

# $\mu$ -hybrid inflation with low reheat temperature and observable gravity waves



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## $\mu$ -hybrid inflation with low reheat temperature and observable gravity waves

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In  $\mu$ -hybrid inflation a nonzero inflaton vacuum expectation value induced by supersymmetry breaking is proportional to the gravitino mass  $m_{3/2}$ , which can be exploited to resolve the minimal supersymmetric standard model  $\mu$  problem. We show how this scenario can be successfully implemented with  $m_{3/2} \sim 1\text{--}100$  TeV and reheat temperature as low as  $10^6$  GeV by employing a minimal renormalizable superpotential coupled with a well-defined nonminimal Kähler potential. The tensor-to-scalar ratio  $r$ , a canonical measure of primordial gravity waves, in most cases is less than or of the order of  $10^{-6}\text{--}10^{-3}$ .

[Rehman, Shafi, Vardag;17]

# Introduction; Minimal supersymmetric standard model

MSSM

Superfields		Spin 0	Spin 1/2	$3_C \times 2_L \times 1_Y$
Squarks, Quarks	$\hat{Q}$	$(\tilde{u}_L \tilde{d}_L)$	$(u_L d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	$\hat{U}^c$	$\tilde{u}_R^*$	$\bar{u}_R$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\hat{D}^c$	$\tilde{d}_R^*$	$\bar{d}_R$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
Sleptons, Leptons	$\hat{L}$	$(\tilde{\nu} \tilde{e}_L)$	$(\nu e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	$\hat{E}^c$	$\tilde{e}_R^*$	$\bar{e}_R$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, Higgsinos	$\hat{H}_u$	$(H_u^+ H_u^0)$	$(\tilde{H}_u^+ \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	$\hat{H}_d$	$(H_d^0 H_d^-)$	$(\tilde{H}_d^0 \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

# $\mu$ -problem of MSSM

$$W_{MSSM} = \widehat{U}^c y_u \widehat{Q} \widehat{H}_u - \widehat{D}^c y_d \widehat{Q} \widehat{H}_d - \widehat{E}^c y_e \widehat{L} \widehat{H}_u + \mu \widehat{H}_u \widehat{H}_d$$

$\mu$  and soft SUSY breaking terms  $\sim$  electroweak scale  $\ll$  planck scale

- forbid the  $\mu$ -term at the tree level, invoke it later as a coupling of some scalar field  $S$  to Higgs  $\longrightarrow \lambda S \widehat{H}_u \widehat{H}_d$
- value of parameter  $\mu$  is linked to mechanism of SUSY breaking
- VEV is determined by minimizing a potential that depends on soft-supersymmetry breaking terms  $\longrightarrow \langle S \rangle \propto m_{3/2}$  gravitino mass
- If we can explain why  $m_{soft} \ll M_P$  then we will also be able to explain why  $\mu$  is of the same order.  $\longrightarrow \mu = \frac{\lambda}{\kappa} \equiv \gamma m_{3/2}$



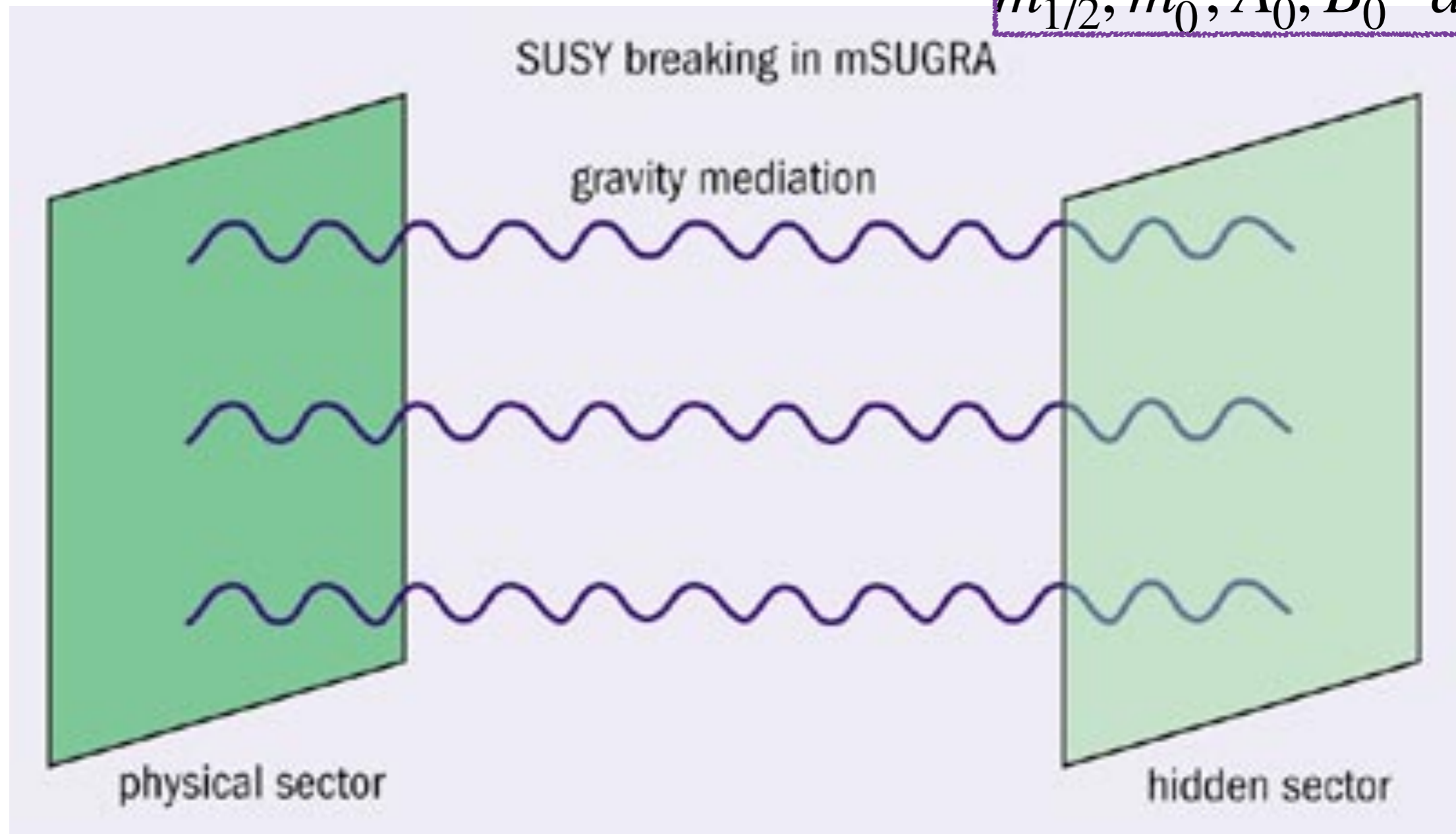
N=1

SUGRA=SUSY + gravity

# Minimal supergravity mSUGRA

- gravity mediates the breaking of SUSY through the existence of a **hidden sector**

$m_{1/2}, m_0^2, A_0, B_0$  and  $\mu$



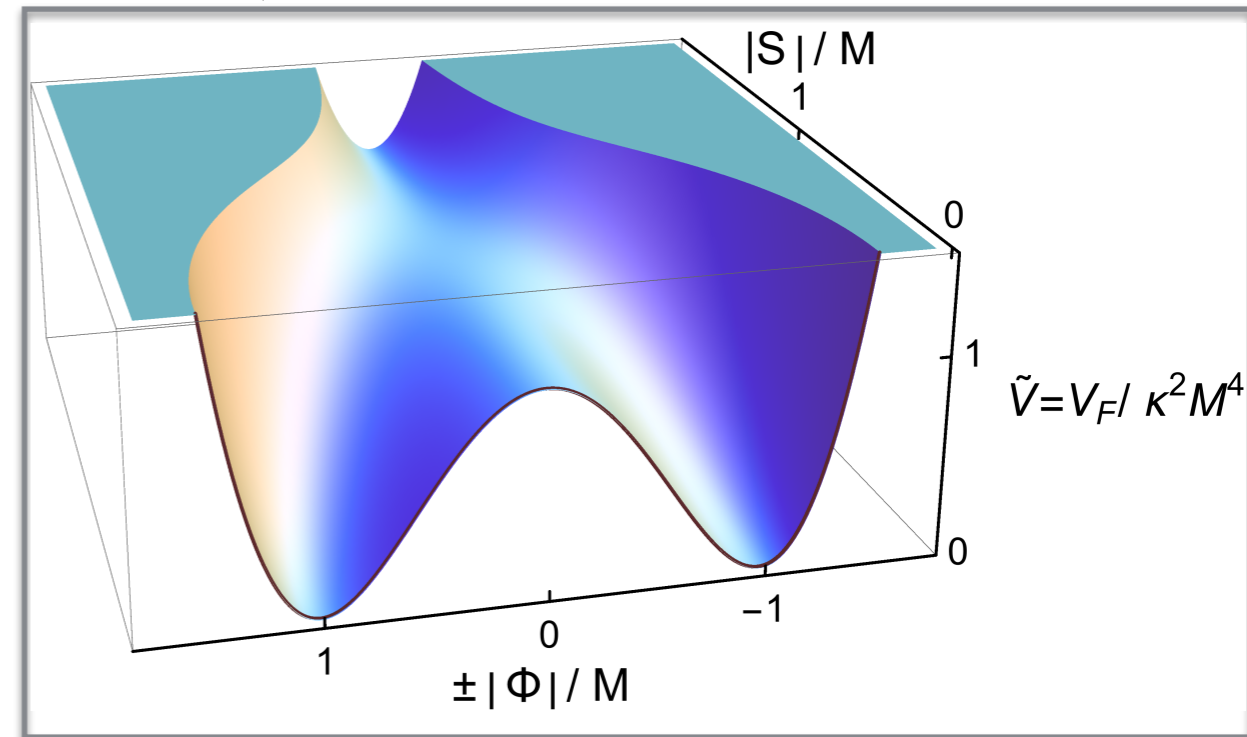
# $\mu$ -hybrid inflation(HI) with minimal Kähler

