Asymptotic Scale Invariance and its Consequences

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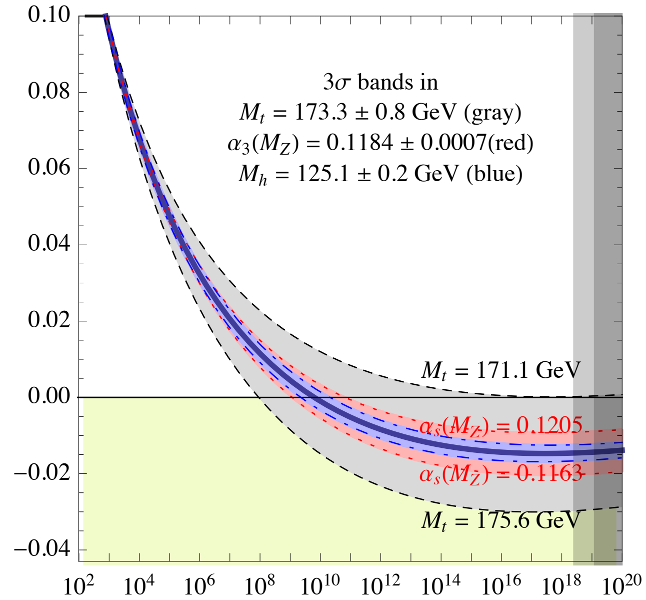
Introduction

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia (2014)

- ✓ SM works very well (to explain collider expt.).
- ✓ Higgs boson with $m_h \simeq 125\,\mathrm{GeV}$
- ✓ SM itself can be extended up to⋯

Running Higgs quartic coupling



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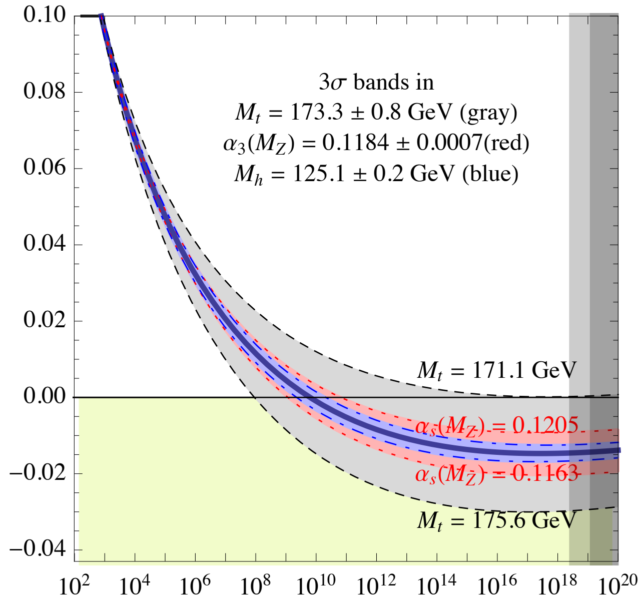
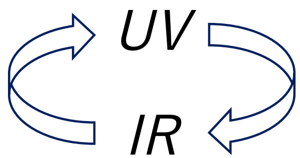


Any information on UV completion?
(extracted from boundary condition)



EW vacuum meta-stability?

☐ New physics can easily alter the running.

motivated by unsolved issues (DM, neutrino mass, BAU, etc.)

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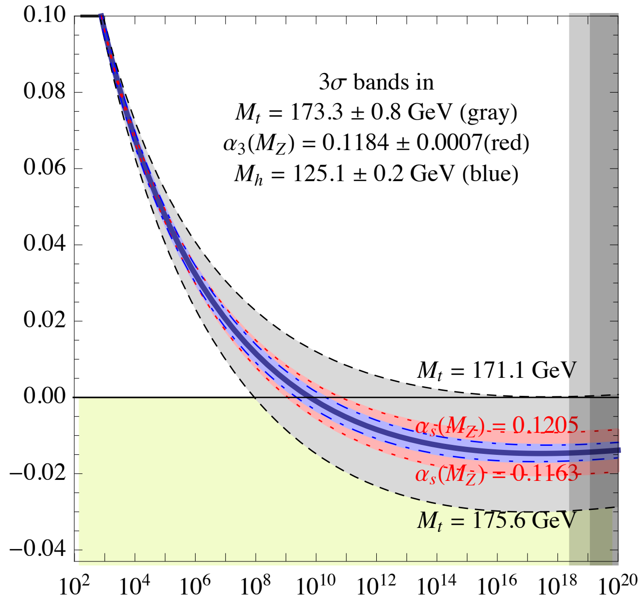

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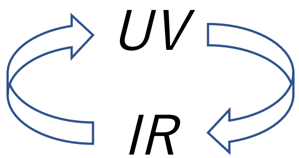


EW vacuum meta-stability?

☐ Impact of **symmetry in low-energy effective theory**

Quantum Scale invariance

(Assumption)

following from unknown UV completion

Outline

Scale invariance (classical/quantum)

Asymptotic SI and vacuum meta-stability issue

without any technical aspects

Self-consistency and Validity of EFT approach

with some technical aspects
(regularization/renormalization)

Summary

Scale invariance (classical)

Invariance under
$$\begin{cases} x^\mu \to \sigma^{-1} x^\mu \\ \Phi(x) \to \sigma^{d_\Phi}\Phi(x) \end{cases}$$

d_Φ : Mass dimension of dynamical fields Φ

Explicit mass scale breaks SI :

$$V = -\frac{\mu_{\rm EW}^2}{2}\,h^2 + \frac{\lambda}{4}\,h^4$$

In the SM of particle physics sector,
SI is broken only by the Higgs mass term.

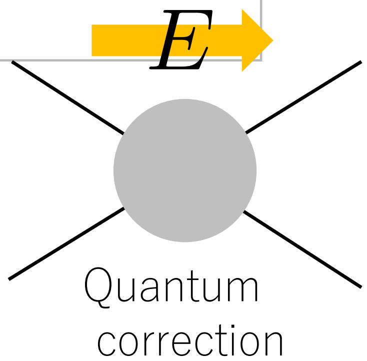
Scale-invariant for $h \gg \mu_{\rm EW}$.
(approximately)

Is SI necessarily anomalous?

