Effects of an early matter dominated era on gravitational waves induced by scalar perturbations

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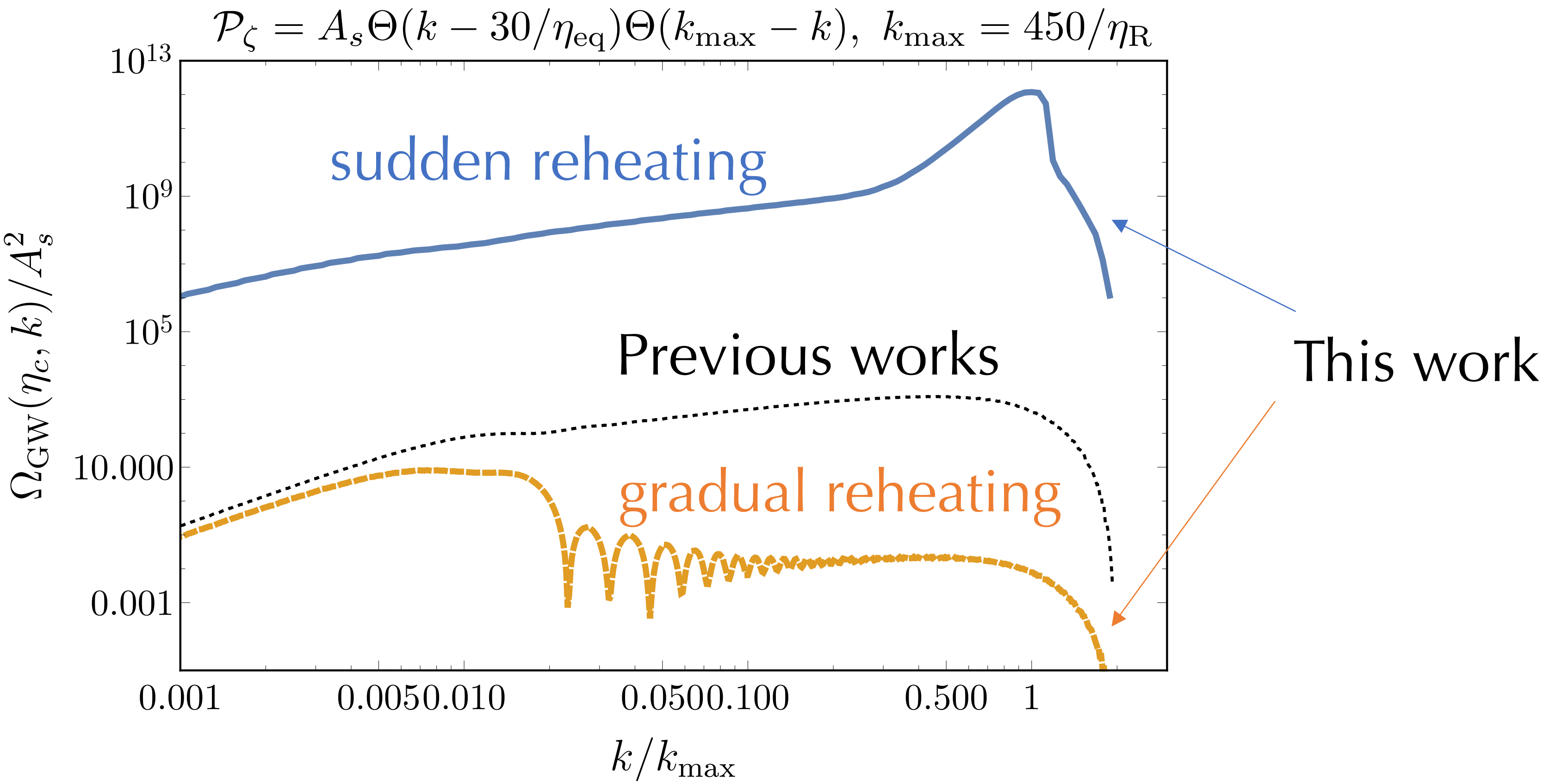
Overview



We revisit the effect of an early matter dominated era on the gravitational waves induced by scalar perturbations.

As a result, we find that the induced GWs strongly depend on how quickly the reheating occurs.

Outline

- Introduction
- eMD effects in gradual reheating scenario
- eMD effects in sudden reheating scenario
- Summary

GWs induced by scalar perturbations

At linear order, scalar perturbations do not induce GWs.

However,

at second order, scalar perturbations can induce GWs.

(Ananda et al 2006, Baumann et al. 2007)

Metric perturbations:

Scalar perturbations (related to curvature perturbations)

$$\mathrm{d}s^2 = a^2\left[-(1+2\Phi)\mathrm{d}\eta^2 + \left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x_i\mathrm{d}x_j\right]$$

Tensor perturbations (GWs)

E.o.M for GWs:

$$h_k^{\lambda\,\prime\prime} + 2\mathcal{H}h_k^{\lambda\,\prime} + k^2 h_k^\lambda = 4\mathcal{S}^\lambda(k,\eta)$$

Source term from second order scalar perturbations

$$\mathcal{S}^\lambda(k,\eta) = \int \frac{\mathrm{d}^3k'}{(2\pi)^3}\mathrm{e}^{\lambda lm}(\hat{k})k'_l k'_m\left[2\Phi_{k'}(\eta)\Phi_{k-k'}(\eta) + \frac{4}{3(1+w)}\left(\Phi_{k'}(\eta)+\frac{\Phi'_{k'}(\eta)}{\mathcal{H}}\right)\left(\Phi_{k-k'}(\eta)+\frac{\Phi'_{k-k'}(\eta)}{\mathcal{H}}\right)\right]$$

Motivations for the induced GWs

The frequencies of the induced GWs correspond to the scales of the scalar perturbations inducing GWs.

Since the wide range of GW frequencies could be observed in the future, we could investigate small-scale (scalar) perturbations using the induced GWs!
$(\lambda \lesssim 1\,\mathrm{Mpc})$

Small-scale perturbations enter the horizon early, the induced GW could be a good probe of the early Universe.

↓

Main question of this work

What effects does the early matter dominated era make on the induced GWs?

The history of the Universe

Reheating

Inflation era

We focus on this!

(Early matter dominated (eMD) era)

Radiation dominated (RD) era

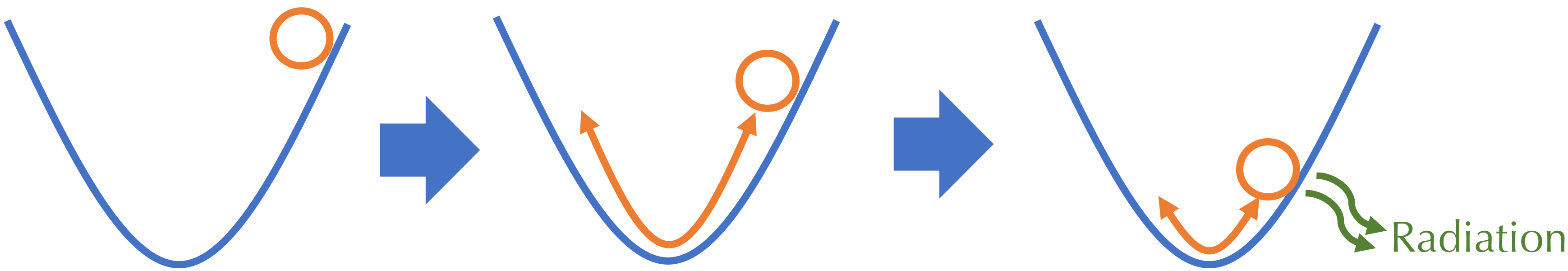
(Late) matter dominated era (from z~3400)

Dark energy dominated era (from z~0.5)

What causes eMD era ?

One scenario: oscillation of inflaton

Chaotic inflation



Inflation occurs the vacuum energy of the inflaton.

