Frame (in)equivalence in QFT and Cosmology

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$$\Gamma[Q] = -log Z[J(Q)] - J(Q)\cdot Q$$

However, many things that we give for granted about QFT are not true when gravity is in the game

Our motivation to study this is Inflation

$$S = \int d^4x \sqrt{g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + F(R, \phi) \right]$$

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For example, in Higgs inflation (Bezrukov & Shaposhnikov, 2008)

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}\left(M_P^2 + \xi\phi^2\right)R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

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This term is 'ugly'

$$\tilde{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu}$$

$$\tilde{\phi} = \tilde{\phi}(\phi)$$

$$\Rightarrow \quad S = \int d^x\sqrt{\tilde{g}}\left[-\frac{M_P^2}{2}R + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \tilde{V}(\tilde{\phi})\right]$$

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Jordan Frame

$$\tilde{g}_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu}$$

$$\tilde{\phi} = \tilde{\phi}(\phi)$$

$$S = \int d^x \sqrt{\tilde{g}} \left[-\frac{M_P^2}{2} \tilde{R} + \frac{1}{2} \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} - \tilde{V}(\tilde{\phi}) \right]$$

Einstein Frame

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In general we need to include Quantum Corrections and thus evaluate the Quantum Effective Action

$$\Gamma[g, \phi]$$

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Classically, it is trivial

$$\Phi, \ \tilde{\Phi}(\Phi)$$

$$\frac{\delta S}{\delta \Phi} = \frac{\delta \tilde{\Phi}}{\delta \Phi} \frac{\delta S}{\delta \tilde{\Phi}}$$

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However, in Quantum Field Theory we need to integrate over off-shell states

$$Z[J] = \int [d\Phi] \, e^{-S[\Phi] - J \cdot \Phi}$$

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Let us consider an example

$$S = \int d^4x \sqrt{g} \left(-\frac{\xi}{2}\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \right)$$

Scale Invariant

$$g_{\mu\nu} \to \alpha^2 g_{\mu\nu}$$

$$\phi \to \alpha^{-1}\phi$$

$$S = \int d^4x \sqrt{\tilde{g}} \left(-\frac{M_P^2}{2}\tilde{R} + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} \right)$$

Shift Invariant

$$\tilde{\phi} \to \tilde{\phi} + \tilde{\alpha}$$

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Scale invariant theories are anomalous $\langle 0 | \mathcal{A} | 0 \rangle \neq 0$

Shift symmetric theories are not anomalous $\langle 0 | \mathcal{A} | 0 \rangle = 0$

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Both Quantum Field Theories are **NOT** equivalent!!!!

Their S-matrices are **different**

Finite terms in the Effective action are the key

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Invariant

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Invariant on-shell

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Not invariant!!!!

$$[d\Phi] = \prod_x \frac{d\Phi(x)}{\sqrt{2\pi}} \sqrt{\det C}$$

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C(x,y) is an ultra-local metric on field space

$$C(x,y) = \Lambda^2 \delta(x-y)$$

Consider a simple free theory

$$S = \int d^4x \, \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

$$Z[0] = \det \left(C^{-1} \Box \right)^{-\frac{1}{2}} = \det \left(\frac{\Box}{\Lambda^2} \right)^{-\frac{1}{2}}$$

