Axion superradiance in rotating neutron stars

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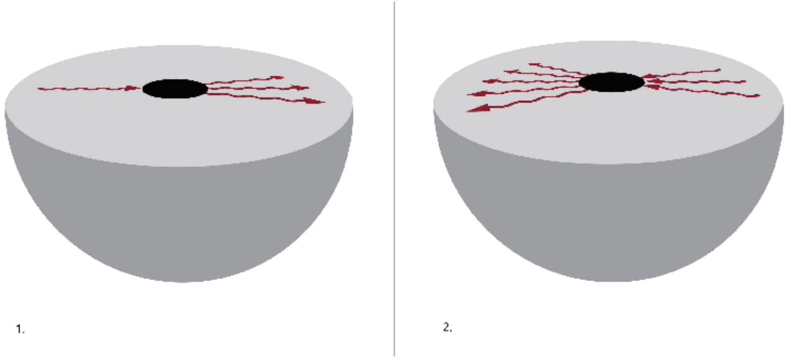
Outline

1. Superradiance
2. Axion-photon superradiance
3. Axion superradiance in neutron stars

Black Hole Superradiance



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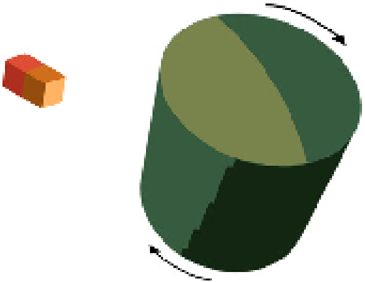
Carrousel Analogy



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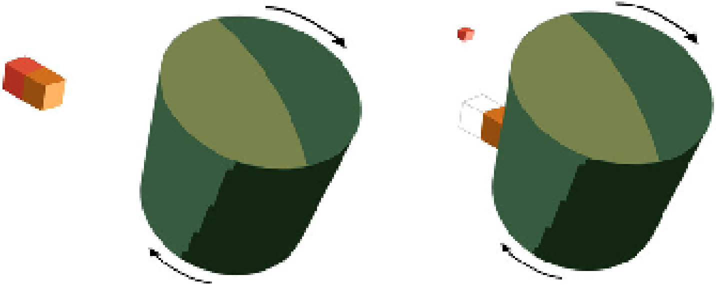

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Axion Black Hole Superradiance

- Axions build up around Kerr black hole from an initial quantum fluctuation.
- 'Bosenova' explosion as axion cloud collapses (Arvanitaki & Dubovsky, 1004.3558)
- Depletion of black hole spin
- Stellar mass BH spin measurements exclude $6 \times 10^{-13}\,\mathrm{eV} < m_a < 2 \times 10^{-11}\,\mathrm{eV}$ for $f_a \gtrsim 10^{13} GeV$. (Arvanitaki, Baryakhtar & Huang, 1411.2263)
- Advanced Ligo will be sensitive to $m_a \lesssim 10^{-10}\,\mathrm{eV}$. (Arvanitaki *et al*, 1604.03958).
- Constraints on the string axiverse mass spectrum (Stott & Marsh, 1805.02016)

Rotational Superradiance in Stars

- No horizion - superradiance in stars relies on non-gravitational dissipative dynamics, which become amplifying due to the star's rotation (Zel'dovich, 1971).

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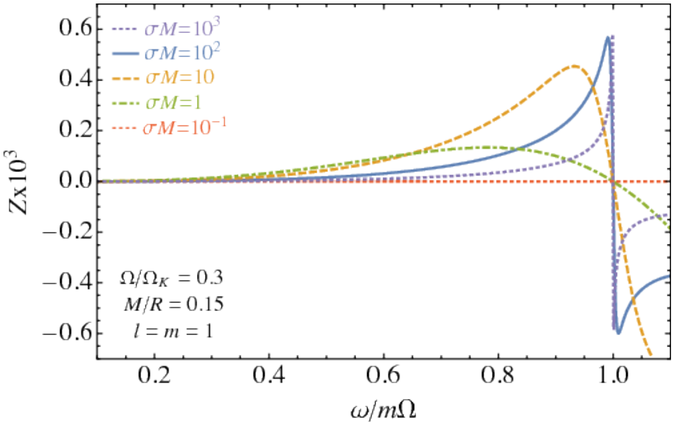
- The effective damping parameter $\alpha\omega'$ becomes negative.
- Superradiance corresponds to a change in sign of the imaginary part of the eigenfrequency Im$[\omega]$.

