Behaviors of two supersymmetry breaking scales in N = 2 supergravity

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arxiv:1907.XXXX

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Background

- Extended (N≧2) supergravity in 4D naturally appears from higher-dimensional supergravity and string compactifications.
- We need to consider its breaking for phenomenological application (e.g., chiral structure)

There are several breaking patterns unlike N = 1 case:

・The vacuum may preserve some supersymmetries (**partial breaking**)

・Even if the full breaking occurs,
some of supersymmetry breaking scales may be **degenerate** or **hierarchical**

➡ They significantly affect particle and cosmological phenomenology

Purpose

Our purpose
＝To clarify the relations between the breaking patterns and input parameters in the theory

We focus on N=2 supergravity (good practice partner)

N=2 (unbroken)
N=1 (partially broken)
N=0 (fully broken)

Talk Plan

- ~~Introduction~~
- Setup
- Gravitino masses
- Scalar potential and stationary conditions
- Behavior of two supersymmetry breaking scales
- Summary

[1]S. Ferrara, L. Girardello, M. Porrati,
Phys. Lett. B 366, 155 (1996) ,Phys. Lett. B 376, 275 (1996)

Setup

- We focus on the model of Ref. [1]

Single vector multiplet (SU(1,1)/U(1))
Single hypermultiplet (SO(4,1)/SO(4))

can realize partial breaking

⬇

$$\text{Vector multiplets}: \ \{z^i, \lambda^{iA}, A^i_\mu\}, \quad \underline{(i = 1, \cdots, n_v)}$$

Multiple generalization

$$\text{Hypermultiplet}: \ \{b^u, \zeta_\alpha\},$$

$$\text{Gravitational multiplet}: \ \{g_{\mu\nu}, \psi^A_\mu, A^0_\mu\}.$$

$(A = 1, 2)$

$(u = 0, \cdots, 3)$

$\alpha = 1, 2$

Setup

- Vector sector

Prepotential: $f(z^i) \qquad (i = 1, \cdots, n_v)$

Kahler potential: $\mathcal{K} = -\log \mathcal{K}_0, \quad \text{where} \quad \mathcal{K}_0 \equiv 2(f + \bar{f}) - (z - \bar{z})^i (f_i - \bar{f}_i).$

- Homogeneous coordinate

$$z^i \to \{X^0, X^i\}$$
$$f(z^i) \to F(X^0, X^i)$$

Electric magnetic duality acts on $\Omega^M(z) = \begin{pmatrix} X^\Lambda(z) \\ F_\Sigma(z) \end{pmatrix}, \quad (\Lambda, \Sigma = 0, 1, \cdots, n_v)$

$$F_\Lambda = \frac{\partial F}{\partial X^\Lambda}$$

Setup

- Hyper sector

  Hyper metric: $h_{uv} = \frac{1}{2(b^0)^2}\delta_{uv},$

  Isometry: $b^m \to b^m + c^m, \quad (m = 1, 2, 3)$

- We gauge the isometry by using vector fields:

  $A_\mu^\Lambda \ (\Lambda = 0, 1, \cdots, n_v)$

- But, there are ambiguities to use electric and magnetic vectors for gauging.

Gauging by “embedding tensor formalism”

B. de Wit, H. Samtleben, M. Trigiante,
JHEP 0509, 016 (2005)

- Introduce formally double vectors

A_μ^Λ : Electric vector $\qquad (\Lambda, \Sigma = 0, 1, \cdots, n_v)$

$A_{\mu\Sigma}$: Magnetic vector

- Introduce gauge couplings (= embedding tensor)

$$\Theta_M{}^m = \begin{pmatrix} \Theta_\Lambda{}^m \\ \Theta^{\Sigma m} \end{pmatrix} = \begin{array}{cl} \begin{pmatrix} \Theta_\Lambda{}^1 & \Theta_\Lambda{}^2 & \Theta_\Lambda{}^3 \\ \Theta^{\Sigma 1} & \Theta^{\Sigma 2} & \Theta^{\Sigma 3} \end{pmatrix} & \begin{array}{l} \longleftarrow \text{Electric gauge coupling} \\ \longleftarrow \text{Magnetic gauge coupling} \end{array} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ b^1 & b^2 & b^3 \end{array} & \end{array}$$

remember
$b^m \to b^m + c^m, \quad (m = 1, 2, 3)$

- Covariant derivative: $D_\mu \equiv \partial_\mu - \underline{A_\mu^\Lambda \Theta_\Lambda{}^m T_m} - \underline{A_{\mu\Sigma} \Theta^{\Sigma m} T_m}$