During the oscillation of an inflaton, the Universe behaves as MD era.

The oscillation amplitude decays due to the expansion and the decay to radiation.

During the oscillation

Time average

$$\langle \rho \rangle = \left\langle \frac{1}{2}\dot{\phi}^2 + \frac{m^2}{2}\phi^2 \right\rangle = m^2 \langle \phi^2 \rangle$$

$$\langle P \rangle = \left\langle \frac{1}{2}\dot{\phi}^2 - \frac{m^2}{2}\phi^2 \right\rangle = 0$$

MD era !
$(w = P/\rho = 0)$

RD era begins.

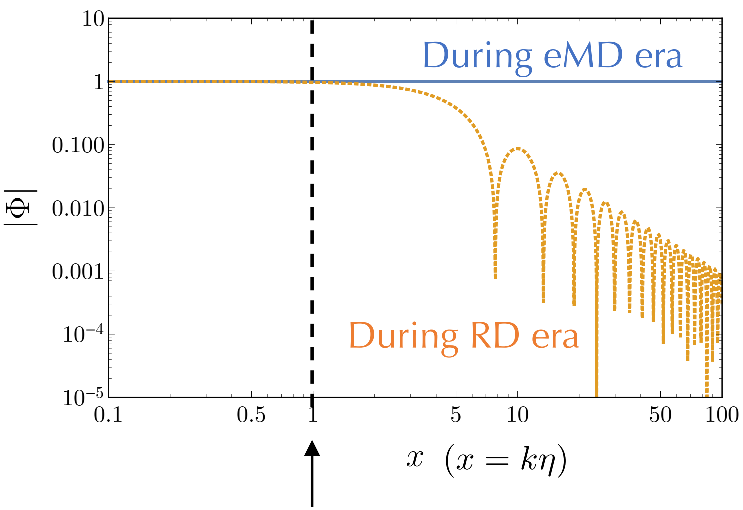
(Other scenario: oscillation of curvaton)

Results in previous works

There are works discussing the effects of an eMD era on the induced GWs.

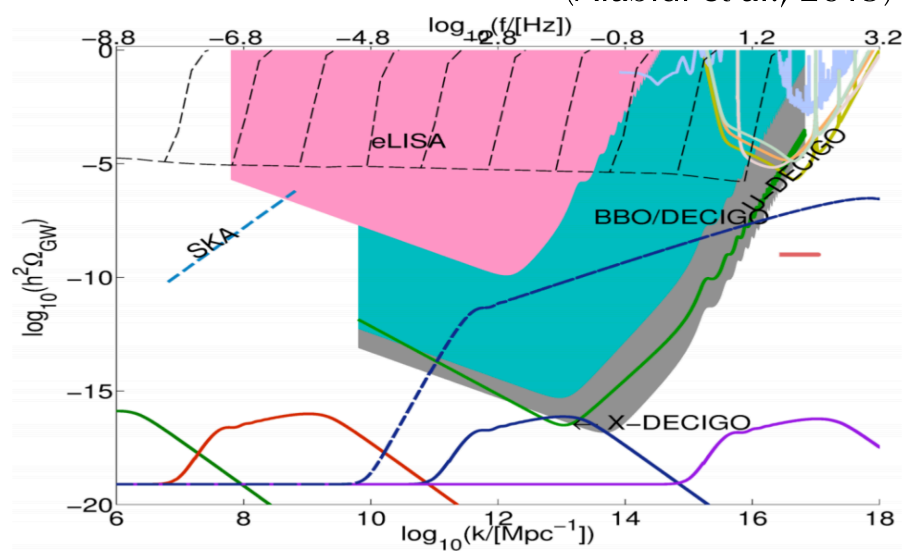
(Assadullahi, Wands, 2009, Alabidi, Kohri, Sasaki, Sendaouda, 2013, Kohri, Terada, 2018)

During an eMD era, the transfer function of the gravitational potential (Φ), the source of GWs, does not decay even on subhorizon because of the growing matter perturbations.



Horizon entry

(Alabidi et al., 2013)



Induced GWs are enhanced?

Implicit assumptions in previous works

In previous works, the authors implicitly assume the reheating occurs suddenly and stop calculation at the reheating.

(Assadullahi, Wands, 2009, Alabidi, Kohri, Sasaki, Sendaouda, 2013)

However,

1. The transition can occur gradually in some realistic situations.
2. Even if the transition occurs suddenly, we need to care about the GWs induced after the transition.

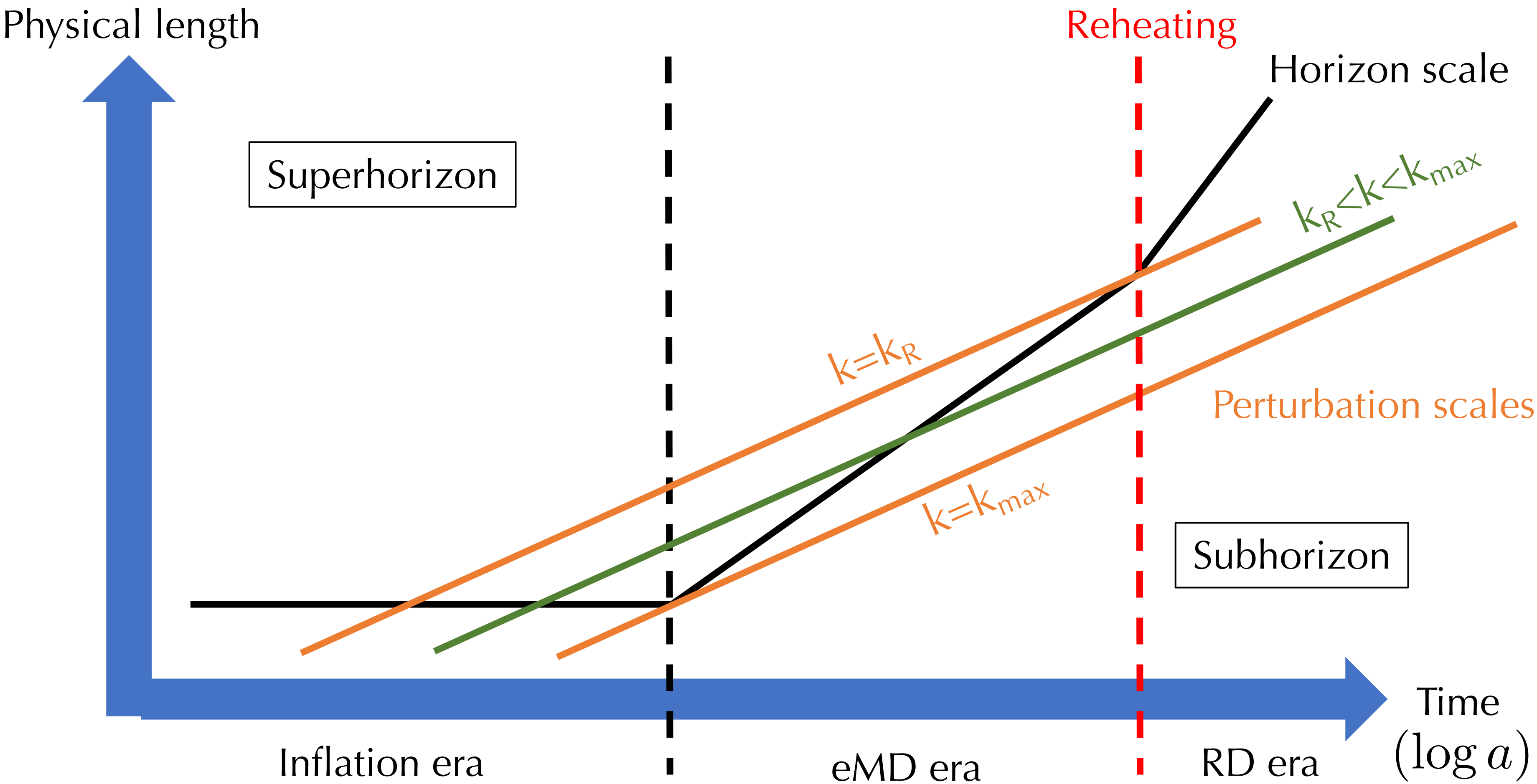
What we do in this work

We revisit the effects of an eMD era on the induced GWs.