Scattering amplitude

(dimensionless)

Energy scale dependent



$$\lambda_{\rm eff} \sim \lambda + \beta \ln(E/\mu)$$

Is SI necessarily anomalous?

Scattering amplitude

(dimensionless)

Energy scale dependent

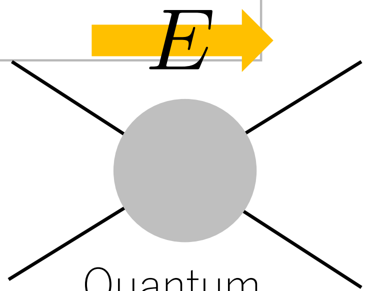
Define a theory

(coupling “constants”)

at some reference scale

(renormalization scale)

$$\mu = e^t \mu_0$$



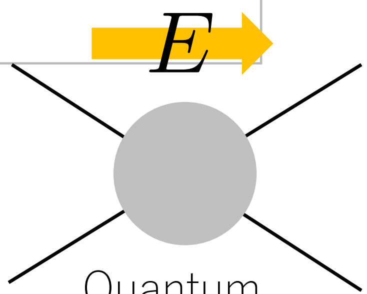
Quantum correction

$$\lambda_{\rm eff} \sim \lambda + \beta \ln(E/\mu)$$

Is SI necessarily anomalous?

Scattering amplitude
(dimensionless)

Energy scale dependent



Quantum correction

Define a theory
(coupling "constants")

at some reference scale
(renormalization scale)

$$\mu = e^t \mu_0$$

Once fixed

Different μ for convenience

Physics unchanged

RG running
of parameters

$$d_t \lambda = \beta$$

$$\lambda_{\rm eff} \sim \lambda + \beta \ln(E/\mu)$$

Is SI necessarily anomalous?

Scattering amplitude
(dimensionless)

Energy scale dependent

Define a theory
(coupling "constants")

at some reference scale

Physical

$$\mu = e^t \mu_0$$

Infinitely many modes in QFT

Regularization with explicit scale

Artifact

Renormalization
with counter terms

Once fixed

Different μ for convenience

Physics unchanged

RG running
of parameters

$$d_t \lambda = \beta$$

SI is broken
at quantum level

Is SI necessarily anomalous?

Not anomalous if there is no explicit scale

➡ **Dynamical** “reference” scale to define a theory (⟺ Different UV completion)

$$\omega = \phi \times f(h/\phi, \ldots)$$

➡ Quantum Scale Inv. with $\beta \neq 0$

F.Englert, C.Truffin, R.Gastmans (1976)

M.Shaposhnikov, D.Zenhausern (2009)

C.Tamarit (2013)

D.M.Ghilencea, Z.Lalak, P.Olszewski (2016)

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Running

⇕

Ratio E/ω changed

F.Englert, C.Truffin, R.Gastmans (1976)

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Asymptotic scale invariance

Assume the simplest form

$$\omega^2 \propto \xi h^2 + \zeta \phi^2 + \cdots$$ Dynamical fields (Exactly SI)

Dimensionless constants

Asymptotic scale invariance

Assume the simplest form

$$\omega^2 \propto \xi h^2 + \zeta\phi^2 + \cdots$$

Dynamical fields

$$\Rightarrow \quad h^2 + \mu_\star^2$$

SM Higgs

- Maybe regarded as a toy model of the exactly SI one.
- No new d.o.f involved.
- $\mu_\star$ is “hidden” at tree-level

Scale-invariant for $h \gg \mu_\star$

Asymptotic scale invariance

Couplings still run : $\beta \neq 0$

What's the consequence of scale invariance?

Coleman-Weinberg one-loop correction

Effective Higgs potential

$$V_{\text{eff}}(h) = \frac{\lambda h^4}{4} + \sum_i (-1)^{F_i} \frac{m_i^4}{(4\pi)^2} \ln \frac{m_i^2}{\omega^2} + \cdots$$

SM particle masses $m_i^2 \propto h^2$

: Relevant energy scale in vacuum diagrams

Reference scale

Compared with

Asymptotic scale invariance

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Reference scale

$$\Rightarrow \quad \frac{V_{\rm eff}}{h^4/4} \equiv \boldsymbol{\lambda}(h) = \lambda + B \ln \frac{h^2}{\omega^2} + \cdots$$

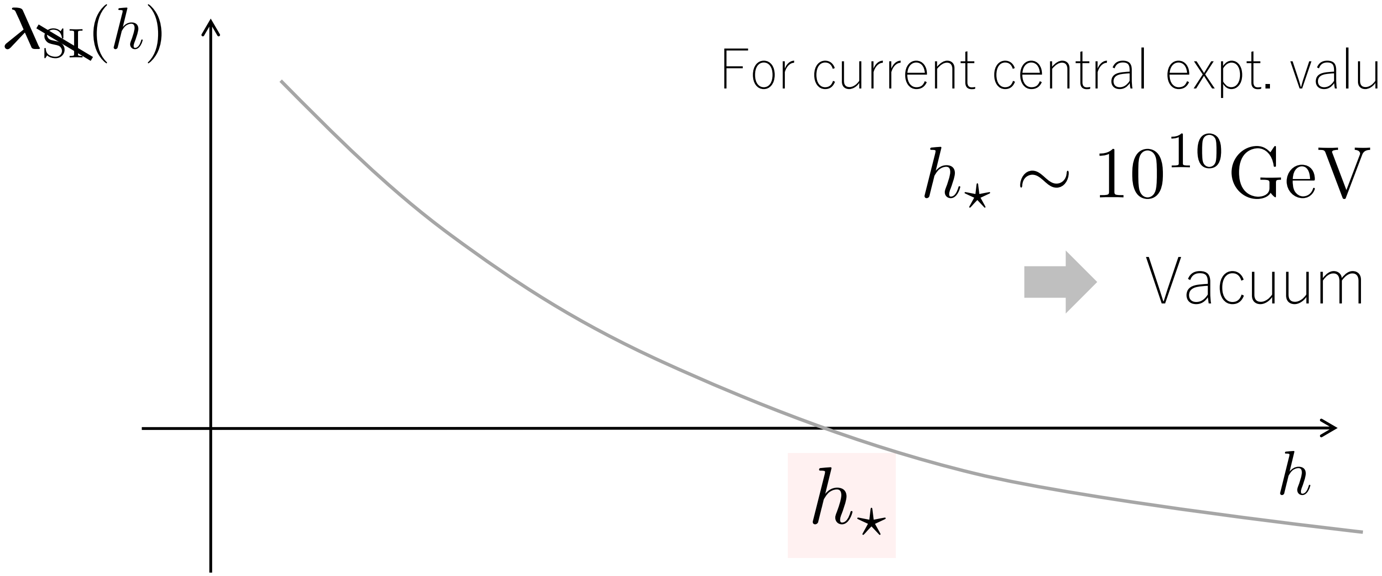
"Running"