Gauging by “embedding tensor formalism”

B. de Wit, H. Samtleben, M. Trigiante,
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- Constraints

$$\begin{aligned}
\Theta_\Lambda{}^1\Theta^{\Lambda 2} - \Theta_\Lambda{}^2\Theta^{\Lambda 1} &= 0, \\
\Theta_\Lambda{}^2\Theta^{\Lambda 3} - \Theta_\Lambda{}^3\Theta^{\Lambda 2} &= 0, \\
\Theta_\Lambda{}^3\Theta^{\Lambda 1} - \Theta_\Lambda{}^1\Theta^{\Lambda 3} &= 0.
\end{aligned}$$

- It allows us to exhaust gauging possibilities

= just choosing parameters of embedding tensor

- Note: Auxiliary 2-form, topological coupling must be introduced, to match the d.o.f.

Gravitino masses

- Order parameters of SUSY breaking = gravitino masses

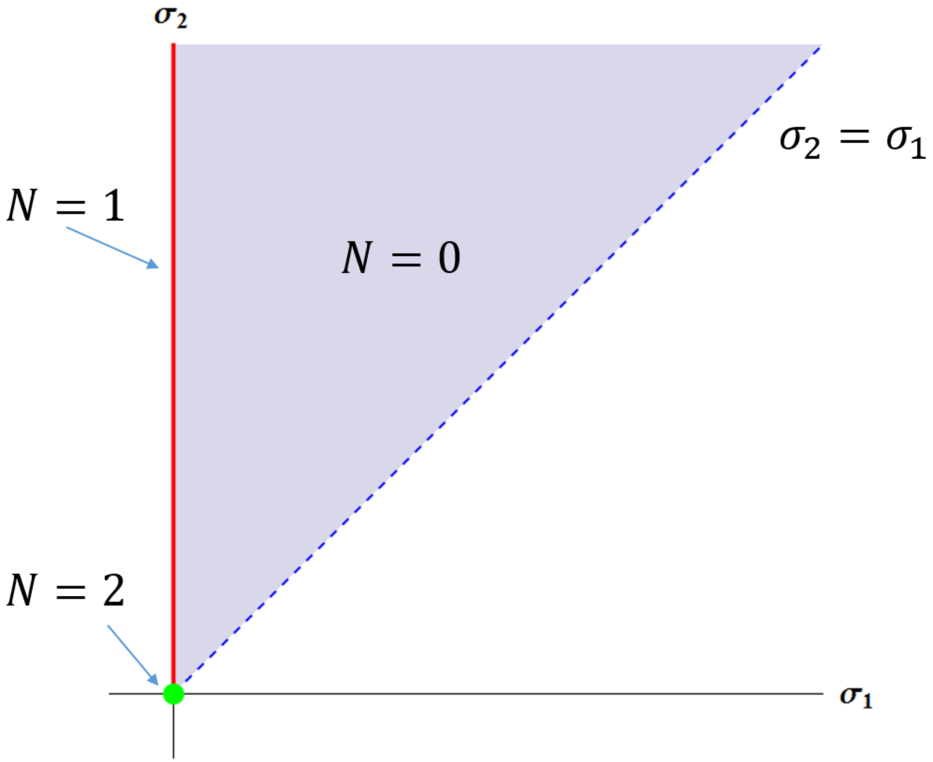
$$m_A = \frac{e^{\mathcal{K}/2}}{b^0}\sigma_A, \quad A = 1, 2.$$

