Relaxation of hierarchy in higher dimensional Starobinsky model

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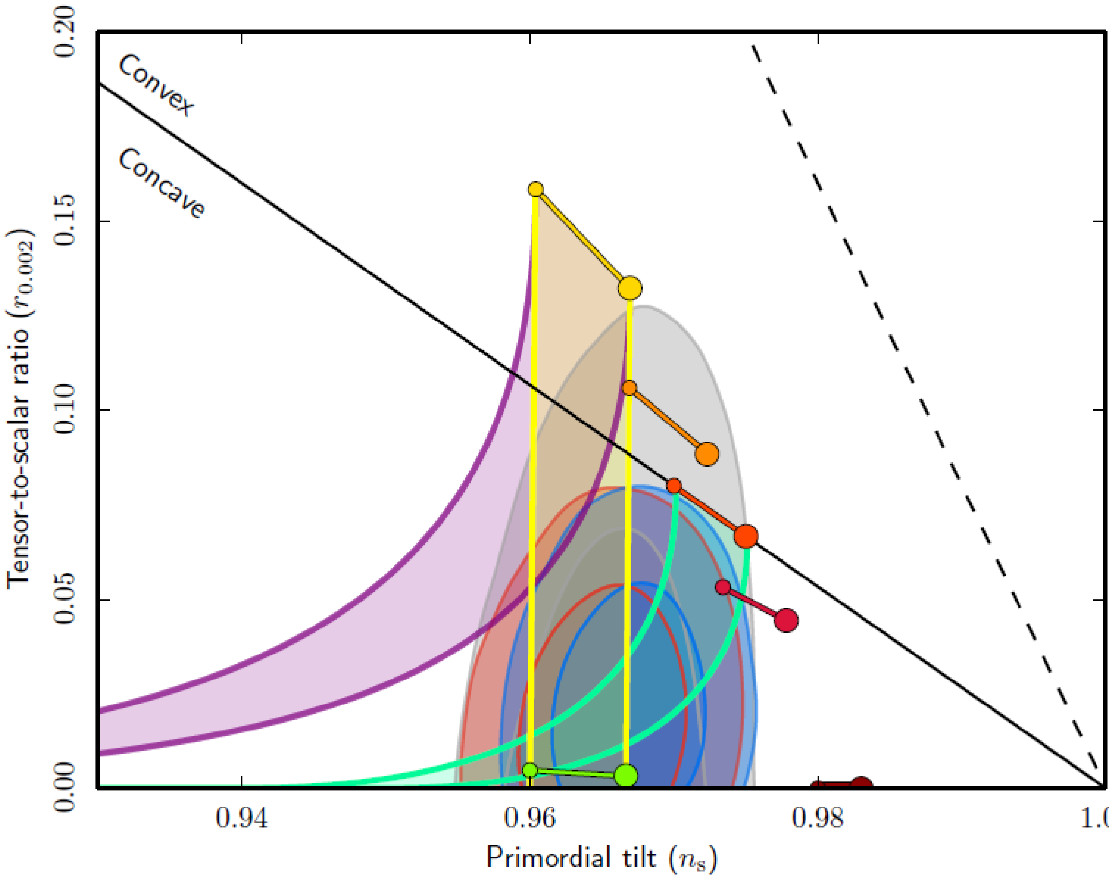
Inflation and Starobinsky model $N_e = 50 \sim 60$

- Inflation[1]
  - *Solve initial problems of standard BB cosmology
  - *Predict observables of CMB
  - *Can be caused by scalar field



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[2]A.A.Starobinsky, Phys.Lett.B (1980)

[3]Planck Collaboration, arXiv:1807.06211

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- **Inflation[1]**

  *Solve initial problems of standard BB cosmology

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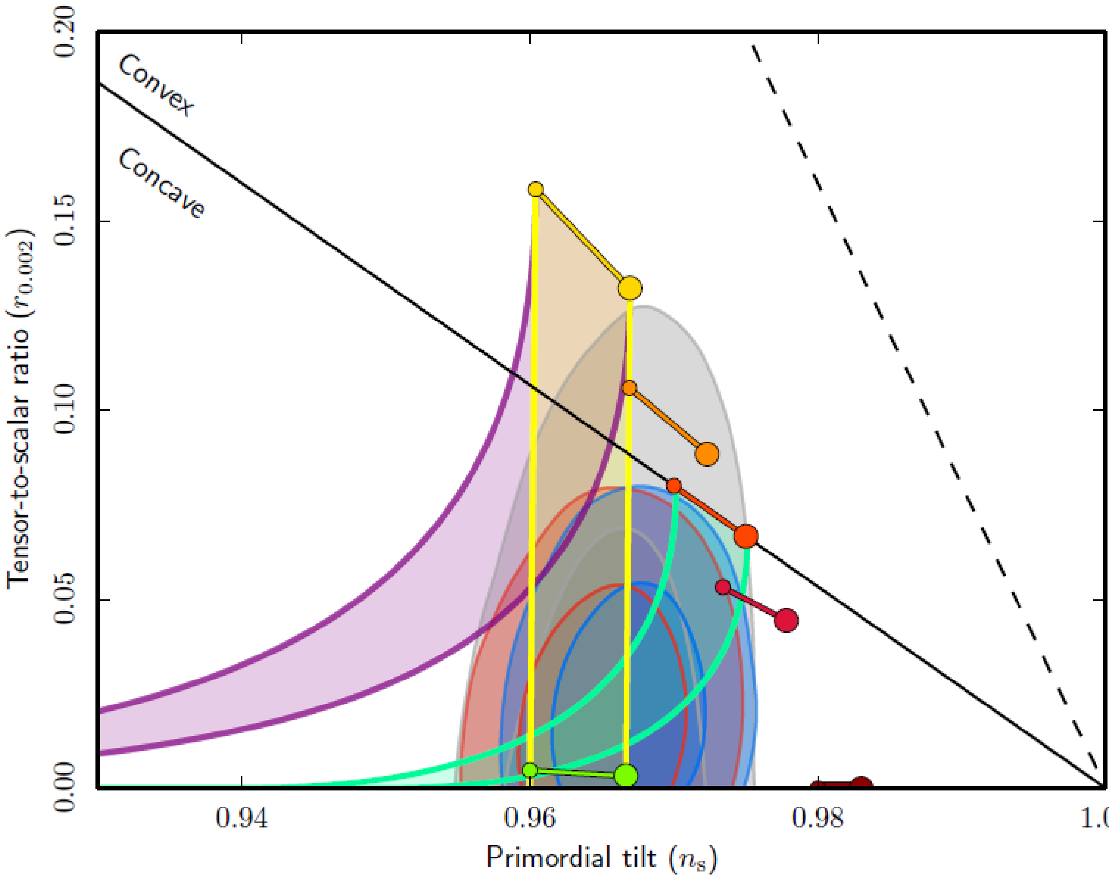
- **Starobinsky model[2]**