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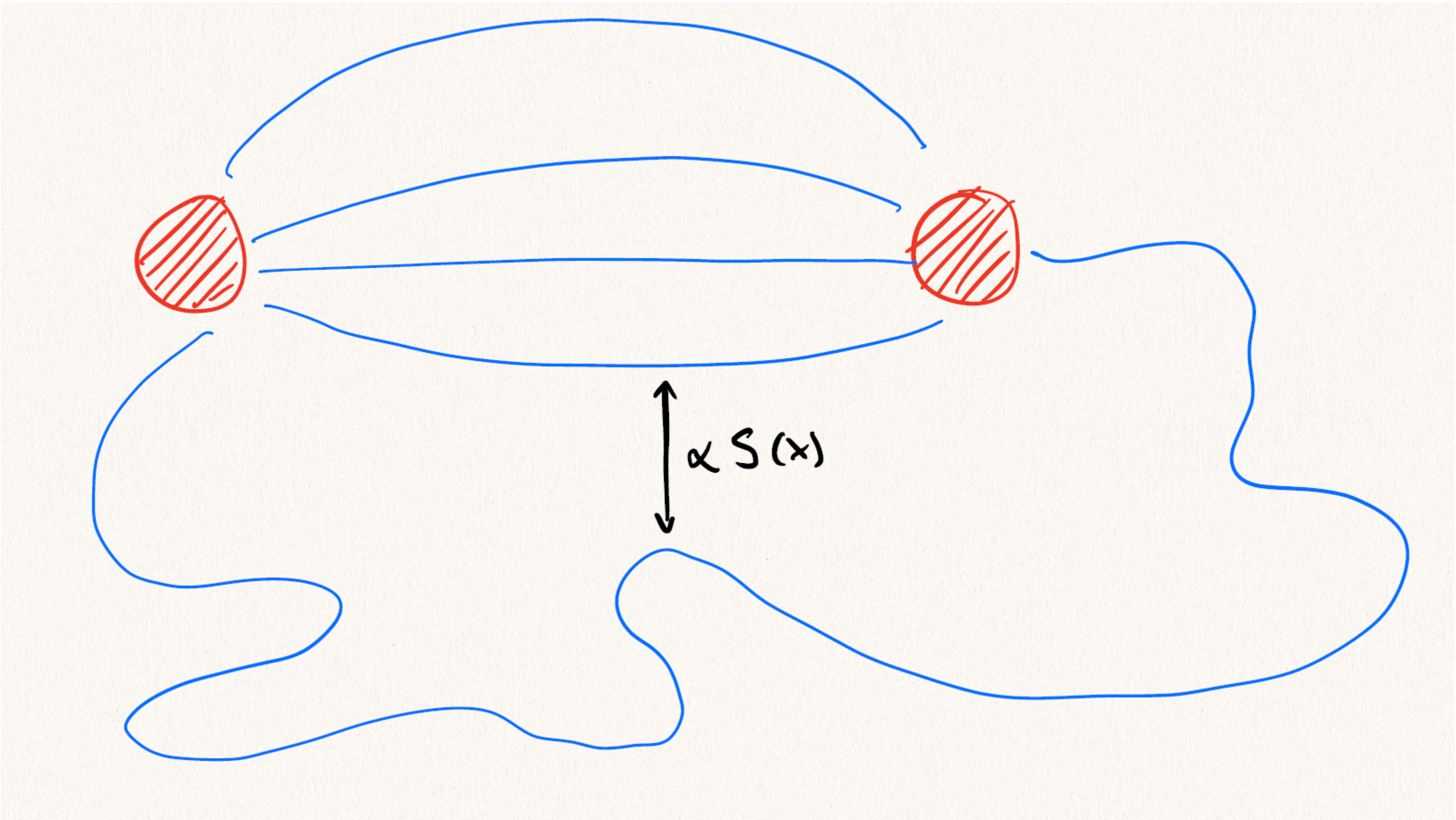
UV cut-off

Consider a simple free theory



This is the key point

In flat space it is enough to define C(x,y) as before

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But in curved space, diff invariance imposes

$$C(x,y) = \Lambda^2 \delta(x-y) \sqrt{g}$$

The field space metric depends on the space-time metric

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But in curved space, diff invariance imposes

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The field space metric C(x,y) then **transforms** when changing frame!!!!

In the Einstein frame

$$\tilde{C}(x, y) = \Lambda^2 \sqrt{\tilde{g}}\, \delta(x - y)$$

In the Jordan frame

$$C(x, y) = \Lambda^2 \sqrt{g}\, \delta(x - y)$$

But if we transform...

In the Einstein frame

$$\tilde{C}(x,y) = \Lambda^2 \sqrt{\tilde{g}}\, \delta(x-y)$$

In the Jordan frame

$$C(x,y) = \Lambda^2 \sqrt{g}\, \delta(x-y)$$

But if we transform...

$$C(x,y)_{\mathrm{Einstein}} = \Lambda^2 \sqrt{\tilde{g}}\, \delta(x-y) \Omega(\phi)^2$$

$$C(x,y)_{\mathrm{Einstein}} \neq \tilde{C}(x,y)$$

<< A recipe for a Quantum Field Theory >>

- Define the physical frame of the theory
- Define the path integral in that frame with a flat metric

$$C(x,y) = \Lambda^2 \delta(x-y)$$

- This defines the theory and its physical consequences
- In any other frame, transform both the action and the integration measure
- This is equivalent to

$$\tilde{\Gamma} = \Gamma - \mathcal{A}$$

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Standard effective action

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Frame discriminant

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- This is equivalent to

$$\mathcal{A} = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g}\, \mathrm{CT}\, \log \Omega$$

This solves the problem in the example

If you start in the Einstein frame...

There is no anomaly

When you transform to Jordan

$$\Gamma = \tilde{\Gamma} - \mathcal{A}$$

Anomaly

Also anomaly

This solves the problem in the example

If you start in the Einstein frame...

There is no anomaly

When you transform to Jordan

$$\Gamma = \tilde{\Gamma} - \mathcal{A}$$

If you start in the Jordan frame...

There is anomaly

When you transform to Einstein

$$\tilde{\Gamma} = \Gamma - \mathcal{A}$$

No anomaly

Anomaly

Conclusions

- A QFT is defined by the action and the integration measure
- Changing variables in the effective action involves transforming the measure
- This transformation induces new finite pieces
- These new pieces modify the effective potential
- There are potential physical effects induced by them
- This is not restricted to conformal rescaling nor to scale invariant theories
- Any field redefinition will produce the same effect


KEEP
CALM
AND
TRUST QFT
but do it well