Relativistic effects in new force search with isotope shifts

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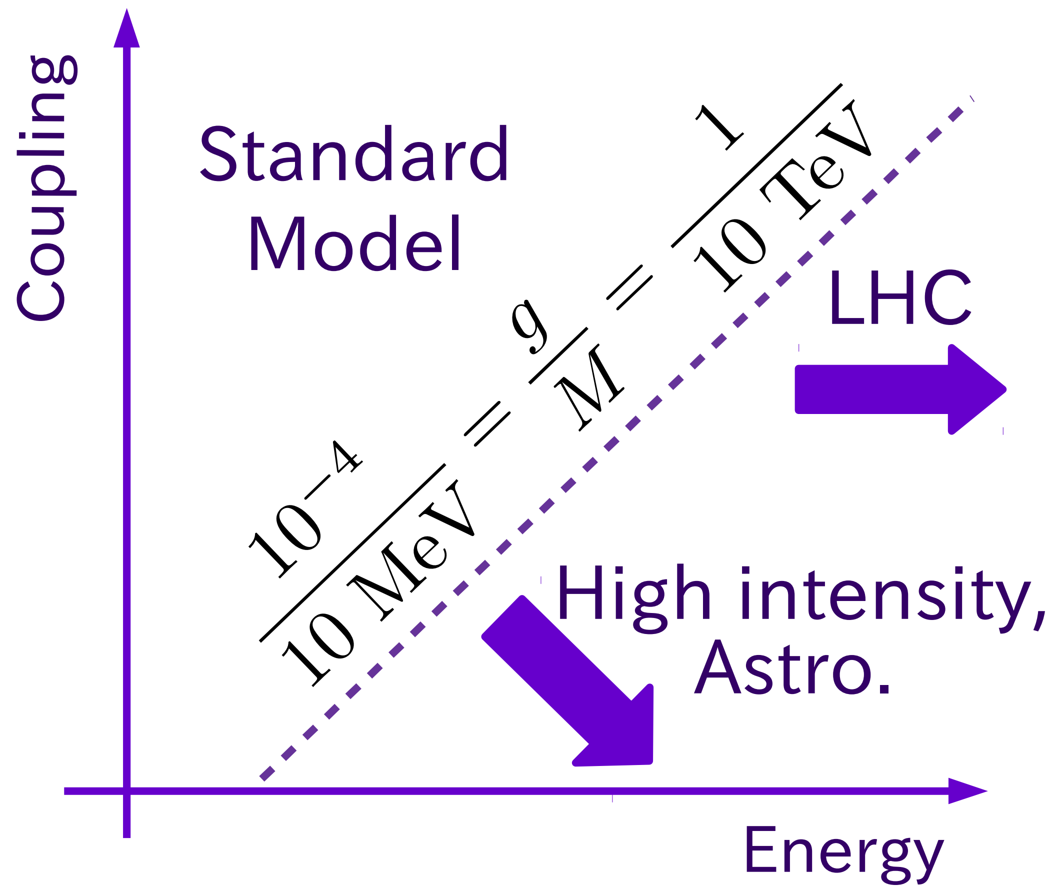
科研費 KAKENHI

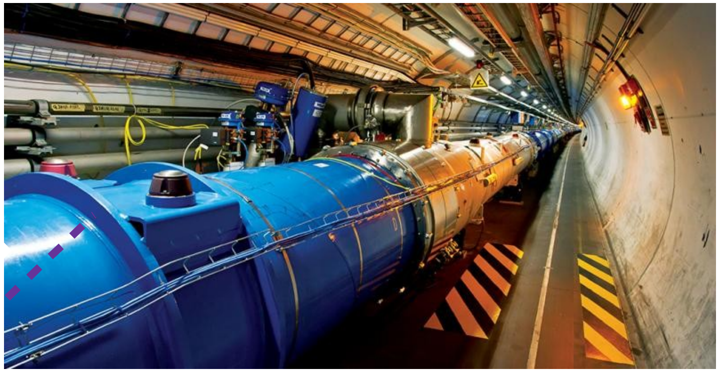
Based on 1710.11443 with K. Mikami & M. Tanaka (Osaka U) and work in preparation with M. Tanaka

