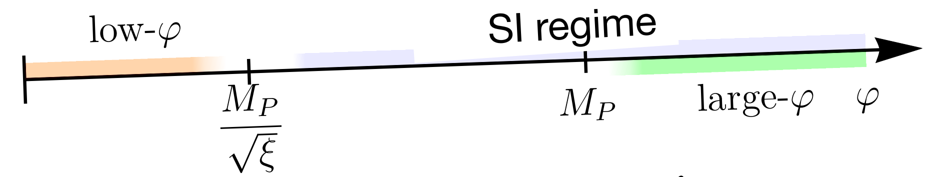
- Physical implications of (singular) instantons
- Correlation functions via (singular) instantons
- …

Thank you!

Just one example: Higgs inflation scenario

Lagrangian of the model: (inspired by Higgs inflation)

$$\frac{\mathscr{L}_J}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + V$$



$$V = \frac{\lambda}{4}\varphi^4$$

to get rid of the non-minimal coupling

to make the kinetic term canonical

Some fields redefinition:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \qquad \Omega^2 = \frac{M_P^2 + \xi\varphi^2}{M_P^2}\,, \qquad \boxed{\varphi = M_P e^{\bar{\varphi}/M_P}}$$

``Einstein frame" Lagrangian:

$$\frac{\mathscr{L}_E}{\sqrt{\tilde{g}}} = -\frac{1}{2}M_P^2\tilde{R} + \frac{1}{2a}(\tilde{\partial}\bar{\varphi})^2 + V\Omega^{-4}$$

$$a \xrightarrow{\text{SI regime}} a_{SI} = \frac{1}{1/\xi + 6}$$

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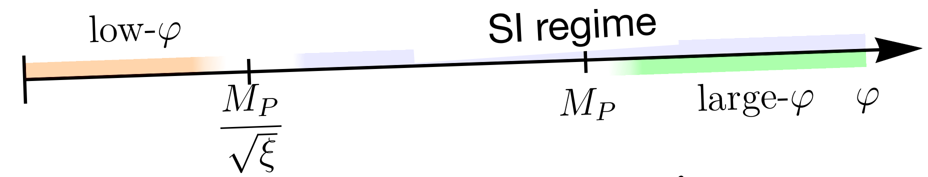

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$$a \xrightarrow{\text{SI regime}} a_{SI} = \frac{1}{1/\xi + 6}$$

Action to vary: $S' = -\,\bar{\varphi}(0)/M_P + S$

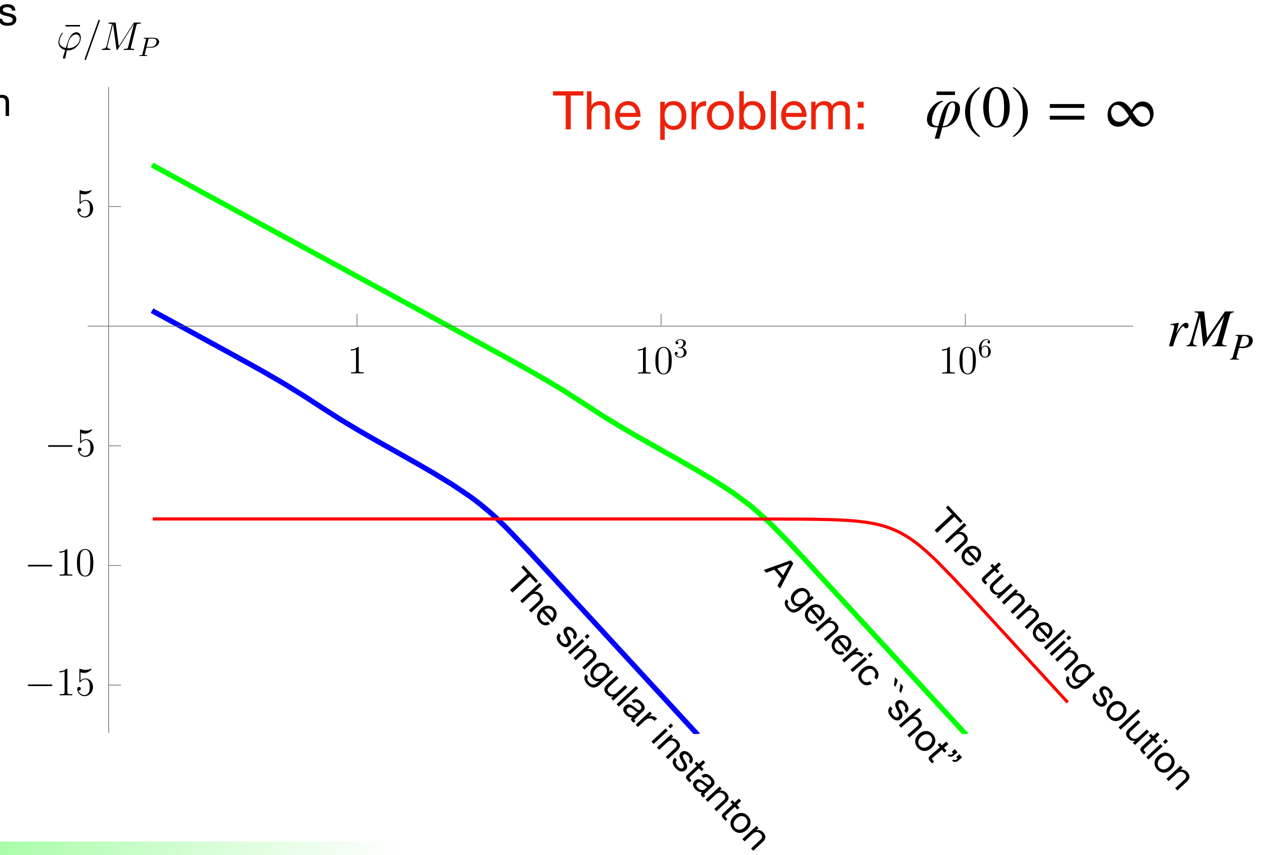
the source provides an additional boundary condition

Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

EoM for $\bar{\varphi}$ in the SI regime: $\dfrac{r^3\bar{\varphi}'}{f a_{SI}} = -\dfrac{1}{M_P}$

Short-distance asymptotics of the instanton: $\bar{\varphi}' \sim M_P r^{-1}$

The problem: $\bar{\varphi}(0) = \infty$



Just one example: Higgs inflation scenario

To cure the problem, let us modify the Lagrangian:

some operator with higher degree of the derivative of φ . For example, take $n = 2$

$$\frac{\mathscr{L}_J}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + V + \delta_n \frac{(\partial\varphi)^{2n}}{(M_P\Omega)^{4n-4}}$$

$$V = \frac{\lambda}{4}\varphi^4$$

Some fields redefinition:

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``Einstein frame” Lagrangian:

$$\frac{\mathscr{L}_E}{\sqrt{\tilde{g}}} = -\frac{1}{2}M_P^2\tilde{R} + \frac{1}{2a}(\tilde{\partial}\bar{\varphi})^2 + V\Omega^{-4}$$

$$\frac{\delta\mathscr{L}_E}{\sqrt{\tilde{g}}} \xrightarrow{\text{SI regime}} \delta_2\frac{(\tilde{\partial}\bar{\varphi})^4}{\xi^2 M_P^4}$$

$$a \xrightarrow{\text{SI regime}} a_{SI} = \frac{1}{1/\xi + 6}$$

Action to vary: $S' = -\bar{\varphi}(0)/M_P + S$

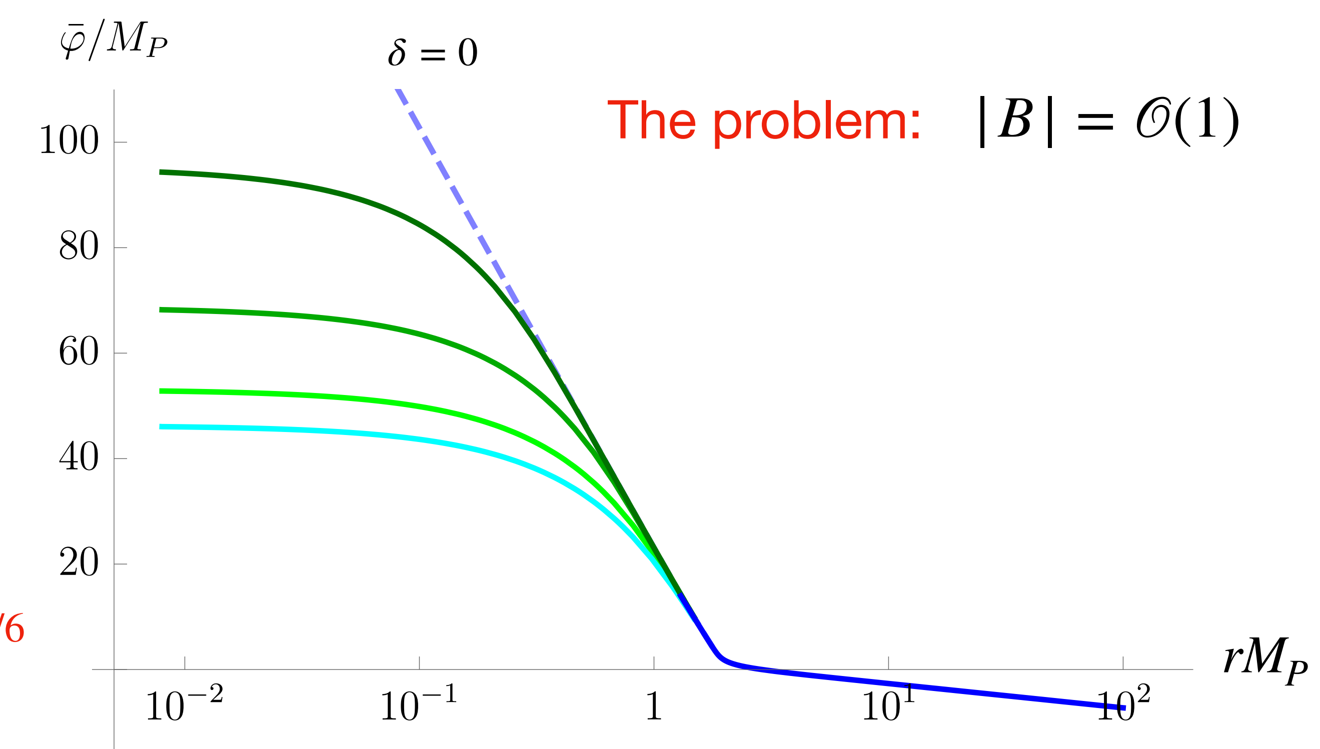
Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

