DYNAMICAL FRICTION IN A SUPERFLUID

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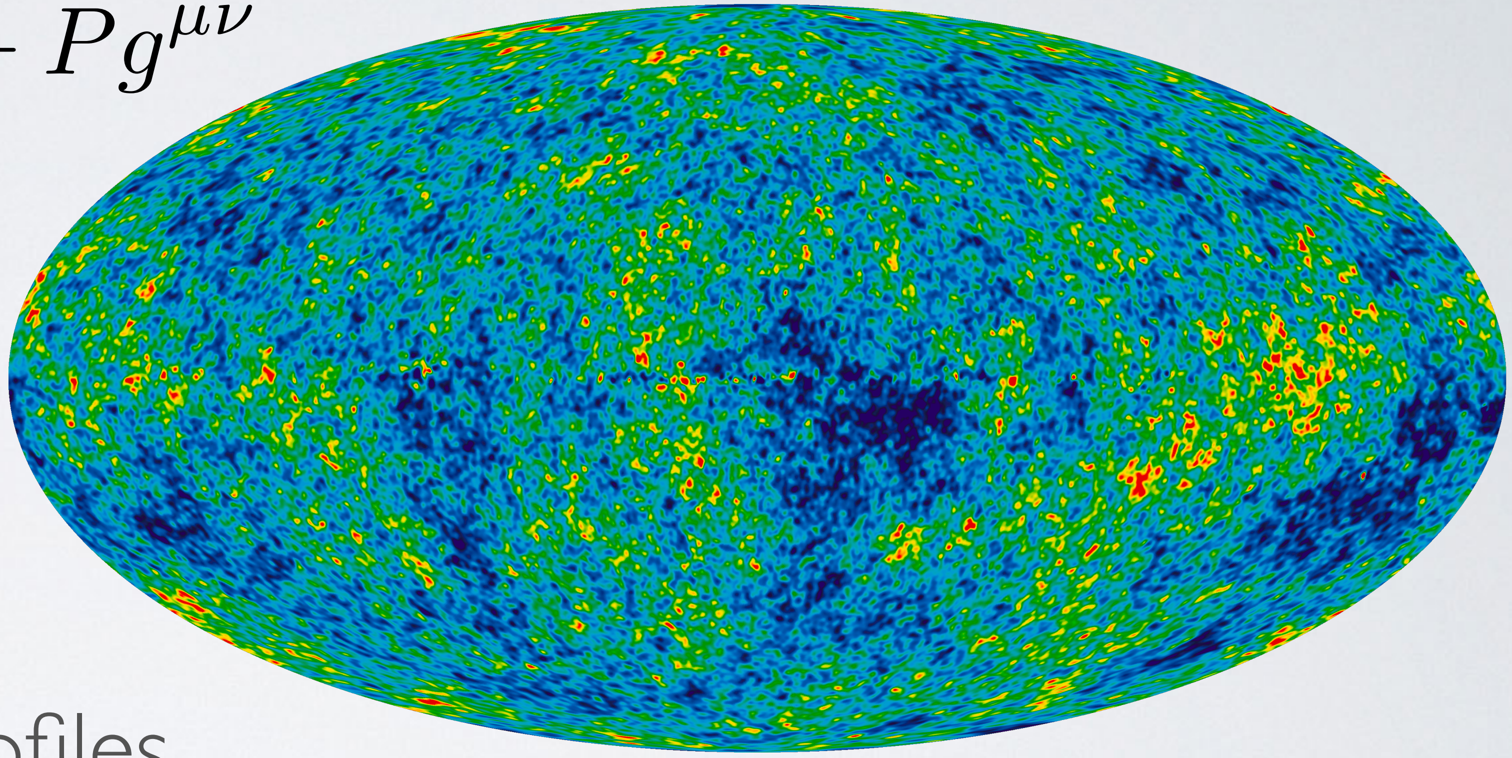
Is dark matter a **particle**, or is it a **fluid**?

On cosmological scales:

$$T^{\mu\nu} = (\rho + P)U^\mu U^\nu + P g^{\mu\nu}$$

$$P \approx 0$$



On galactic scales…?

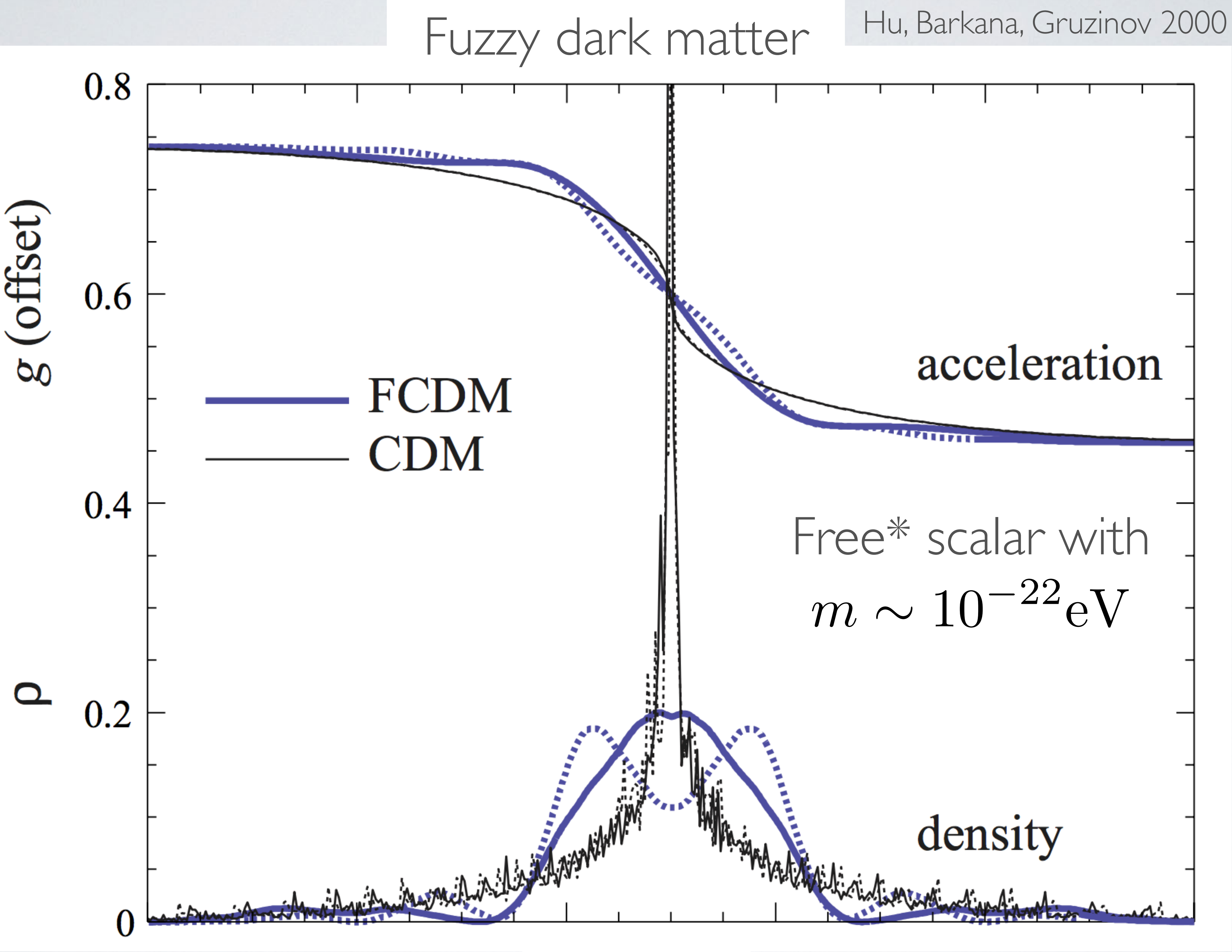
Non-cuspy density profiles

Baryonic mass correlated to circular velocity

Is dark matter a **particle**, or is it a **fluid**?