$$\frac{M_P^2}{2}\int d^4x\sqrt{-g}\left(R+\frac{1}{2M^2}R^2\right)$$

$\Longleftrightarrow$ Einstein gravity＋Scalar field



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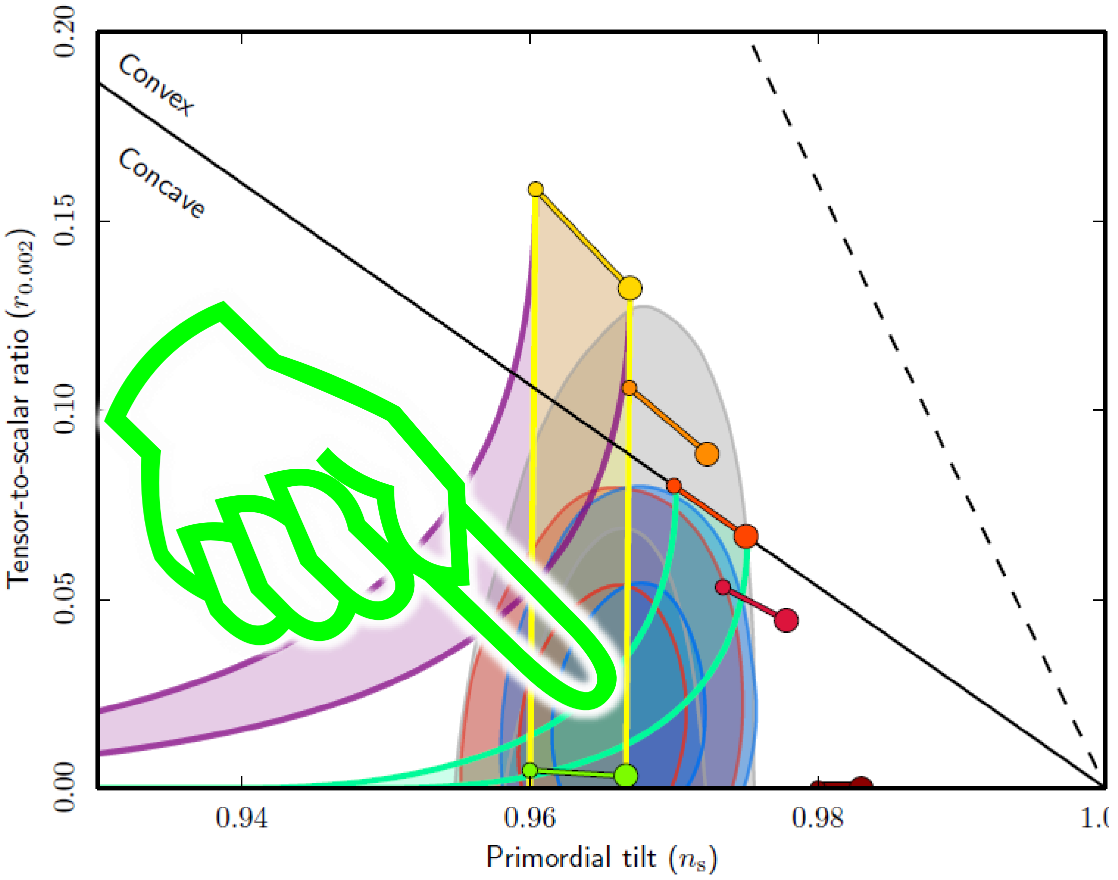
⟺ Einstein gravity + Scalar field

$$\begin{cases} n_s - 1 = -\dfrac{2}{N_e} + \mathcal{O}(N_e^{-2}) \\ r = \dfrac{12}{N_e} + \mathcal{O}(N_e^{-3}) \end{cases}$$



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The origin of Starobinsky model

- Starobinsky model

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The origin of Starobinsky model

- String/M theory effective action [4]

$$\frac{M_{(D)}^{D-2}}{2}\int d^D x\sqrt{-g^{(D)}}\times\left(R^{(D)}+\sum_{m=2}^{\infty}\mathcal{L}^{(m)}(R^{(D)}_{\mu\nu\rho\sigma},g^{(D)}_{\mu\nu})\right)$$

m-th order of curvature

(not Lovelock[5] in general)

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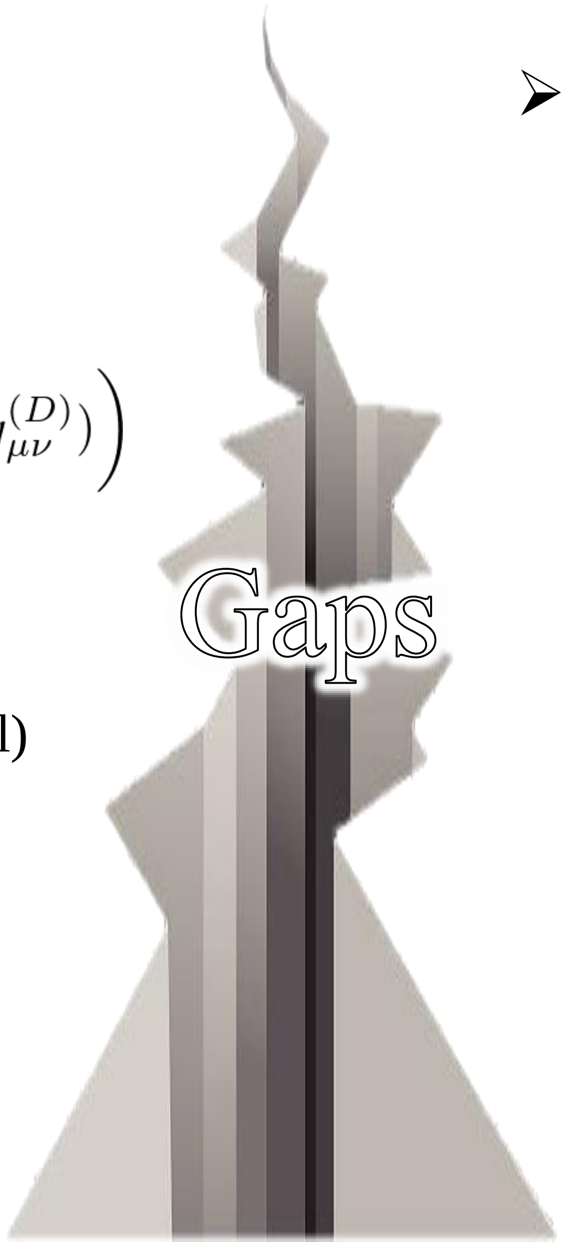