We consider two cases:

1. Gradual reheating, whose time scale is comparable to the Hubble time scale at that time.
2. Sudden reheating, whose time scale is much shorter than the Hubble time scale at that time.

Evolutions of perturbations



To spotlight the effects of an eMD era,
we focus on the perturbations entering horizon during eMD era ($k_R < k < k_{max}$).

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Gradual reheating

We consider the case where the field dominating the Universe during an eMD era (inflaton or curvaton) decays to radiation with a constant decay rate Γ.

$$(a^3 \rho_m \propto \mathrm{e}^{-\Gamma t})$$

In this gradual reheating scenario, the situation is similar to the decaying dark matter scenario.

E.o.M in decaying DM scenario (Poulin, Serpico, Lesgourgues, 2018)

$$\rho_m' = -(3\mathcal{H} + a\Gamma)\rho_m,$$

$$\rho_r' = -4\mathcal{H}\rho_r + a\Gamma\rho_m$$

ρ_m : Energy density of matter

ρ_r : Energy density of radiation

δ : Energy density perturbation

$\theta \ (\equiv ik_j v^j)$: Velocity divergence

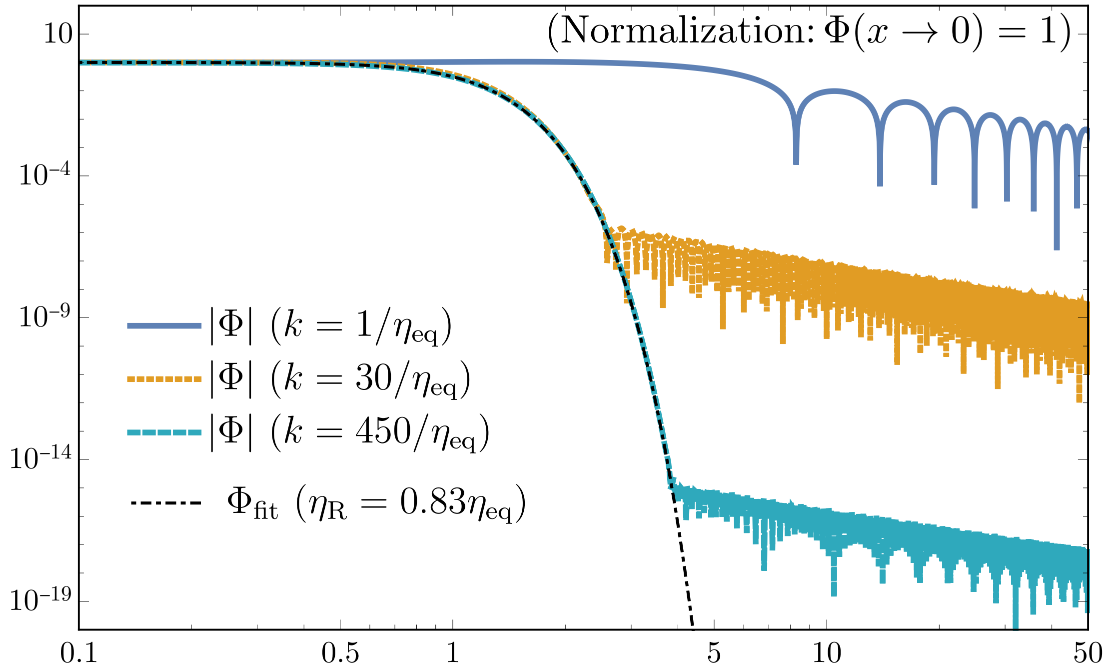
$$\delta_m' = -\theta_m + 3\Phi' - a\Gamma\Phi,$$

$$\theta_m' = -\mathcal{H}\theta_m + k^2\Phi,$$

$$\delta_r' = -\frac{4}{3}(\theta_r - 3\Phi') + a\Gamma\frac{\rho_m}{\rho_r}(\delta_m - \delta_r + \Phi),$$

$$\theta_r' = \frac{k^2}{4}\delta_r + k^2\Phi - a\Gamma\frac{3\rho_m}{4\rho_r}\left(\frac{4}{3}\theta_r - \theta_m\right)$$

Perturbations in gradual reheating



On subhorizon scale,

$$k^2\Phi \simeq \frac{3}{2}\mathcal{H}^2\left(\frac{\rho_m}{\rho_{\rm tot}}\delta_m + \frac{\rho_r}{\rho_{\rm tot}}\delta_r\right)$$

During eMD era, δ_m ($\propto a$) grows on subhorizon unlike δ_r .

Even after $\eta_{\rm eq}$, Φ is mainly determined by $\rho_m\delta_m$ for a while.

We naively expect

$$\Phi \propto \rho_m \propto \exp\left(\int^\eta d\bar{\eta}\, a\Gamma\right)$$

If we take $\eta_{\rm R} = 0.83\eta_{\rm eq}$, the fitting formula fits the numerical result during the exponential decay very well!

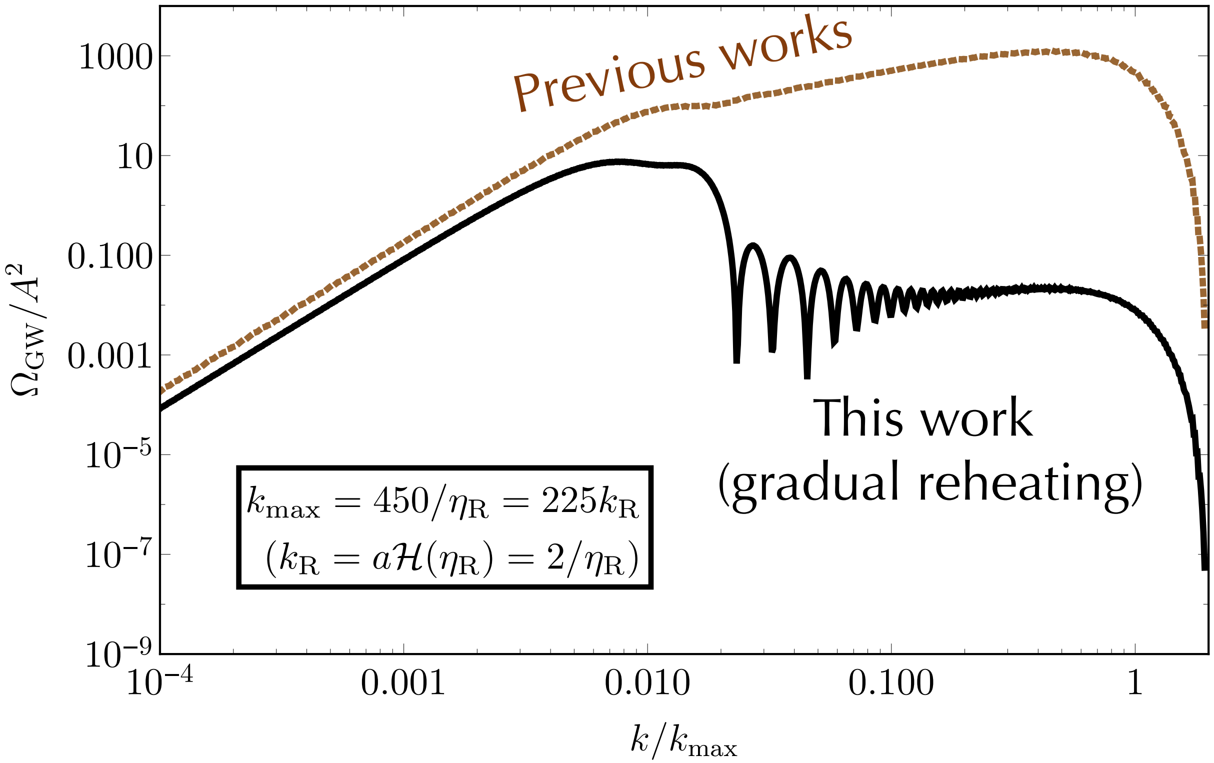
Fitting formula

$$\Phi_{\rm fit} \equiv \exp\left(\int^\eta d\bar{\eta} a(\bar{\eta})\Gamma\right) \qquad (a(\eta_{\rm R})\Gamma = \mathcal{H}(\eta_{\rm R}) = 2/\eta_{\rm R})$$