Superradiance

For superradiance we need:

- Rotation
- Bound states - i.e. massive particle for gravitational bound state
- Dissipation

Example: Dark photons in neutron stars



$Z := \frac{|A_{\text{out}}|^2}{|A_{\text{in}}|^2} - 1.$

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Can the massive particle and the dissipation be in different sectors that talk to eachother?

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- *Photons* dissipate energy into the neutron star magnetosphere via the magnetosphere's bulk conductivity.
- Axions and photons mix, so these effects together lead to a superradiant instability in the neutron star magnetosphere.

The axion-photon system

$$\mathcal{L} \supset \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{a\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} - A_\mu j^\mu\right],$$

$$j^\mu = \sigma F^{\mu\nu}u_\nu + \rho u^\mu,$$

giving the linearised equations of motion:

$$\Box\phi + \mu^2\phi = -\frac{g_{a\gamma\gamma}}{2}F_{\mu\nu}\tilde{F}_B^{\mu\nu},$$

$$\partial_\mu F^{\mu\nu} - \sigma F^{\nu\mu}u_\mu = -g_{a\gamma\gamma}\left(\partial_\mu\phi\right)\tilde{F}_B^{\mu\nu}.$$

The axion-photon system

In Lorenz gauge:

$$\begin{aligned}
\Box\phi + \mu^2\phi &= -g_{a\gamma\gamma}\left[\nabla A^0 + \dot{\mathbf{A}}\right]\cdot\mathbf{B} \\
\Box A^0 &= -g_{a\gamma\gamma}\nabla\phi\cdot\mathbf{B} - \sigma\mathbf{u}\cdot\left[\nabla A^0 + \dot{\mathbf{A}}\right] \\
\Box\mathbf{A} &= g_{a\gamma}\dot{\phi}\mathbf{B} - \sigma\left[\nabla A^0 + \dot{\mathbf{A}}\right] + \sigma\mathbf{u}\times[\nabla\times\mathbf{A}]
\end{aligned}$$

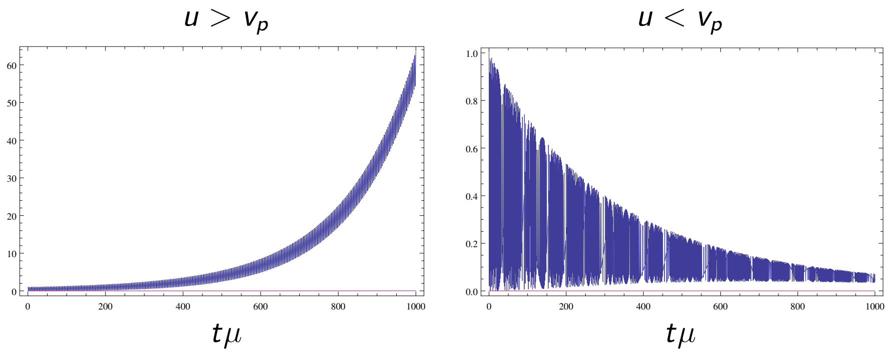
For a neutron star magnetosphere, $\mathbf{B}\sim 10^{10}\,\mathrm{G}$, $0.01\Omega \lesssim \sigma \lesssim 100\Omega$ (Li *et al*, 1107.0979).

Warm up: 1D Example

- Consider a 1D fluid flow with $\mathbf{A} \cdot \mathbf{u} = 0, \quad \mathbf{B} \cdot \mathbf{k} = 0$.
- Introduce a sound speed: $\Box\phi \to \ddot{\phi} - c_s^2 \nabla^2 \phi$.
- The unperturbed axion sound speed is $v_p(k)^2 = (c_s^2 + \mu^2/k^2)$.
- We find:

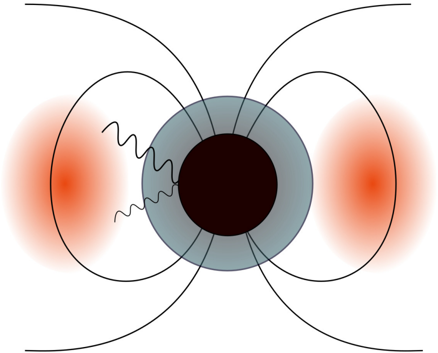
$$\ddot{\phi} + k^2 c_s^2 \phi + \mu^2 \phi = -g_{a\gamma\gamma} \dot{A}_{\parallel} B$$
$$\ddot{A}_{\parallel} + k^2 A_{\parallel} + \sigma u^0 \dot{A}_{\parallel} + i\sigma(\mathbf{k} \cdot \mathbf{u}) A_{\parallel} = g_{a\gamma\gamma} \dot{\phi} B$$

Warm up: 1D Example



The axion (blue) and photon component (red) (arbitrary units) for the mode $|\mathbf{k}| = 3\mu$. The left- and right-hand plots correspond to sound speeds $c_s = 0.01$ and $c_s = 0.7$, respectively. The other parameter values are $Bg_{a\gamma\gamma}/\mu = 0.3$, $u = 0.5$ and $\sigma/\mu = 3$.