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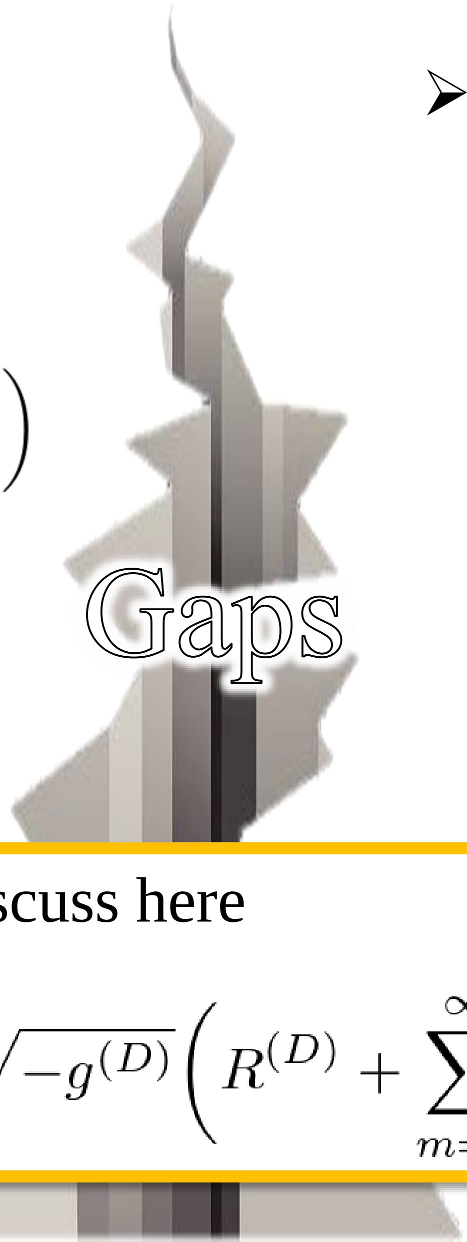

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- Toy model we will discuss here

$$\frac{M_{(D)}^{D-2}}{2}\int d^Dx\sqrt{-g^{(D)}}\left(R^{(D)} + \sum_{m=2}^{\infty}\frac{\lambda_m}{mM^{2m-2}}(R^{(D)})^m\right)$$

Starobinsky model in higher dimensions[6]

$$\frac{M_{(D)}^{D-2}}{2}\int d^Dx\sqrt{-g^{(D)}}\left(R^{(D)}+\frac{1}{nM^{2n-2}}(R^{(D)})^n\right)$$

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S.V.Ketov and H.Nakada, Phys.Rev.D (2017) S.P.Otero, F.G.Pedro, and C.Wieck, JHEP (2017)

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$D = 2n$

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Legendre-Weyl translation

Einstein Gravity + scalar field in D dimensions

Compactify extra dimensions with a form flux

Einstein Gravity + scalar field in 4 dimensions

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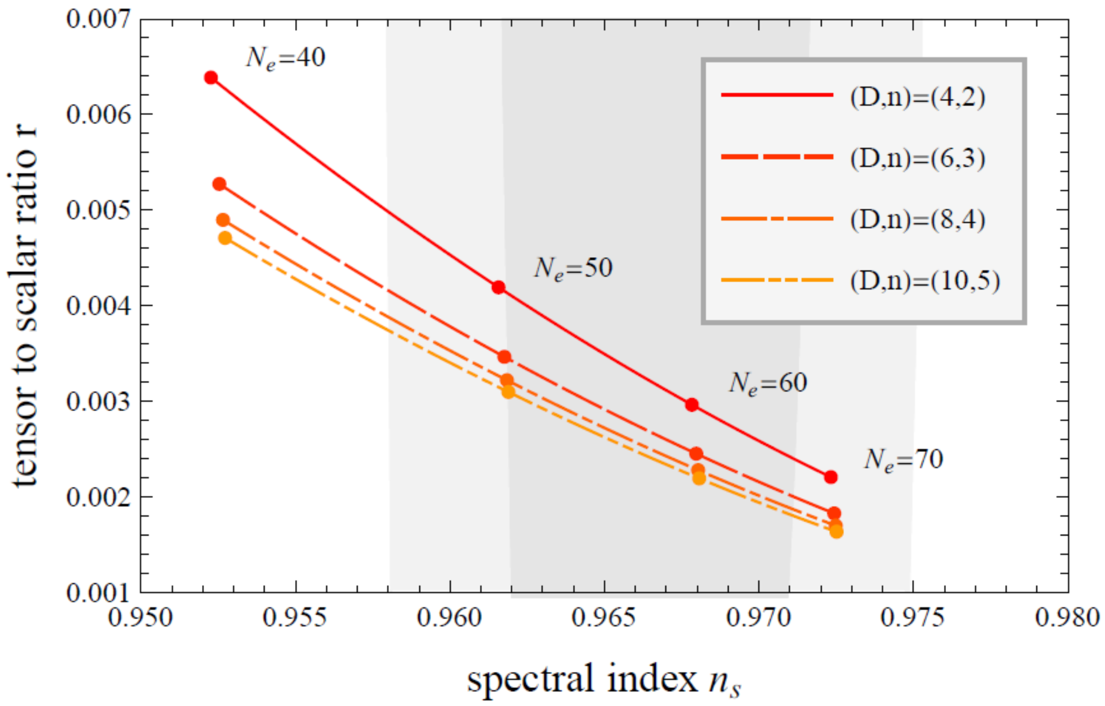

$$\begin{cases} n_s - 1 = -\dfrac{2}{N_e} + \mathcal{O}(N_e^{-2}) \\ r = \dfrac{4(2n-1)}{n-1} N_e^{-1} + \mathcal{O}(N_e^{-3}) \end{cases}$$

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What we will do

$$\frac{M_{(D)}^{D-2}}{2}\int d^Dx\sqrt{-g^{(D)}}\left(R^{(D)}+\ F^{(poly)}(R^{(D)})\right)$$

