Quantum Walks and Neutrinos

Farhan Tanvir Chowdhury

Quantum Structures, Information and Control
Aberystwyth University

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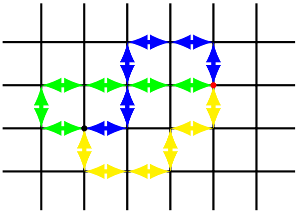
Schuster Building, The University of Manchester



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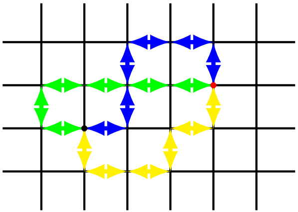

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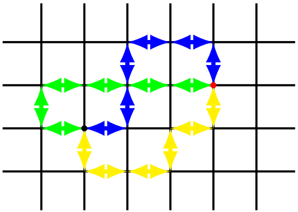
- **Quantum:** Walker traverses path formed of superposition of random shifts on some composite $\mathcal{H}$ based on flipping quantum coin, typical basis: $|\uparrow\rangle = (1, 0)^T$ and $|\downarrow\rangle = (0, 1)^T$.

Quantum Walks: What and why?



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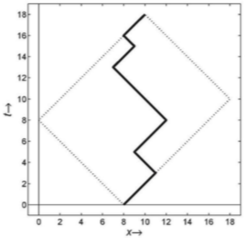
Quantum Walks (QWs) analogous to Feynman's Checkerboard for 1+1D Dirac Equation:

$$i\hbar \frac{\partial \psi}{\partial t} = mc^2 \sigma_x \psi - i\hbar \sigma_z \frac{\partial \psi}{\partial x} \quad (1)$$

- Feynman's original 1+1D checkerboard has been extended to higher dimensions, and **also used** to simulate neutrino mixing and neutrino oscillations by Petr Jizba in 2015 paper.
- QWs very useful for simulating relativistic quantum phenomena, like Graphene Carrier Density and **Relativistic Transport**.



Checkerboard Diagram Source and Peter Jizba's 2015 Paper DOI: 10.1088/1742-6596/626/1/012048

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Quantum Walks: Discrete and Continuous

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Particle starts at origin denoted by position state $|0\rangle$ and if quantum coin is set as:

$$\begin{pmatrix} \cos\epsilon & -i\sin\epsilon \\ -i\sin\epsilon & \cos\epsilon \end{pmatrix}, \tag{2}$$

Then for $\epsilon \to 0$, it has been shown (rigorously) that we can recover the Dirac Equation:

$$(i\gamma^\mu \partial_\mu + m)\psi = 0 \tag{3}$$

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- In arXiv:quant-ph/0606050, Strauch connects DTQWs to CTQWs.

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- Only left handed (Dirac) neutrinos interact within SM, and we consider these.
- We now look at simulating (using DTQWs) standard neutrino oscillations in matter, using framework of Molfetta et al (arXiv:1607.00529v2).

Neutrino Model

- Introduce flavour states $\tilde{\Psi}_\alpha$ $(\alpha = e, \mu, \tau)$ related to the mass eigenstates by unitary transformation:

$$\tilde{\Psi}_\alpha(t, x) = \sum_i R_{\alpha i}\, \Psi_i(t, x), \tag{4}$$

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- Suppose $|\nu_\alpha\rangle$ produced at $t = 0$, then at t the neutrino state evolves according to

$$|\nu_\alpha\rangle_t = e^{-iHt} \sum_{i=1}^{3} R^*_{\alpha i}\, |\nu_i\rangle = \sum_i |\nu_i\rangle e^{-iE_i t}\, R^*_{\alpha i}. \tag{5}$$

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So at t, the initial neutrino can be detected as any flavour ν_β with R being:

$$R = \begin{pmatrix} \cos\phi_{12} & \sin\phi_{12} \\ -\sin\phi_{12} & \cos\phi_{12} \end{pmatrix} \tag{6}$$

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- The transition probability for $\nu_\alpha \to \nu_\beta$:

$$P(\nu_\alpha \to \nu_\beta; t) = |\sum_{i=1}^{3} R_{\beta i}\, e^{-iE_i\ t}\, R^*_{\alpha i}|^2 \tag{7}$$

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$$\begin{bmatrix} \psi^1_{j+1,p} \\ \dots \\ \psi^n_{j+1,p} \end{bmatrix} = \left(\bigoplus_{h=1,n} SQ^h_\epsilon \right) \begin{bmatrix} \psi^1_{j,p} \\ \dots \\ \psi^n_{j,p} \end{bmatrix} \tag{8}$$

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- S is a spin-dependent translation operator and Q^h_ϵ is the quantum coin:

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- Introducing a unitary transformation R analogous to earlier slide, we get:

$$\tilde{\Psi}_{j+1,p} = R \left(\bigoplus_{h=1,n} S Q^h_\epsilon \right) R^\dagger \tilde{\Psi}_{j,p} \tag{10}$$

- Assuming existence of a continuous limit imposes following constraint on the coin:

$$\lim_{\epsilon\to 0}\left[R\left(\bigoplus_{h=1,n} SQ_{\epsilon}^{h}\right)R^{\dagger}\right] = I_{2n}. \qquad (11)$$

with $Q^i = I_2$ as $\epsilon \to 0$ and $RR^\dagger = I_{2n}$ by definition.