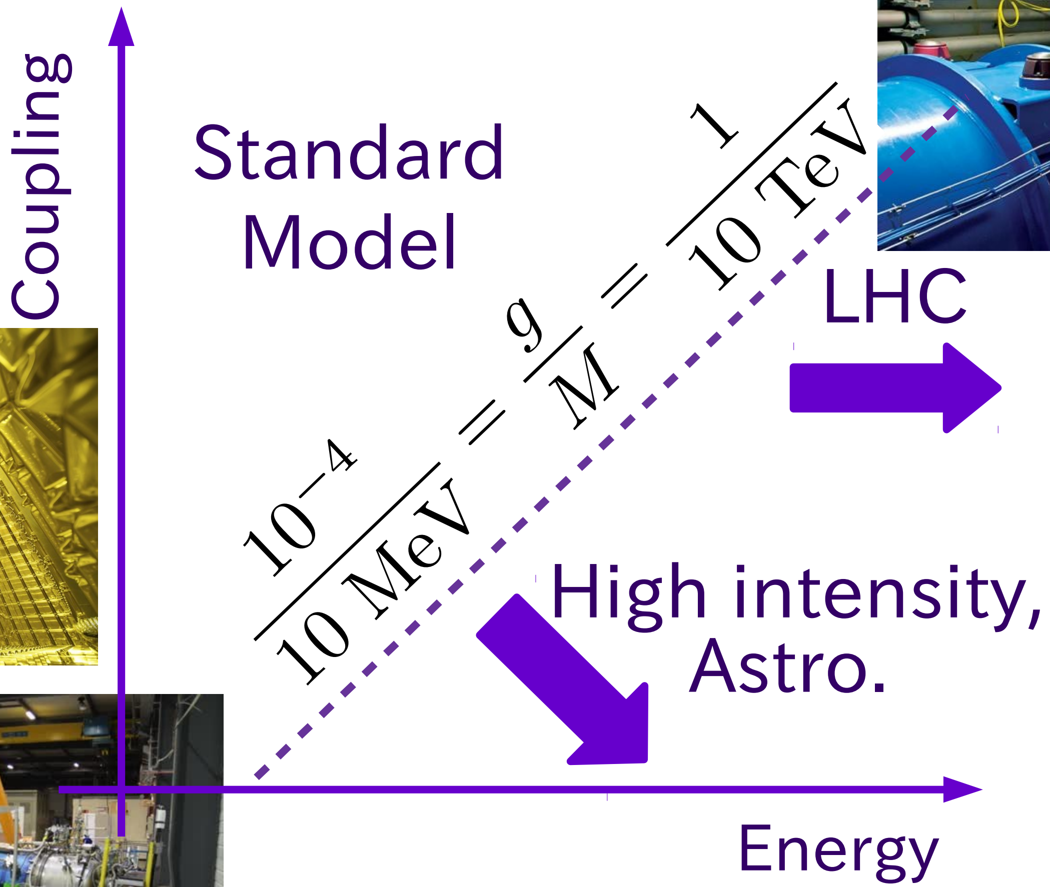
Physics of light new boson

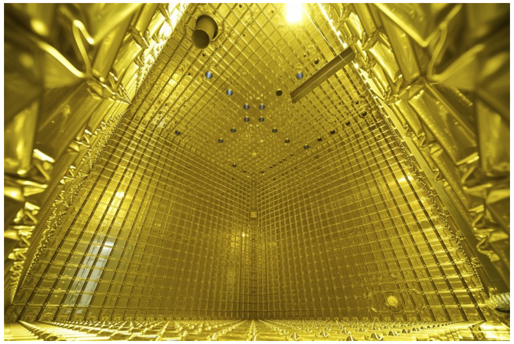


- The light bosons may appear in many observations.

Physics of light new boson







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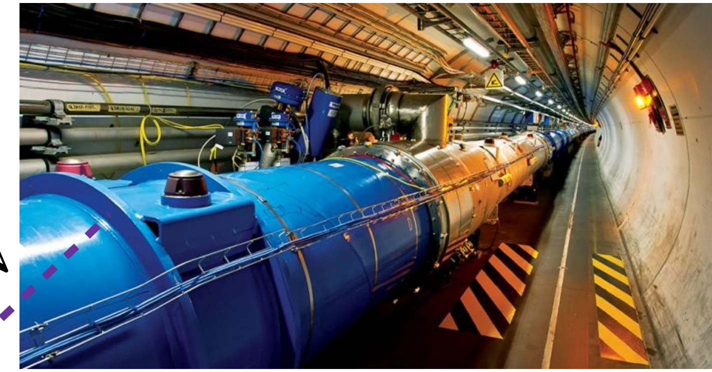
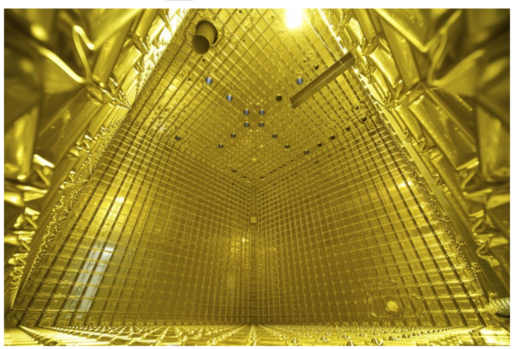
Physics of light new boson





Coupling

Standard Model

$$\frac{10^{-4}}{10\ \mathrm{MeV}} = \frac{g}{M} = \frac{1}{10\ \mathrm{TeV}}$$

LHC

High intensity, Astro.

Energy

- The light bosons may appear in many observations.

Atomic clocks

- Atomic spectroscopy with an extreme precision.
  - ☺ Error of the atomic clocks $O(10^{-15}–10^{-18})$.

$$^{87}\mathrm{Sr}: \ \ 429\ 228\ 004\ 229\ 873.4\ \mathrm{Hz}$$

(From Wikipedia:atomic clock)

c.f.) the electron g-2 is $O(10^{-10})$.