Fuzzy dark matter

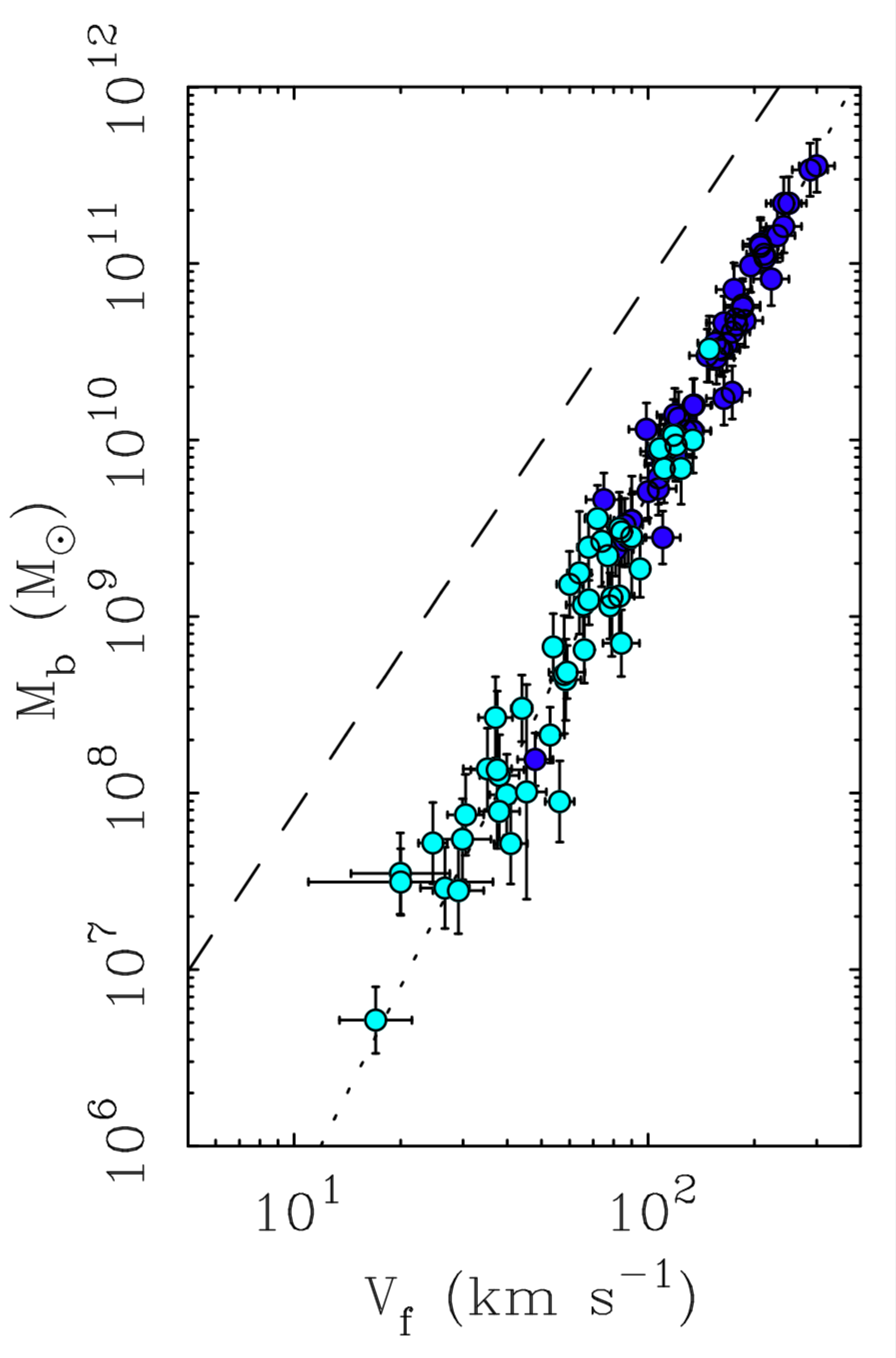
Hu, Barkana, Gruzinov 2000



Free* scalar with

$m \sim 10^{-22}\text{eV}$

Is dark matter a **particle**, or is it a **fluid**?