$$\sigma_1 = X_+ - X_-, \quad \sigma_2 = X_+ + X_-,$$

$$X_\pm \equiv \frac{1}{\sqrt{2}}\sqrt{|\alpha|^2 + |\beta|^2 + |\gamma|^2 \pm |\alpha^2 + \beta^2 + \gamma^2|}.$$

$$\begin{aligned}
\alpha &\equiv \Theta_M{}^1 \Omega^M = (\Theta_\Lambda{}^1 X^\Lambda + \Theta^{\Lambda 1} F_\Lambda),\\
\beta &\equiv \Theta_M{}^2 \Omega^M = (\Theta_\Lambda{}^2 X^\Lambda + \Theta^{\Lambda 2} F_\Lambda),\\
\gamma &\equiv \Theta_M{}^3 \Omega^M = (\Theta_\Lambda{}^3 X^\Lambda + \Theta^{\Lambda 3} F_\Lambda),
\end{aligned}$$

Electric magnetic



Behaviour of gravitino masses

$$m_A = \frac{e^{\mathcal{K}/2}}{b^0}\sigma_A, \quad A = 1, 2.$$

- Only one isometry gauging: $\alpha \neq 0$ and $\beta = \gamma = 0,$

$$\Rightarrow \quad \sigma_1 = \sigma_2 = |\alpha|.$$

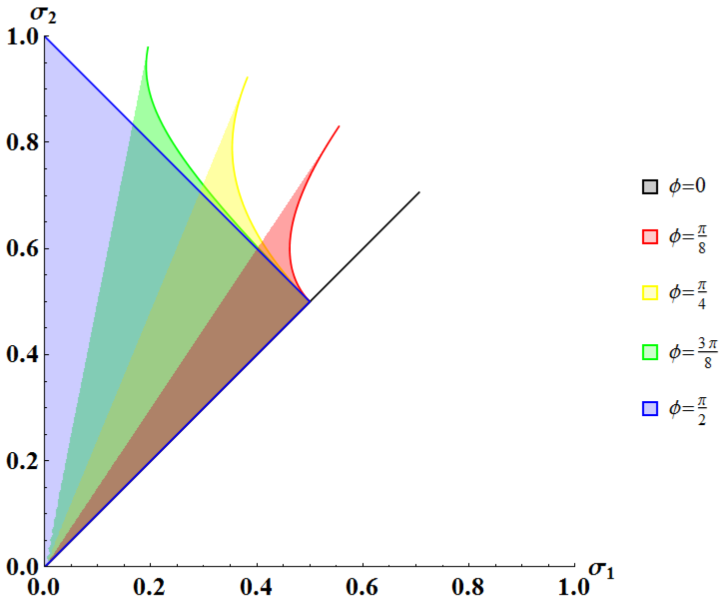
degenerate breaking scale

- Two isometry gauging: $\alpha, \beta \neq 0$ and $\gamma = 0,$

$$\sigma_1 = X_+ - X_-, \quad \sigma_2 = X_+ + X_-,$$

$$X_\pm = \frac{1}{\sqrt{2}}\sqrt{|\alpha|^2 + |\beta|^2 \pm \sqrt{|\alpha|^4 + |\beta|^4 + 2|\alpha|^2|\beta|^2 \cos 2\phi}},$$

$$\phi \equiv \arg\alpha - \arg\beta$$



$\nabla\alpha\cdot\overline{\nabla\beta} = g^{i\bar{j}}\nabla_i\alpha\bar{\nabla}_{\bar{j}}\bar{\beta}$ and $|\nabla\alpha|^2 = \nabla\alpha\cdot\overline{\nabla\alpha}$

Scalar potential

- Scalar potential:

$$V = \frac{e^{\mathcal{K}}}{(b^0)^2}(-|\alpha|^2 - |\beta|^2 - |\gamma|^2 + |\nabla\alpha|^2 + |\nabla\beta|^2 + |\nabla\gamma|^2).$$

- Stationary conditions:

$$\frac{\partial V}{\partial b^0} = 0 \quad \Longrightarrow \quad -|\alpha|^2 - |\beta|^2 - |\gamma|^2 + |\nabla\alpha|^2 + |\nabla\beta|^2 + |\nabla\gamma|^2 = 0,$$