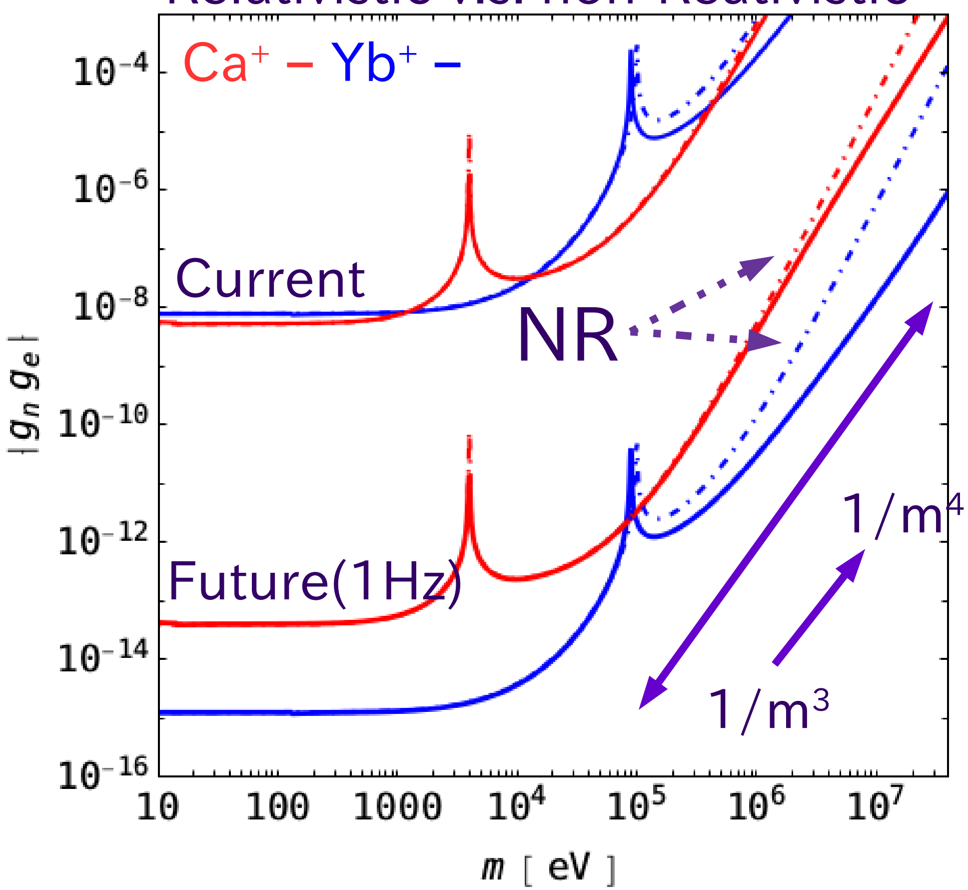
$$\frac{g_e - 2}{2} = \begin{cases} -0.001\,159\,652\,180\,73(28)_{\mathrm{EX}} \\ -0.001\,159\,652\,181\,64(76)_{\mathrm{TH}} \end{cases}$$

  - ☹ The calculation of the spectrum is too difficult.

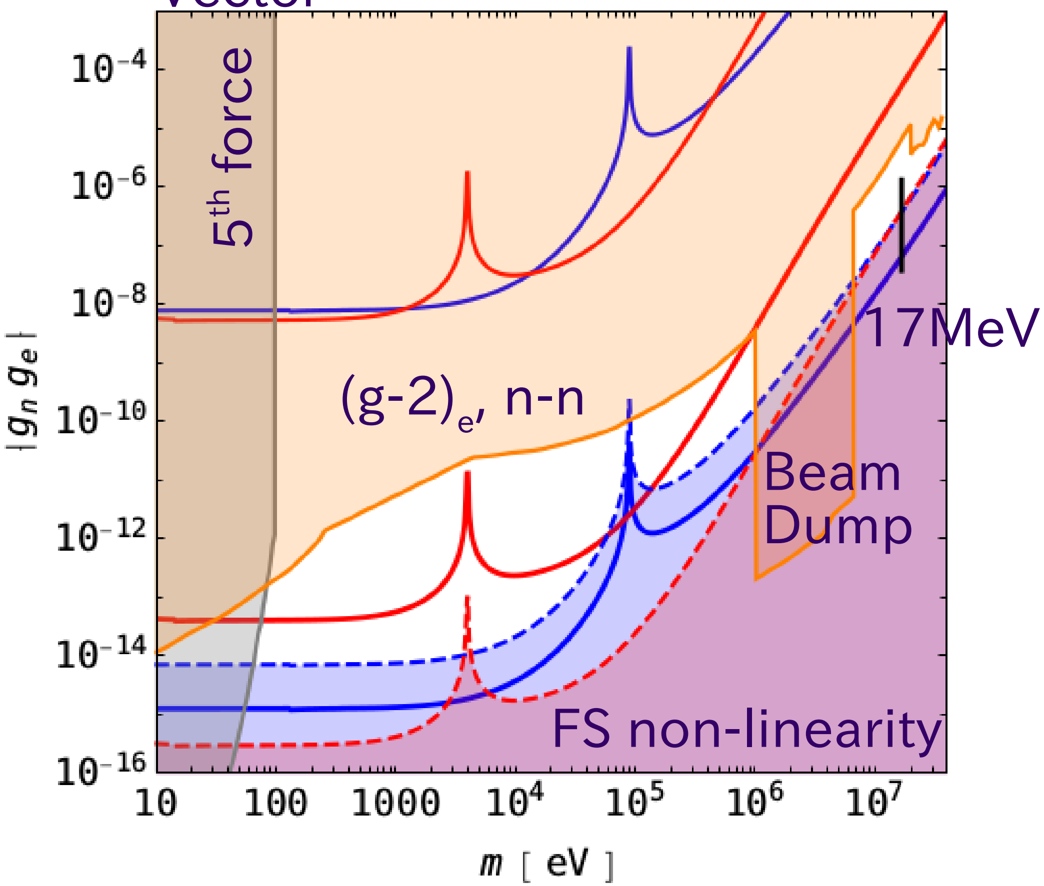
- Reduce the uncertainty with a linear relation.
- The new constraints on **the light new boson**.