Superfluid dark matter

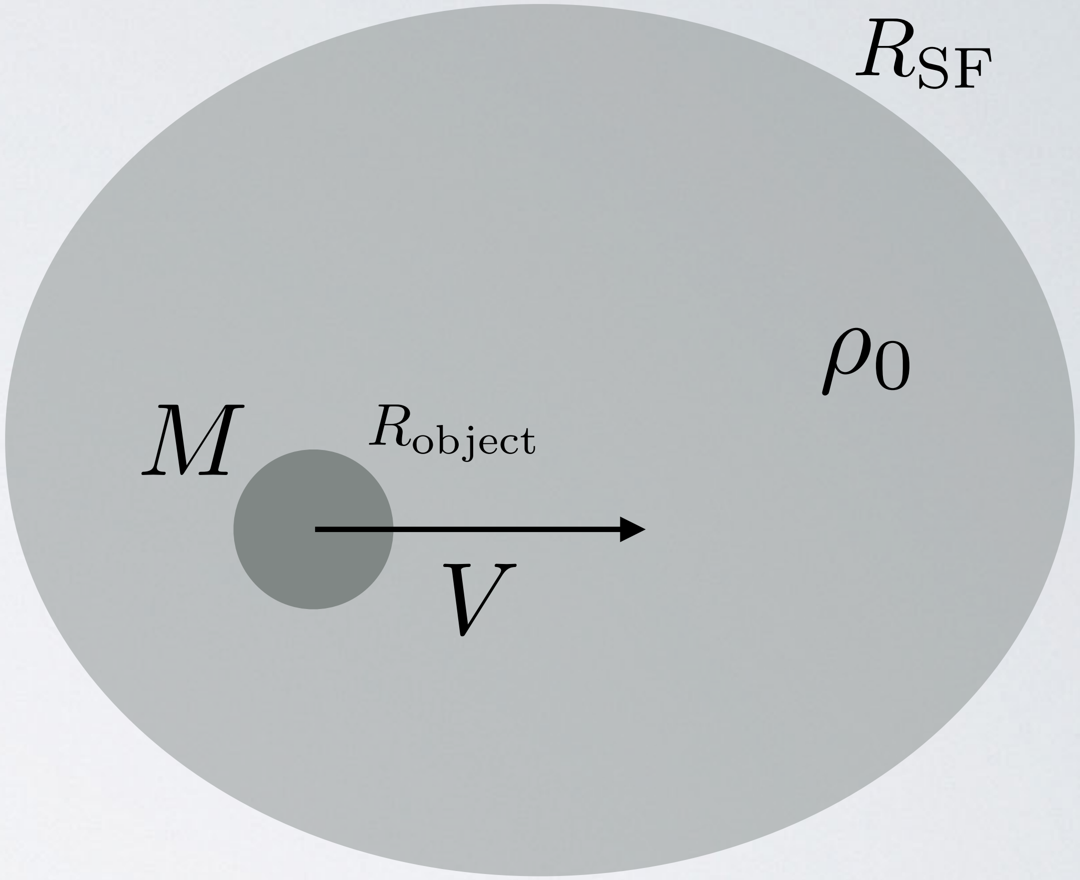
Berezhiani & Khoury 2015

DM forms a superfluid condensate around galaxies

Phonons mediate a fifth force that reproduces the Baryonic Tully-Fisher Relation

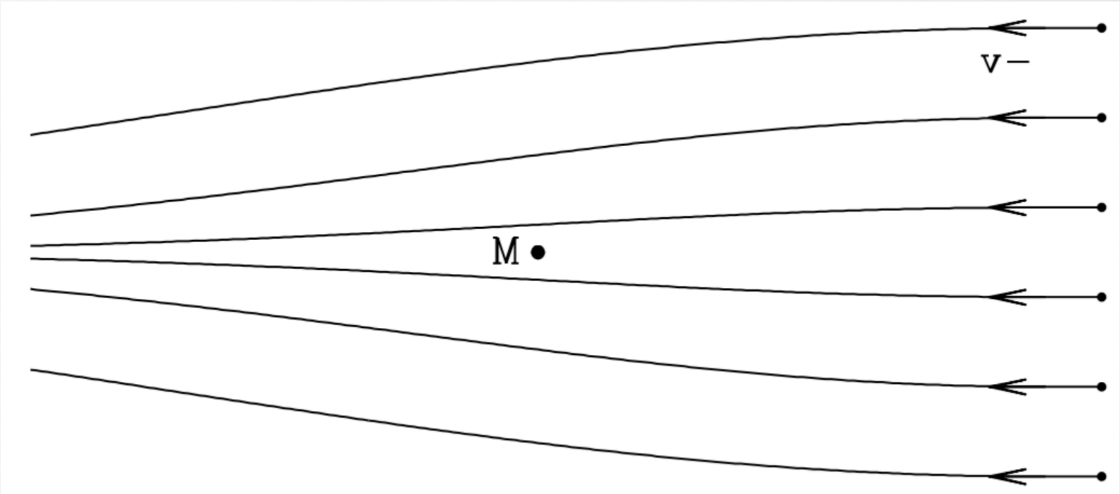
How to distinguish particle dark matter from dark matter as a fluid?

Dynamical friction



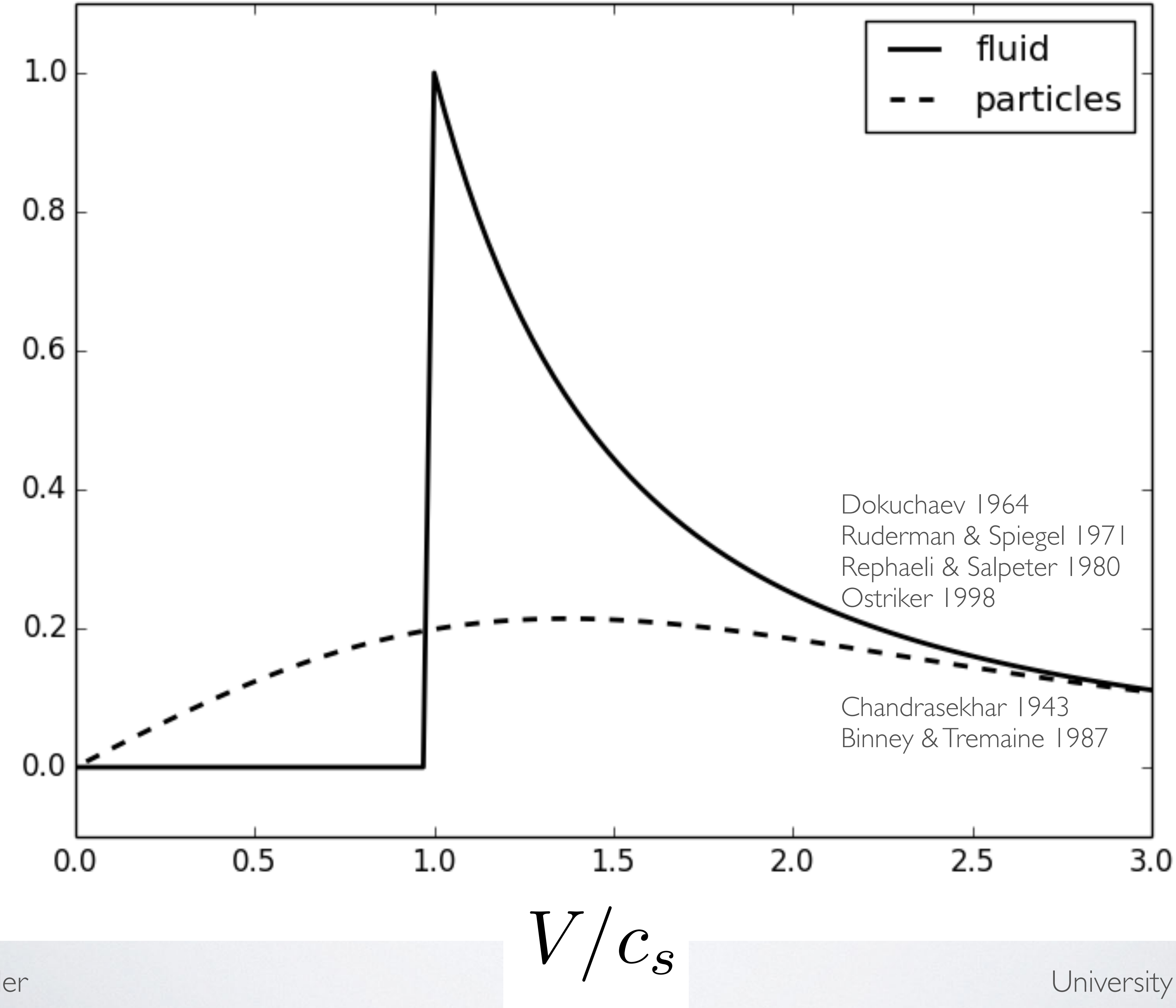
(i) A perturber moves through a uniform cloud of particles

