The universe as a condensate of spacetime atoms

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Review: arXiv:1602.08104 (with L. Sindoni)
Toy model: arXiv:1712.07266 (with E. Adjei, W. Wieland)
Inhomogeneities: arXiv:1709.01095 (with D. Oriti), arXiv:1811.10639

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Incompleteness of classical cosmology

The standard picture of modern cosmology suggests that the Universe started at a singularity of the spacetime manifold where densities, energies, etc. diverge and conventional physics breaks down. The singularity suggests that *quantum gravity is needed to account for the beginning of the Universe*.

(There are other ways of avoiding the singularity, such as (classically) modifying gravity or introducing exotic matter, but generic inflationary spacetimes are singular in the past [Borde, Guth, Vilenkin 2003].)

Quantum gravity should account for the **initial conditions** for the early universe, in particular show the emergence of a regime in which usual quantum field theory (QFT) on curved spacetime techniques apply.

Quantum cosmology models built to justify the usual (Bunch-Davies) initial conditions remain controversial (e.g., [Feldbrugge, Lehners, Turok 2017]).

Matrix models

Archetypal example of "emergence" of continuum geometry from combinatorics: matrix models for two-dimensional (Riemannian) geometry.

Define an action for a Hermitian $N \times N$ matrix M by

$$S(M) = \frac{1}{2}\mathrm{Tr}(M^2) - \frac{g}{\sqrt{N}}\mathrm{Tr}(M^3) \equiv \frac{1}{2}M_{ij}M_{ji} - \frac{g}{\sqrt{N}}M_{ij}M_{jk}M_{ki}$$

and define a partition function

$$Z = \int dM\, e^{-S(M)} = \sum_{\Gamma} g^{V_\Gamma} N^{\chi} =: \sum_{\Delta} e^{\frac{4\pi}{G}\chi(\Delta) - \frac{\Lambda}{G}A(\Delta)}\,,$$

expanded in Feynman graphs (vacuum bubbles) Γ or dual triangulated 2-manifolds Δ. This gives a sum over discrete geometries, in which $\chi = 2$ (spheres) dominate in the continuum limit $N \to \infty$, weighted by the 2d Einstein-Hilbert action.

Group field theory

Group field theories generalise this idea of "geometry from combinatorics" to higher dimensions (in particular four spacetime dimensions). Define

$$\varphi : \mathrm{SU}(2)^4 \to \mathbb{C}\,, \quad \varphi(g_1, g_2, g_3, g_4) = \varphi(g_1 h, g_2 h, g_3 h, g_4 h)\ \forall h \in \mathrm{SU}(2)$$

then, for an appropriate $S[\varphi, \bar{\varphi}]$,

$$Z = \int \mathcal{D}\varphi\, \mathcal{D}\bar{\varphi}\ \exp\left(-S[\varphi, \bar{\varphi}]\right) = \sum_{\Gamma} \frac{\lambda^{N(\Gamma)}}{\mathrm{sym}(\Gamma)} Z[\Gamma]$$

generates a sum over 4d simplicial complexes Γ weighted by an $Z[\Gamma]$, itself a sum over the discrete geometries associated to Γ.

Main challenges on the right-hand side are controlling the sum over Γ and taking a continuum limit. Address these by working in terms of φ!

A group field theory condensate

Perturbation theory around $\varphi = 0$ corresponds to 4d geometries made from very few "building blocks". For a continuum to emerge, the number of building blocks should be as large as possible.

This suggests [SG, Oriti, Sindoni 2013] studying perturbations around a different phase in which

$$\langle\varphi(g_1,\ldots,g_4)\rangle =: \sigma(g_I) \neq 0\,.$$

The mean field $\sigma(g_I)$ is analogous to the "condensate wavefunction" in a **Bose-Einstein condensate** in condensed matter physics. It describes a homogeneous configuration of building blocks in the same quantum state [SG, Oriti, Sindoni 2014].