Preliminary results

Relativistic v.s. non-Reativistic



Vector



Plan

- Introduction
- The linearity and its violation
  - The field shift and its higher order (SM)
  - The particle shift (New Physics)

  ↓

  The relativistic effects

  ↓

- The constraint of a light new boson
- Conclusion

Isotope shift and the linearity

- Isotope shifts follow a linearity.

$$\delta H_{A'A} = \delta K_{A'A} + \delta V_{A'A}$$

$$\delta \nu = G\,\delta\mu + F\,\delta\langle r^2\rangle$$

Isotope dependence.

Wave function dependence.

- Linearity for isotope pairs.

1963: W. H. King

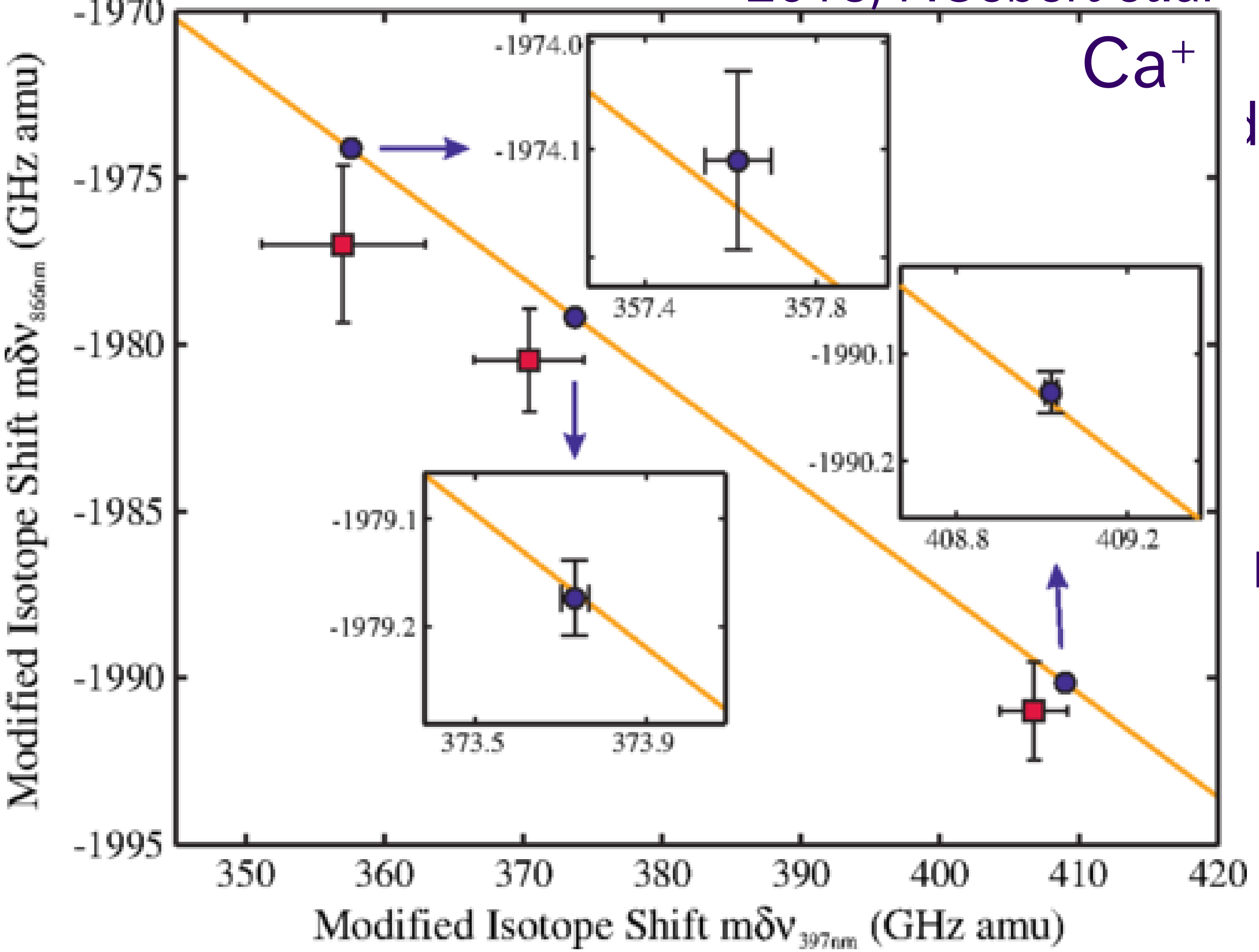
$$\frac{\delta\nu_2}{\delta\mu} = \frac{F_2}{F_1}\frac{\delta\nu_1}{\delta\mu} + \left(G_2 - \frac{F_2}{F_1}G_1\right)$$

Constant for isotope pairs.

Isotope shift and the linearity

- Isotope shifts follow a linearity.

2015, F.Gebert et.al



ıence.

H. King

Isotope shift and the linearity

non

does not

- Isotope shifts follow a linearity.

$$\delta H_{A'A} = \delta K_{A'A} + \delta V_{A'A}$$

Isotope dependence.

$$\delta\nu = G\,\delta\mu + F\,\delta\langle r^2\rangle + X$$

NLO corrections
Yukawa potential

Wave function dependence.

