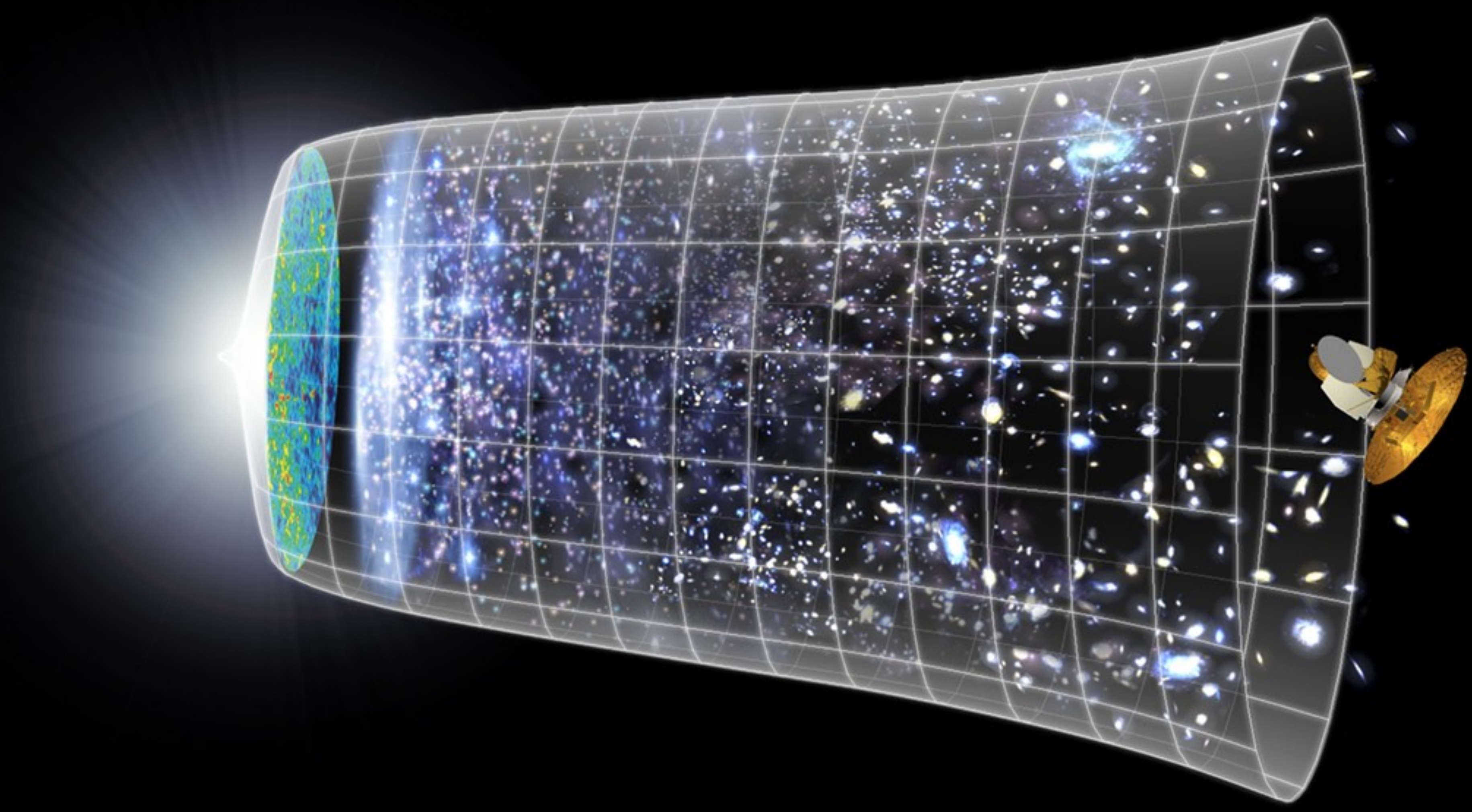
Bouncing Universe from Nothing

Takahiro Terada
(KEK, JSPS fellow)

Hiroki Matsui, Fuminobu Takahashi, TT,
Phys. Lett. B 795 (2019) 152, arXiv:1904.12312 [gr-qc]