$\sigma(g_I)$ should satisfy the (nonlinear, nonlocal) field equations of a group field theory. Simplify these by assuming an "isotropic form"

$$\sigma(g_I) = \sum_j \sigma_j \, \mathrm{D}^j(g_I)$$

Dynamical condensates [Oriti, Sindoni, Wilson-Ewing 2016; SG 2016]

Dynamics is defined with respect to a matter clock field ϕ, which becomes an additional argument of the GFT field; the equation of motion is then, e.g.

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \bar{\sigma}_j^4(\phi) = 0\,.$$

The total 3-volume at "scalar time" ϕ is given by $V(\phi) = \sum_j V_j |\sigma_j(\phi)|^2$.

In the simplest case, where there is only a single j component and GFT interactions can be neglected, we have

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) = 0 \quad \Rightarrow \quad V(\phi) \stackrel{\phi \to \pm\infty}{\to} |\alpha^\pm|^2 \exp\left(\pm 2\sqrt{\frac{B_j}{A_j}}\phi\right)\,.$$

The GFT condensate solution for $V(\phi)$ corresponds to a **quantum bounce** which interpolates between the classical contracting and expanding solutions

$$V(\phi) = V_0 \exp(\pm\sqrt{12\pi G}\phi)\,.$$

Toy model for GFT condensates

A simple model for GFT condensate cosmology [Adjei, SG, Wieland 2018] consists of creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ for a single GFT mode, with dynamics given by a *squeezing* Hamiltonian

$$\hat{\mathcal{H}} = \frac{\mathrm{i}}{2}\lambda\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)\,.$$

As is well known in quantum optics, the number of quanta in a squeezed state $|\phi\rangle \equiv \exp(-\mathrm{i}\phi\hat{\mathcal{H}})|0\rangle$ is

$$\langle\phi|\hat{a}^\dagger\hat{a}|\phi\rangle = \sinh^2(\lambda\phi) \sim \frac{1}{4}e^{2|\lambda\phi|}$$

which is exactly a classical "bounce" solution for $\lambda = \sqrt{3\pi G}$. By applying a Legendre transform to the full (free) GFT action one can show that the GFT Hamiltonian is also of this squeezing type [Wilson-Ewing 2018].

Towards cosmological phenomenology

In order to describe a more realistic universe, we extend the formalism from exactly homogeneous to slightly inhomogeneous universes. This is possible in a GFT formalism for quantum gravity coupled to *four* free, massless scalar fields.

There is now a local 3-volume element $V(\phi^I)$ for each spacetime point. One can compute fluctuations in this observable through the 2-point function

$$\langle\delta\hat{V}(\phi^I)\delta\hat{V}(\phi'^I)\rangle\,,\quad \delta\hat{V}(\phi^I)=\hat{V}(\phi^I)-\langle\hat{V}(\phi^I)\rangle\,,$$

finding a non-zero power spectrum due to vacuum fluctuations [SG, Oriti 2018]
This can be converted into the observationally relevant power spectrum of the gauge-invariant curvature perturbation variable ζ, where one finds [SG 2019]

$$\Delta^2_\zeta(k)\sim 0.01\left(\frac{m_{\rm Pl}V}{j^{3/2}M^4}\right)k^3\,.$$

Spectral index $n_s=4$ consistent with semiclassical QFT on curved spacetime!

Summary

- Fundamental challenge for theoretical cosmology to explain the emergence of a semiclassical universe described by QFT on curved spacetime; addressing it might explain initial conditions, or constrain possible early universe scenarios.

- **Group field theories** are a generalisation of matrix models in which there are additional group-theoretic degrees of freedom associated to discrete geometry. Proposal that macroscopic universe corresponds to a *condensate* in GFT.

- Simple approximations to dynamical equations for the condensate mean field can correctly reproduce a flat cosmology with massless scalar matter, resolving the classical singularity by a bounce. Interactions modify the resulting cosmological dynamics (e.g., φ^4 leads to dust, φ^6 to Λ)

- New proposal for generating inhomogeneities through vacuum fluctuations in GFT, rather than in a separate scalar field. Consistency with usual semiclassical treatment shown in one example. Needs to be generalised!

Thank you!