Minkowski vacuum

$$\frac{\partial V}{\partial z^i} = 0 \quad \Longrightarrow \quad e^{2\mathcal{K}}(\bar{\nabla}_{\bar{j}}\bar{\alpha}\bar{\nabla}_{\bar{k}}\bar{\alpha} + \bar{\nabla}_{\bar{j}}\bar{\beta}\bar{\nabla}_{\bar{k}}\bar{\beta} + \bar{\nabla}_{\bar{j}}\bar{\gamma}\bar{\nabla}_{\bar{k}}\bar{\gamma})g^{j\bar{j}}g^{k\bar{k}}f_{ijk} = 0.$$

Behavior of two supersymmetry breaking scales in explicit models

- Single vector multiplet $(n_v = 1)$

Stationary condition

$$((\bar{\nabla}_{\bar{z}}\bar{\alpha})^2 + (\bar{\nabla}_{\bar{z}}\bar{\beta})^2 + (\bar{\nabla}_{\bar{z}}\bar{\gamma})^2)(g^{z\bar{z}})^2 f_{zzz} = 0.$$

$$\text{Case A}: \quad f_{zzz} = 0.$$

$$\text{Case B}: \quad (\nabla_z\alpha)^2 + (\nabla_z\beta)^2 + (\nabla_z\gamma)^2 = 0,$$

Case A $\quad f_{zzz} = 0.$

- Simplest choice of prepotential: $f = z,$
- Gauging:

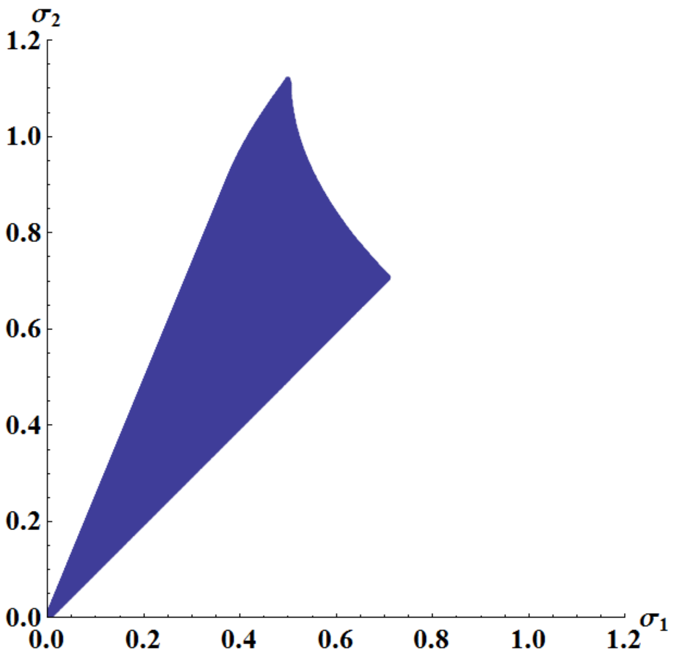
$$(i) \quad \Theta_M^{\ m} = \begin{pmatrix} \boxed{E_1} & \boxed{E_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad \sigma_1 = \sigma_2 = \sqrt{E_1^2 + E_2^2}, \quad \text{degenerate}$$

$$(ii) \quad \Theta_M^{\ m} = \begin{pmatrix} \boxed{E_1} & 0 & 0 \\ 0 & \boxed{E_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} \sigma_1 &= \sqrt{E_1^2 + E_2^2|z|^2 - 2E_1E_2\mathrm{Im}z}, \\ \sigma_2 &= \sqrt{E_1^2 + E_2^2|z|^2 + 2E_1E_2\mathrm{Im}z}, \end{aligned}$$

$$(iii) \quad \Theta_M^{\ m} = \begin{pmatrix} \boxed{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \boxed{M} & 0 \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} \sigma_1 &= E - M, \\ \sigma_2 &= E + M, \end{aligned}$$

Case A $f_{zzz} = 0.$

(ii)