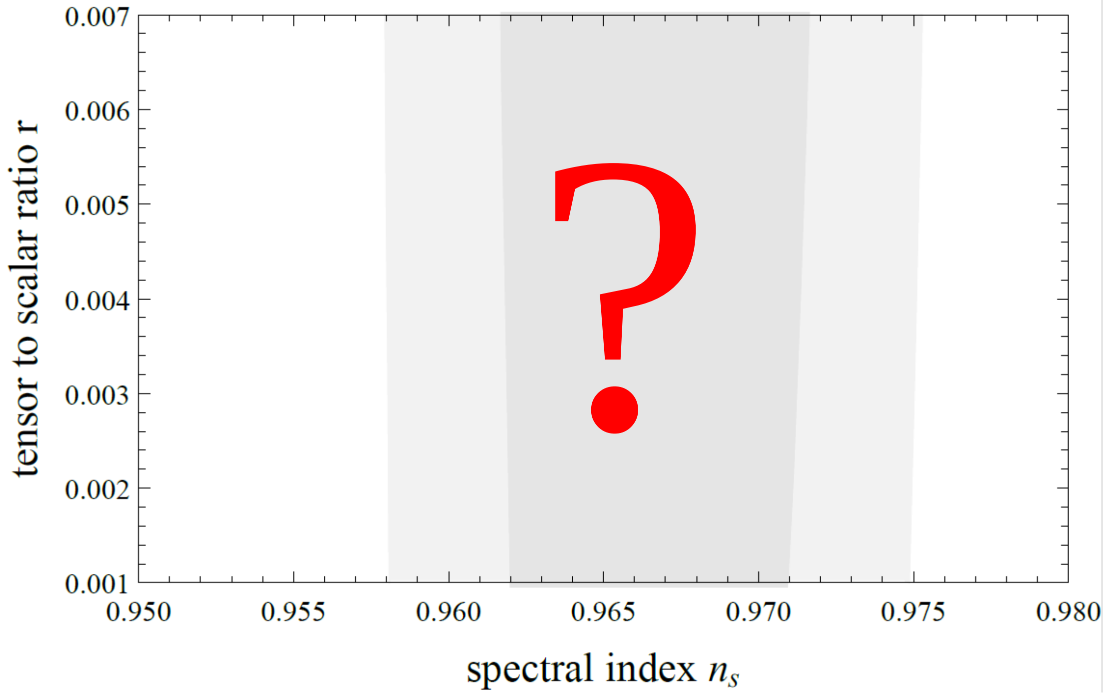
Legendre-Weyl translation

Einstein Gravity + scalar field in D dimensions

~~Compactify extra dimensions with a form flux~~

Assume compactification, for simplicity

Einstein Gravity + scalar field in 4 dimensions



$$\begin{cases} n_s - 1 = ? \\ r = ? \end{cases}$$

Other possibilities ?

$$\frac{M_{(D)}^{D-2}}{2}\int d^Dx\sqrt{-g^{(D)}}\left(R^{(D)}+\frac{1}{nM^{2n-2}}(R^{(D)})^n\right)\quad D\neq 2n$$

c.f. S.Kaneda, S.V.Ketov and N.Watanabe, Class.Quant.Grav. (2010)
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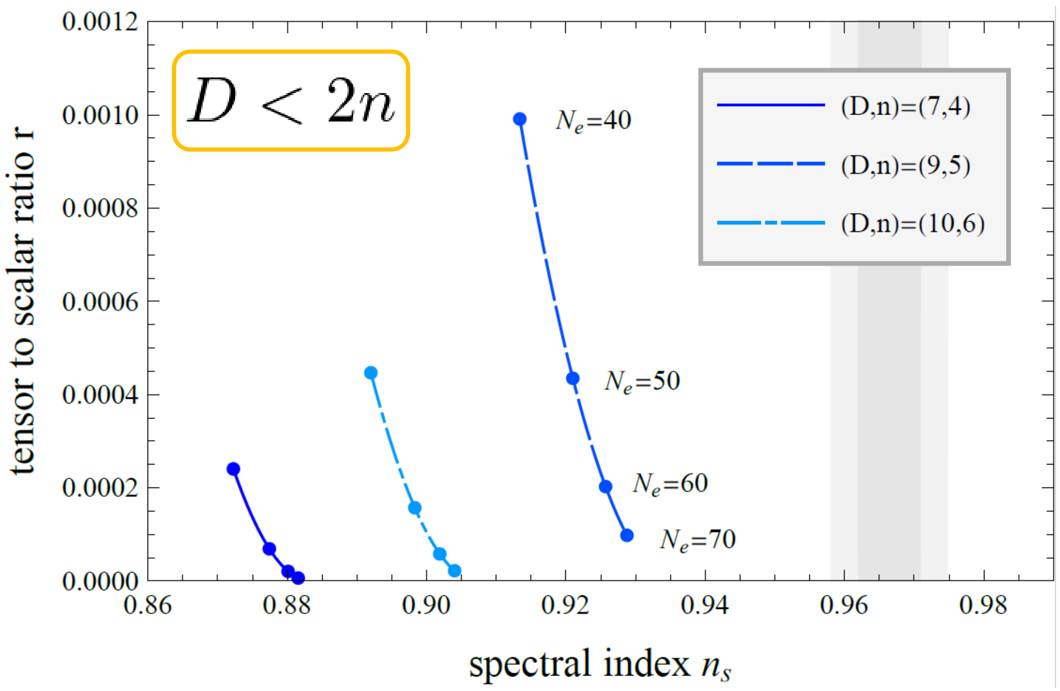