A unique renormalizable superpotential  $W$ ,

gauge singlet super field

$$W = \kappa S(\Phi\bar{\Phi} - M^2) + \lambda S H_u H_d,$$

waterfall fields



$$K_c = |S|^2 + |\Phi|^2 + |\bar{\Phi}|^2 + |H_u|^2 + |H_d|^2$$

minimal Kähler potential

Inflationary potential  $V$ ,

$$V(x) \simeq \kappa^2 M^4 \left( 1 + \mathcal{N} \frac{k^2}{8\pi^2} F_\kappa(x) + \frac{\lambda^2}{4\pi^2} F_\lambda(y) + \frac{1}{2} \left( \frac{M}{m_P} \right) x^4 + a \frac{m_{3/2}}{\kappa M} x + \left( \frac{m_S}{\kappa M} \right)^2 x^2 \right)$$

radiative

SUGRA

soft

# Cosmology with gravitinos

gauge fermion of supergravity

spin-3/2 superpartner of graviton (spin-2)

# Gravitino problem [1]

- interacts 'gravitationally' → decays late; or
- if gravitino is lightest supersymmetric particle (LSP); then it is decayed into by next-to-LSP (or NLSP), very late

disastrous effect on BBN

stringent bounds on reheat temperature

gravitino lifetime

see Fig. 1 of [3]

$$\tau_{3/2} \simeq 1.6 \times 10^4 \left( \frac{1 \text{ TeV}}{m_{3/2}} \right)^3$$

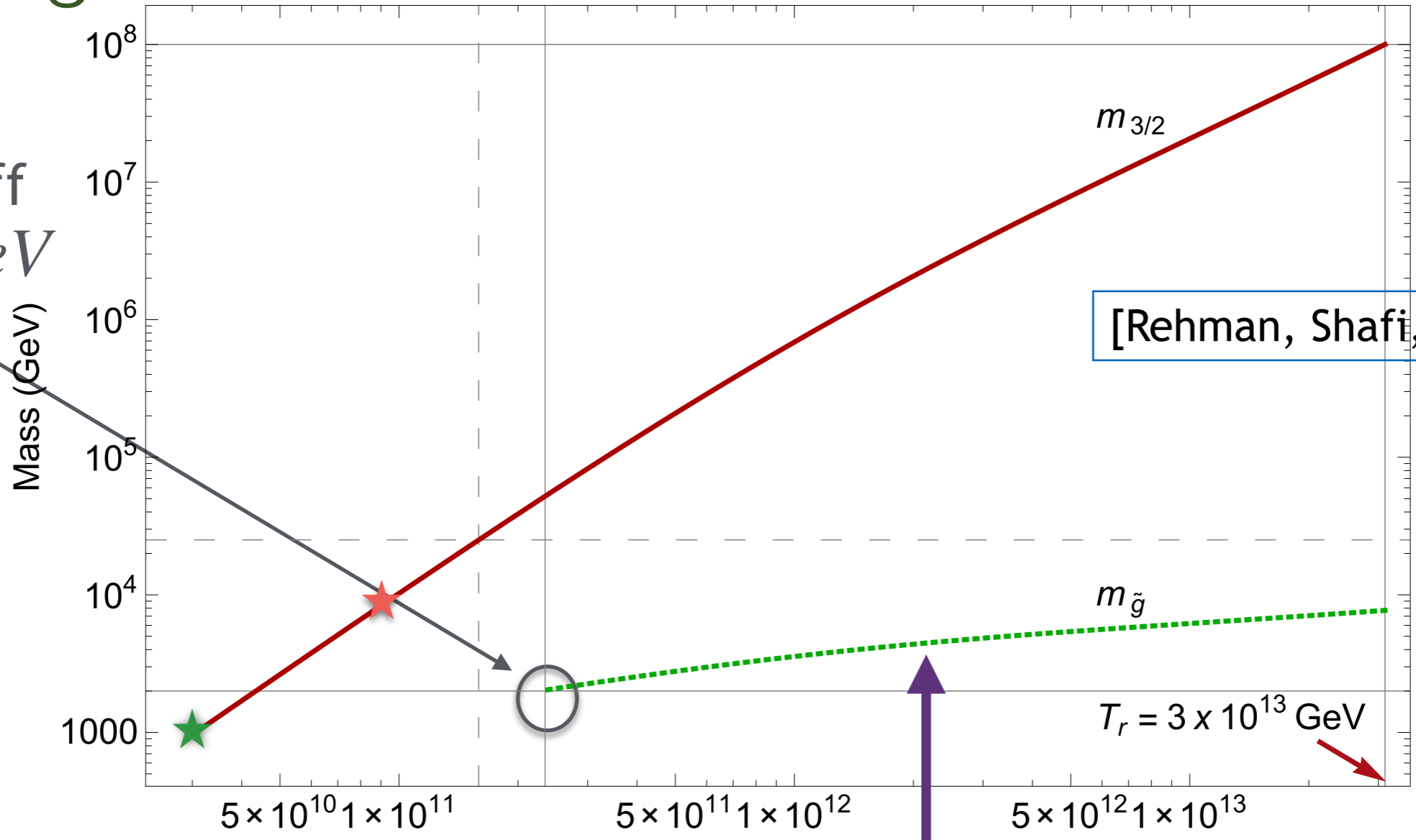
Three possibilities: [2]

- stable LSP gravitino
- unstable long-lived gravitino with  $m_{3/2} < 25 \text{ TeV}$  →  $\tau_{3/2} \gtrsim 1 \text{ s}$
- unstable short-lived gravitino with  $m_{3/2} > 25 \text{ TeV}$  →  $\tau_{3/2} \lesssim 1 \text{ s}$

# 1. LSP gravitino

minimal Kähler potential

LHC cutoff  
 $m_{\tilde{g}} \gtrsim 2 \text{ TeV}$



[Rehman, Shafi, Vardag;17]

$$\Omega_{3/2} h^2 = 0.23 \left( \frac{T_r}{10^{10} \text{ GeV}} \right) \left( \frac{1 \text{ eV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{2 \text{ TeV}} \right)^2$$

[4]

$$\Omega_{DM} h^2 = 0.11$$

[5]