(ii) An overdensity builds up behind the perturber



(iii) The gravitational attraction of the overdensity slows the perturber

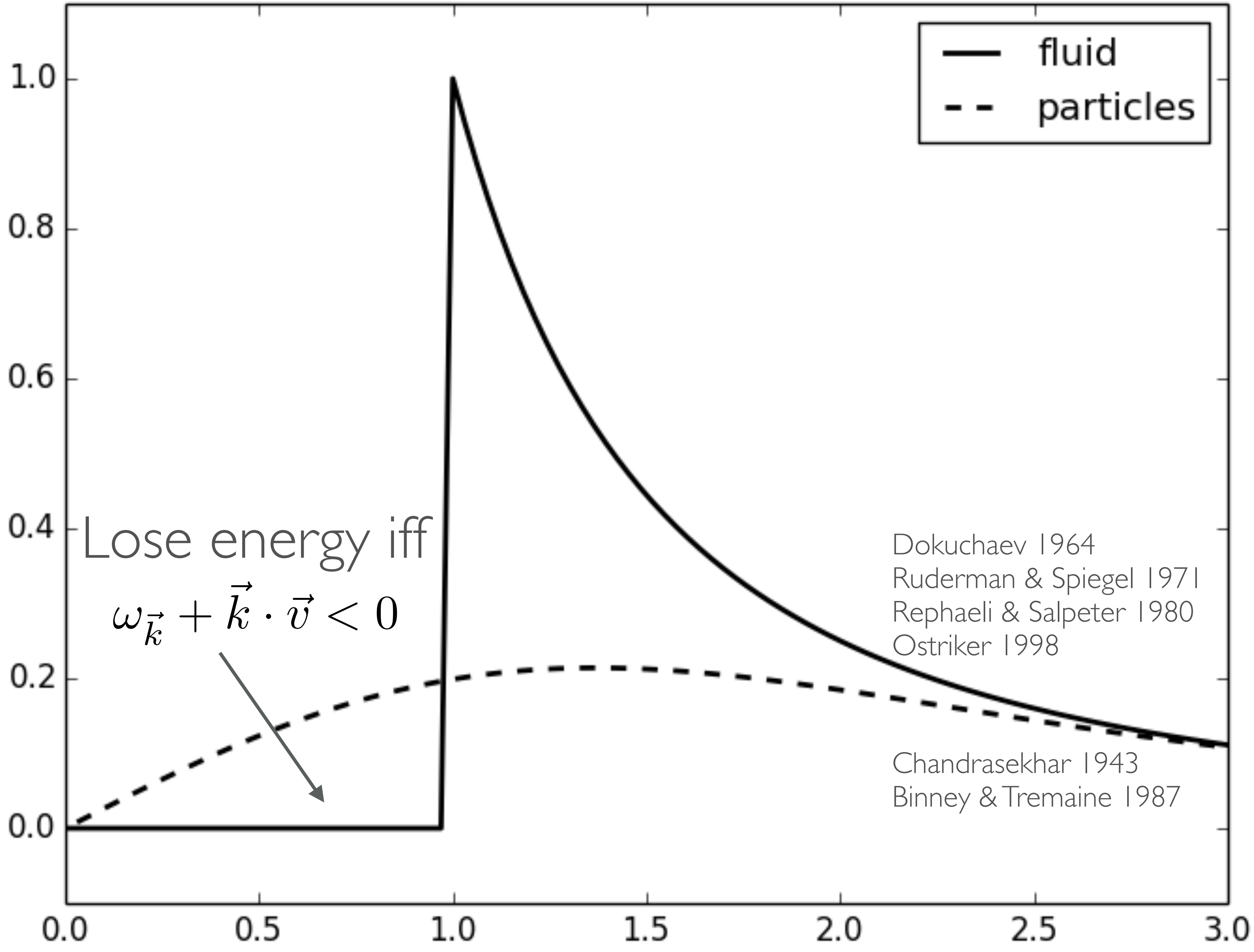
Dynamical friction force $[4\pi(GM)^2\rho_0 c_s^{-2}]$



Dokuchaev 1964
Ruderman & Spiegel 1971
Rephaeli & Salpeter 1980
Ostriker 1998

Chandrasekhar 1943
Binney & Tremaine 1987

Dynamical friction force $[4\pi(GM)^2\rho_0 c_s^{-2}]$



Lose energy iff

$\omega_{\vec{k}} + \vec{k}\cdot\vec{v} < 0$

Dokuchaev 1964
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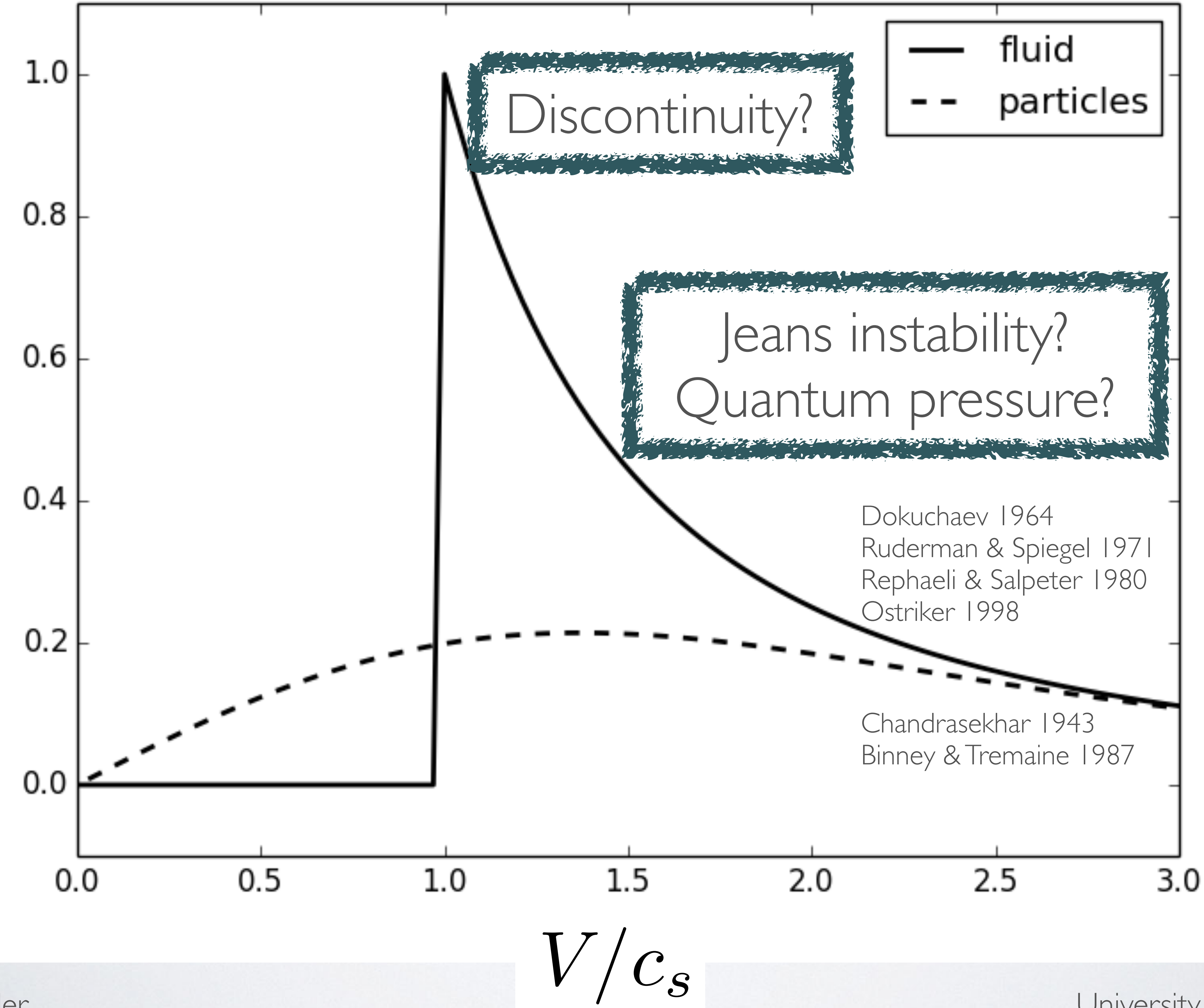
V/c_s

Dynamical friction force $[4\pi(GM)^2\rho_0 c_s^{-2}]$



Discontinuity?

Jeans instability?
Quantum pressure?