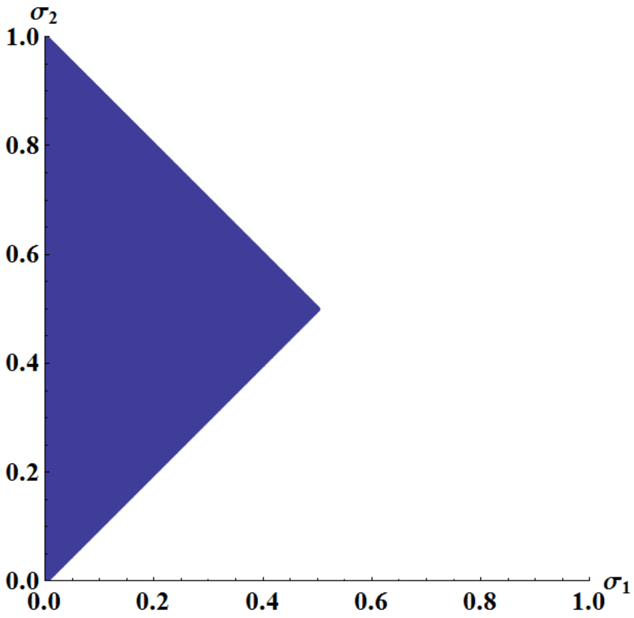
$$\sigma_1 = \sqrt{E_1^2 + E_2^2|z|^2 - 2E_1E_2\mathrm{Im}z},$$

$$\sigma_2 = \sqrt{E_1^2 + E_2^2|z|^2 + 2E_1E_2\mathrm{Im}z},$$

$$z = 1 + i$$

Always leads to full breaking

(iii)



$$\sigma_1 = E - M, \quad \sigma_2 = E + M,$$

Partial breaking can be covered

Case B $f_{zzz} \neq 0, \quad \text{or} \quad (\nabla_z \alpha)^2 + (\nabla_z \beta)^2 + (\nabla_z \gamma)^2 = 0$

- Prepotential:

$$f = az + bz^2 + cz^3, \quad c \neq 0$$

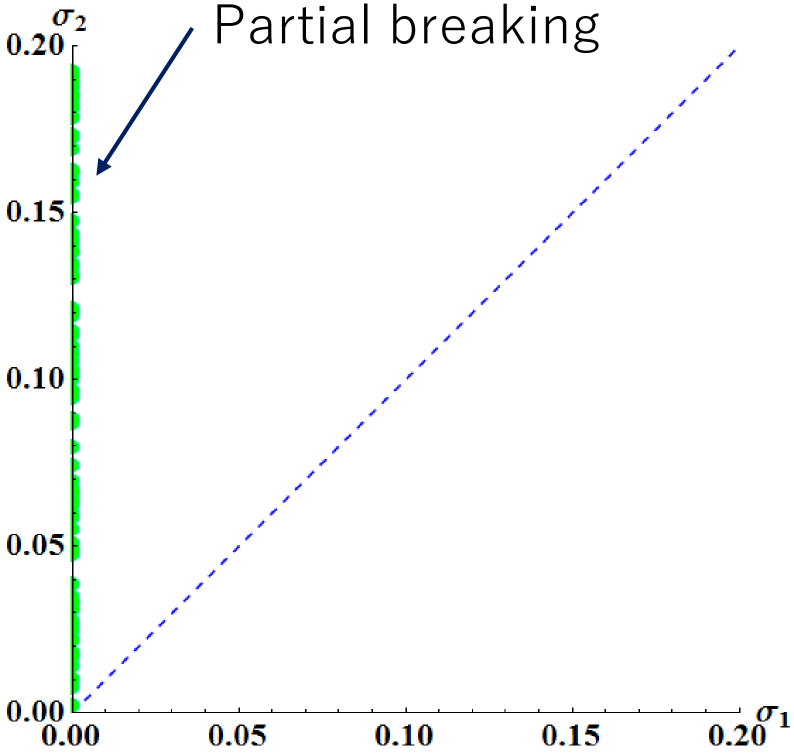
- Gauging:

$$\Theta_M^{\ m} = \begin{pmatrix} E_1 & E_3 & 0 \\ E_2 & 0 & 0 \\ 0 & 0 & 0 \\ M & 0 & 0 \end{pmatrix}$$