No LSP gravitino

- gravitino is heavier than gluino for all values above LHC cutoff



$$m_{3/2} < 25 \text{ TeV}$$

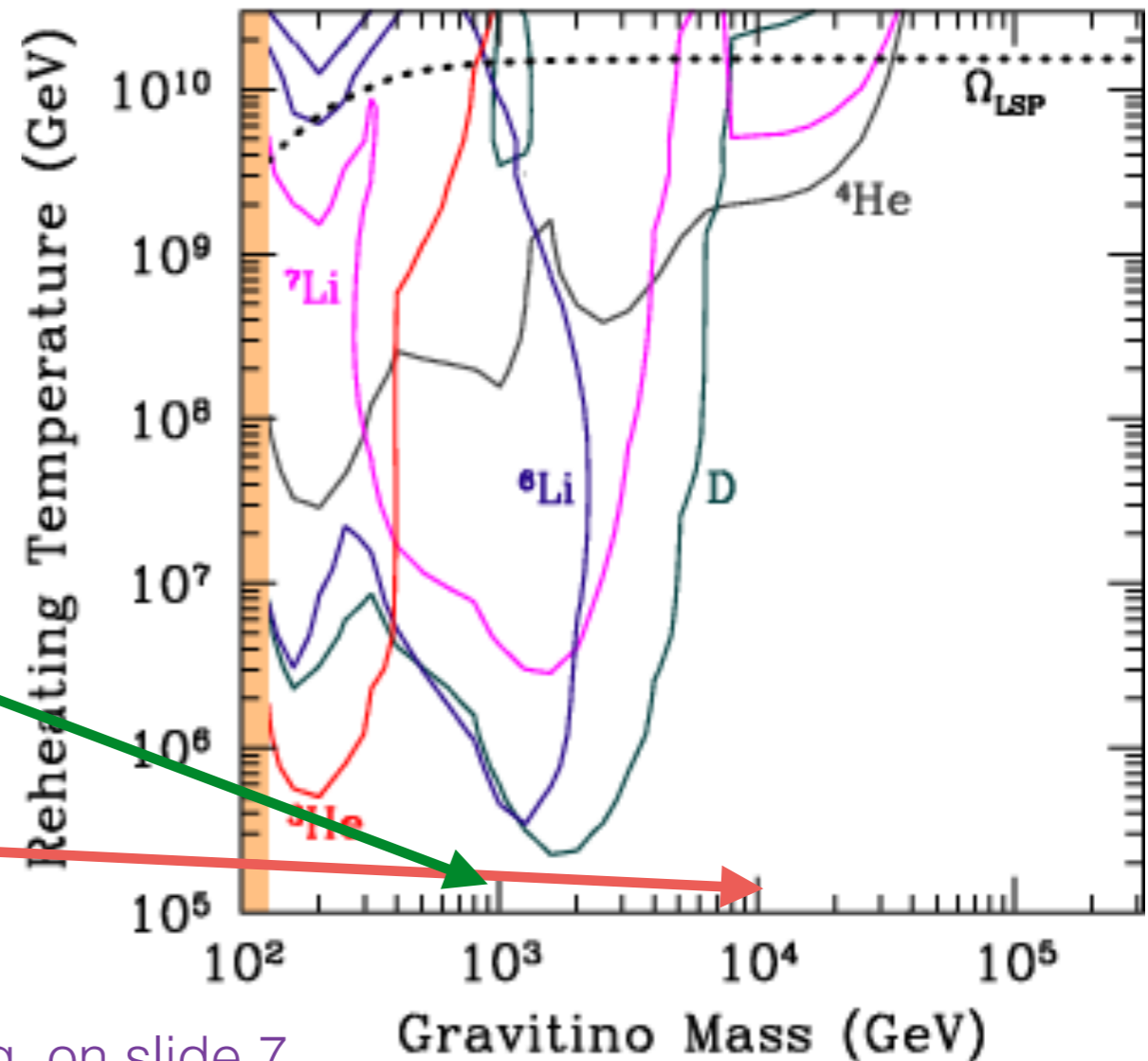
minimal Kähler potential

## 2. Unstable long-lived gravitino

gravitino decay right after BBN which can adversely affect light nuclei abundance and ruin success of BBN

BBN constraints

Fig.2 of [3]



see ★ on Fig. on slide 7

BBN limits:

$$T_r \lesssim 3 \times (10^5 - 10^6) \text{ GeV}$$

$m_{3/2} \sim 1 \text{ TeV}$

$$T_r \lesssim 2 \times 10^9 \text{ GeV}$$

$m_{3/2} \sim 10 \text{ TeV}$

$T_r$  from inflationary constraints is  $3 \times 10^{10} \text{ GeV}$  and  $10^{11} \text{ GeV}$  for 1 TeV and 10 TeV gravitino mass

Unstable long-lived gravitino is inconsistent with the BBN bounds.

$$m_{3/2} > 25 \text{ TeV}$$

minimal Kähler potential

LSP neutralino constraints

### 3 (a). Unstable short-lived gravitino

- constraints on LSP neutralino

$$m_{\tilde{\chi}_1^0} \gtrsim 18 \text{ GeV} \quad [6]$$

- gravitino decays into LSP neutralino

[3]

$$\Omega_{\tilde{\chi}_1^0} h^2 \simeq 2.8 \times 10^{11} \times Y_{3/2} \left( \frac{m_{\tilde{\chi}_1^0}}{1 \text{ TeV}} \right)$$

- and the gravitino yield

$$Y_{3/2} \simeq 2.3 \times 10^{-12} \left( \frac{T_r}{10^{10} \text{ GeV}} \right)$$

[3]

- LSP neutralino density produced by gravitino decay should not exceed the observed DM relic density

[2]

$$m_{\tilde{\chi}_1^0} \lesssim 18 \left( \frac{10^{11} \text{ GeV}}{T_r} \right)$$

- Inconsistent with **LSP neutralino constraints**



$$m_{3/2} > 25 \text{ TeV}$$

minimal Kähler potential

3(b). Unstable short-lived gravitino; with LSP neutralino in thermal equilibrium

- neutralino abundance is independent of gravitino yield
- *bound from gravitino life time* (for a typical value of the freeze-out temperature)

$$\tau_{3/2} \lesssim 10^{-11} \text{ sec} \left( \frac{1 \text{ TeV}}{m_{\tilde{\chi}_1^0}} \right)^2.$$

- *bound on  $m_{3/2}$*  by comparing with gravitino life time eq.(slide 8)

$$m_{3/2} \gtrsim 10^8 \text{ GeV} \left( \frac{m_{\tilde{\chi}_1^0}}{2 \text{ TeV}} \right)^{2/3}$$

[2]

split SUSY

# Conclusion for $\mu$ -HI with minimal Kähler potential<sup>[2]</sup>

- minimal Kähler  $\rightarrow T_r > 10^{11}$  GeV
- no LSP gravitino
- $m_{3/2}$  sufficiently large  $\rightarrow$  LSP is in thermal equilibrium  
when gravitino decay  $\rightarrow m_{3/2} > 10^8$  GeV [2]
- successful  $\mu$ -HI leads to *split supersymmetry*.

# $\mu$ -hybrid inflation with nonminimal Kähler

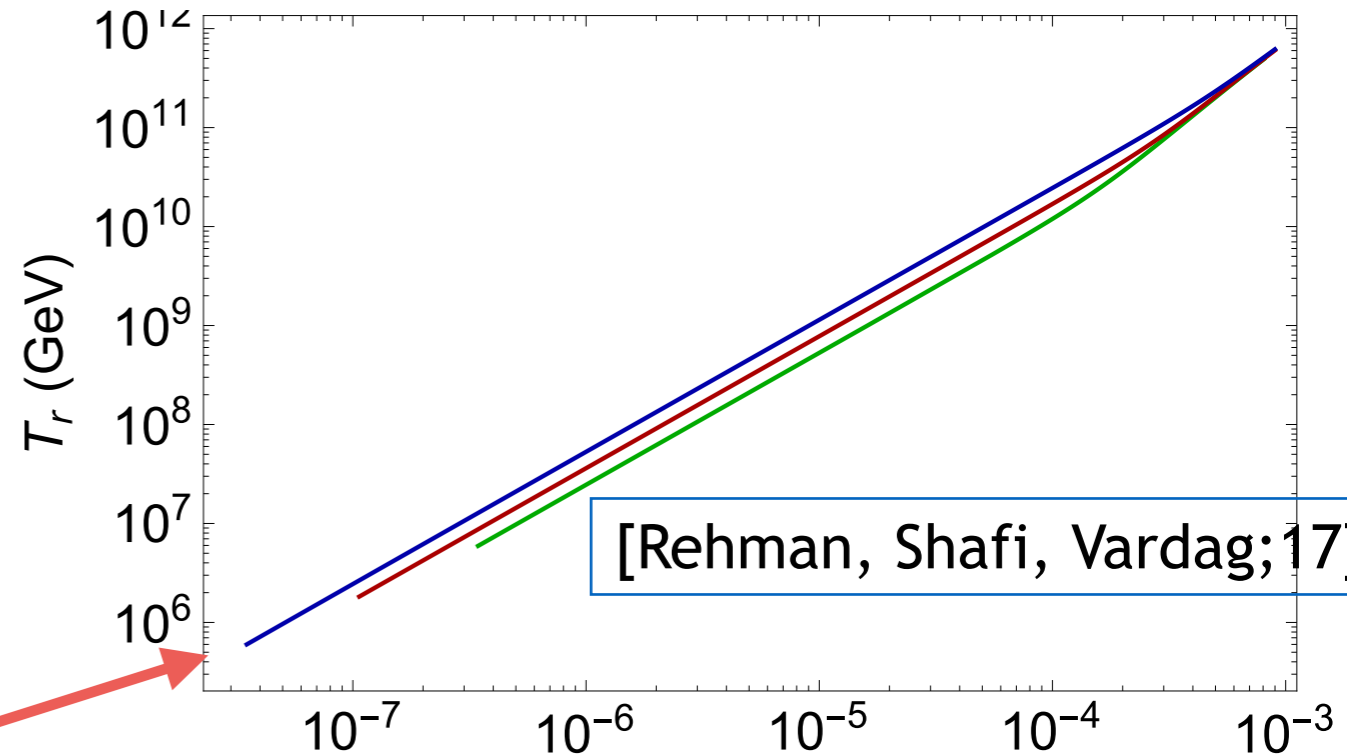
$$K = K_c + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{SS} \frac{|S|^6}{6m_P^4} + \dots$$

nonminimal Kähler potential

$$V(x) \simeq \kappa^2 M^4 \left( 1 + \mathcal{N} \frac{k^2}{8\pi^2} F_\kappa(x) + \frac{\lambda^2}{4\pi^2} F_\lambda(y) + \frac{\gamma_S}{2} \left( \frac{M}{m_P} \right) x^4 + \kappa_S \left( \frac{M}{m_P} \right)^2 x^2 + a \frac{m_{3/2}}{\kappa M} x + \left( \frac{m_S}{\kappa M} \right)^2 x^2 \right)$$

modified SUGRA corrections

nonminimal Kähler potential decreases  $T_r$



$T_r \gtrsim (6 \times 10^6, 2 \times 10^6, 6 \times 10^5) \text{ GeV}$  for  $m_{3/2} \sim (1, 10, 100) \text{ TeV}$