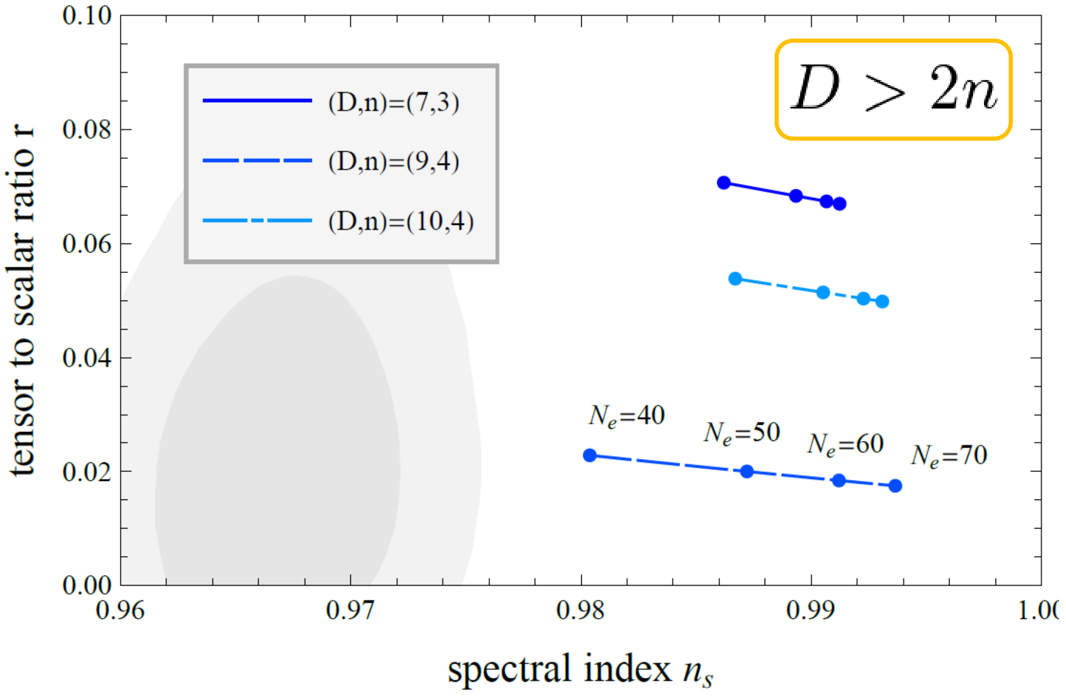

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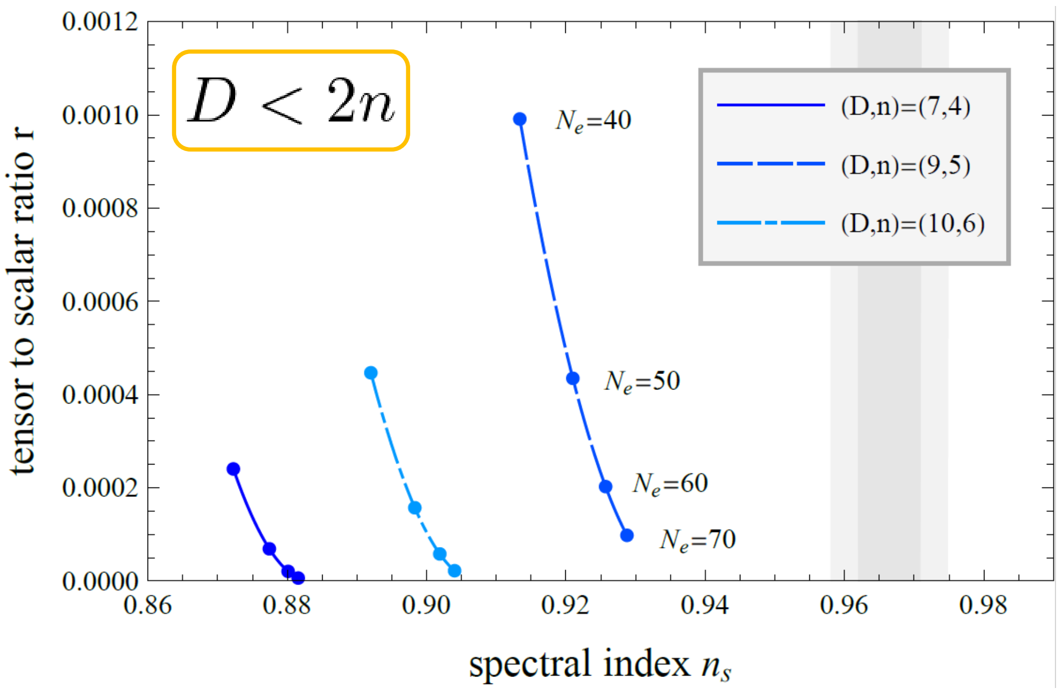

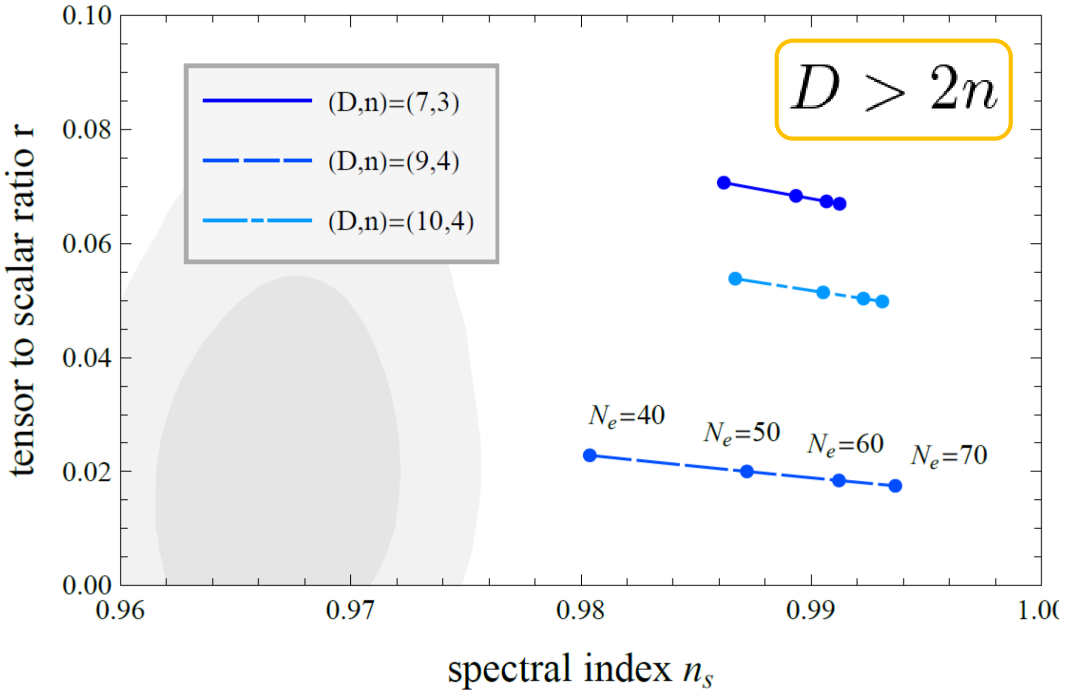

$4 \leq \forall D \leq 10$

Inflation successfully occurs when and only when $D = 2n$

Effect of an additional term –analytical-

$D = 2n$ $\quad m \neq n$

$$\frac{M_{(D)}^{D-2}}{2}\int d^D x\sqrt{-g^{(D)}}\left(R^{(D)} + \frac{1}{nM^{2n-2}}(R^{(D)})^n + \frac{\lambda}{mM^{2m-2}}(R^{(D)})^m\right)$$

c.f. Q.G.Huang, JCAP (2014)

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expected to appear
in high energy physics

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expected to appear in high energy physics

perturbatively

$$\begin{cases} n_s - 1 = -\dfrac{2}{N_e} \left[1 + \bigcirc \, \lambda N_e^{\frac{m-1}{n-1}} \right] + \mathcal{O}\left(N_e^{-2}\right) \\ r = \dfrac{4(2n-1)}{n-1} N_e^{-2} \left[1 + \bigcirc \, \lambda N_e^{\frac{m-1}{n-1}} \right] + \mathcal{O}\left(N_e^{-3}\right) \end{cases}$$

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- There is non-negligible correction if $\dfrac{1}{N_e} < \lambda N_e^{\frac{m-1}{n-1}} (\ll 1)$
- The correction becomes small as $n = D/2$ becomes large $\qquad \because 1 < N_e$