Introduction

Cosmological History



NASA/WMAP Science Team

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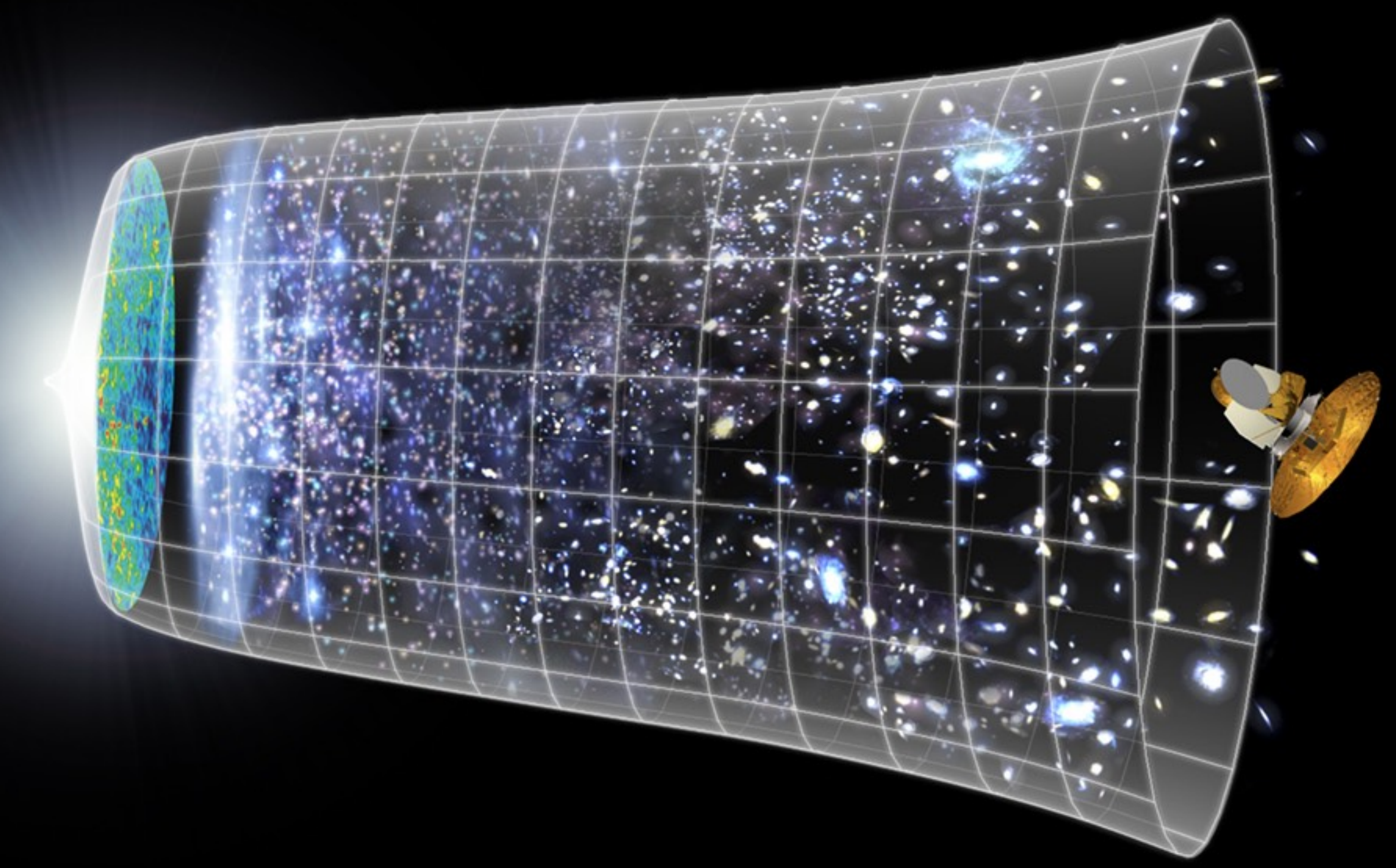


Always Expanding?

NASA/WMAP Science Team

Usually, yes.

For a flat universe,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$$

Friedmann eq.

$$H^2 = \frac{\rho}{3}$$

Energy density

Hubble parameter

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

(reduced Planck unit: $8\pi G = 1$)

As long as $\rho > 0$, it keeps expanding.

Universe can contract

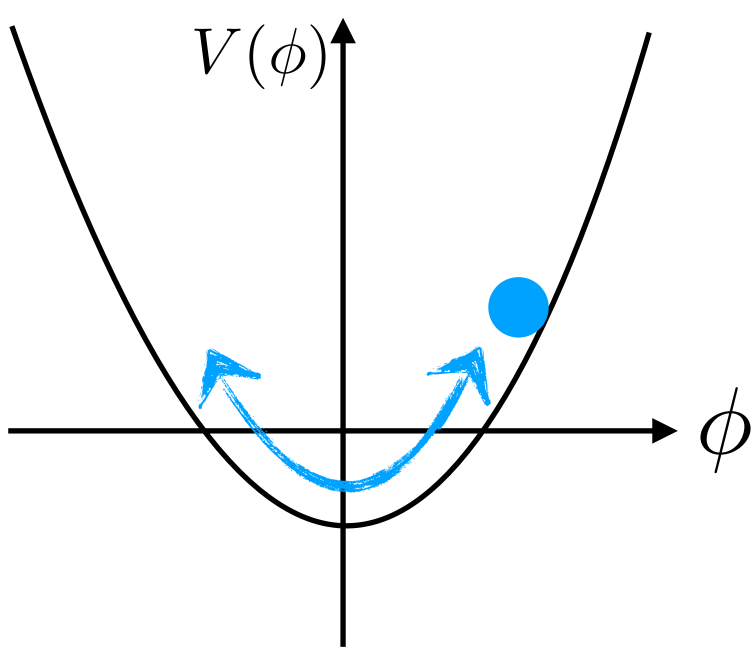
If $\rho = 0$ (i.e., $H = 0$) **is realized, the universe starts to contract.**