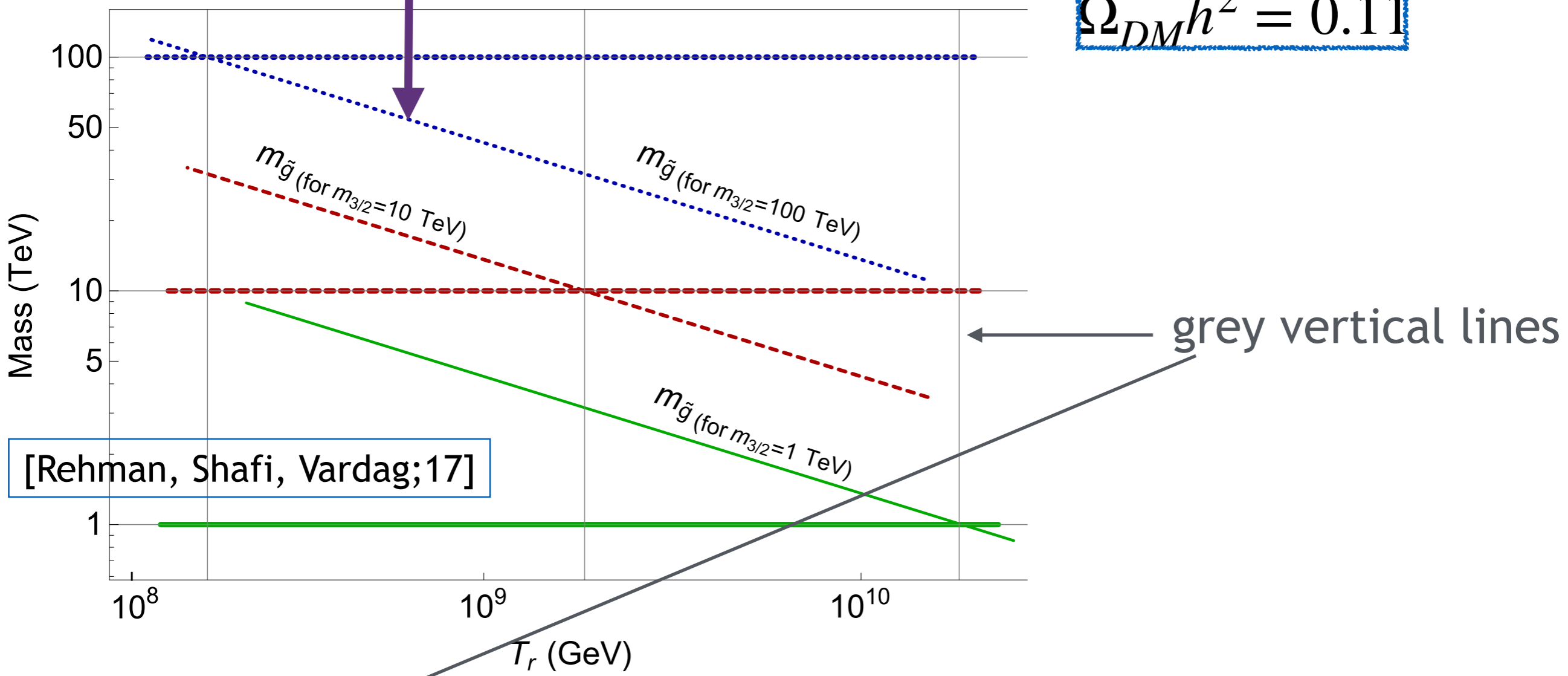


# 1. LSP gravitino

$$\Omega_{3/2} h^2 = 0.23 \left( \frac{T_r}{10^{10} \text{ GeV}} \right) \left( \frac{1 \text{ TeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{2 \text{ TeV}} \right)^2$$

$$\Omega_{DM} h^2 = 0.11$$

see slide 7



[Rehman, Shafi, Vardag;17]

$T_r \lesssim 2 \times 10^{10}, 10^9, 10^8 \text{ GeV}$  for  $m_{3/2} \sim (1, 10, 100) \text{ TeV}$

LSP gravitino scenario can be consistently realized





$$m_{3/2} < 25 \text{ TeV}$$

nonminimal Kähler potential

## 2. Unstable long-lived gravitino

BBN constraints

- gravitino decay after BBN  $\rightarrow$  adversely affect light nuclei abundance  $\rightarrow$  ruin success of BBN

from inflationary constraints  $T_r < 6 \times 10^6 \text{ GeV}$  and  $2 \times 10^6 \text{ GeV}$  for 1 TeV and 10 TeV gravitino masses  
see   on slide 14

- BBN bounds

see slide 8  $\rightarrow$

$$T_r \lesssim 3 \times (10^5 - 10^6) \text{ GeV} \quad m_{3/2} \sim 1 \text{ TeV}$$

$$T_r \lesssim 2 \times 10^9 \text{ GeV} \quad m_{3/2} \sim 10 \text{ TeV}$$

This scenario, is marginally ruled out for 1 TeV gravitino mass but sits comfortably within BBN for 10 TeV gravitino mass

$$m_{3/2} > 25 \text{ TeV}$$

Nonminimal Kähler potential

### 3. Unstable short-lived gravitino

LSP  
neutralino  
constraints

- heavy gravitino  $\rightarrow$  shorter lifetime;
- Expt. constraints on LSP neutralino

$$m_{\tilde{\chi}_1^0} \gtrsim 18 \text{ GeV}$$

- upper bound on LSP neutralino:

see slide 9

$$m_{\tilde{\chi}_1^0} \lesssim (18 - 10^5) \text{ GeV for } 10^{11} \text{ GeV} \gtrsim T_r \gtrsim 6 \times 10^5 \text{ GeV}$$

*nonLSP  $m_{3/2} \sim 100 \text{ TeV}$  holds :  $10^6 \text{ GeV} \lesssim T_r \lesssim 10^{11} \text{ GeV}$*