Asymptotic scale invariance

When SI is ***anomalous***,

$$\boldsymbol{\lambda}_{\cancel{\mathrm{SI}}}(h) = \lambda + B \ln \frac{h^2}{\mu^2} + \cdots$$

Explicit breaking



For current central expt. value of m_t ,

$$h_\star \sim 10^{10}\mathrm{GeV}$$

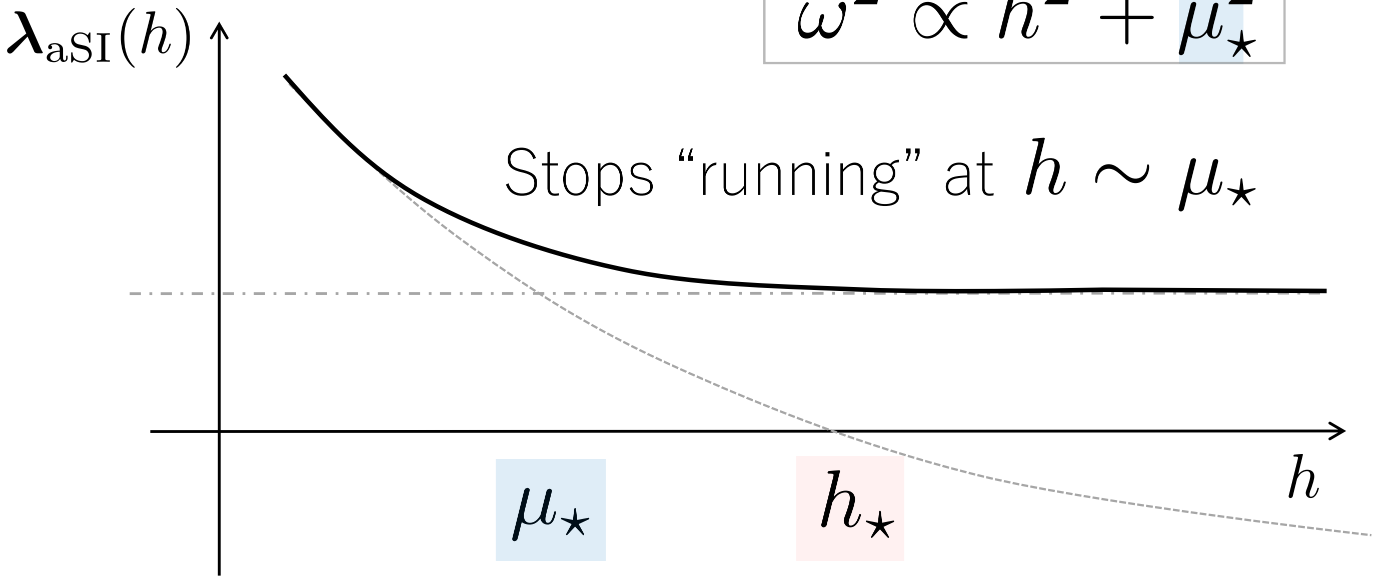
➡ Vacuum meta-stability

Asymptotic scale invariance

$E/\omega \Rightarrow$ Constant

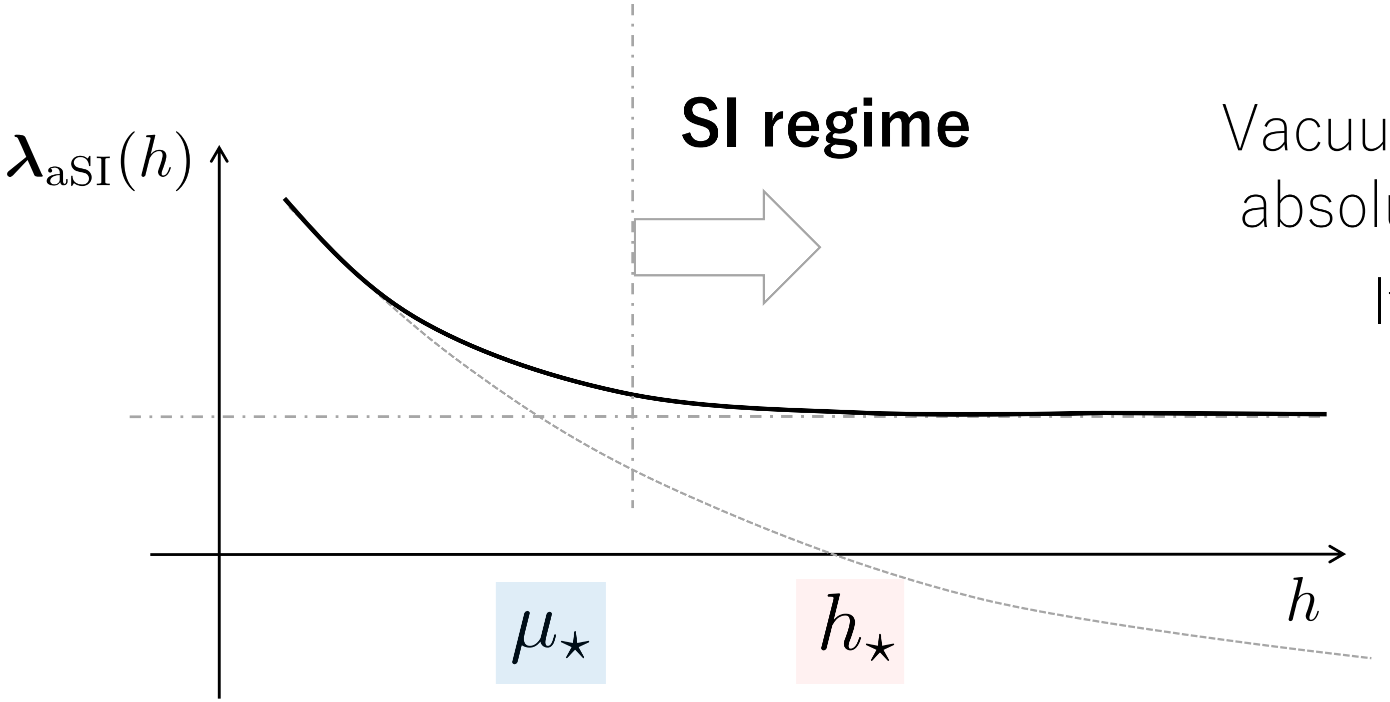
$$\boldsymbol{\lambda}_{\mathrm{aSI}}(h) = \lambda + B \ln \frac{h^2}{\omega^2} + \cdots \quad \Rightarrow \quad \boldsymbol{\lambda}_{\mathrm{aSI}}(\infty) = \boldsymbol{\lambda}_{\cancel{\mathrm{SI}}}(\mu_\star)$$