Dokuchaev 1964
Ruderman & Spiegel 1971
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Ostriker 1998

Chandrasekhar 1943
Binney & Tremaine 1987

Hydrodynamical description of superfluid dynamical friction

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G} - |\partial\Psi|^2 - m^2|\Psi|^2 - \frac{\lambda}{2}|\Psi|^4\right)$$

Non-relativistic limit: $\Psi = \frac{\psi}{\sqrt{2m}}\mathrm{e}^{-imt}$

$$\mathrm{i}\partial_t\psi = -\frac{\Delta}{2m}\psi + \frac{\lambda}{4m^2}|\psi|^2\psi + m\Phi\psi$$

Gross-Pitaevskii

Cast GP into hydro form:

$$\psi = \sqrt{\frac{\rho}{m}}\mathrm{e}^{\mathrm{i}\theta} \qquad P = \frac{\lambda}{8m^4}\rho^2 \qquad \vec{v} = \frac{\vec{\nabla}\theta}{m}$$

"Quantum pressure"

$$\begin{aligned} \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) &= 0\,; \\ \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} &= -\frac{1}{\rho}\vec{\nabla}P - \vec{\nabla}\Phi + \frac{1}{2m^2}\vec{\nabla}\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right) \end{aligned}$$

Hydrodynamical description of superfluid dynamical friction

$$
\begin{aligned}
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\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} &= -\frac{1}{\rho} \vec{\nabla} P - \vec{\nabla} \Phi + \frac{1}{2m^2} \vec{\nabla} \left(\frac{\Delta \sqrt{\rho}}{\sqrt{\rho}} \right)
\end{aligned}
$$

Linearize: $\rho = \rho_0 (1 + \alpha)$ $c_s^2 = \frac{\partial P}{\partial \rho}\Big|_{\rho_0}$

$$\ddot{\alpha} - c_s^2 \Delta \alpha + \frac{1}{4m^2} \Delta^2 \alpha = \Delta \phi$$

$$\Delta \phi = 4\pi G \left(\rho_{\rm ext} + \rho_0 \alpha \right)$$

Combine:

$$\ddot{\alpha} - c_s^2 \Delta \alpha - m_g^2 \alpha + \frac{1}{4m^2} \Delta^2 \alpha = 4\pi G \rho_{\rm ext}$$

Tachyonic mass

$$m_g^2 \equiv 4\pi G \rho_0$$

Quantum pressure

Perturber sourcing overdensity

Hydrodynamical description of superfluid dynamical friction

$$\ddot{\alpha} - c_s^2 \Delta\alpha - m_g^2 \alpha + \frac{1}{4m^2}\Delta^2\alpha = 4\pi G \rho_{\rm ext}$$

$$\rho_{\rm ext}(x) = M\delta(x)\delta(y)\delta(z - Vt)$$

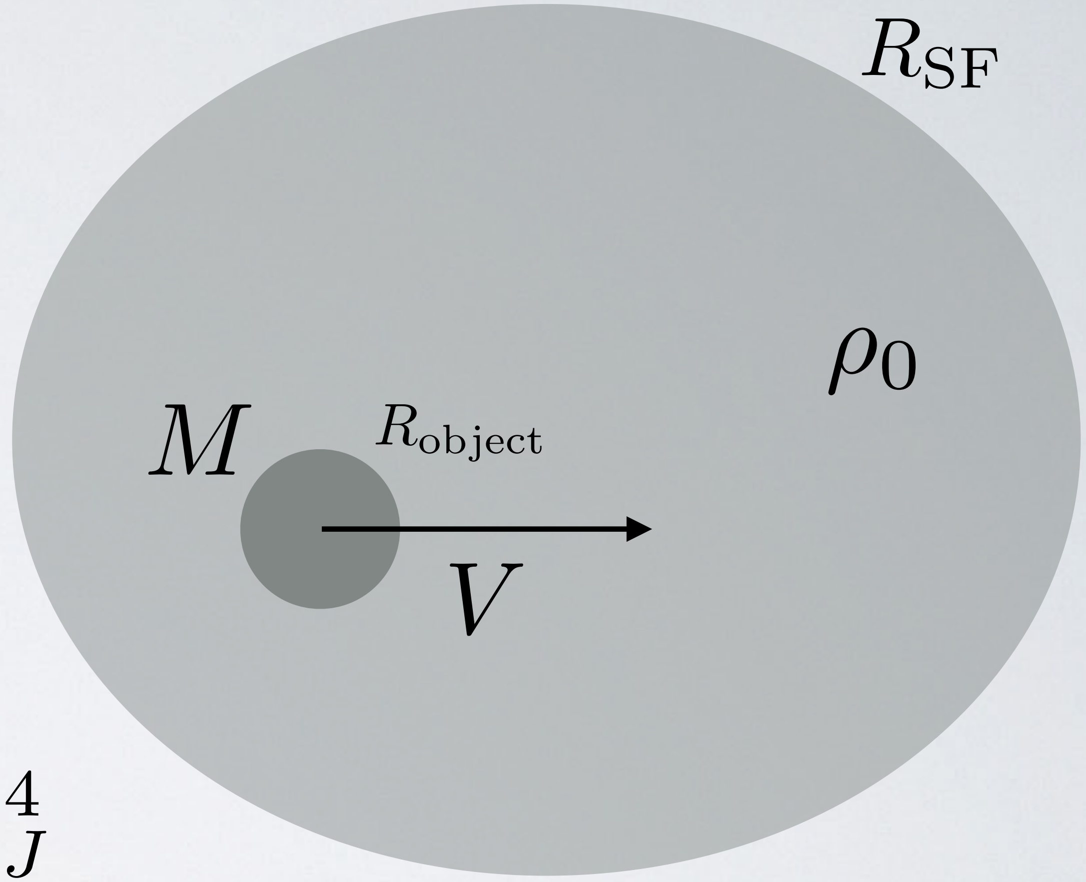
Dynamical friction is the work done by gravity: $F = \frac{M}{V}\dot{\phi}_\alpha$

$$\phi_\alpha(k_0, \vec{k}) = -\frac{4\pi G\rho_0}{\vec{k}^2}\alpha(k_0, \vec{k})$$

$$\alpha(k_0, \vec{k}) = -\frac{4\pi G\rho_{\rm ext}(k)}{k_0^2 - \omega_k^2}$$

$$F = \frac{M}{V}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \mathrm{e}^{\mathrm{i}k_0 t - \mathrm{i}\vec{k}\cdot\vec{x}}\, \mathrm{i}k_0\, \frac{4\pi G\rho_0}{\vec{k}^2}\, \frac{4\pi G\rho_{\rm ext}(k)}{k_0^2 - \omega_k^2}$$

Low-momentum cutoffs



Cutoff I: size of superfluid cloud

$$k_{\rm min} = 2\pi R_{\rm SF}^{-1}$$

Cutoff II: Jeans scale

$$k_{\rm min} = k_{\rm J}$$
$$\omega_{k_{\rm J}}^2 \equiv 0 = -m_g^2 + c_s^2 k_J^2 + \frac{k_J^4}{4m^2}$$