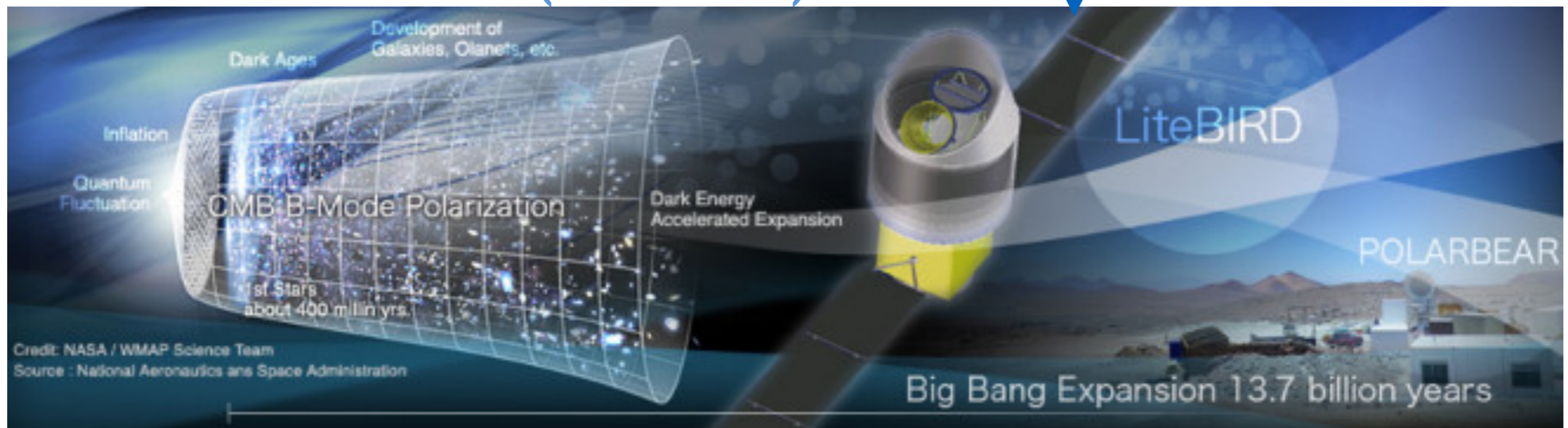
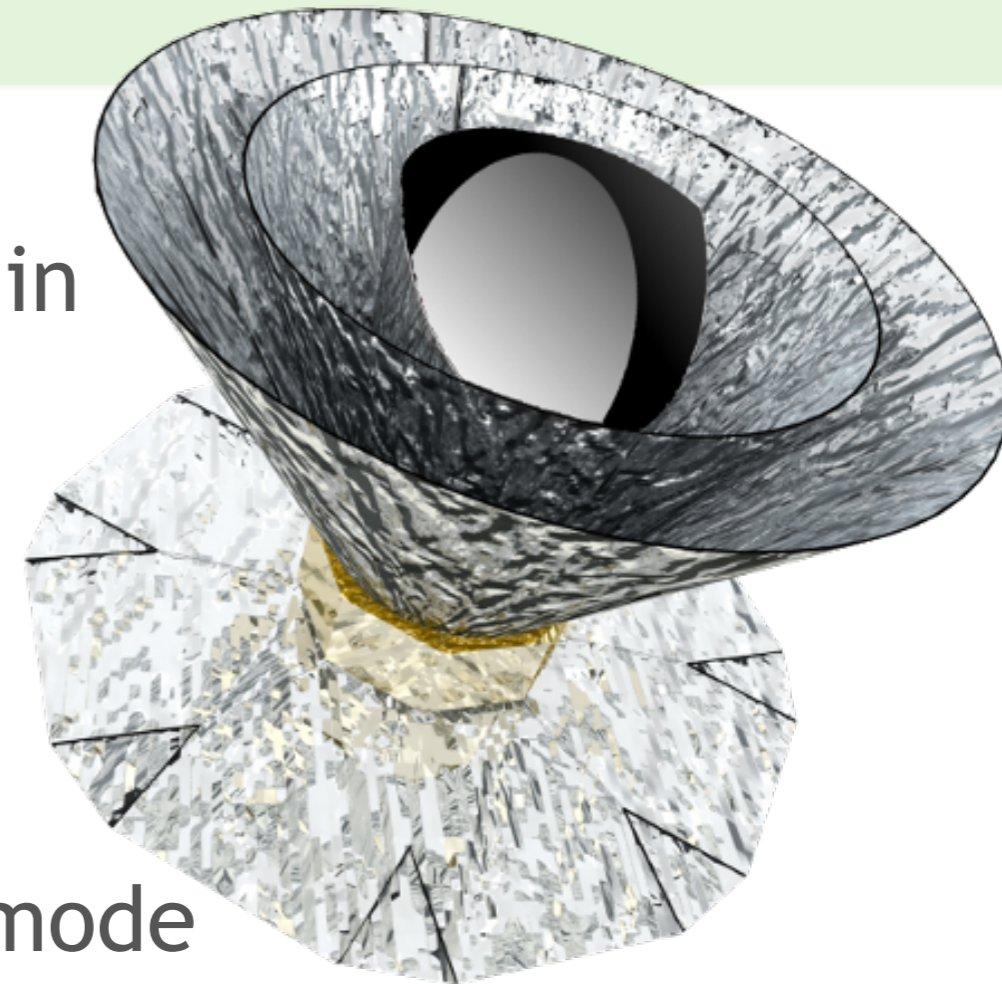


# Observable gravity waves



# Primordial gravity waves (PGW)

- PGW are gravitational waves observed in cosmic microwave background (CMB)
- Polarized Radiation Imaging and Spectroscopy Mission (PRISM) [7] →
- Lite(light) satellite for the study of B-mode polarization and Inflation from cosmic background Radiation Detection (LiteBIRD) [8] →

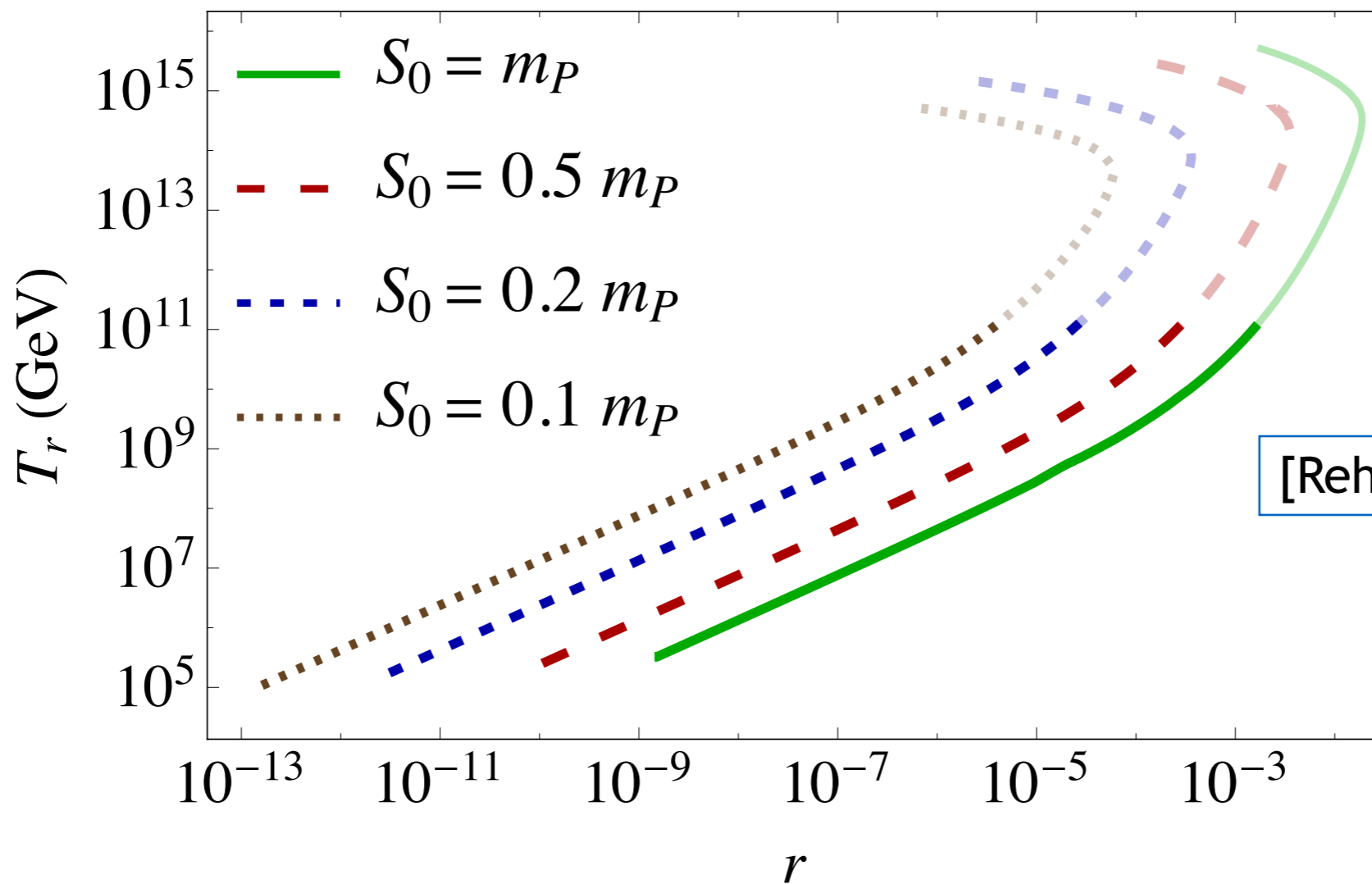




# Measure of PGW $r$

**PRISM** will measure  $r \sim 5 \times 10^{-4}$  [7]

**LiteBIRD** will provide a precision of  $\delta r < 0.001$  [8]



Upper bound range on the tensor-to-scalar ratio  $r < 10^{-6} - 10^{-3}$

# Conclusion for $\mu$ -HI with nonminimal Kähler potential

- Successful  $\mu$ -HI is realized with resolution of the  $\mu$ -problem via **nonminimal Kähler potential**.
- brings  $T_r \sim 10^6 - 10^7$  GeV
- compatible with BBN constraints and **TeV-scale** SUSY
- Upper bound range on the tensor-to-scalar ratio  
 $r < 10^{-6} - 10^{-3}$