$$\omega^2 \propto h^2 + \mu_\star^2$$



Asymptotic scale invariance

$V_{\rm eff}(h) \propto h^4$ for $h \gtrsim \mu_\star$ as if there were no quantum correction



Vacuum absolute stability

If $\mu_\star < h_\star$

Self-consistency

Regularization
Renormalization } both respect SI for $h \gg \mu_\star$.

Asymptotic SI is manifest
at each order of perturbative computation.

Self-consistency

Regularization
Renormalization
} both respect SI for $h \gg \mu_\star$.

Dimensional regularization $(n = 4 - 2\varepsilon)$

$$\omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

: mass dimension n

$$\omega^2 \propto \mu_\star^2 + h^2$$

$$\omega^{\frac{2\varepsilon}{1-\varepsilon}} = \omega\Big|_{\rm BG}^{\frac{2\varepsilon}{1-\varepsilon}} \times \left(1 + \frac{2\varepsilon h}{\mu_\star^2 + h^2}\delta h + \cdots\right)$$

Fluctuation

$$h \to h + \delta h$$

Background

Self-consistency

Regularization
Renormalization } both respect SI for $h \gg \mu_\star$.

Dimensional regularization $(n = 4 - 2\varepsilon)$

$$\omega^2 \propto \mu_\star^2 + h^2$$

$$\omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$: Non-renormalizable

→ Non-polynomial operators for renormalization

$$\omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{h^{4+2k}}{(\mu_\star^2 + h^2)^k}$$

→ Asymptotically scale-invariant

$(h^{1/d_h})^n$ for $h \gg \mu_\star$

Self-consistency

Regularization
Renormalization } both respect SI for $h \gg \mu_\star$.

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→ Non-polynomial operators for renormalization

Up to what energy scale is this **effective theory** valid?

Validity of EFT approach

Non-polynomial operators $\frac{h^{4+2k}}{(\mu_\star^2 + h^2)^k}$ required

Unitarity bound

N-particle amplitude

$$\mathcal{M}_N \sim E^{4-N}$$

at most

J.M.Cornwall, D.N.Levin, G.Tiktopoulos (1974)