Dirac Equation for Relativistic Flavour Neutrinos

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- With above, the DTQW recovers standard Dirac Equations of the form:

$$\left[\partial_t - \left(\bigoplus_{h=1,n} \sigma_z \right) \partial_x - i\mathcal{M} \right] \tilde{\Psi}(t,x) = 0, \tag{12}$$

describing relativistic n-flavour neutrinos in 1+1D.

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- For 3 flavour neutrinos, the corresponding transition probability can be derived as:

$$P(\nu_\alpha \to \nu_\beta; t) = |\sum_k \tilde{\psi}_k^{\alpha *}(0) \tilde{\psi}_k^{\beta}(t)|^2. \quad (13)$$

Simulation Example: Vacuum Neutrinos

- Consider $\{e, \mu, \tau\}$ generations in vacuum, for which R recovers the PMNS mixing matrix and depends on 3 real parameters:

$$R = e^{i\phi_{\mu\tau}\lambda_7} e^{i\phi_{e\tau}\lambda_5} e^{i\phi_{e\mu}\lambda_2} \tag{14}$$

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- Observe oscillatory behaviour of 3 flavour neutrinos starting from a pure electron-neutrino initial state:

$$\tilde{\psi}_k^{i*}(0) = \frac{1}{\sqrt{n}} \sum_{p=0}^{n-1} e^{-i(k-k_0)x_p} \otimes (1, 0, 0, 0, 0, 0)^\top \tag{15}$$

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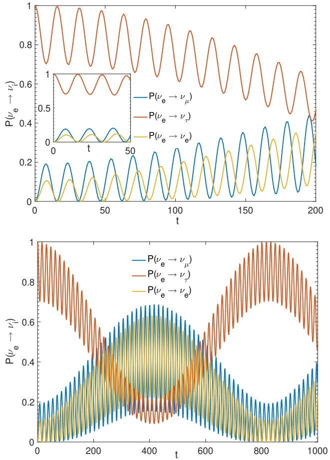
- Time evolution of the probability $P(\nu_\alpha \to \nu_\beta; t)$, $\beta \in \{e, \tau, \mu\}$ of a 3 flavour neutrino oscillation in vacuum, simulated by a QW shown on right for 200 and 1000 time steps. **Values:** ${\Delta m^2}_{e\mu} = 0.003$ rad, ${\Delta m^2}_{\mu\tau} = 0.32$ rad, ${\Delta m^2}_{e\tau} = 0.31$ rad, $\phi_{12} = 0.34$ rad, $\phi_{13} = 0.54$, $\phi_{23} = 0.45$ rad and $k_0 = 100$.



[Molfetta and Pérez 2016]

Simulation Example: Matter Neutrinos

- To accommodate matter effects, DTQW modified to:

$$\Psi_{j+1,p} = V_p R \left(\bigoplus_{h=1,2} SQ_{\epsilon}^{h} \right) R^{\dagger} \Psi_{j,p}, \tag{16}$$

where a position-dependent phase V_p is introduced and:

$$V_p = \mathrm{diag}(e^{i\epsilon\rho_p}, 1) \otimes \mathbb{I}_2 \tag{17}$$

- Consider one mixing angle and a 2-dimensional matrix R:

$$R = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \otimes \mathbb{I}_2 \tag{18}$$

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$$i\partial_t \tilde{\Psi}(t,x) - \mathcal{H}_m \tilde{\Psi}(t,x) = 0, \qquad (19)$$

$$\mathcal{H}_m = i\left(\sigma_z \otimes \mathcal{I}\right)\partial_x - \mathcal{M} + \gamma^5 \mathbb{I}_4 \rho(x) \qquad (20)$$

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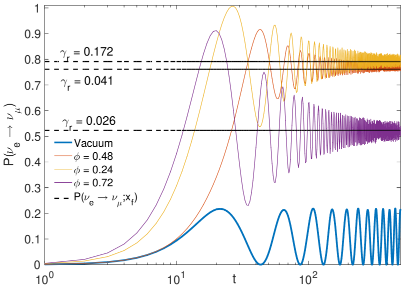
The QW can mimic the time evolution of 2 neutrino flavours in matter with a linear density and for $\gamma_r \ll 1$ in 125 time steps. The dashed line (black) represents the asymptotic crossing probability for different adiabaticity parameters γ_r.



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- QWs can be used to develop stable numerical schemes, even for classical computers.
- QWs extremely useful for simulating simple discrete toy models of physical phenomena that preserves symmetries, so important for tackling foundational questions in Physics.

Conclusion, Parting Thoughts and Future Directions

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- Questions?