Use whichever cutoff whose scale is higher:

$$k_{\rm min} = \max\left(2\pi R_{\rm SF}^{-1}, k_{\rm J}\right)$$

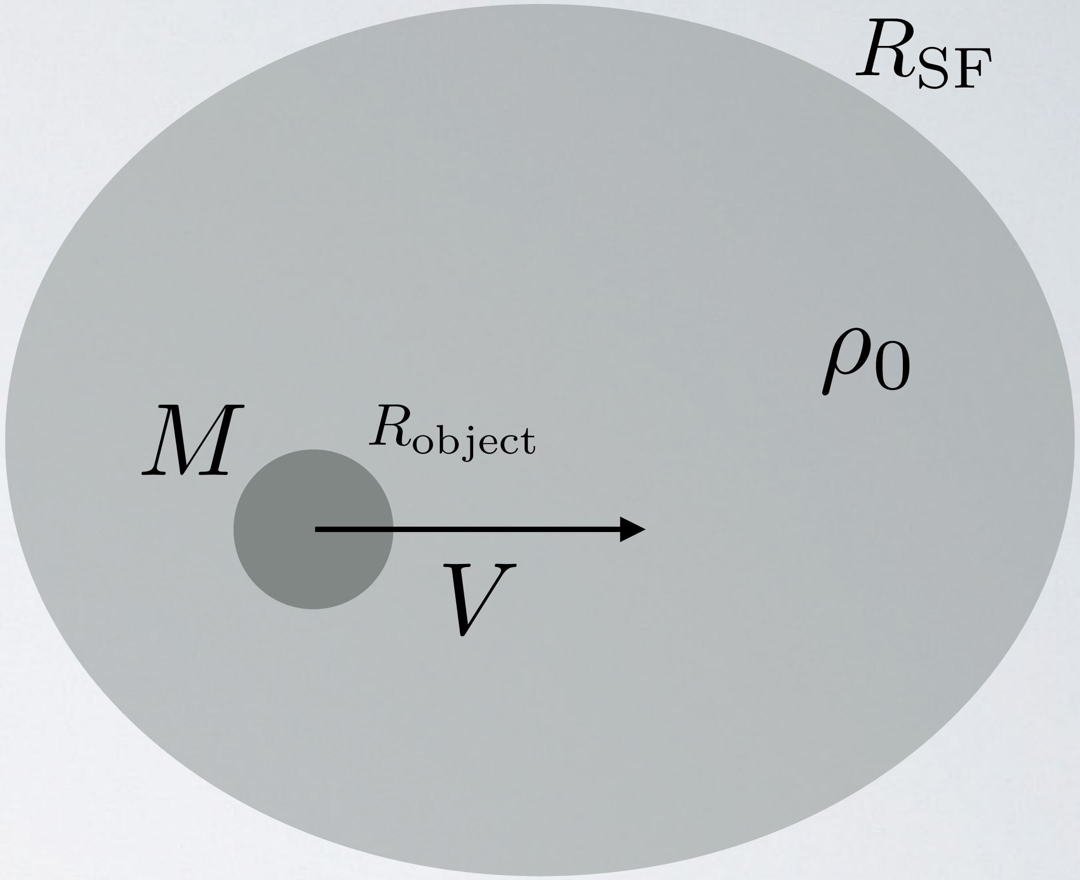
Hydrodynamical description of superfluid dynamical friction

High-momentum cutoffs

Cutoff I: size of perturber

(or linearization breaks down)

$$k_{\mathrm{max}} = 2\pi R_{\mathrm{object}}^{-1}$$



"Cutoff" II: integrand is non-zero

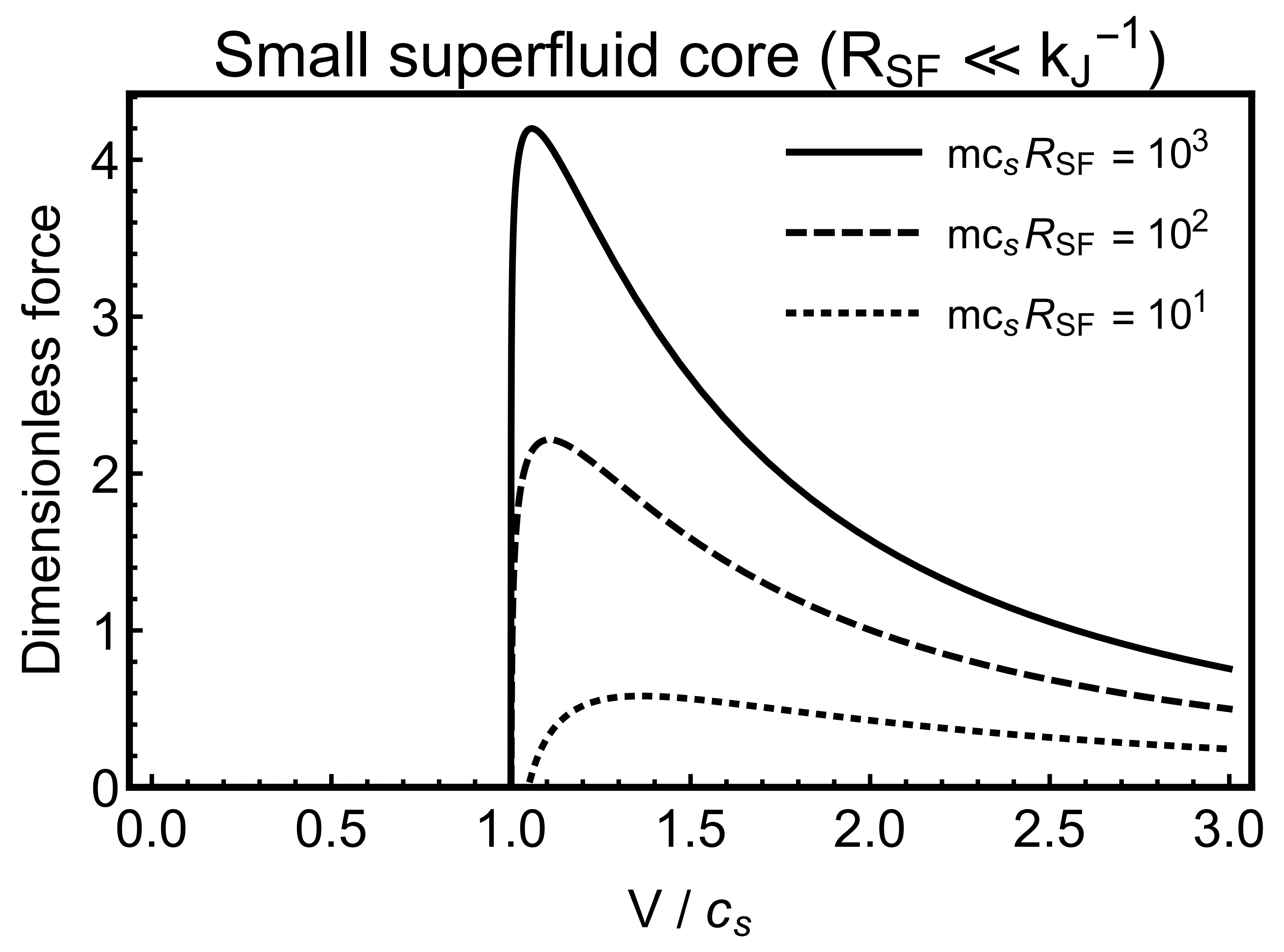
$$F = -\frac{4\pi G^2 M^2 \rho_0}{V^2} \int \frac{\mathrm{d}k}{k} \, \mathrm{d}\cos\theta \, \mathrm{e}^{\mathrm{i}\omega_k t - \mathrm{i}kVt\cos\theta} \delta\left(\frac{\omega_k}{kV} - \cos\theta\right)$$