$$= \begin{cases} \exp\left(-\frac{2}{3}\left(\frac{\eta}{\eta_{\rm R}}\right)^3\right) & (\eta < \eta_{\rm R}) \\ \exp\left(-2\left(\left(\frac{\eta}{\eta_{\rm R}}\right)^2 - \frac{\eta}{\eta_{\rm R}} + \frac{1}{3}\right)\right) & (\eta \geq \eta_{\rm R}) \end{cases}$$

Results

GWs induced by curvature perturbations with $\mathcal{P}_\zeta = A\,\Theta(k - 30/\eta_{\rm eq})\Theta(k_{\rm max} - k)$



The induced GWs are suppressed compared to the previous results.

Main reason

A damping of tensor perturbation occurs during the gradual transition.

Why does the damping occur?

E.o.M for tensor perturbations:

$$h_{\boldsymbol{k}}^{\lambda\,''} + 2\mathcal{H}h_{\boldsymbol{k}}^{\lambda\,'} + k^2 h_{\boldsymbol{k}}^{\lambda} = 4\mathcal{S}_{\boldsymbol{k}}^{\lambda}$$

In $\eta \ll \eta_R$, the source is almost constant and the tensor perturbations on subhorizon scale become

$$h_{\boldsymbol{k}}^{\lambda} \simeq \frac{4\mathcal{S}_{\boldsymbol{k}}^{\lambda}}{k^2}$$

In $\eta_R \lesssim \eta \lesssim 2\eta_R$, the source decays gradually and the tensor perturbations follows the decay for a while. During this phase, the damping occurs.

$$h_{\boldsymbol{k}}^{\lambda} \simeq \frac{4\mathcal{S}_{\boldsymbol{k}}^{\lambda}}{k^2}$$ (The source and h are damping.)

Note: This damping was not taken into account in previous works.

In $2\eta_R \lesssim \eta$, the source decays rapidly and the time derivative of the tensor perturbations dominates the E.o.M. Then, tensor perturbations decouple from the source.

$$h_{\boldsymbol{k}}^{\lambda} \simeq A_{\rm dec}\frac{a_{\rm dec}}{a}e^{ik\eta}$$ (Freely propagating GWs)

Amplitude at the decoupling

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Toy model for sudden reheating

Here, we consider the sudden reheating scenario, where the transition time scale is much shorter than the Hubble time scale at that time.

Toy model:

$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\tau^2 + \frac{\lambda}{4}\tau^2\chi^2 + \frac{c}{2}M\phi\chi^2$$

ϕ : Dominant field (such as inflaton or curvaton)

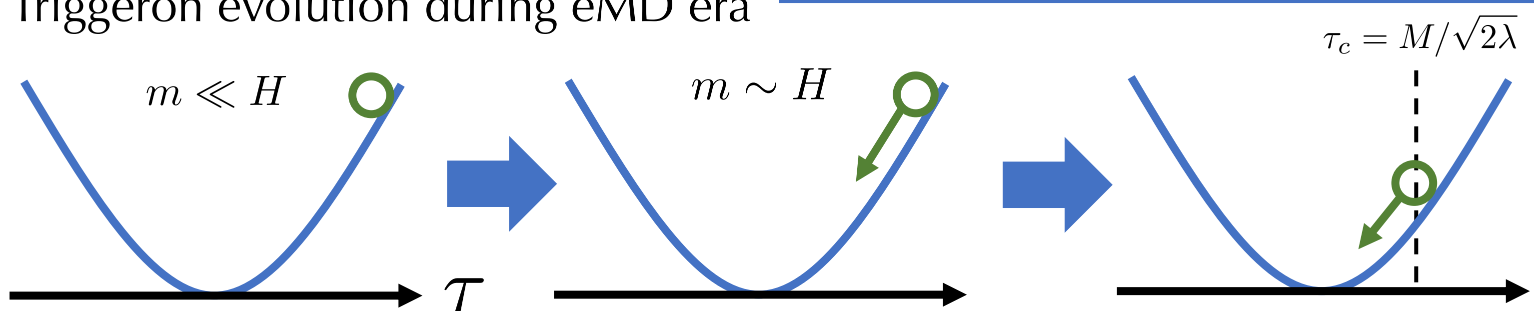
τ : Trigger field ("triggeron")

χ : Field denoting the daughter particles of ϕ

χ has τ-dependent mass. $(m^2_{\chi,\mathrm{eff}} = \langle \lambda\tau^2/2 \rangle)$

We take M≫m and initial value of τ as τ_0 ≫M.

Triggeron evolution during eMD era

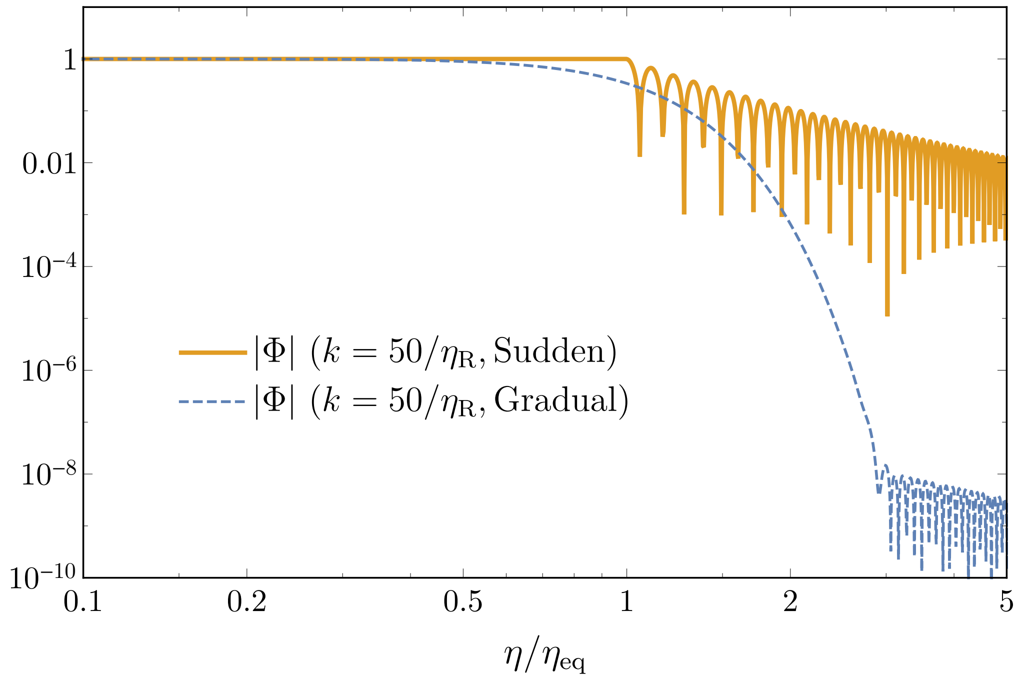


At first, the decay of ϕ to χ is prohibited (M<2$m_{\chi,\,\mathrm{eff}}$).