Non

- Linearity for isotope pairs.

2016, C. Delaunay et. al

$$\frac{\delta\nu_2}{\delta\mu} = \frac{F_2}{F_1}\frac{\delta\nu_1}{\delta\mu} + G_2 - \frac{F_2}{F_1}G_1 + \left(X_2 - \frac{F_2}{F_1}X_1\right)/\delta\mu$$

Constant for isotope pairs.

Field shift

Def: $\int d\vec{r}\,\left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right)\delta V(\vec{r})$

$$-Z\alpha\int d\vec{r'}\,\frac{\delta\rho(\vec{r'})}{|\vec{r}-\vec{r'}|}$$

Expand

$$\propto \int_0^\infty dr' \int_0^{r'} dr\, r^2 \sum_k \xi_k r^k \left(r' - \frac{r'^2}{r}\right)\delta\rho(r')$$

$$\delta\langle r^k\rangle = \int d\vec{r}\, r^k \delta\rho(r)$$

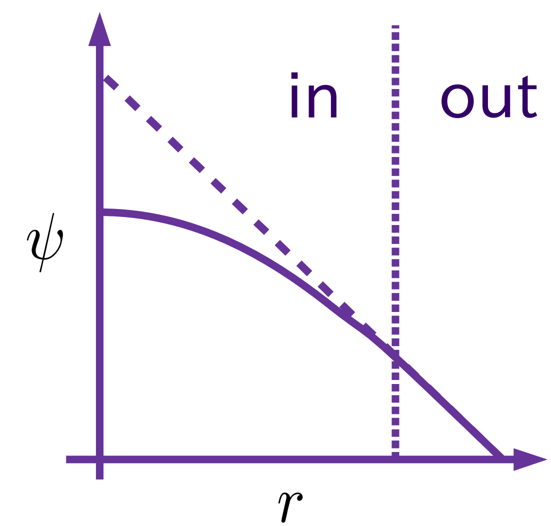
1969, E. C. Seltzer

$$= Z\alpha \sum_k \frac{\xi_k}{(k+3)(k+2)}\delta\langle r^{k+2}\rangle$$

- NLO field shift

$$\delta\nu = G\delta\mu + F\delta\langle r^2\rangle + \tilde{F}\delta\langle r^4\rangle + \cdots$$

$$|\psi|^2 \sim \xi_0 + \xi_2 r^2 + \cdots$$



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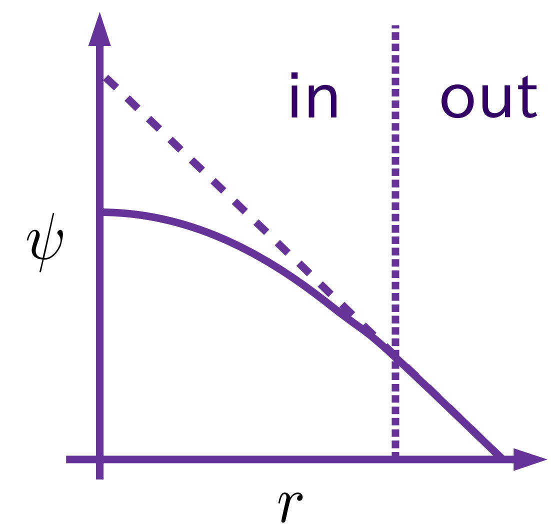

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Field shift

Def: $\int d\vec{r}\, \left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right) \delta V(\vec{r})$ ← $-Z\alpha \int d\vec{r'}\, \frac{\delta\rho(\vec{r'})}{|\vec{r}-\vec{r'}|}$

Expand

$$\propto \int_0^\infty dr' \int_0^{r'} dr\, r^2 \sum_k \xi_k r^k \left(r' - \frac{r'^2}{r}\right) \delta\rho(r')$$

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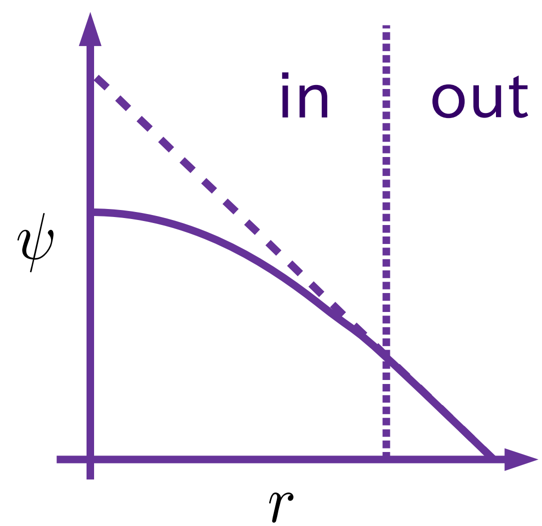