Integrand is only non-zero for modes $k < k_\star$ $\quad \frac{\omega_{k_\star}}{k_\star V} \equiv 1$

$$k_{\mathrm{max}} = \min\left(2\pi R_{\mathrm{object}}^{-1}, k_\star\right)$$

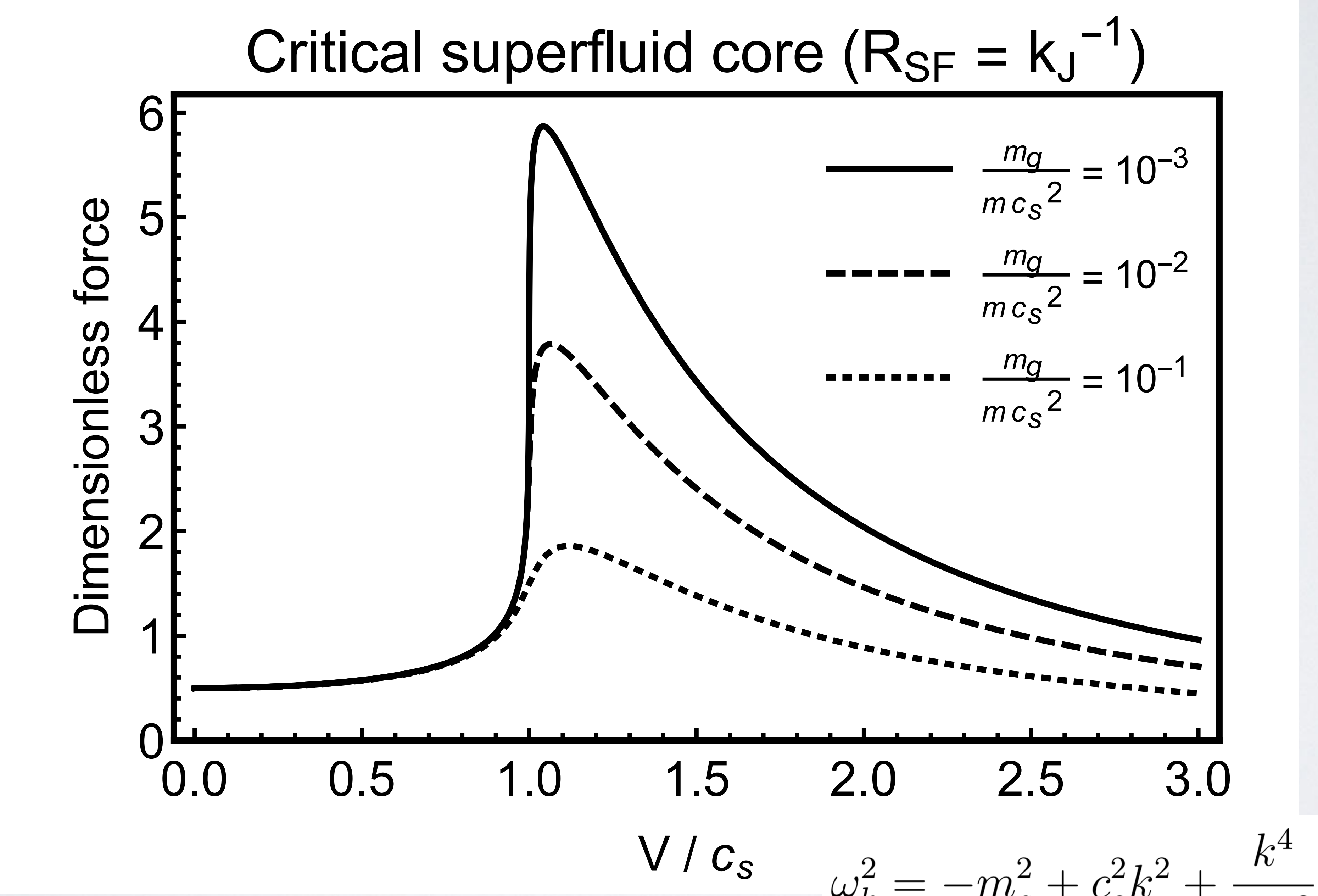
Results I: small superfluid core

$$|F| = \frac{4\pi G^2 M^2 \rho_0}{V^2} \ln\left(\frac{\min\left(2\pi R_{\mathrm{object}}^{-1}, k_\star\right)}{\max\left(2\pi R_{\mathrm{SF}}^{-1}, k_{\mathrm{J}}\right)}\right)$$



Results II: large superfluid core

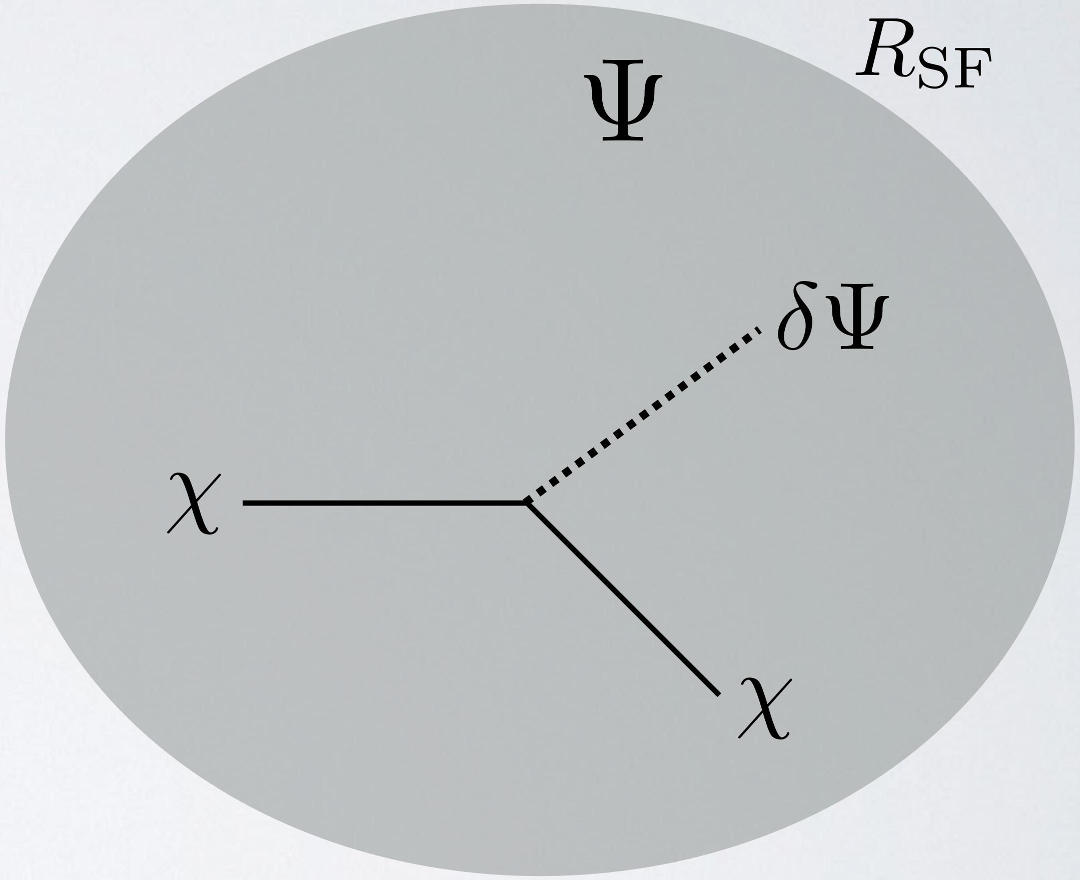
$$|F| = \frac{4\pi G^2 M^2 \rho_0}{V^2} \ln\left(\frac{\min\left(2\pi R_{\mathrm{object}}^{-1}, k_\star\right)}{\max\left(2\pi R_{\mathrm{SF}}^{-1}, k_{\mathrm{J}}\right)}\right)$$



$$\omega_k^2 = -m_g^2 + c_s^2 k^2 + \frac{k^4}{4m^2}\,.$$

Quasiparticle description of superfluid dynamical friction

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\pi G}R \underbrace{- |\partial\Psi^2| - m^2|\Psi|^2 - \frac{\lambda}{2}|\Psi|^4}_{\text{Superfluid}} \underbrace{- \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\chi^2}_{\text{Perturber}}\right)$$