τ starts to roll at m~H.

When τ passes by τ_c, the decay channel of ϕ opens and reheating occurs instantaneously.

Evolutions in sudden reheating



There is no damping in Φ during the transition in the sudden reheating scenario.
After the transition, perturbations that enters the horizon during eMD era oscillates with the time scale shorter than the Hubble time at that time ($k\eta \gg 1$).

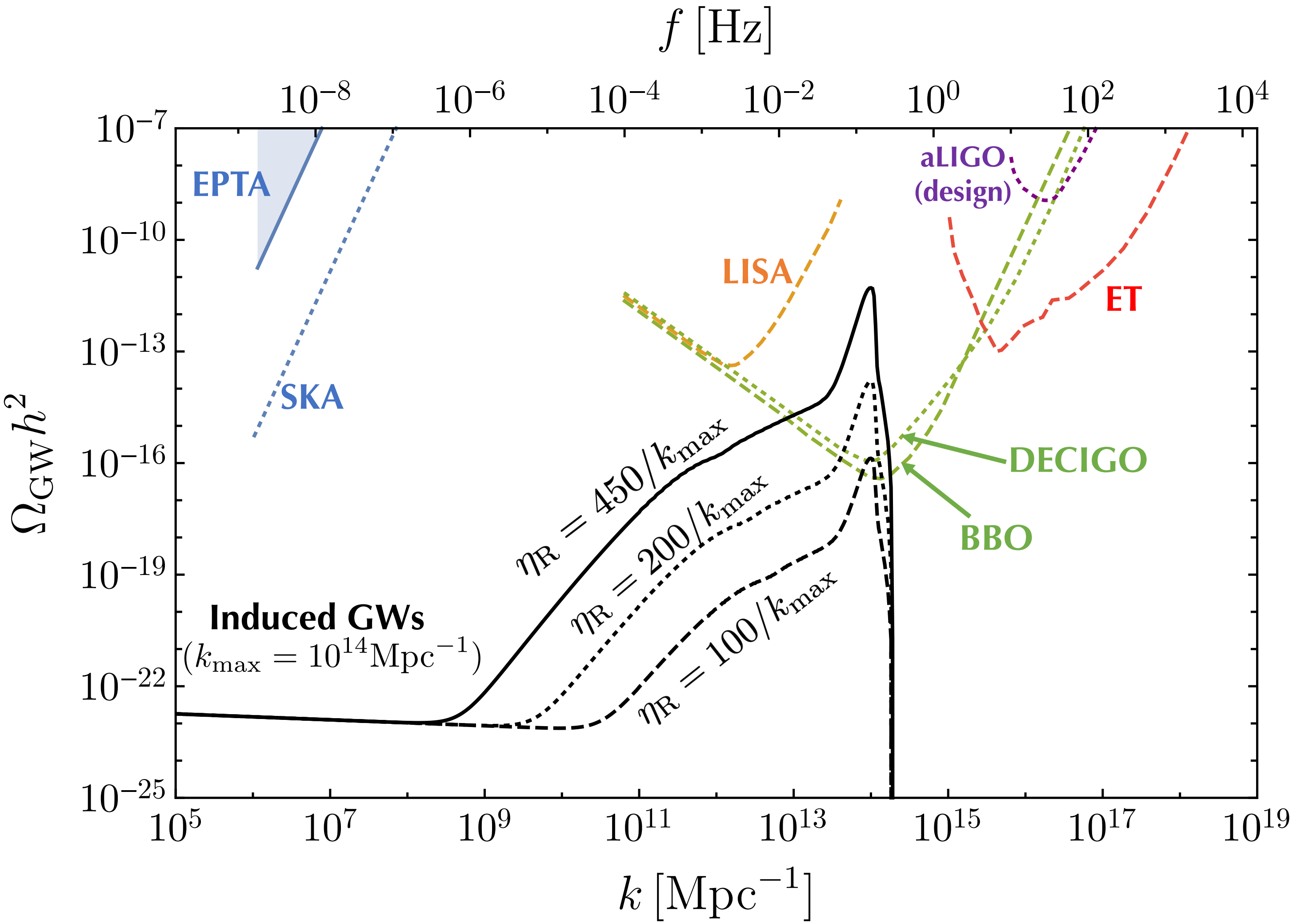
Enhance GWs !

Source term

Dominant $(\Phi'/\mathcal{H}|_{\eta=\eta_{\rm R}} \sim k\eta_{\rm R}\Phi)$

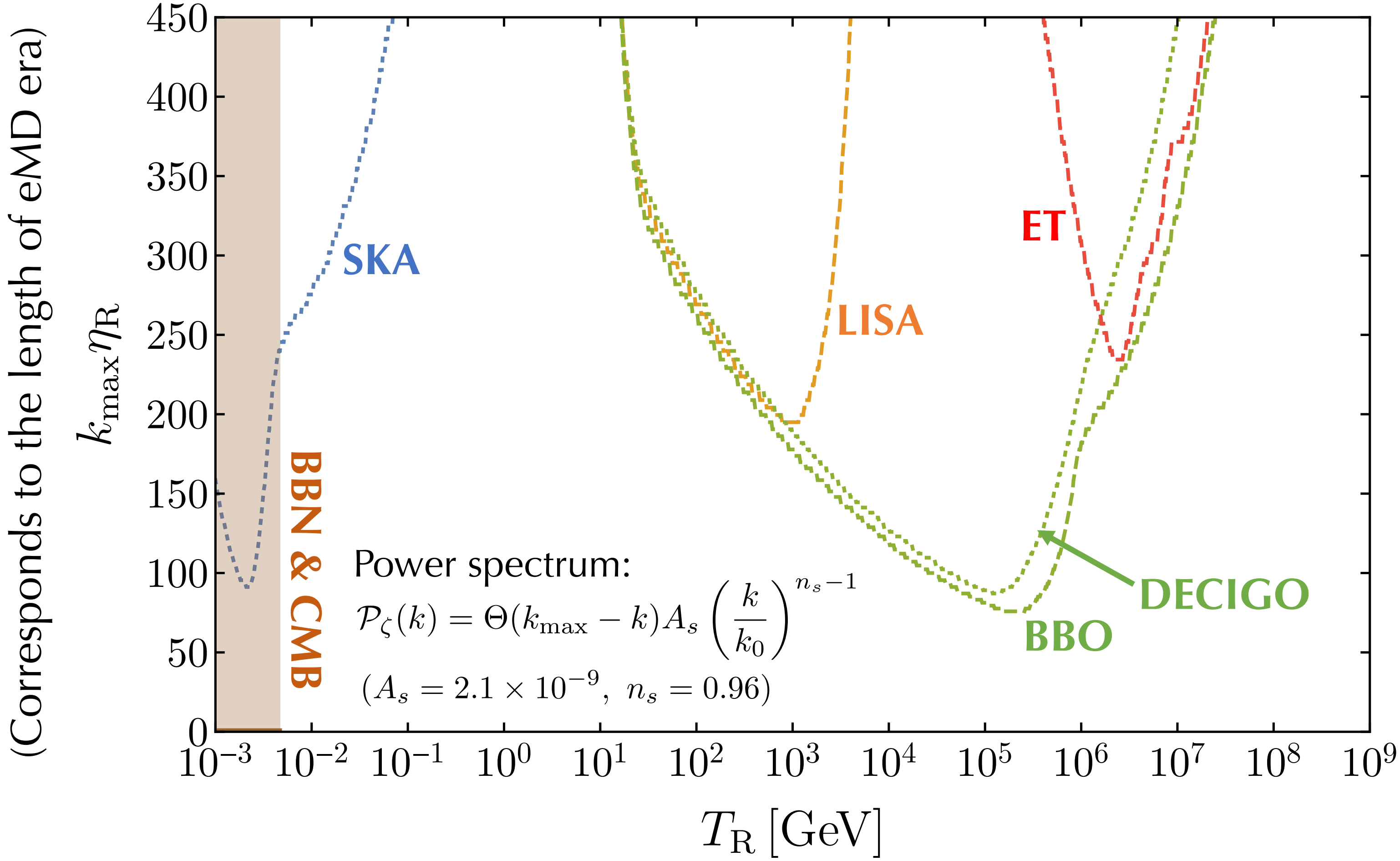
$$\mathcal{S}^{\lambda}(k,\eta) = \int \frac{\mathrm{d}^3k'}{(2\pi)^3} \mathrm{e}^{\lambda lm}(\hat{k}) k'_l k'_m \left[2\Phi_{k'}(\eta)\Phi_{k-k'}(\eta) + \frac{4}{3(1+w)}\left(\Phi_{k'}(\eta) + \frac{\Phi'_{k'}(\eta)}{\mathcal{H}}\right)\left(\Phi_{k-k'}(\eta) + \frac{\Phi'_{k-k'}(\eta)}{\mathcal{H}}\right)\right]$$