Tree unitarity violation

at $\Lambda \sim \sqrt{\mu_\star^2 + h^2}$

Strong coupling

or

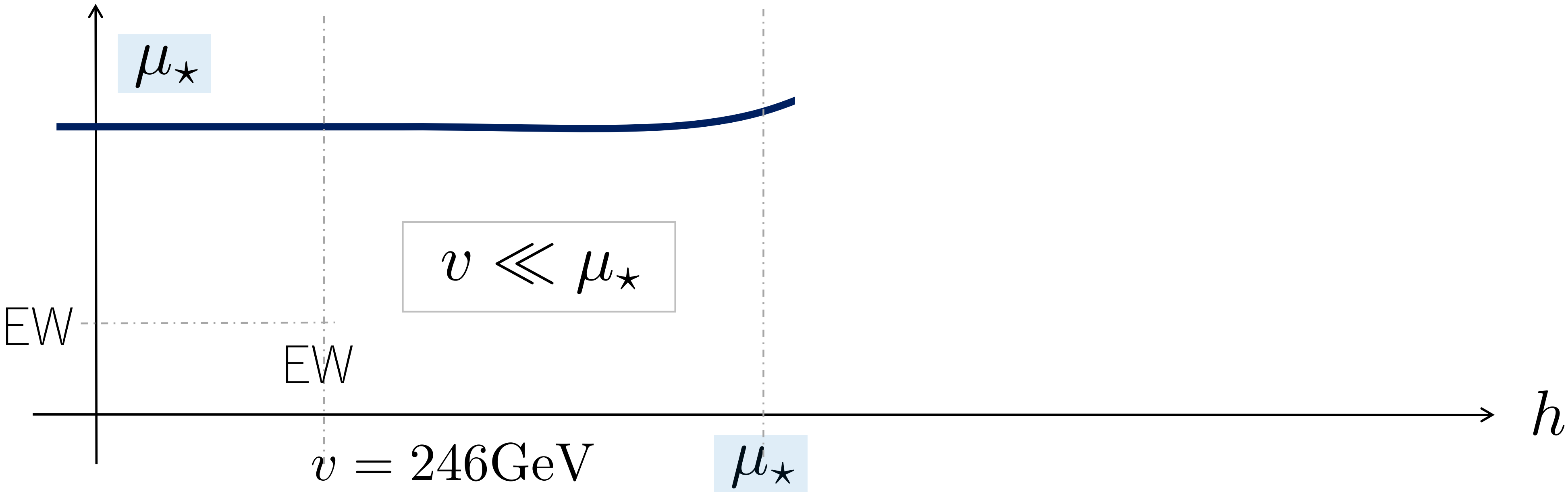
New physics

Validity of EFT approach

Non-polynomial operators $\dfrac{h^{4+2k}}{(\mu_\star^2 + h^2)^k}$ required

F.Bezrukov, A.Magnin, M.Shaposhnikov, S.Sibiryakov (2010) Higgs inflation

$\Lambda \sim \sqrt{\mu_\star^2 + h^2}$: Field dependent



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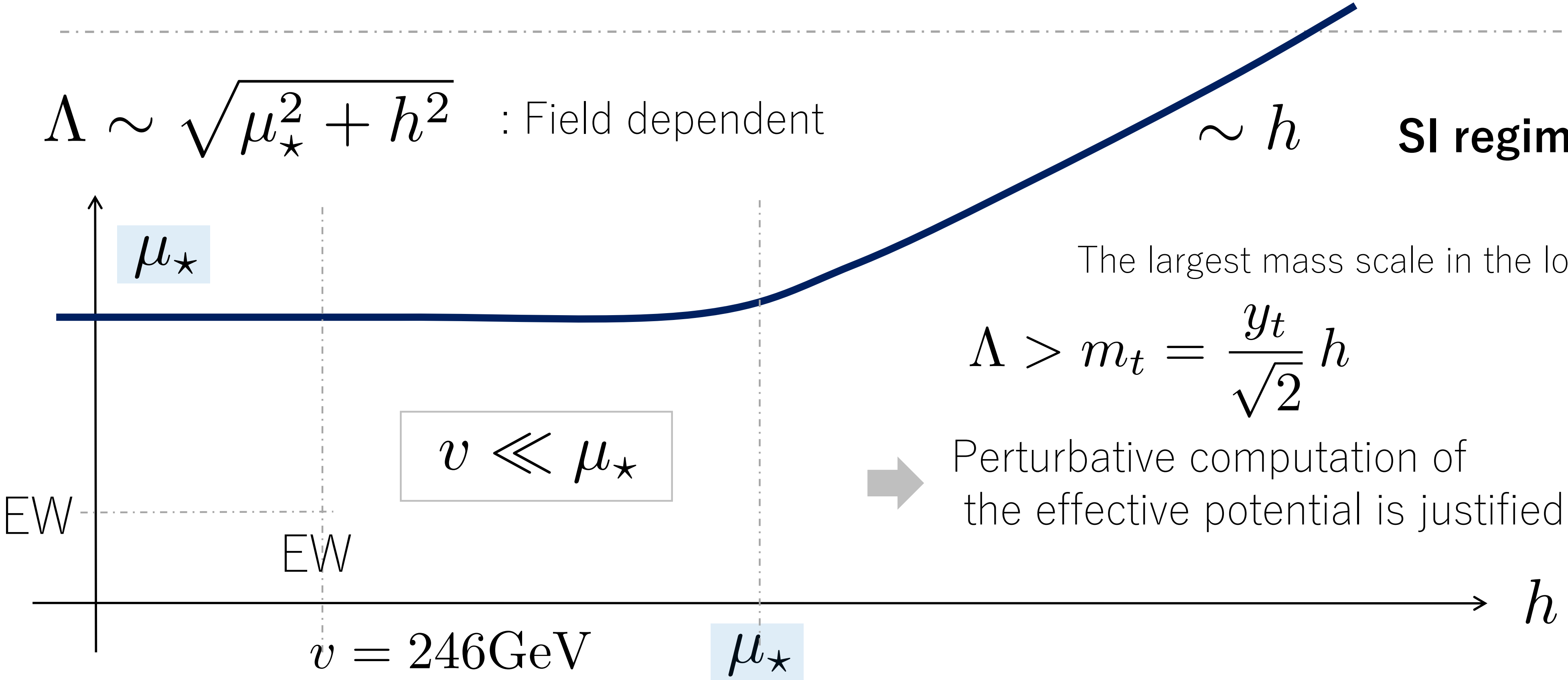
$$\Lambda \sim \sqrt{\mu_\star^2 + h^2}$$: Field dependent



The largest mass scale in the loops

$$\Lambda > m_t = \frac{y_t}{\sqrt{2}} h$$

➡ Perturbative computation of the effective potential is justified.

Some comments

Tree unitarity violation scale

$$\Lambda \sim \sqrt{\mu_\star^2 + h^2}$$

$\neq$

Heavy particle

$$M \sim \Lambda$$

Cf. Planck scale

as tree unitarity violation scale

➡ does not necessarily mean large radiative correction to Higgs mass.

F.Bezrukov, M.Shaposhnikov (2007)

J.Garcia-Bellido, J.Rubio, M.Shaposhnikov, D.Zenhausern (2011) Higgs-Dilaton model

Asymptotic SI in ***Higgs inflation*** (prescription 1)

➡ Asymptotically flat potential (Einstein frame)

$$\omega^2 \propto M_{\mathrm{P,eff}}^2 = M_{\mathrm{P}}^2 + \xi h^2$$

Effective Planck mass in Jordan frame

In general, $\omega^2 \not\propto M_{\mathrm{P,eff}}^2$

➡ More variety of Higgs potential shapes (before asymptotic flatness)

Summary

Asymptotically scale-invariant model

- ✓ Quantum Scale invariance for $h \gg \mu_\star$.
- ✓ Perturbative realization within EFT with dimensional regularization.
- ✓ Field dependent tree unitarity violation scale $\Lambda \sim \sqrt{\mu_\star^2 + h^2}$.
- ✓ Effective potential is computable without knowing UV completion.

Cosmology based on effective potential

- ✓ Asymptotic scale invariance can be responsible for
  absolute stability of our EW vacuum.