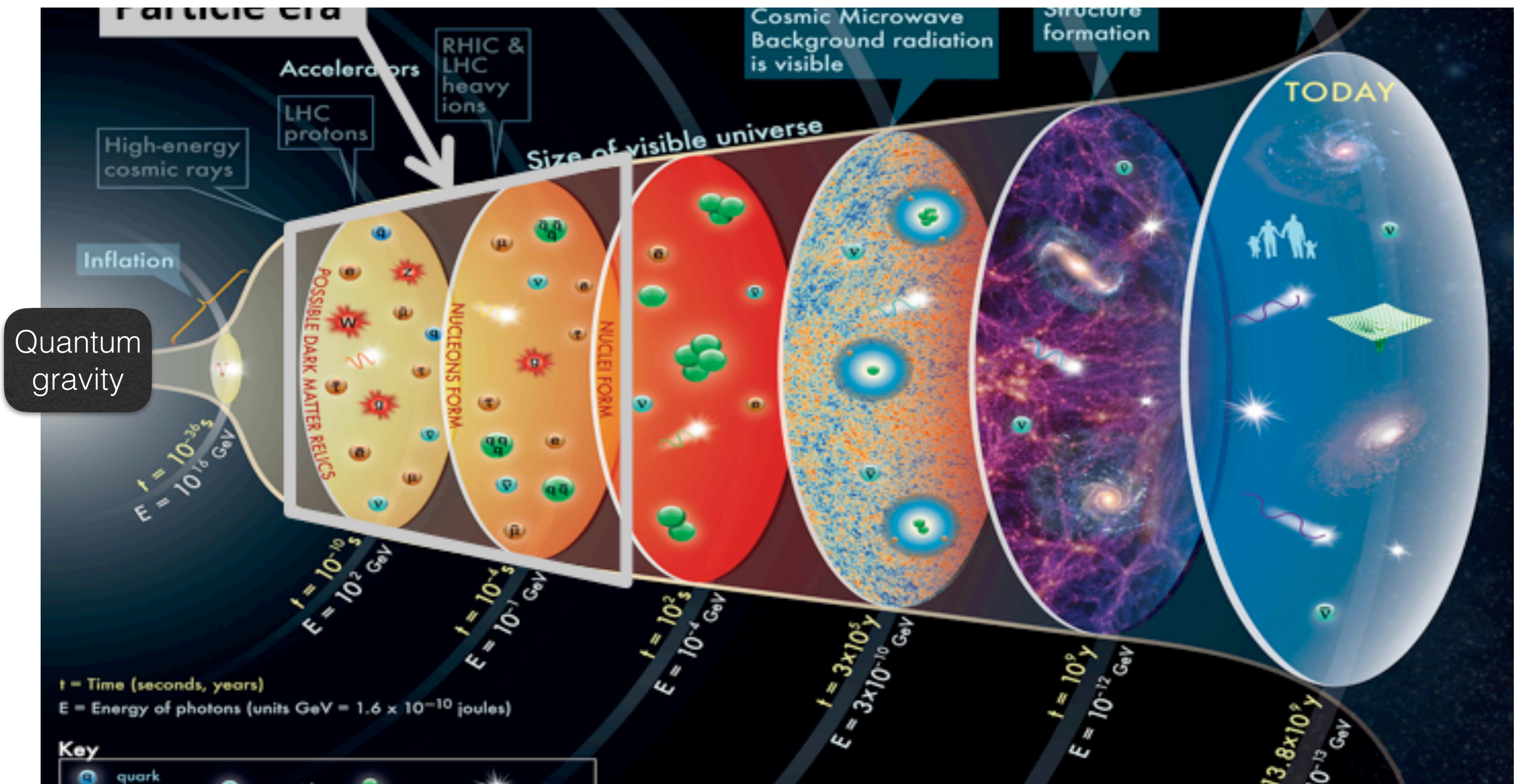
# References

- [1] PLB **138** (1984) 265; PLB **145** (1984) 181
- [2] Proc.Sci.PLANCK (2015) 121 [arXiv:1506.01410]
- [3] PRD **78** (2008) 065011
- [4] NP **B606** (2001) 518; NP **B790** (2008) 336E
- [5] Astrophys. J. Suppl. **208** (2013) 20
- [6] PLB **562** (2003) 18
- [7] PRISM Collaboration (2013) [arXiv:1306.2259]
- [8] J. Low. Temp. Phys. **176** (2014) 733

# Backup Slides

# Early universe and cosmological Inflation

Initial exponential expansion that universe underwent (proposed in order to explain big bang puzzles)





# Cosmological time line

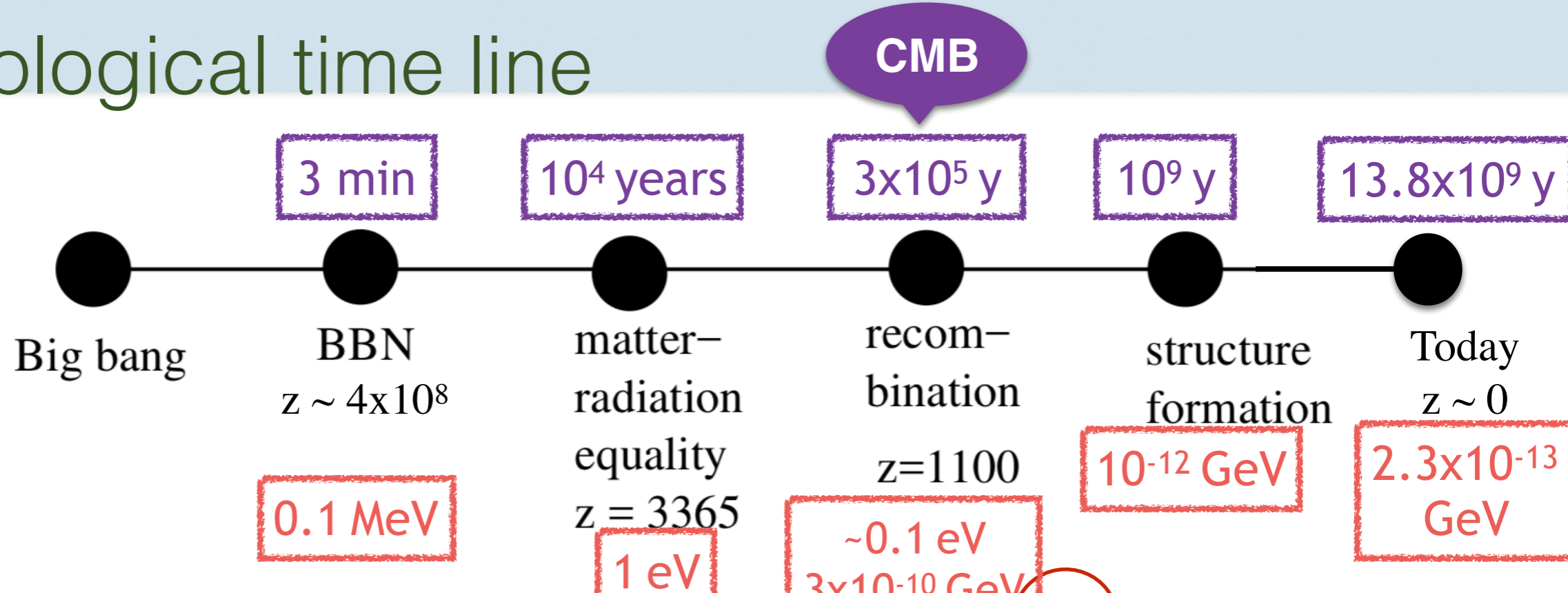


FIG. 1. The cosmological timeline, *ca.* 1970.