Induced GWs



$$\mathcal{P}_\zeta(k) = \Theta(k_{\rm max} - k) A_s \left(\frac{k}{k_0}\right)^{n_s - 1} \quad A_s = 2.1 \times 10^{-9}, \quad n_s = 0.96$$

Future constraints on reheating temperature



The future observations could constrain the wide range of reheating temperature in the sudden reheating scenario.

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What we did

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However,

1. The transition can occur gradually in some realistic situations.
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What we did in this work

We have revisited the effects of an eMD era on the induced GWs.

We have considered two cases:

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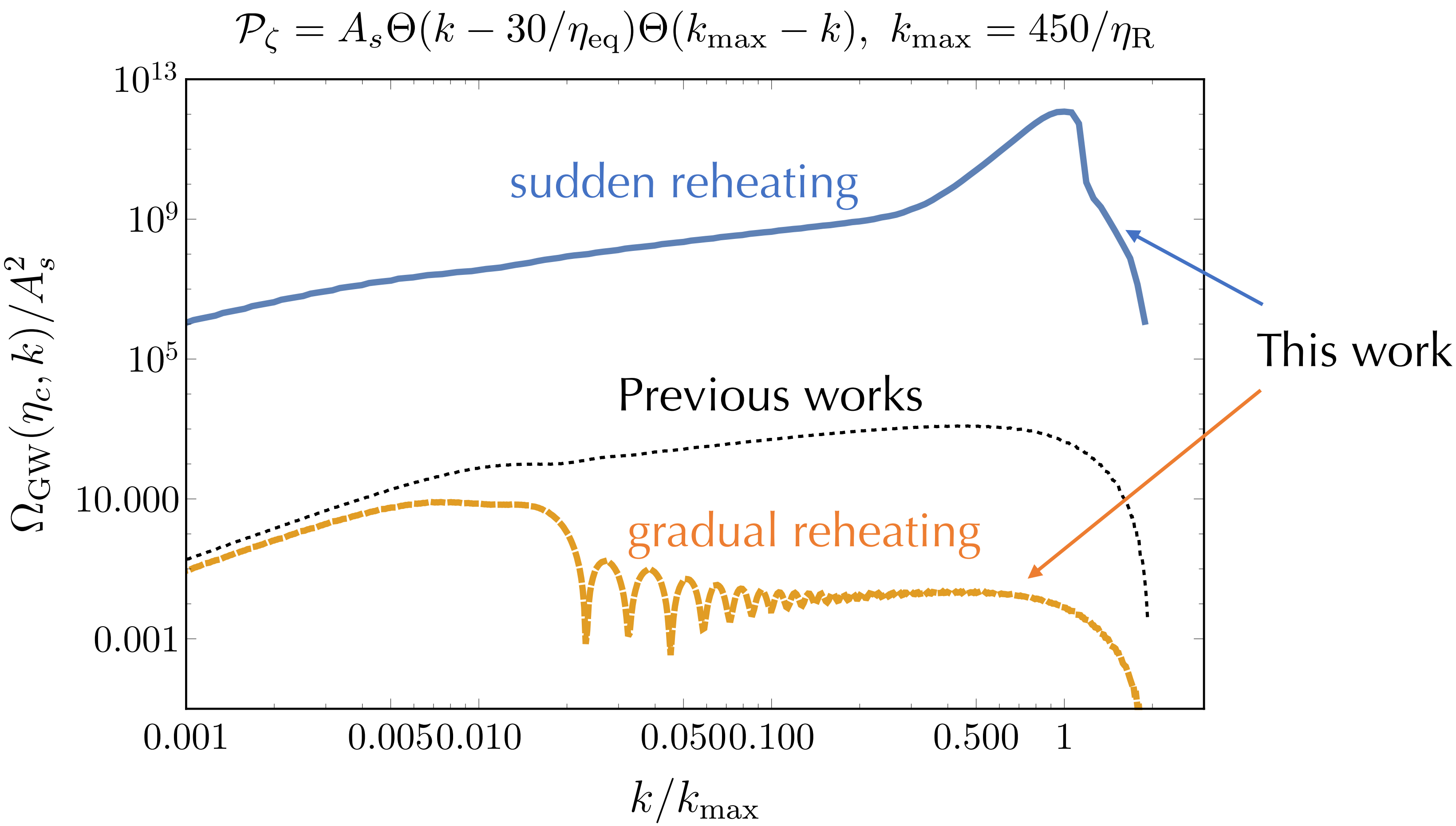
Results of this work



As a result, we have found that the induced GWs strongly depend on how quickly the reheating occurs.

Backup

Foregrounds

In future, stochastic GWs of astrophysical origin or cosmological origins that are different from the induced GWs, such as those from phase transition, may be detected.
In that case, the constraints would become weak.

Foreground example:

Around PTA target frequency
GWs from supermassive-black-hole binaries

Around LISA target frequency
GWs from galactic white-dwarf binaries

Around BBO target frequency
GWs from neutron star binaries

We may use non-Gaussianity of the induced GWs to discriminate them from other sources. (Bartolo et al., 2018)

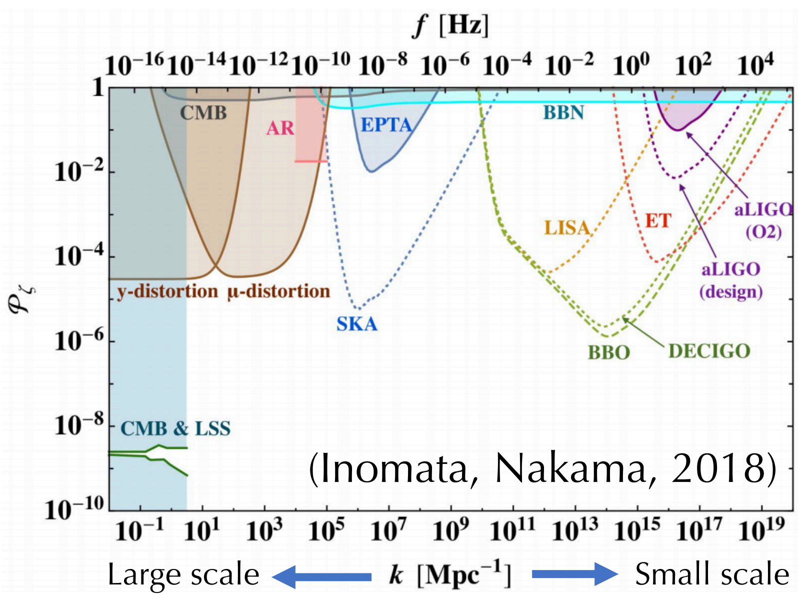
Motivations for the induced GWs

The frequencies of the induced GWs correspond to the scale of the scalar perturbations inducing GWs.