Thank you

Asymptotic scale invariance

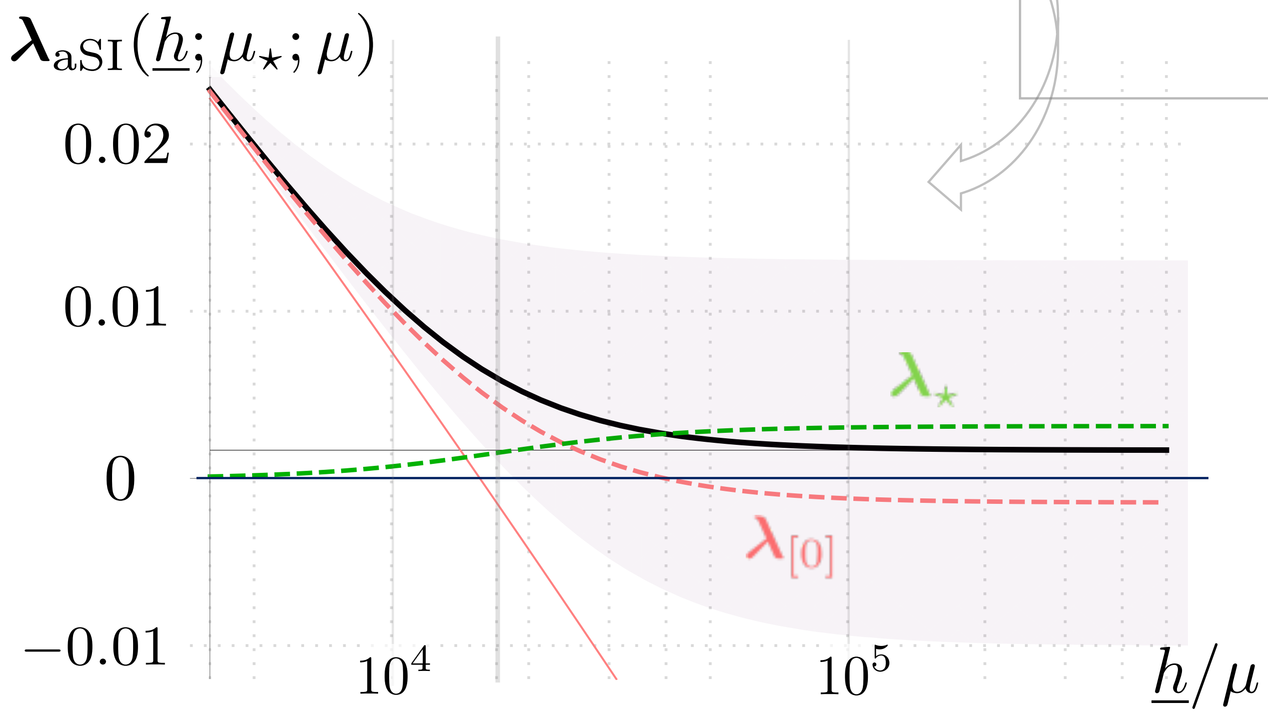
$$V_{\text{eff}}(h) = \sum_{k=0}^{\infty} \frac{\boldsymbol{\lambda}_{[k]}}{4} \frac{h^{4+2k}}{(\mu_\star^2 + h^2)^k}$$

Two-loop computation with assumption

No non-polynomial operator at tree-level

Correction with higher power k

More loop-suppressed



Asymptotic value

$$\sum_{k=0}^{\infty} \boldsymbol{\lambda}_{[k]}$$

Finiteness (scale invariance) constraints UV completion.

Asymptotic SI and Higgs inflation

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\mathrm{P,eff}}^2}{2} R - \frac{g^{\mu\nu}}{2} \partial_\mu h \partial_\nu h - V(h) + \cdots$$

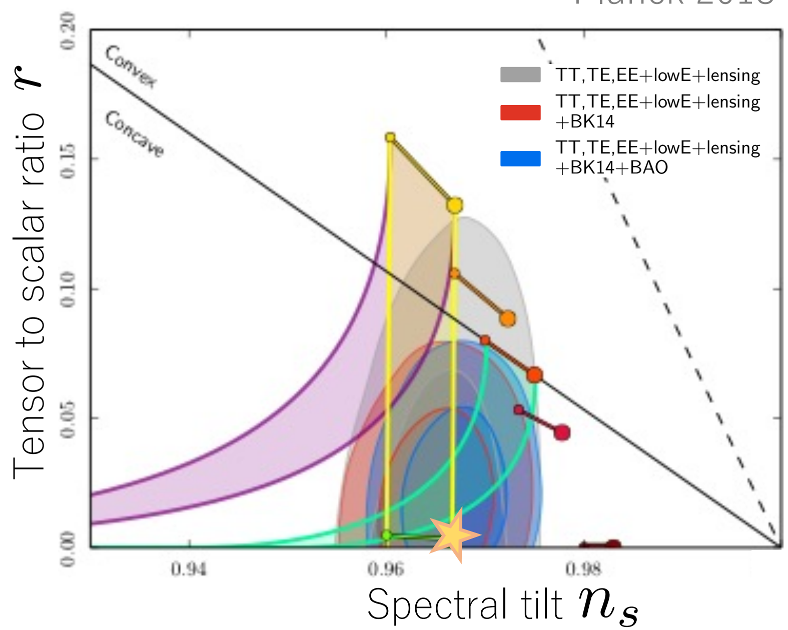
Effective Planck mass $\quad M_{\mathrm{P,eff}}^2 = M_{\mathrm{P}}^2 + \xi h^2$

F.Bezrukov, M.Shaposhnikov (2007)

$\xi \sim 10^4 \sqrt{\lambda}$ Large non-minimal coupling

$\Rightarrow A_s \simeq 2.2 \times 10^{-9}$

Planck 2018



Asymptotic SI and Higgs inflation

$$\frac{\lambda h^4}{4} \Rightarrow \omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

Renormalization prescription	$\omega^2 \propto$
I	$M_{\rm P}^2 + \xi h^2 = M_{\rm P,eff}^2$
II	$M_{\rm P}^2$ (constant)

F.Bezrukov, M.Shaposhnikov (2007)