Effect of an additional term –numerical#1-

$D = 2n$ $\quad$ $m = n + 1$

$$\frac{M_{(D)}^{D-2}}{2} \int d^D x \sqrt{-g^{(D)}} \left(R^{(D)} + \frac{1}{nM^{2n-2}} (R^{(D)})^n + \frac{\lambda}{mM^{2m-2}} (R^{(D)})^m \right)$$

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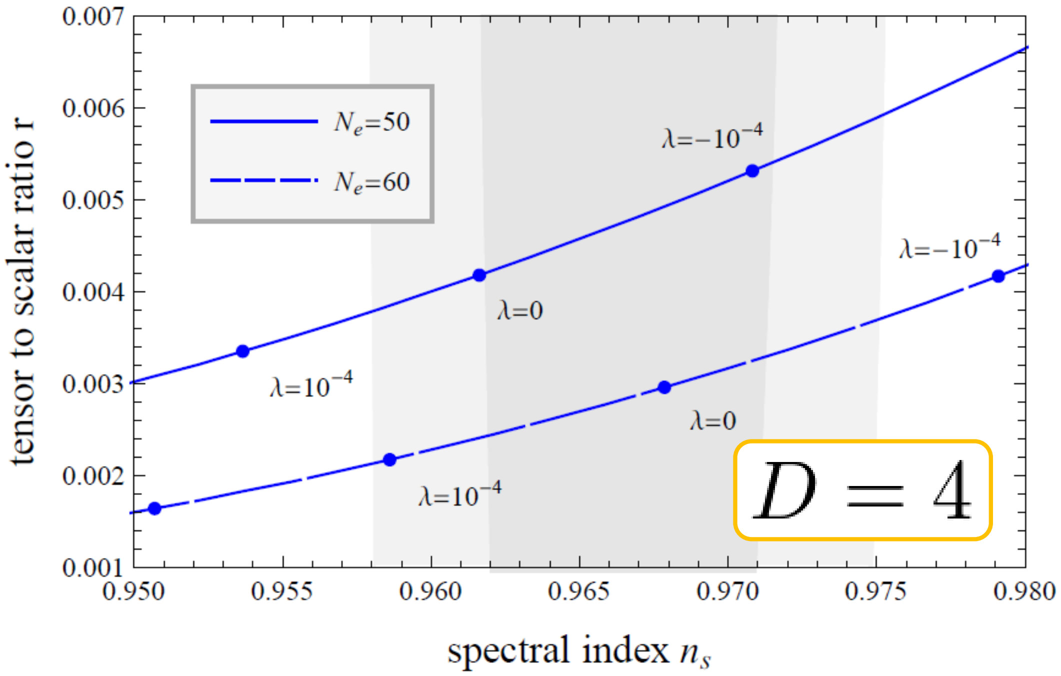

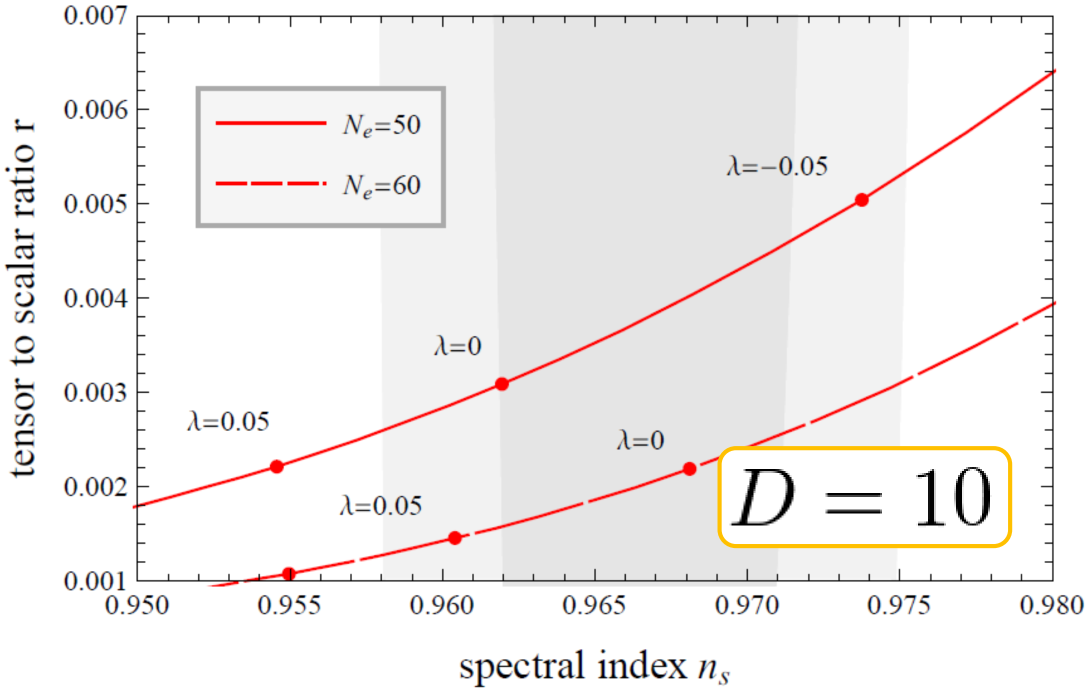

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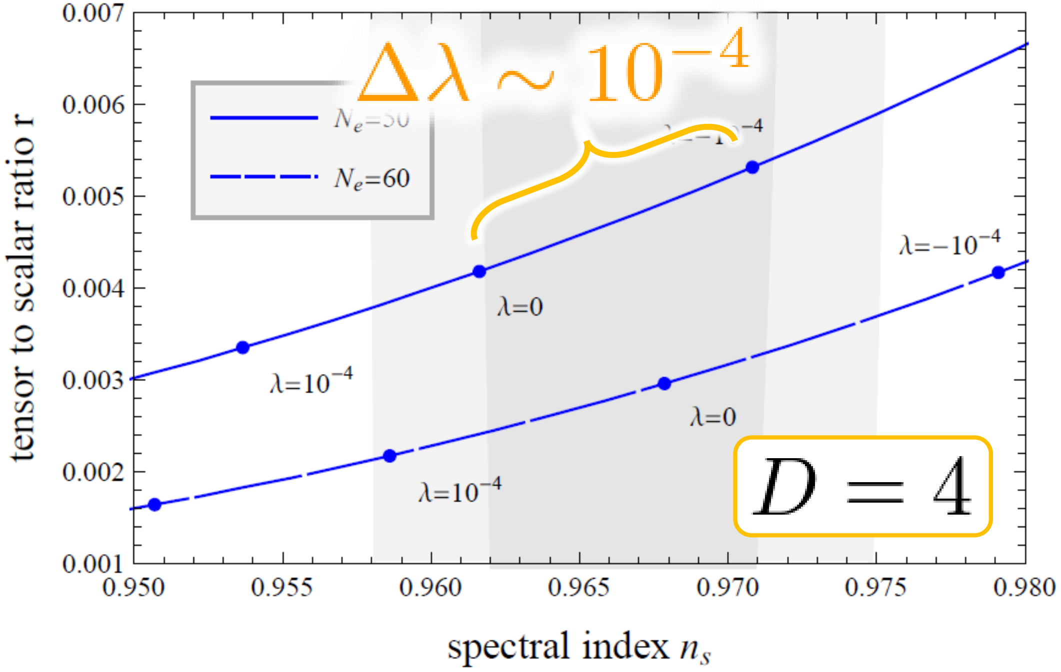