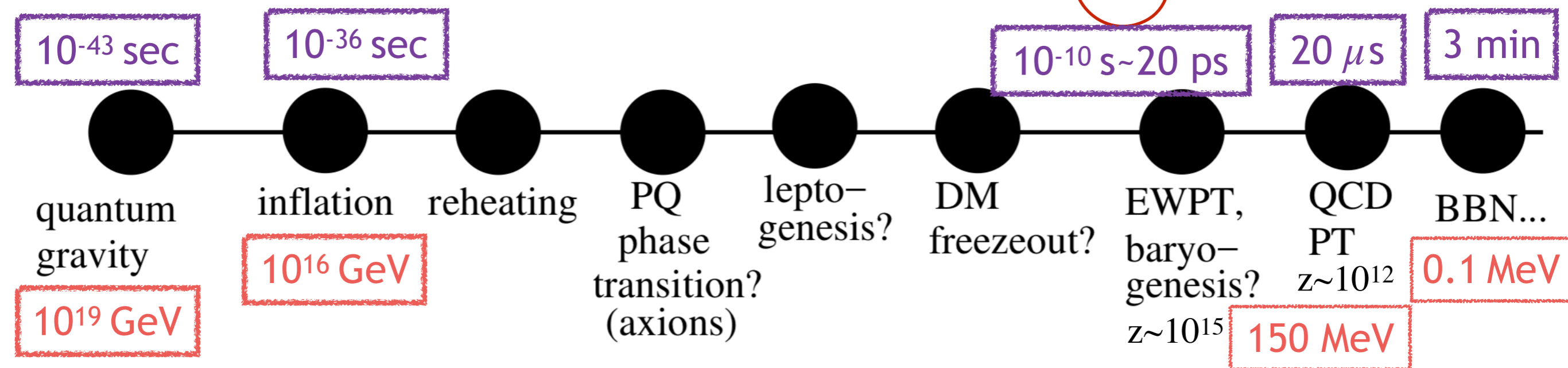
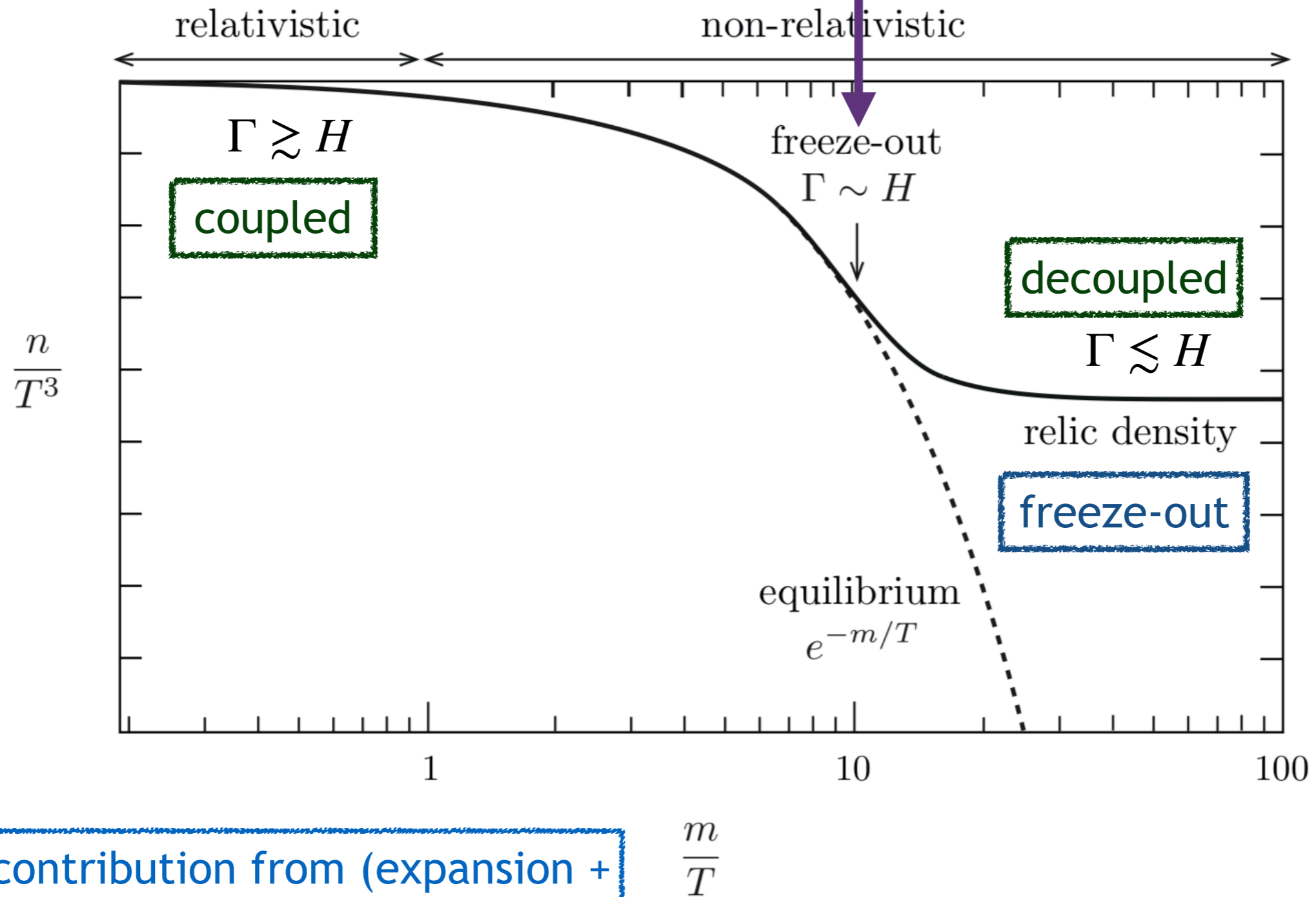


FIG. 2. The cosmological timeline, *ca.* 2018.

# Freeze out

Particle abundance no longer follows equilibrium trajectory, it has decoupled from the rest of the universe.



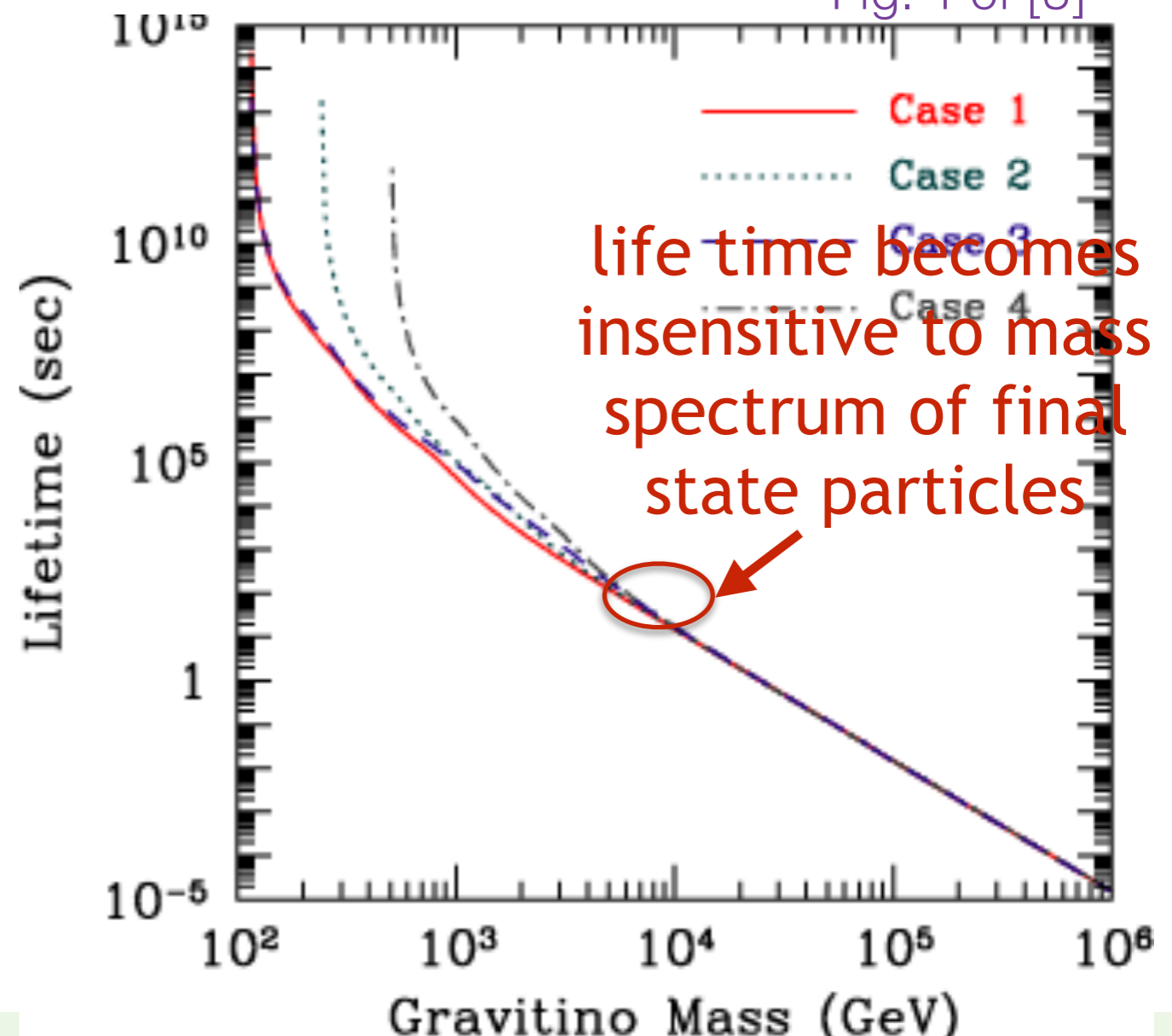
# Gravitino life time

Feynman rules for gravitino interactions → gravitino production cross-section → gravitino decay rate → gravitino number density produced subsequent to inflation → linear in max. reheat temperature