$(\lambda \lesssim 1\,\mathrm{Mpc})$

We can investigate small-scale (scalar) perturbations using the induced GWs!

(Observations of CMB and LSS cannot investigate small-scale perturbations due to the Silk damping and the non-linear growth of perturbations)



The future and current GW projects can investigate the power spectrum of curvature perturbations.

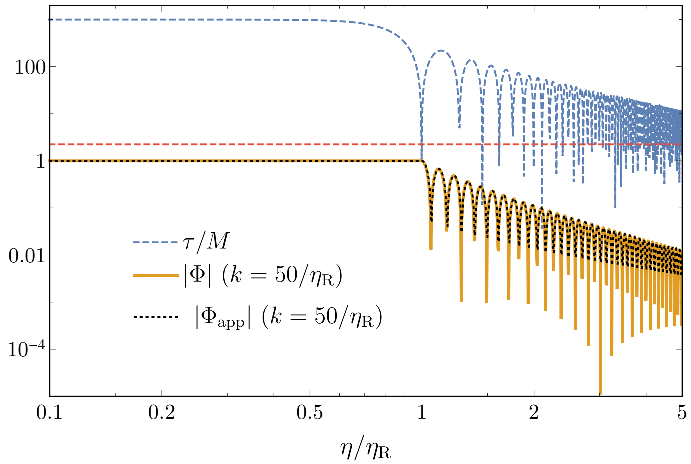
(The left figure is just an advertisement of our previous work, arXiv: 1812.00674.)

Since small-scale perturbations enter the horizon early, the induced GW could be a good probe of the early Universe.

Main question of this work

What kind of effects does the early matter dominated era make on the induced GWs?

Evolutions in sudden reheating



$$\Phi_\mathrm{app}(x, x_\mathrm{R}) = \begin{cases} 1 & (\text{for } \eta \leq \eta_\mathrm{R}) \\ A(x_\mathrm{R})\mathcal{J}(x) + B(x_\mathrm{R})\mathcal{Y}(x) & (\text{for } \eta \geq \eta_\mathrm{R}) \end{cases}$$

J and Y are the independent solutions of the following equation for Φ during RD era.

$$\Phi'' + 3(1+w)\mathcal{H}\Phi' + wk^2\Phi = 0 \quad (w = 1/3)$$

$$\mathcal{J}(x) = \frac{3\sqrt{3}\, j_1\left(\frac{x - x_\mathrm{R}/2}{\sqrt{3}}\right)}{x - x_\mathrm{R}/2} \qquad j_1(x) = \frac{\sin x - x\cos x}{x^2}$$

$$\mathcal{Y}(x) = \frac{3\sqrt{3}\, y_1\left(\frac{x - x_\mathrm{R}/2}{\sqrt{3}}\right)}{x - x_\mathrm{R}/2} \qquad y_1(x) = -\frac{\cos x + x\sin x}{x^2}$$

A and B are determined so that Φ and Φ' are continuous at η_R.

$$A(x_\mathrm{R}) = \frac{1}{\mathcal{J}(x_\mathrm{R}) - \frac{\mathcal{Y}(x_\mathrm{R})}{\mathcal{Y}'(x_\mathrm{R})}\mathcal{J}'(x_\mathrm{R})}$$

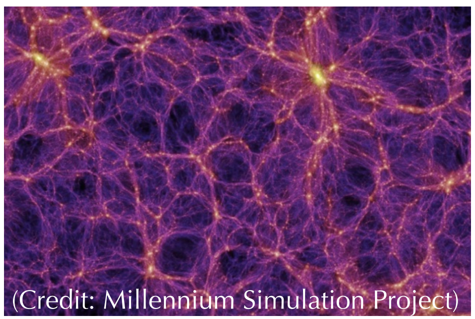
$$B(x_\mathrm{R}) = -\frac{\mathcal{J}'(x_\mathrm{R})}{\mathcal{Y}'(x_\mathrm{R})}A(x_\mathrm{R})$$

Scalar perturbation in Cosmology

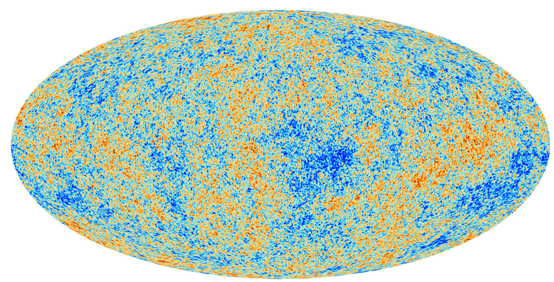
Scalar perturbations induce the perturbations of energy density ($\delta\rho/\rho$).

Scalar perturbations are origins of many things!

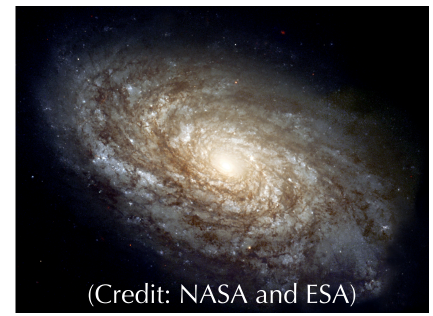
Examples


(Credit: Millennium Simulation Project)

Large Scale Structure


(Credit: ESA and the Planck Collaboration)

CMB perturbations


(Credit: NASA and ESA)

Galaxies

From the observations, we already know the amplitude of the scalar perturbations.

$$\mathcal{P}_\zeta = 2.1 \times 10^{-9} \quad \text{(Planck 2018)}$$
$$(\delta\rho/\rho \sim 10^{-5})$$

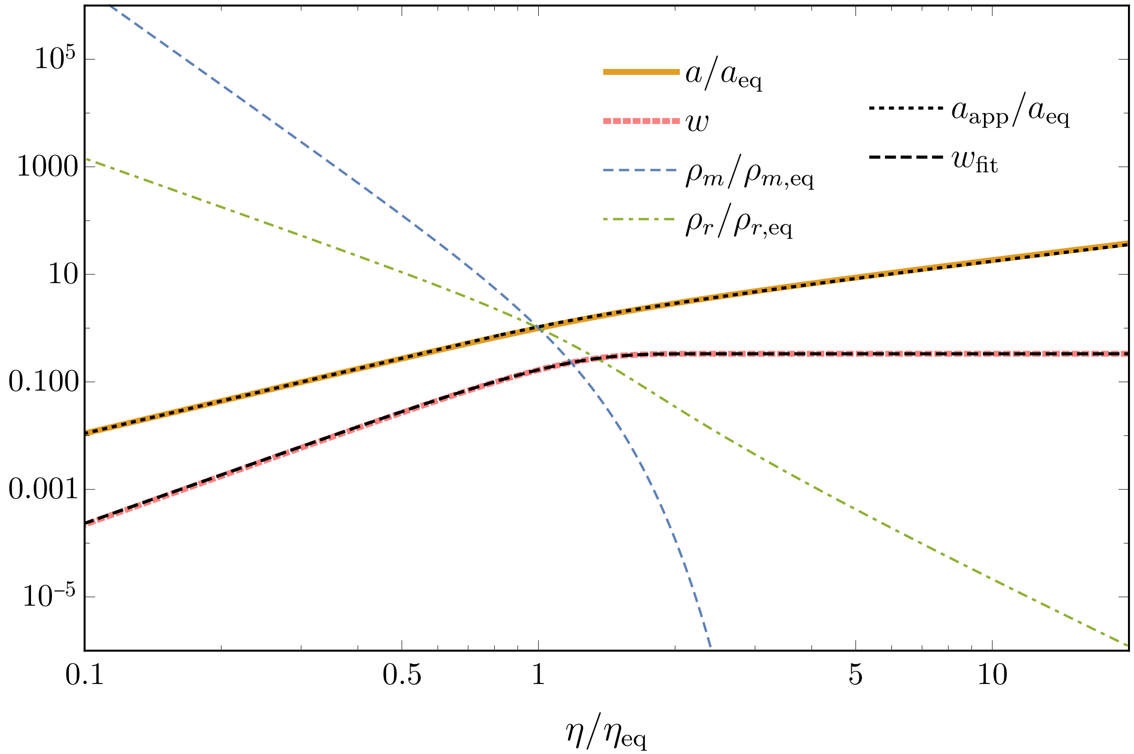
$$\mathcal{P}_\zeta = \frac{9}{4}\mathcal{P}_\Phi$$

curvature perturbation   scalar perturbation

Scalar perturbations originate from vacuum fluctuations of an inflaton during the inflation era.