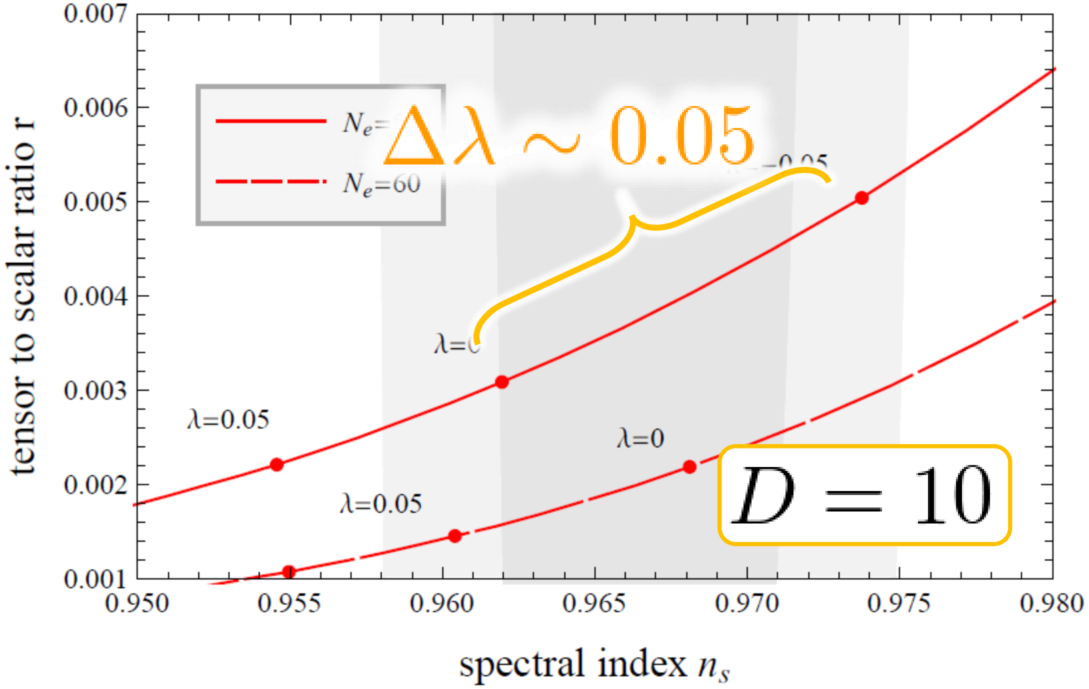

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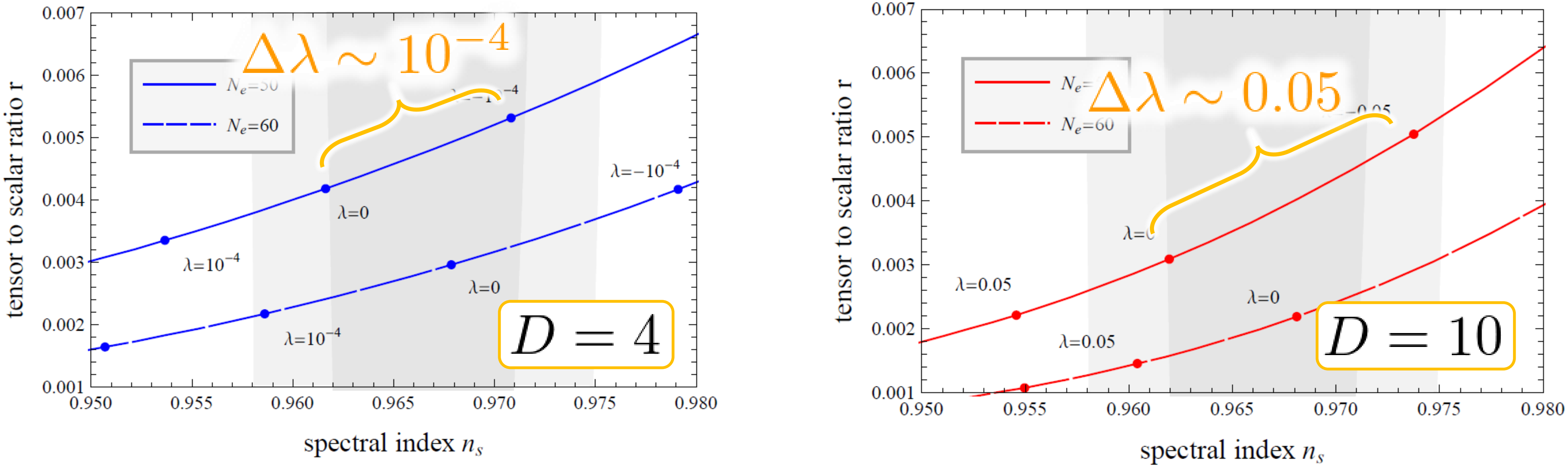

Constraint on λ is relaxed in higher dimensions

Effect of an additional term –numerical#2-

From $n_s = 0.9665 \pm 0.0038$

[3]Planck Collaboration, arXiv:1807.06211

c.f. Q.G.Huang, JCAP (2014)
T. Asaka et al., PTEP 2016 (2016)