EoM for $\bar{\varphi}$ in the SI regime: $\dfrac{4\delta}{M_P^4}\dfrac{\rho^3\bar{\varphi}'^3}{f^3} + \dfrac{r^3\bar{\varphi}'}{fa_{SI}} = -\dfrac{1}{M_P}$

$\delta = \delta_2/\xi^2$

Short-distance asymptotics of the instanton: $\bar{\varphi}' \sim M_P^2\delta^{-1/6}$

The problem: $|B| = \mathscr{O}(1)$



Different colors for different δ

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some polynomial operator

$$\frac{\mathscr{L}_J}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}F(\varphi/M_P)(\partial\varphi)^2 + V + \delta_n \frac{(\partial\varphi)^{2n}}{(M_P\Omega)^{4n-4}}$$

$$V = \frac{\lambda}{4}\varphi^4$$

Some fields redefinition:

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``Einstein frame'' Lagrangian:

$$\frac{\mathscr{L}_E}{\sqrt{\tilde{g}}} = -\frac{1}{2}M_P^2\tilde{R} + \frac{1}{2a'}(\tilde{\partial}\bar{\varphi})^2 + V\Omega^{-4}$$

SI regime

$$\frac{\delta\mathscr{L}_E}{\sqrt{\tilde{g}}} \longrightarrow \delta_2 \frac{(\tilde{\partial}\bar{\varphi})^4}{\xi^2 M_P^4}$$

large-φ regime

$$a' \longrightarrow a_{HE} \gg a_{SI}$$

Action to vary: $S' = -\bar{\varphi}(0)/M_P + S$

Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

EoM for $\bar{\varphi}$ in the HE regime: $\frac{4\delta}{M_P^4}\frac{\rho^3\bar{\varphi}'^3}{f^3} + \frac{r^3\bar{\varphi}'}{f a_{HE}} = -\frac{1}{M_P}$

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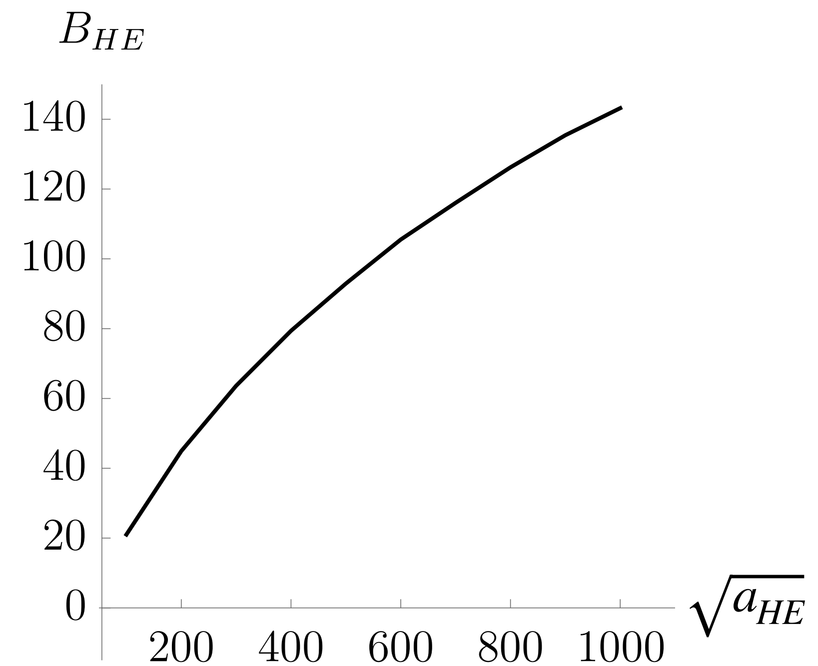

Here $\xi \sim 10^3$, $\delta \sim 10^{-8}$

The result: $B_{HE} \sim \sqrt{a_{HE}}$ $\quad B \approx B_{HE}$ $\quad \langle\varphi\rangle \sim M_P e^{-B}$