Background in gradual reheating

The evolutions of background quantities.



$$w \equiv P/\rho$$

$$\rho_m = \rho_r \text{ at } \eta = \eta_{\rm eq}$$

The fitting formula for a:

$$\frac{a_{\rm app}(\eta)}{a_{\rm app}(\eta_{\rm R})} = \begin{cases} \left(\frac{\eta}{\eta_{\rm R}}\right)^2 & (\eta < \eta_{\rm R}) \\ 2\frac{\eta}{\eta_{\rm R}} - 1 & (\eta \geq \eta_{\rm R}) \end{cases}$$

The fitting formula for w:

$$w_{\rm fit} = \frac{1}{3}\left(1 - \exp\left(-0.7\left(\frac{\eta}{\eta_{\rm eq}}\right)^3\right)\right)$$

In the above figure, we take $\eta_{\rm R} = 0.83\eta_{\rm eq}$.

Formulas we use to calculate induced GWs

Summary of the formulas we use: (Kohri, Terada, 2018)

$$\Omega_{\mathrm{GW}}(\eta,k) = \frac{\rho_{\mathrm{GW}}(\eta,k)}{\rho_{\mathrm{tot}}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$$

Energy density of the induced GWs

$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(v,u,k\eta)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

Power spectrum of the induced GWs

Here, we assume that the scale factor is given as, (because this fits the numerical result well)

$$\frac{a_{\mathrm{app}}(\eta)}{a_{\mathrm{app}}(\eta_{\mathrm{R}})} = \begin{cases} \left(\frac{\eta}{\eta_{\mathrm{R}}}\right)^2 & (\eta < \eta_{\mathrm{R}}) \\ 2\frac{\eta}{\eta_{\mathrm{R}}} - 1 & (\eta \geq \eta_{\mathrm{R}}) \end{cases}$$

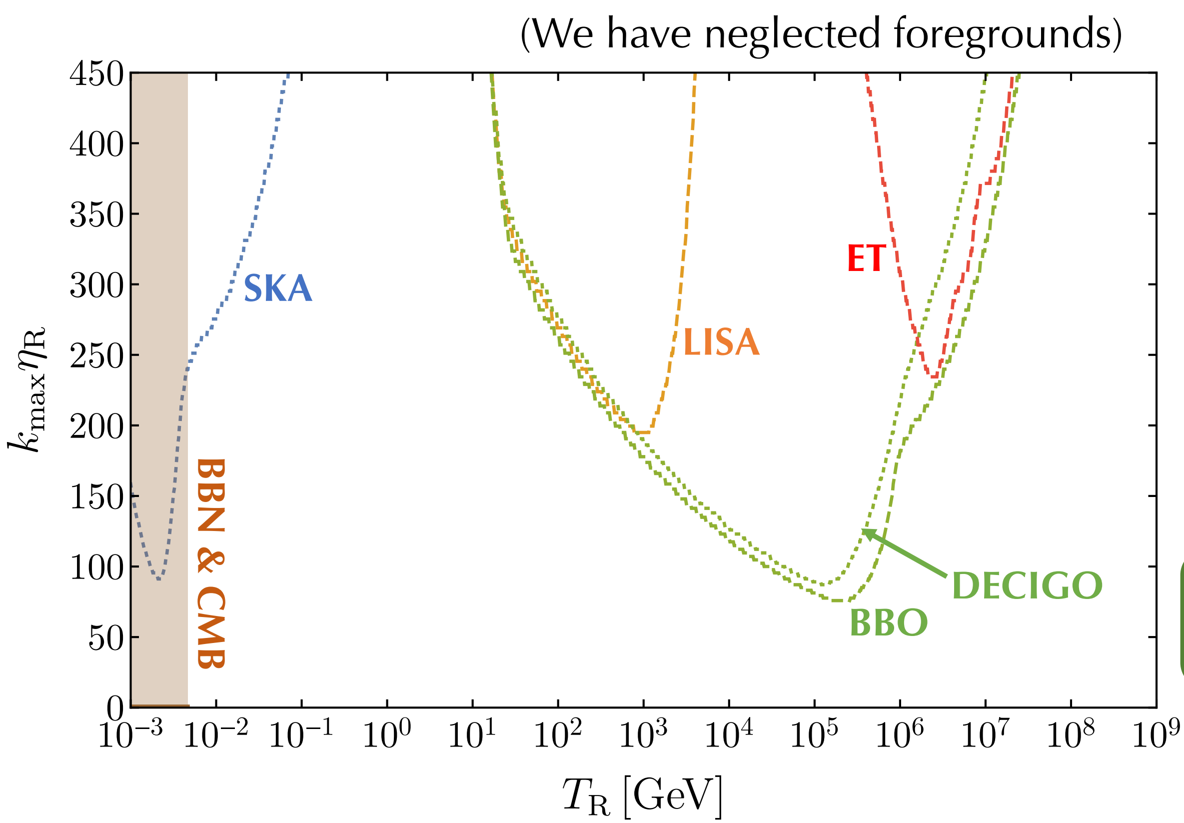
Then we can express I as $(x = k\eta, x_{\mathrm{R}} = k\eta_{\mathrm{R}})$

$$\begin{aligned} I(u,v,x,x_{\mathrm{R}}) &= \int_0^{x_{\mathrm{R}}} \mathrm{d}\bar{x} \left(\frac{1}{2(x/x_{\mathrm{R}})-1}\right)\left(\frac{\bar{x}}{x_{\mathrm{R}}}\right)^2 kG_k^{\mathrm{eMD}\to\mathrm{RD}}(\eta,\bar{\eta})f(u,v,\bar{x},x_{\mathrm{R}}) \\ &+ \int_{x_{\mathrm{R}}}^{x} \mathrm{d}\bar{x} \left(\frac{2(\bar{x}/x_{\mathrm{R}})-1}{2(x/x_{\mathrm{R}})-1}\right) kG_k^{\mathrm{RD}}(\eta,\bar{\eta})f(u,v,\bar{x},x_{\mathrm{R}}) \\ &\equiv I_{\mathrm{eMD}}(u,v,x,x_{\mathrm{R}}) + I_{\mathrm{RD}}(u,v,x,x_{\mathrm{R}}) \end{aligned}$$

$(\Phi(x\to 0) = 1,\ \Phi'(x) = \partial\Phi(x)/\partial\eta)$

$$\boxed{f(u,v,\bar{x},x_{\mathrm{R}}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}}$$

Future constraints on reheating temperature



Power spectrum:

$$\mathcal{P}_\zeta(k) = \Theta(k_{\max} - k) A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$(A_s = 2.1 \times 10^{-9},\ n_s = 0.96)$

Signal to noise ratio:

(Allen & Romano, 1997)

Induced GWs

$$\rho = \sqrt{2T_{\rm obs}} \left[\int_{f_{\min}}^{f_{\max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm GW,eff}(f)}\right)^2\right]^{1/2}$$

Observation time

$T_{\rm obs} = 20{\rm yr}$ for SKA

$T_{\rm obs} = 1{\rm yr}$ for otehrs

Effective noise spectrum of each project

ρ=1 is taken in the figure.

The future observations could constrain the wide range of reheating temperature in the sudden reheating scenario.