Axion superradiance in neutron stars



Schematic illustration of the instability. The axion boundstate (orange) mixes with a photon mode which is then amplified by scattering off the rotating magnetosphere (grey). The photon energy is then deposited back into the axion sector.

3D Case

$$
\begin{aligned}
\Box\phi + \mu^2\phi &= -g_{a\gamma\gamma}\left[\nabla A^0 + \dot{\mathbf{A}}\right]\cdot\mathbf{B} \\
\Box A^0 &= -g_{a\gamma\gamma}\nabla\phi\cdot\mathbf{B} - \sigma\mathbf{u}\cdot\left[\nabla A^0 + \dot{\mathbf{A}}\right] \\
\Box\mathbf{A} &= g_{a\gamma}\dot{\phi}\mathbf{B} - \sigma\left[\nabla A^0 + \dot{\mathbf{A}}\right] + \sigma\mathbf{u}\times[\nabla\times\mathbf{A}]
\end{aligned}
$$

- 3D geometry
- Bound states
- Rotation of star removes need for sound speed

Perturbation theory

We re-write our equations of motion as a time-independent Schrodinger equation:

$$[H(\sigma_*) + V(\sigma_M, g_{a\gamma\gamma})] \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix} = \omega^2 \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix}$$

$$H = \begin{pmatrix} -\frac{d^2}{dr_*^2} + U(r), & 0 & 0 \\ 0 & -\nabla^2 & 0 \\ 0 & 0 & -\nabla^2 \end{pmatrix}$$

Perturbation theory

$$V = V_A + V_{a\gamma\gamma}$$

$$V_{a\gamma\gamma} = ig_{a\gamma\gamma} \begin{pmatrix} 0 & \mathbf{B}(\hat{x}) \cdot \hat{p} & -\omega \mathbf{B}(\hat{x}) \\ \mathbf{B}(\hat{x}) \cdot \hat{p} & 0 & 0 \\ \omega \mathbf{B}(\hat{x}) & 0 & 0 \end{pmatrix}$$

$$V_A = i\sigma_M(\hat{x}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{u}(\hat{x}) \cdot \hat{p} & -\omega \mathbf{u}(\hat{x}) \\ 0 & \hat{p} & -\omega - \mathbf{u}(\hat{x}) \times \hat{p} \end{pmatrix}$$

Perturbation theory

Use quantum mechanical perturbation theory to find the eigen-frequencies of the axion-photon system as a double expansion in σ and $g_{a\gamma\gamma}$:

$$\omega = \omega_{\ell n} + \omega^{(1)} + \omega^{(2)} + \omega^{(3)} + \cdots$$

Axion superradiance in neutron stars

$$\text{Im}\left[\omega_{\ell mn}\right] \simeq \pi g_{a\gamma\gamma}^2 B^2 \sigma_{\mathrm{M}} \left((m\Omega - \omega) - \omega S[\ell, \ell+1, m]^2 \right) \frac{(R_{\mathrm{LC}}\omega)^{2\ell+3} - (R\omega)^{2\ell+3}}{32\omega^3} \alpha^{2\ell+5} \mathcal{F}_{\ell n}$$

$$V_A = i\sigma_M(\hat{x}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{u}(\hat{x}) \cdot \hat{p} & -\omega \mathbf{u}(\hat{x}) \\ 0 & \hat{p} & -\omega - \mathbf{u}(\hat{x}) \times \hat{p} \end{pmatrix}$$

Axion superradiance in neutron stars

- We typically find superradiant timescales $\tau = \frac{1}{\mathrm{Im}[\omega_{\ell mn}]}$ a few orders of magnitude higher than the neutron star spin down time.
- Therefore, we do not expect this process to be observable.
- Our result is an example of a more general phenomenon which can arise when there is an instability in the plasma sector.
- For axion modes which couple to an unstable mode of the neutron star, one could in principle find similar instabilities.