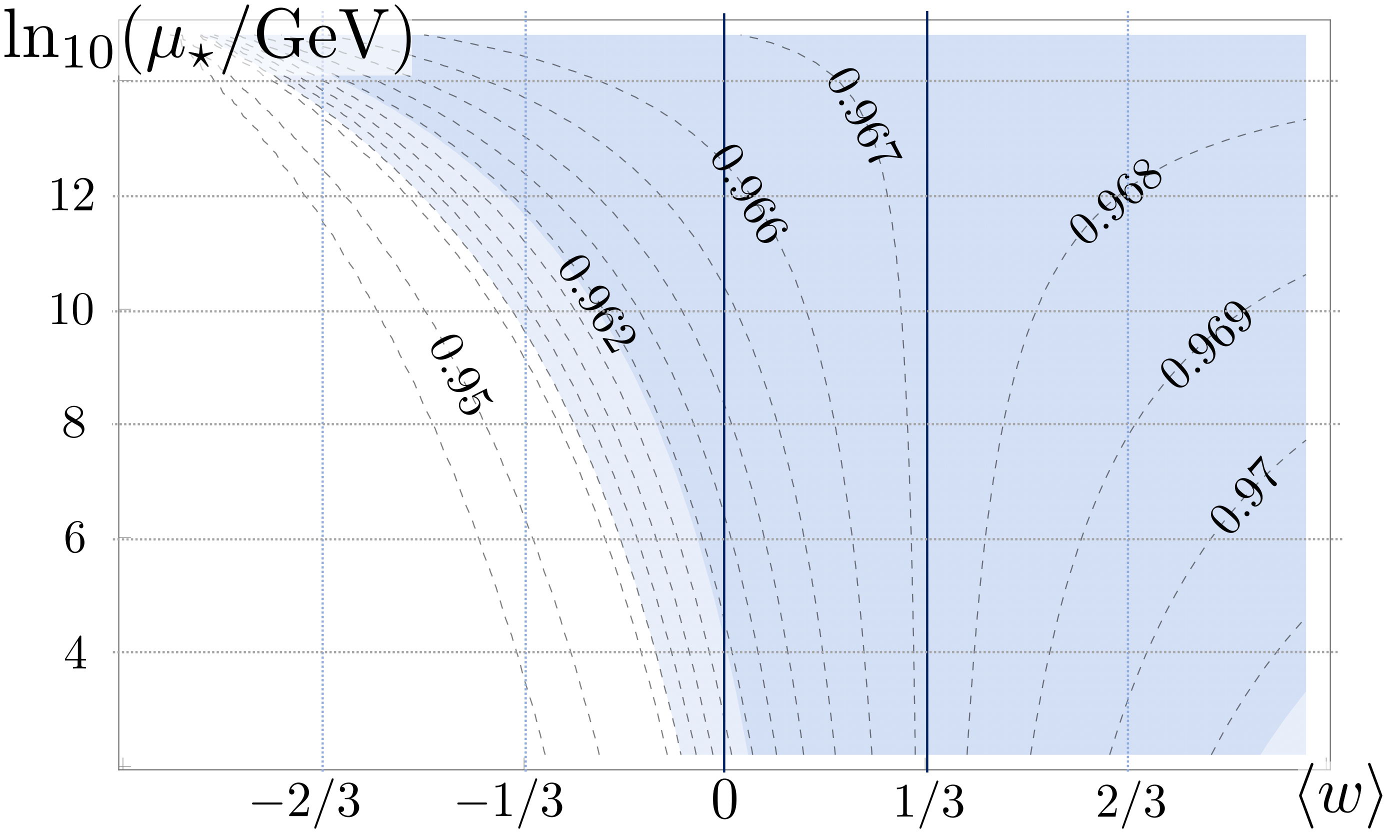
A.O.Barvinsky, A.Y.Kamenshchik, A.A.Starobinsky (2008)