$$\dot{H} = -\frac{1}{2}(\rho + P)$$

Null Energy Condition (NEC): $\rho + P \geq 0$

For example,

it is possible for the scalar field with a negative potential.



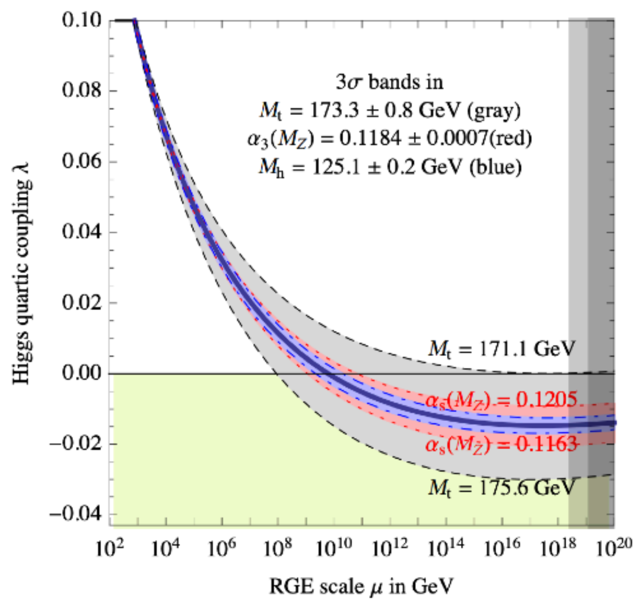
However, it **keeps contracting**,

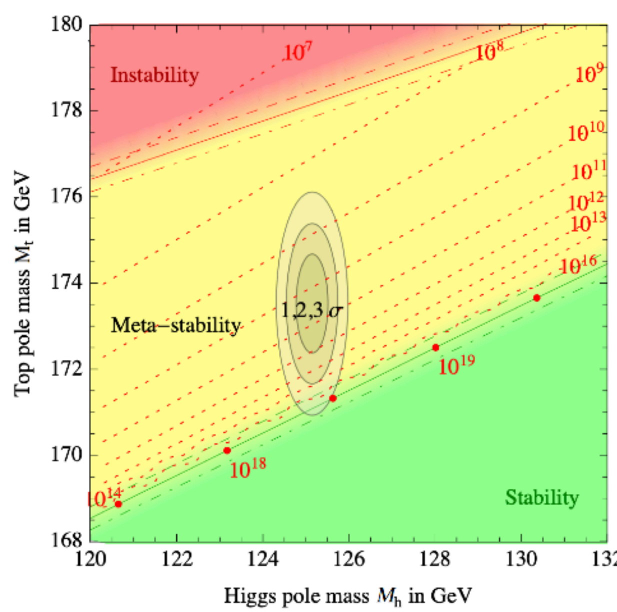
eventually leading to a **big crunch**.

[Linde, hep-th/0110195] [Felder et al., hep-th/0202017]

Negative Potential

Standard Model





[Buttazzo et al., 1307.3536]

Supergravity

R-symmetry breaking, negative semi-definite

$$V = e^K \left(g^{\bar{j}i} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2 \right)$$