Perturbation theory

$$\omega = \omega_{\ell n} + \omega^{(1)} + \omega^{(2)} + \omega^{(3)} + \cdots$$

$$\omega^{(1)} = \frac{1}{2\omega_{\ell n}} \langle \phi_{\ell mn} | V | \phi_{\ell mn} \rangle = 0$$

$$\omega^{(2)} = \frac{1}{2\omega_{\ell n}} \sum_{\ell', m', i} \int d\left[\omega^2\right] \frac{\left\langle \phi_{\ell mn} \left| V_{a\gamma\gamma}\left(\omega_{\ell n}\right) \right| A^{(i)}_{\ell' m'}(\omega) \right\rangle \left\langle A^{(i)}_{\ell' m'}(\omega) \left| V_{a\gamma\gamma}\left(\omega_{\ell n}\right) \right| \phi_{\ell mn} \right\rangle}{\omega_{\ell n}^2 - \omega^2} \in \mathbb{R}$$

Perturbation theory

$$\delta\omega^{(3)}_{\ell mn} = \frac{1}{2\omega_{\ell n}}$$
$$\sum_{\ell_{1,2},m_{1,2},i,j} \int d\left[\omega^2_{1,2}\right] \frac{\left\langle \phi_{\ell mn}|V|A^{(i)}_{\ell_1 m_1}(\omega_1)\right\rangle\left\langle A^{(i)}_{\ell_1 m_1}(\omega_1)|V|A^{(j)}_{\ell_2 m_2}(\omega_2)\right\rangle\left\langle A^{(j)}_{\ell_2 m_2}|V|\phi_{\ell mn}\right\rangle}{\left(\omega^2_{\ell n}-\omega^2_1\right)\left(\omega^2_{\ell n}-\omega^2_2\right)}$$

$$\delta\omega^{(3)}_{\ell mn} = \frac{\pi^2}{8\omega_{\ell n}} \sum_{\ell_{1,2},m_{1,2}} \sum_{i,j}$$
$$\left\langle \phi_{\ell mn}\left|V_{a\gamma\gamma}\right| A^{(i)}_{\ell_1 m_1}\left(\omega_{\ell n}\right)\right\rangle \left\langle A^{(i)}_{\ell_1 m_1}\left|V_A\right| A^{(j)}_{\ell_2 m_2}\left(\omega_{\ell_2}\right)\right\rangle \left\langle A^{(j)}_{\ell_2 m_2}\left|V_{a\gamma\gamma}\right| \phi_{\ell mn}\right\rangle$$

Perturbation Theory

$\phi_{\ell mn} \sim$ Laguerre polynomial $\times$ spherical harmonic
$A_{\ell m}(\omega) \sim$ Bessel function$(r\omega)\times$ spherical harmonic

$$\text{Im}\left[\omega_{\ell mn}\right] \simeq$$
$$\pi g_{a\gamma\gamma}^2 B^2 \sigma_{\rm M} \left((m\Omega - \omega) - \omega S[\ell, \ell+1, m]^2\right) \frac{(R_{\rm LC}\omega)^{2\ell+3} - (R\omega)^{2\ell+3}}{32\omega^3} \alpha^{2\ell+5} \mathcal{F}_{\ell n}$$

Magnetosphere Conductivity

- Drude model $\sigma_{\rm Drude}(\omega) = \frac{n_e e^2 \tau_{\rm coll}}{m_e(1 - i\tau_{\rm coll}\omega)}$ leads to very high conductivities.
- The bulk conductivity of the magnetosphere, as experienced by fields with stellar wavelengths, captures the inability of plasma to completely screen $\mathbf{E} \cdot \mathbf{B}$ (Li *et al*, 1107.0979) and parametrises the departure from the ideal magnetohydrodynamics or "force-free" condition $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$.
- $0.01\Omega \lesssim \sigma \lesssim 100\Omega$ implies approximate balance between Coriolis and Lorentz forces.

Plasma Mass

- Plasma mass from $\mathrm{Im}(\sigma_{\mathrm{Drude}})$.
- Study of 1D case suggests that the inclusion of a plasma mass decreases the superradiant growth rate but does not affect the threshold for superradiance.
- To leading order in $\mathrm{Im}(\sigma_{\mathrm{Drude}})$, the plasma mass does not affect $\mathrm{Im}(\omega)$.

Plasma Mass

- The axial photon mode is described by the equation (Cardoso *et al*, 1704.06151):

$$\left[-\frac{d^2}{dr^2}+\frac{\ell(\ell+1)}{r^2}-i\sigma(\omega-m\Omega)\right](rA_{\ell m})=\omega^2(rA_{\ell m}). \qquad (1)$$

- In the superradiant regime $\omega < m\Omega$, the the plasma mass will become tachyonic.