For an unstable gravitino, the lifetime is, [3]

$$\tau_{3/2} \simeq 1.6 \times 10^4 \left( \frac{1 \text{ TeV}}{m_{3/2}} \right)^3$$

Fig. 1 of [3]

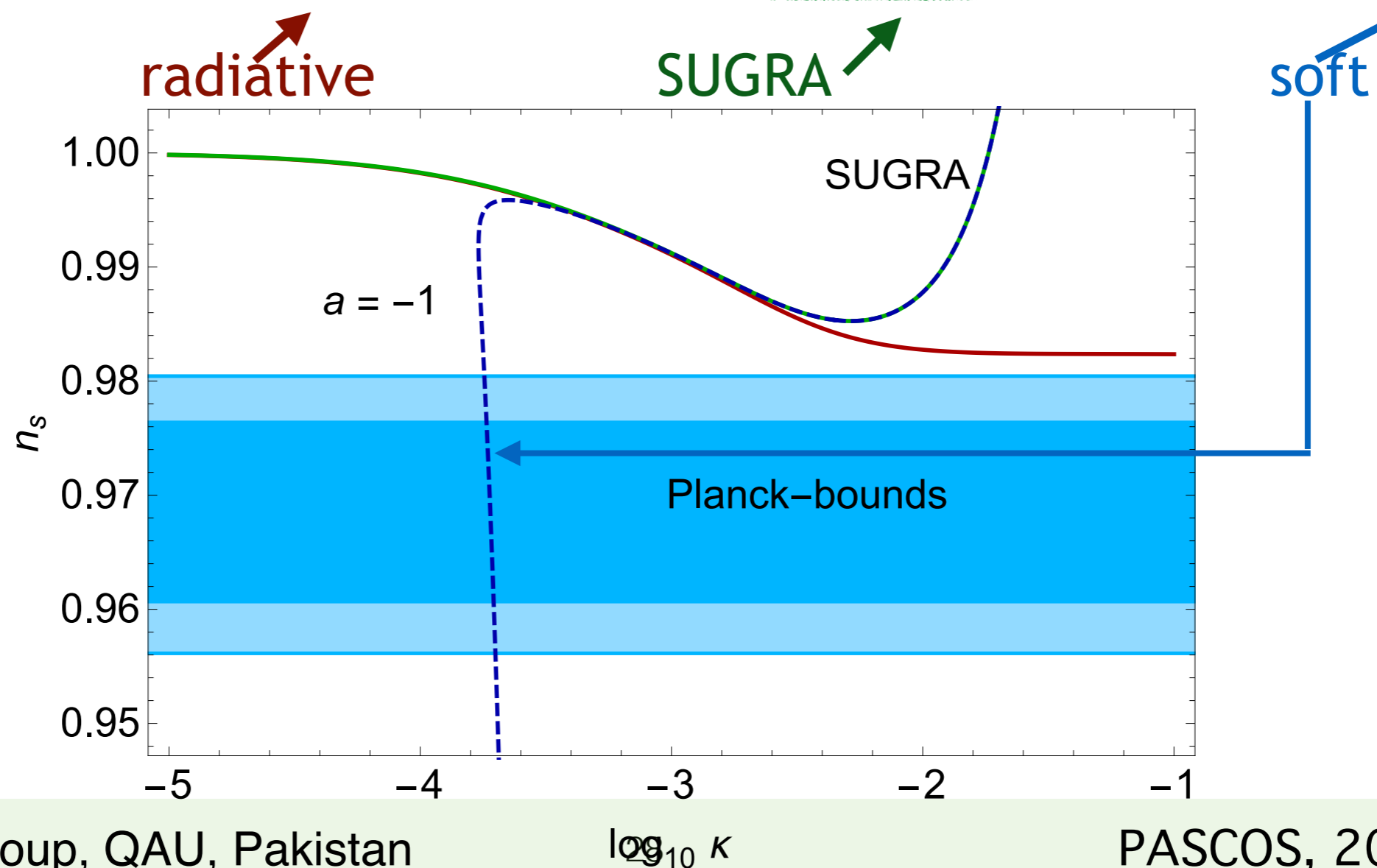


# Significance of 'a-term'

- **Solving particle physics problem:** Inflaton field acquires non-zero vev due to soft SUSY breaking terms.

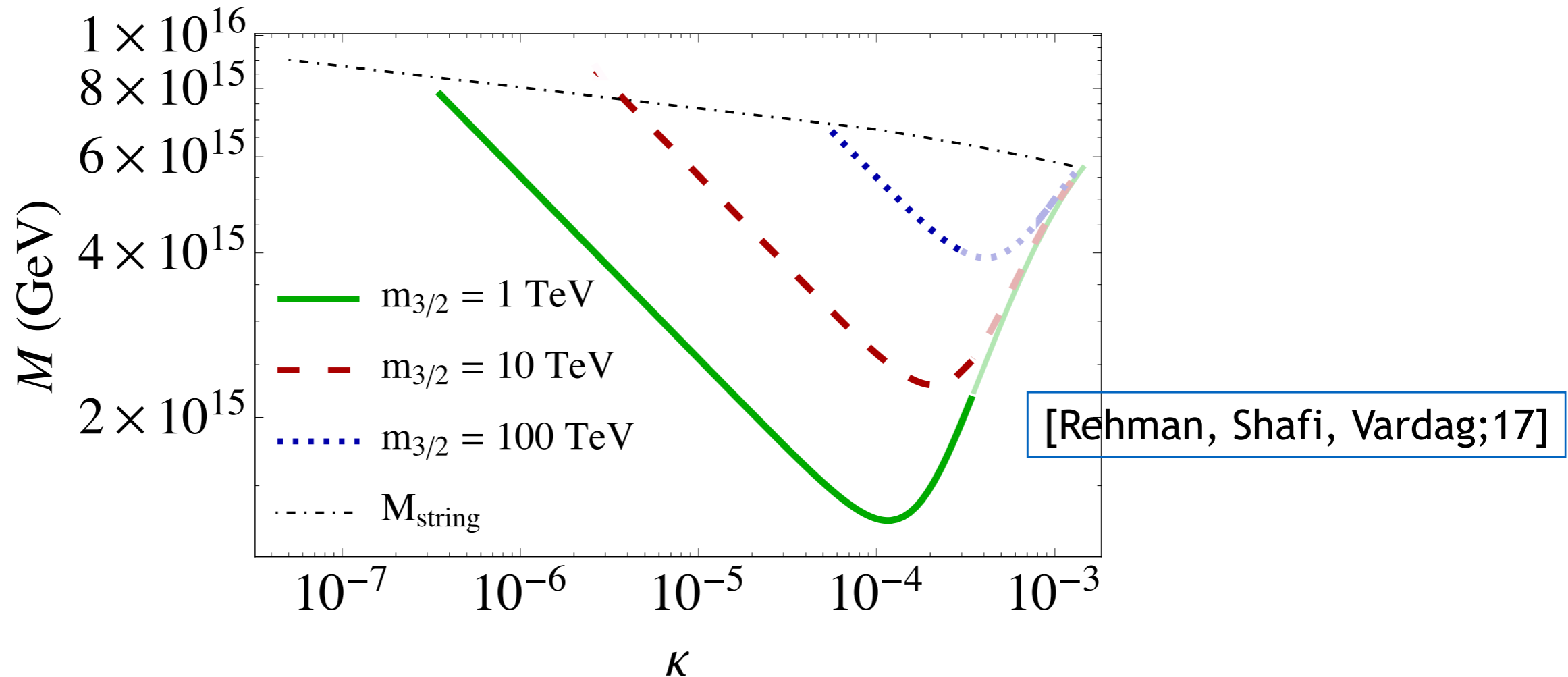
- **Solving cosmological problem:** coefficient of soft term 'a' plays a significant role and brings  $n_s$  within PLANCK bounds

$$V(x) \simeq \kappa^2 M^4 \left( 1 + \mathcal{N} \frac{k^2}{8\pi^2} F_\kappa(x) + \frac{\lambda^2}{4\pi^2} F_\lambda(y) + \frac{1}{2} \left( \frac{M}{m_P} \right) x^4 + a \frac{m_{3/2}}{\kappa M} x + \left( \frac{m_S}{\kappa M} \right)^2 x^2 \right)$$



# Cosmic strings

Cosmic strings arise from the breaking of  $U(1)_{B-L}$  at the end of inflation.



Allowed range of  $r$  permissible by cosmic string bounds is suppressed and unlikely to be observed in future experiments

However, if we avoid the cosmic strings bound, by employing the shifted hybrid inflation, then the range of  $r < 10^{-6} - 10^{-3}$  mentioned earlier is testable in the foreseeable future.