▶ NLO field shift

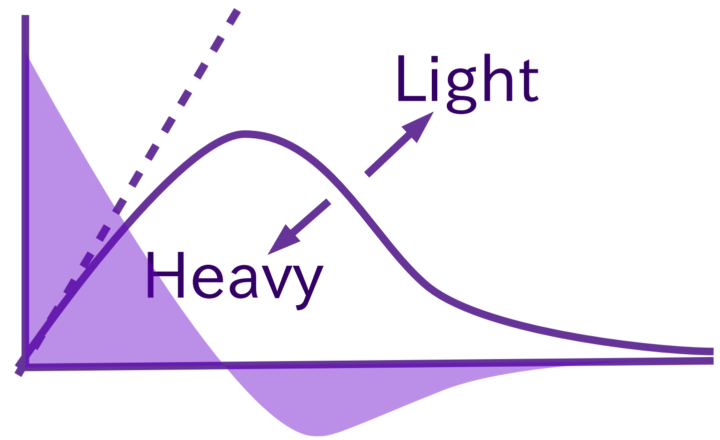
$$\delta\nu = G\delta\mu + F\delta\langle r^2\rangle + \tilde{F}\delta\langle r^4\rangle + \cdots$$

$$|\psi|^2 \sim \xi_0 + \xi_2 r^2 + \cdots$$

Particle shift

Def: $\int d\vec{r}\, \left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right) (A' - A) \frac{g_n g_e}{4\pi} \frac{e^{-mr}}{r}$

- Sensitive to the **e-n coupling**



- For a heavy mediator

$$= (A' - A) \frac{g_n g_e}{4\pi} \sum_k \frac{k!}{m^{k+2}} \xi_k$$

- $\delta\nu = G\delta\mu + F\left(\delta\langle r^2\rangle + c_0/m^2\right)$ ← Keep the linearity

$$+\tilde{F}\left(\delta\langle r^4\rangle + c_2/m^4\right) + \cdots$$

↑ Non-linearity

Relativistic effects

Atomic scale ← Mediator → Nuclear scale

Flat 1/m⁴

$$|\psi|^2 \sim r^{2k}(\xi_0 + \xi_1 r + \xi_2 r^2 + \cdots)$$

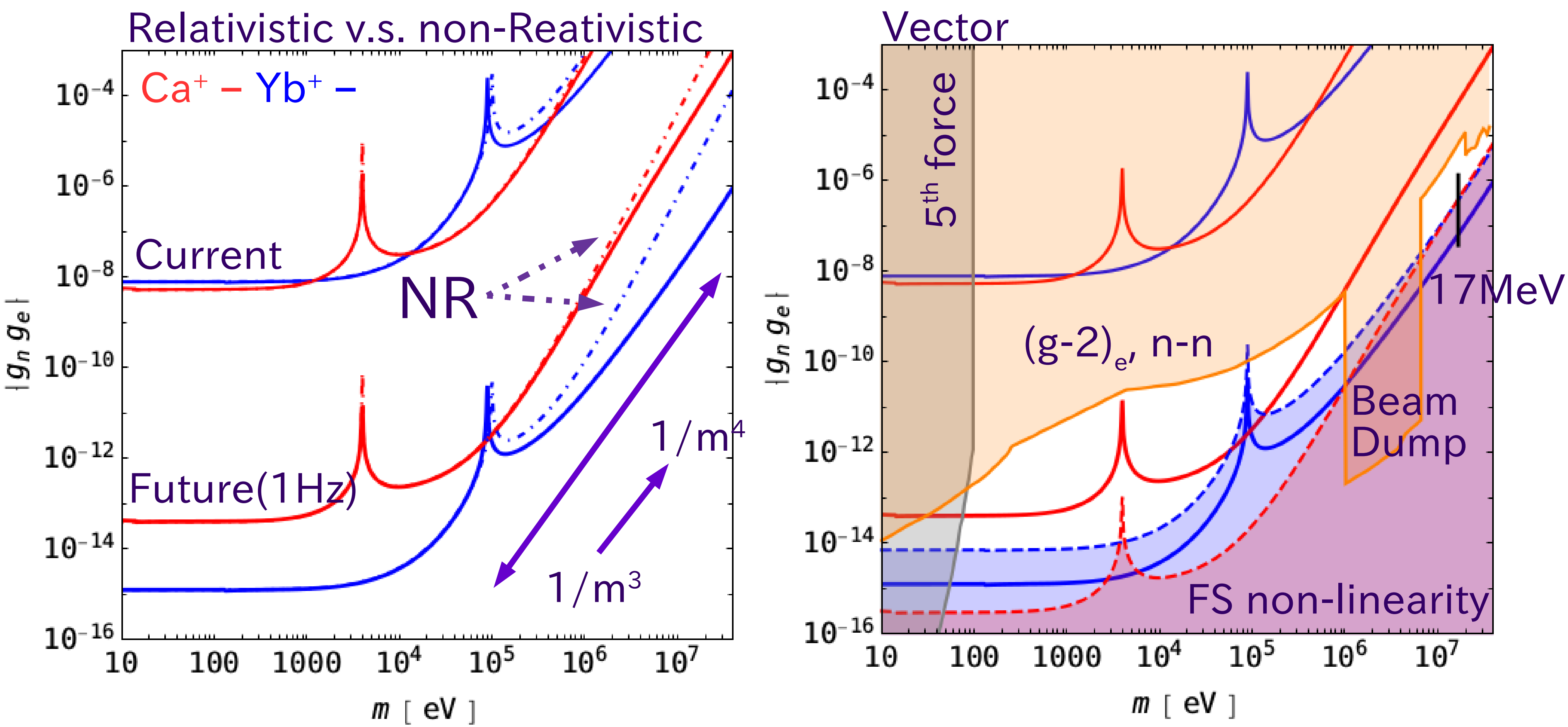
$$k = \begin{cases} l & \text{: non-relativistic} \\ j - 1/2 & \text{: relativistic} \end{cases}$$

$F = |\psi(0)|^2 = \xi_0$

$$\frac{\delta\nu_2}{\delta\mu} = \frac{F_2}{F_1}\frac{\delta\nu_1}{\delta\mu} + G_2 - \frac{F_2}{F_1}G_1 + \left(X_2 - \frac{F_2}{F_1}X_1\right)/\delta\mu$$

- S → P & F → S : $F_1 = F_2$ ➡ $1/m^4$
- $S_{1/2}$ → $P_{1/2}$ & $F_{5/2}$ → $S_{1/2}$: $F_1 \neq F_2 (\neq 0)$ ➡ $1/m^3$

Sensitivity and constraints



- Sensitivity to heavier mediator is improved for Yb^{+}.
- Higher order field shift limits the future sensitivity.