$$\Rightarrow \quad \mu_\star^2 + h^2 \propto M_P^2 + \xi_\star h^2$$

$$\xi_\star = M_{\rm P}^2/\mu_\star^2 \neq \xi$$

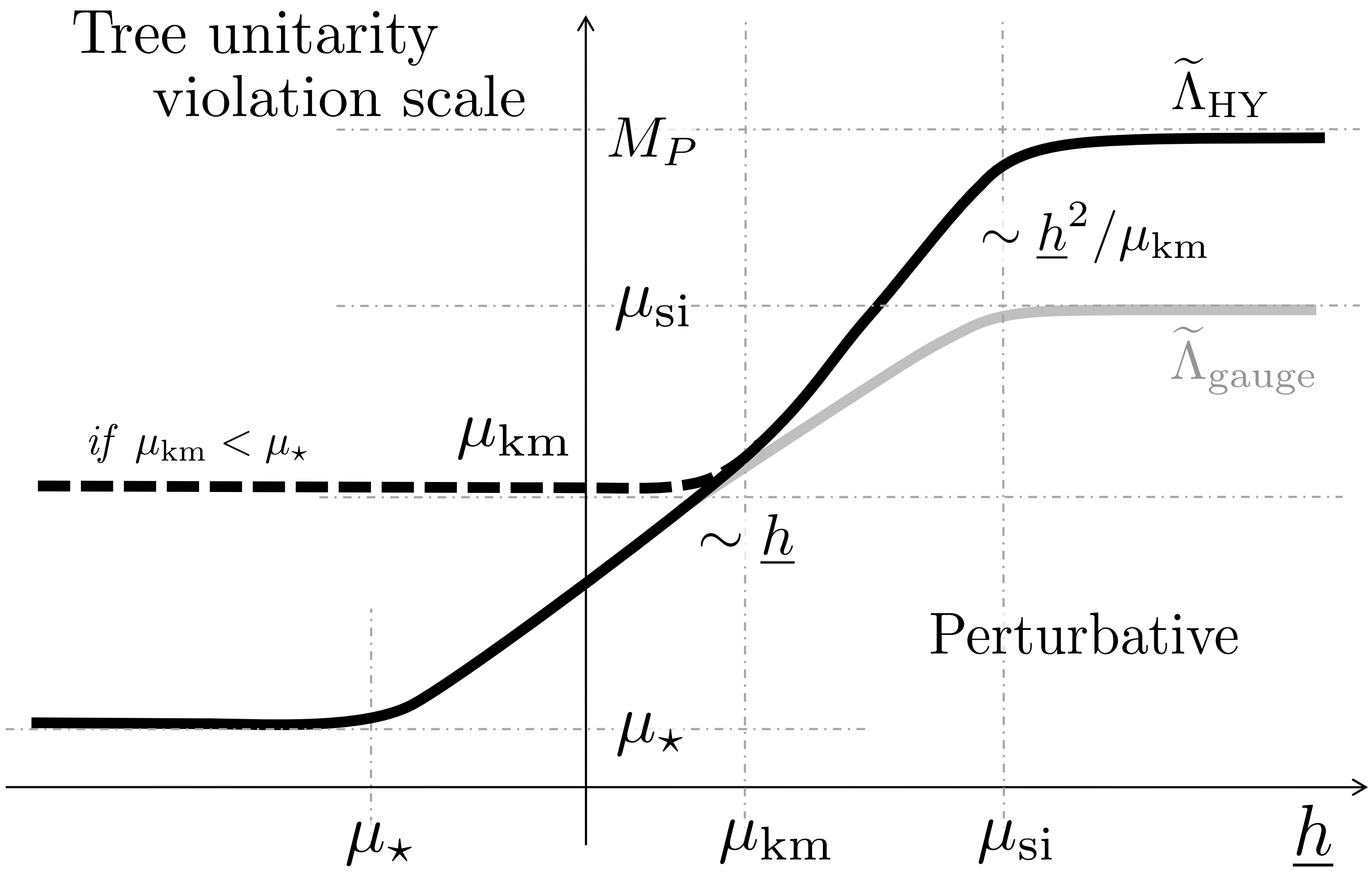
Asymptotic SI and Higgs inflation

Contour plot of spectral tilt n_s



e-folding-averaged equation of state during high-energy phase

Validity of EFT approach with nonminimal coupling



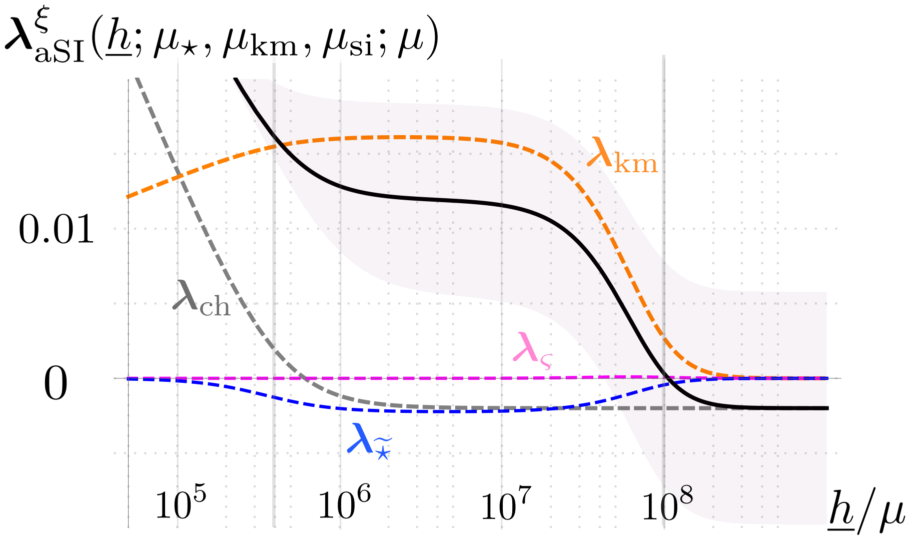
$\mu_{\rm si} \sim M_{\rm P}/\sqrt{\xi}$

$\mu_{\rm km} \sim M_{\rm P}/\xi$

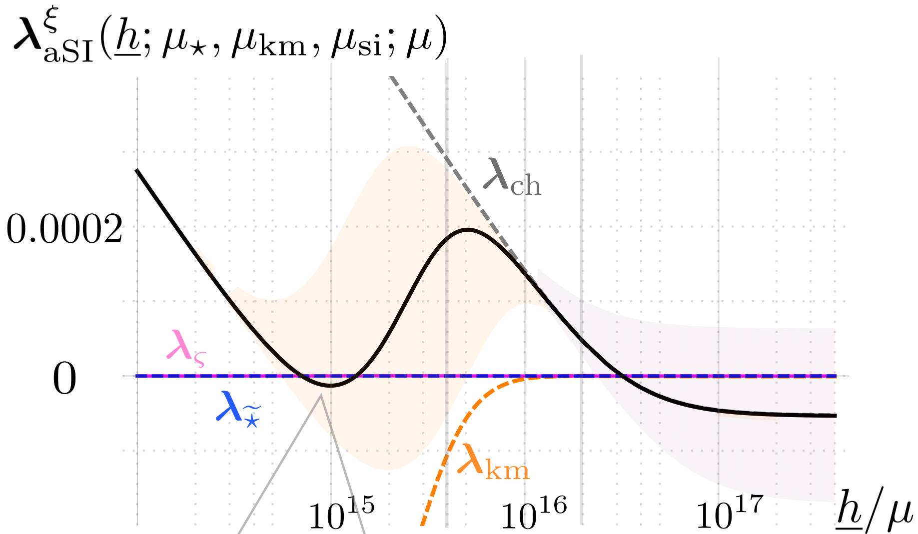
for $\xi \gg 1$

$\boldsymbol{\lambda}$ stops "running" before/after it jumps

$\mu_\star \ll \mu_{\rm km}$



$\mu_{\rm km} \ll \mu_\star$



$h \sim \mu_{\rm km}$

Higgs-graviton mixing becomes significant. (Jordan frame) ➡ Jump

Jump makes Higgs inflation possible even with vacuum meta-stability.

F.Bezrukov, J.Rubio, M.Shaposhnikov (2015)