	$D = 2n = 4$		$D = 2n = 10$	
$m = 2$	(4D Starobinsky term)		$(PB) \lesssim \lambda \lesssim (PB)$ $-0.9 \lesssim \lambda \lesssim (PB)$	$(N_e = 50)$ $(N_e = 60)$
$m = 3$	$-9.5 \times 10^{-5} \lesssim \lambda \lesssim -1.3 \times 10^{-5}$ $-2.3 \times 10^{-5} \lesssim \lambda \lesssim +5.4 \times 10^{-5}$	$(N_e = 50)$ $(N_e = 60)$	$+1.1 \times 10^{-1} \lesssim \lambda \lesssim (PB)$ $-4.7 \times 10^{-1} \lesssim \lambda \lesssim +4.9 \times 10^{-1}$	$(N_e = 50)$ $(N_e = 60)$
$m = 4$	$-5.3 \times 10^{-7} \lesssim \lambda \lesssim -0.7 \times 10^{-7}$ $-1.1 \times 10^{-7} \lesssim \lambda \lesssim +2.5 \times 10^{-7}$	$(N_e = 50)$ $(N_e = 60)$	$+0.5 \times 10^{-1} \lesssim \lambda \lesssim +8.4 \times 10^{-1}$ $-2.9 \times 10^{-1} \lesssim \lambda \lesssim +1.6 \times 10^{-1}$	$(N_e = 50)$ $(N_e = 60)$
$m = 5$	$-4.3 \times 10^{-9} \lesssim \lambda \lesssim -0.6 \times 10^{-9}$ $-0.8 \times 10^{-9} \lesssim \lambda \lesssim +1.7 \times 10^{-9}$	$(N_e = 50)$ $(N_e = 60)$	(10D Starobinsky term)	
$m = 6$	$-4.2 \times 10^{-11} \lesssim \lambda \lesssim -0.6 \times 10^{-11}$ $-0.6 \times 10^{-11} \lesssim \lambda \lesssim +1.4 \times 10^{-11}$	$(N_e = 50)$ $(N_e = 60)$	$-3.8 \times 10^{-2} \lesssim \lambda \lesssim -0.4 \times 10^{-2}$ $-1.1 \times 10^{-2} \lesssim \lambda \lesssim +3.3 \times 10^{-2}$	$(N_e = 50)$ $(N_e = 60)$
$m = 7$	$-4.5 \times 10^{-13} \lesssim \lambda \lesssim -0.6 \times 10^{-13}$ $-0.5 \times 10^{-13} \lesssim \lambda \lesssim +1.2 \times 10^{-13}$	$(N_e = 50)$ $(N_e = 60)$	$-6.4 \times 10^{-3} \lesssim \lambda \lesssim -0.7 \times 10^{-3}$ $-1.7 \times 10^{-3} \lesssim \lambda \lesssim +5.0 \times 10^{-3}$	$(N_e = 50)$ $(N_e = 60)$
$m = 8$	$-5.1 \times 10^{-15} \lesssim \lambda \lesssim -0.6 \times 10^{-15}$ $-0.5 \times 10^{-15} \lesssim \lambda \lesssim +1.1 \times 10^{-15}$	$(N_e = 50)$ $(N_e = 60)$	$-1.4 \times 10^{-3} \lesssim \lambda \lesssim -0.2 \times 10^{-3}$ $-0.4 \times 10^{-3} \lesssim \lambda \lesssim +0.1 \times 10^{-3}$	$(N_e = 50)$ $(N_e = 60)$

Effect of an additional term –numerical#2-

From $n_s = 0.9665 \pm 0.0038$

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T. Asaka et al., PTEP 2016 (2016)

	$D = 2n = 4$	$D = 2n = 10$
$m = 2$	(4D Starobinsky term)	(PB) -0.9 $\lvert\lambda\rvert \lesssim \mathcal{O}(1)$ $(N_e = 50)$ $(N_e = 60)$
$m = 3$	-9.5×10^{-5} $(N_e = 50)$, -2.3×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-5})$	$+1.1 \times 10^{-1}$ $(N_e = 50)$, -4.7×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-1})$
$m = 4$	-5.3×10^{-7} $(N_e = 50)$, -1.1×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-7})$	$+0.5 \times 10^{-1}$ $(N_e = 50)$, -2.9×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-1})$
$m = 5$	-4.3×10^{-9} $(N_e = 50)$, -0.8×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-9})$	(10D Starobinsky term)
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$m = 7$	-4.5×10^{-13} $(N_e = 50)$, -0.5×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-13})$	-6.4×10^{-3} $(N_e = 50)$, -1.7×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-3})$
$m = 8$	-5.1×10^{-15} $(N_e = 50)$, -0.5×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-15})$	-1.4×10^{-3} $(N_e = 50)$, -0.4×10 $(N_e = 60)$; $\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-3})$

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	$D=2n=4$			$D=2n=10$		
$m=2$	(4D Starobinsky term)			(PB) -0.9	$\lvert\lambda\rvert \lesssim \mathcal{O}(1)$	$(N_e=50)$ $(N_e=60)$
$m=3$	-9.5×10 -2.3×10	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-5})$	$(N_e=50)$ $(N_e=60)$	$+1.1\times 10^{-}$ $-4.7\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-1})$	$(N_e=50)$ $(N_e=60)$
$m=4$	-5.3×10 -1.1×10	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-7})$	$(N_e=50)$ $(N_e=60)$	$+0.5\times 10^{-}$ $-2.9\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-1})$	$(N_e=50)$ $(N_e=60)$
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$m=6$	$-4.2\times 10^{-}$ $-0.6\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-11})$	$(N_e=50)$ $(N_e=60)$	$-3.8\times 10^{-}$ $-1.1\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-2})$	$(N_e=50)$ $(N_e=60)$
$m=7$	$-4.5\times 10^{-}$ $-0.5\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-13})$	$(N_e=50)$ $(N_e=60)$	$-6.4\times 10^{-}$ $-1.7\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-3})$	$(N_e=50)$ $(N_e=60)$
$m=8$	$-5.1\times 10^{-}$ $-0.5\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-15})$	$(N_e=50)$ $(N_e=60)$	-1.4×10^{-3} $-0.4\times 10^{-}$	$\lvert\lambda\rvert \lesssim \mathcal{O}(10^{-3})$	$(N_e=50)$ $(N_e=60)$

Hierarchical tuning is relaxed in higher dimensions

Summary

- In order to investigate the origin of Starobinsky action,

  it is important to consider $R^{(D)} + \sum_{m=2}^{\infty} \frac{\lambda_m}{mM^{2m-2}}(R^{(D)})^m$ inflation.

- However there is a few researches besides $R^{(D)} + \frac{1}{nM^{2n-2}}(R^{(D)})^n \quad (D = 2n)$ model.

- We clarify that

  * $R^{(D)} + \frac{1}{nM^{2n-2}}(R^{(D)})^n \quad (D \neq 2n)$ model can not cause successful inflation

  *In $R^{(D)} + \frac{1}{nM^{2n-2}}(R^{(D)})^n + \frac{\lambda}{mM^{2m-2}}(R^{(D)})^m \quad (D = 2n, m \neq n)$ model,

  the additional term affects the prediction of CMB observables.

  *This requires hierarchical tuning of λ .

  *However the hierarchy is relaxed in higher dimensions.

- This results may make it easier to construct Starobinsky-like model

  from the viewpoint of high energy physics.