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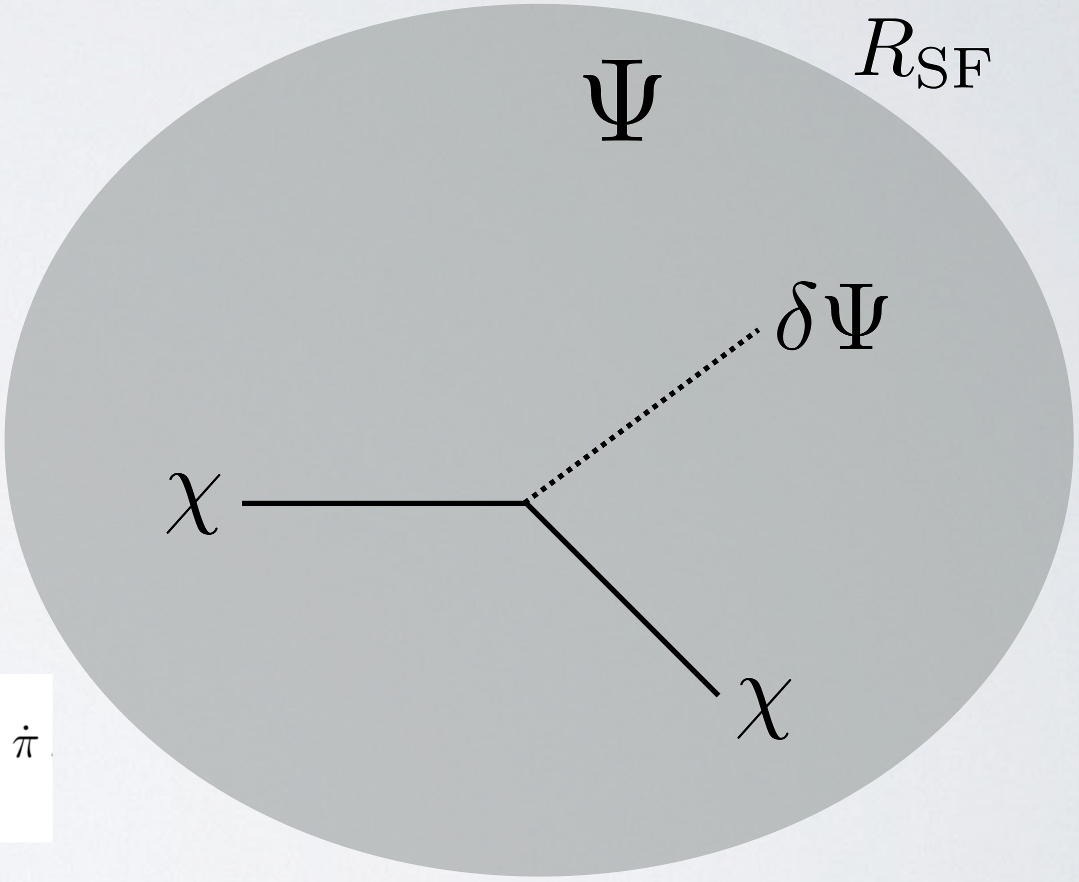
Perturb the condensate:

$$\Psi = (v+h)\mathrm{e}^{\mathrm{i}\sqrt{m^2+\lambda v^2}t+\mathrm{i}\pi}$$

Effective theory of phonons:

$$\mathcal{L} = \frac{1}{2}\dot{\pi}^2 + \frac{1}{2}\pi\left(m_g^2 + c_s^2\Delta - \frac{\Delta^2}{4m^2}\right)\pi - 4\pi G M^2\sqrt{\rho_0}\chi^2\frac{1}{\sqrt{\Delta\left(m_g + c_s^2\Delta - \frac{\Delta^2}{4m^2}\right)}}\dot{\pi}$$



Rate of energy loss: $|F| = \frac{\dot{E}}{V} = \int \omega_k\,\mathrm{d}\Gamma \longrightarrow \boxed{|F| = \frac{4\pi G^2M^2\rho_0}{V^2}\ln\left(\frac{\min\left(2\pi R_{\mathrm{object}}^{-1}, k_\star\right)}{\max\left(2\pi R_{\mathrm{SF}}^{-1}, k_{\mathrm{J}}\right)}\right)}$

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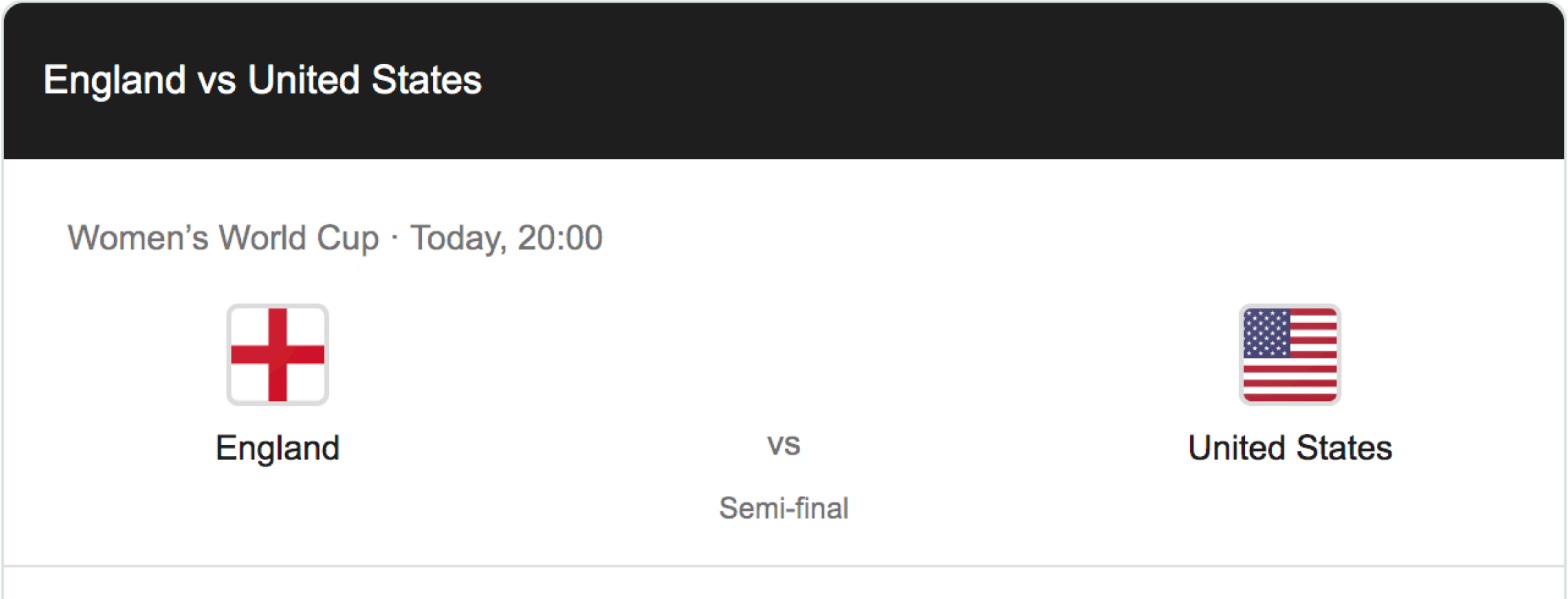
$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\pi G}R - |\partial\Psi^2| - m^2|\Psi|^2 - \frac{\lambda}{2}|\Psi|^4 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\chi^2\right)$$



England vs United States

Women's World Cup · Today, 20:00

England

vs

United States

Semi-final

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