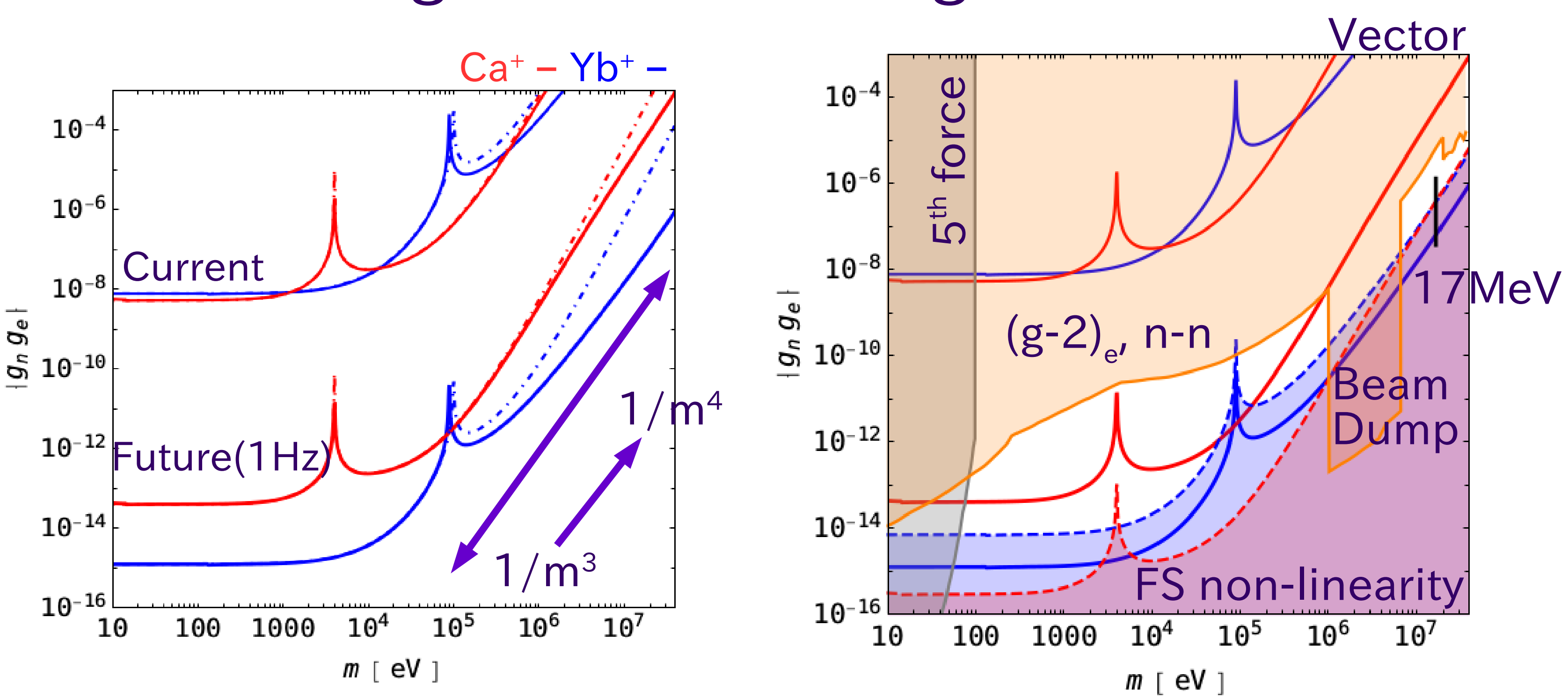
Conclusion

Precision spectroscopy + Linearity of isotopes

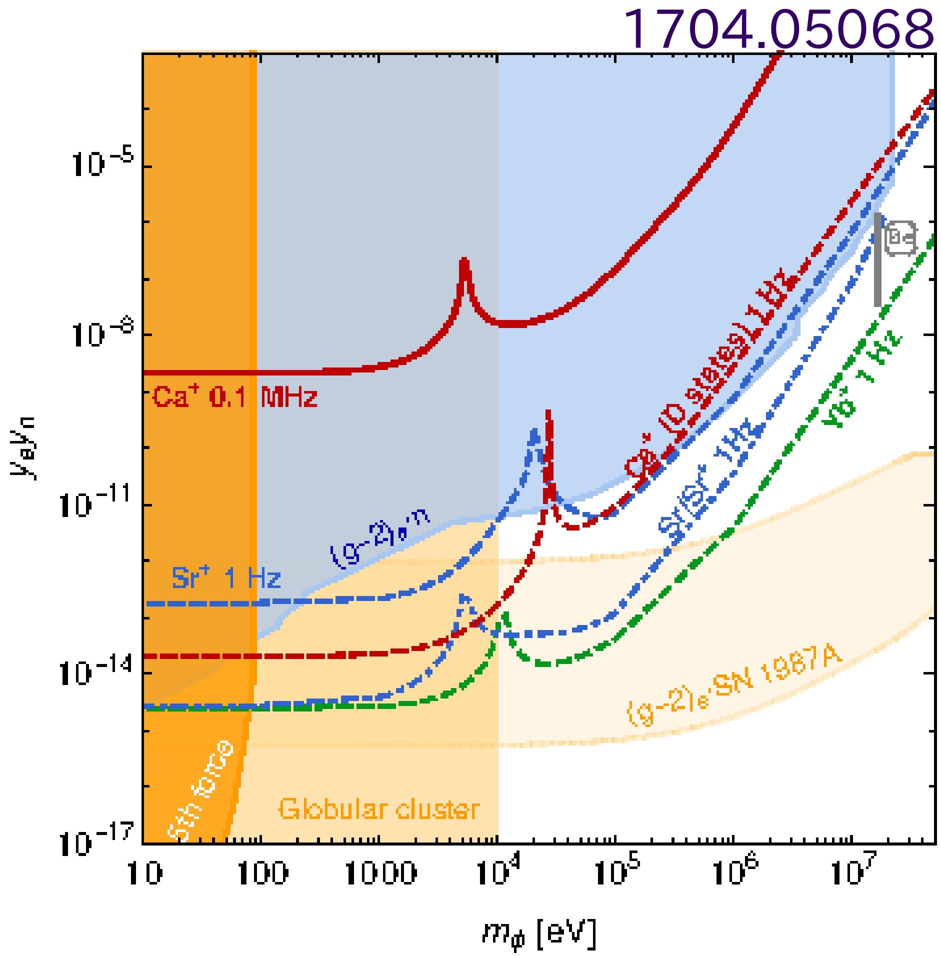
▼

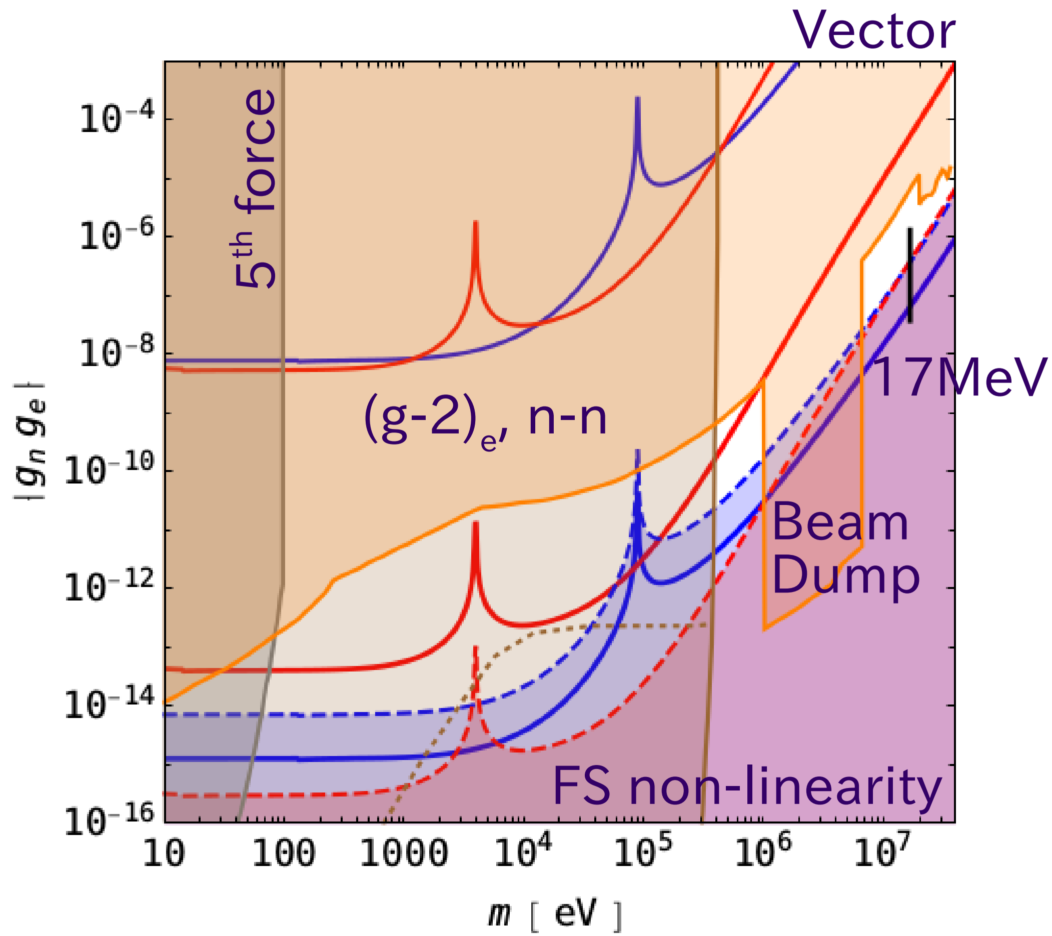
New physics as the non-linearity

- The scaling law at the heavy region.
- The SM background of the higher order field shift.



Some comments





- The stellar cooling has large uncertainty.
- Our result is smooth because of the analytic study.