- Stationary condition

$$z = -\frac{b}{3c} - i\frac{E_2}{6cM},$$

$$a = \frac{b^2}{3c} - \frac{E_3}{M} - \frac{E_2^2}{12cM^2} + i\left(-\frac{E_1}{M} + \frac{bE_2}{3cM}\right)$$



Partial breaking

Q. Is it general ?

$$\sigma_1 = X_+ - X_-, \quad \sigma_2 = X_+ + X_-,$$

$$X_\pm \equiv \frac{1}{\sqrt{2}}\sqrt{|\alpha|^2 + |\beta|^2 + |\gamma|^2 \pm |\alpha^2 + \beta^2 + \gamma^2|}.$$

- PB occurs when $X_+ = X_-$

$$\Downarrow$$

$$\alpha^2 + \beta^2 + \gamma^2 = 0.$$

- By using supergravity Ward identity & Minkowski vacuum condition,

$$\alpha^2 + \beta^2 + \gamma^2 = 0. \quad \Longleftrightarrow \quad (\nabla_z\alpha)^2 + (\nabla_z\beta)^2 + (\nabla_z\gamma)^2 = 0,$$

Stationary condition of case B!!

Partial breaking always occurs whenever $f_{zzz} \neq 0$ in single vector multiplet

Multiple vector multiplets

- Example: $f(z_1, z_2) = z_1^2 z_2.$

$$\Theta_M{}^m = \begin{pmatrix} \Theta_0{}^1 & \Theta_0{}^2 & 0 \\ \Theta_1{}^1 & 0 & 0 \\ 0 & \Theta_2{}^2 & 0 \\ 0 & 0 & 0 \\ \Theta^{11} & 0 & 0 \\ 0 & \Theta^{22} & 0 \end{pmatrix}$$

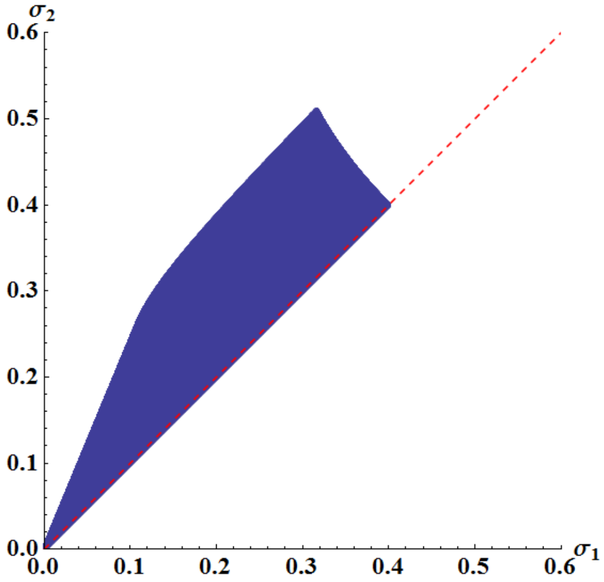
- One of the solution for the stationary condition

$$z_1 = z_2 = 1 + i,$$

$$\Theta_0{}^2 = 2\Theta^{22},$$

$$\Theta_0{}^1 = \Theta_2{}^2 = \Theta^{11} = 0,$$

Full breaking occurs even when $f_{ijk} \neq 0$



Summary

- Our purpose = To clarify the relations between the breaking patterns and <u>input parameters</u> in the theory

  Prepotential & gauge coupling (embedding tensor)

- In single vector multiplet case,

  $\text{Case A}: \quad f_{zzz} = 0.$ ⟶ Various breaking patterns

  $\text{Case B} \quad f_{zzz} \neq 0, \quad \text{or} \quad (\nabla_z \alpha)^2 + (\nabla_z \beta)^2 + (\nabla_z \gamma)^2 = 0$

  ⟶ Only partial breaking

- In multiple vector multiplets case, we can have full breaking even when $f_{ijk} \neq 0$
- It would be interesting to consider string setup and see the behavior of other mass spectra.