SUSY breaking, positive semi-definite

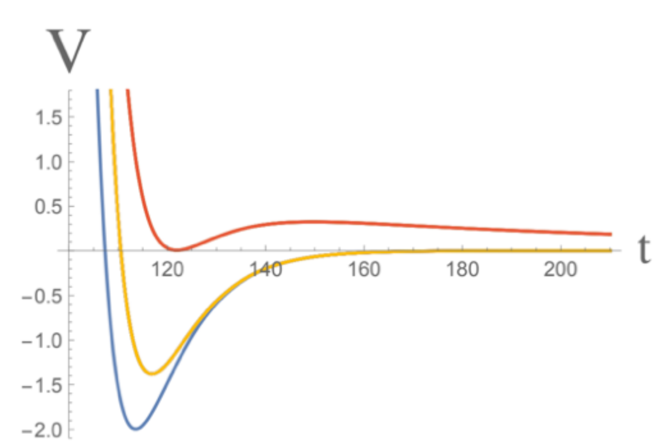
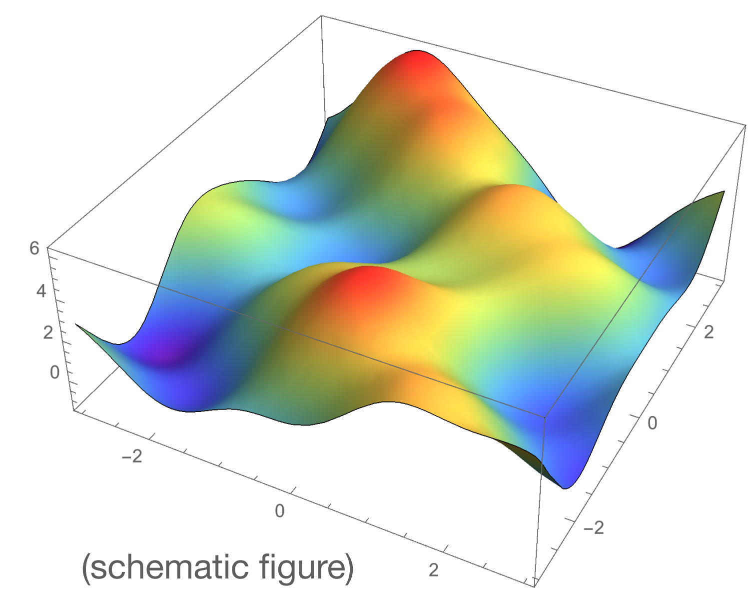


figure from [Kallosh et al., 1808.09428]

String Landscape



(schematic figure)

De Sitter vacuum construction is nontrivial in superstring theory.

(cf. Swampland de Sitter conjecture)
[Obied et al., 1806.08362]

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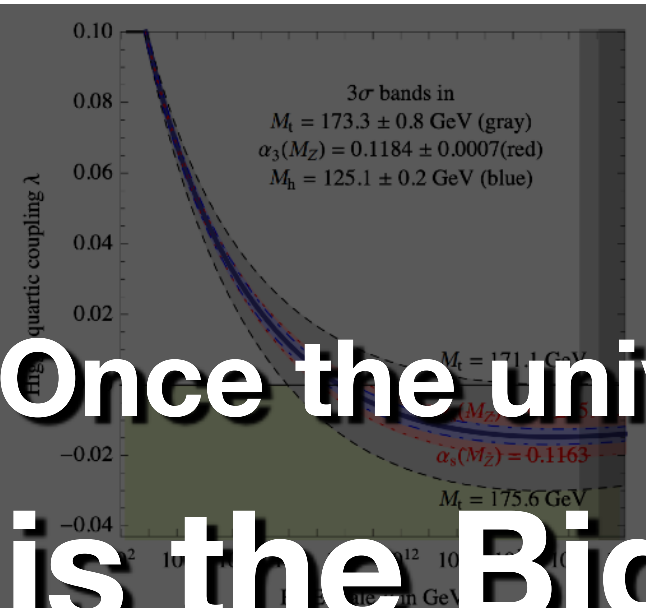

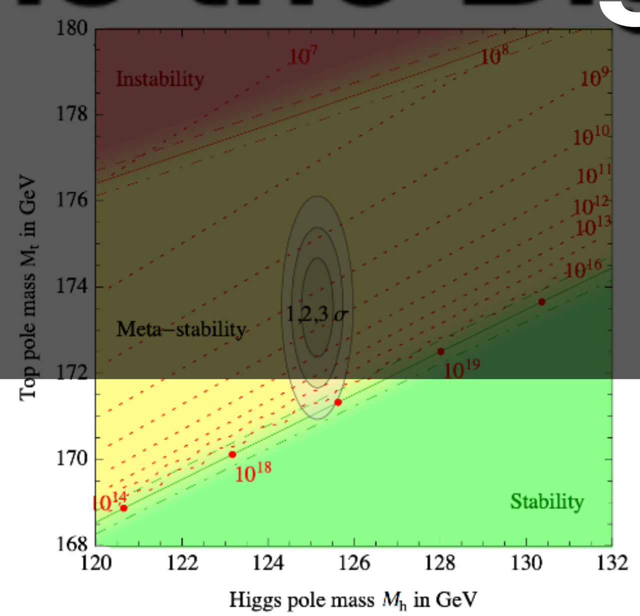

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Supergravity

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negative semi-definite

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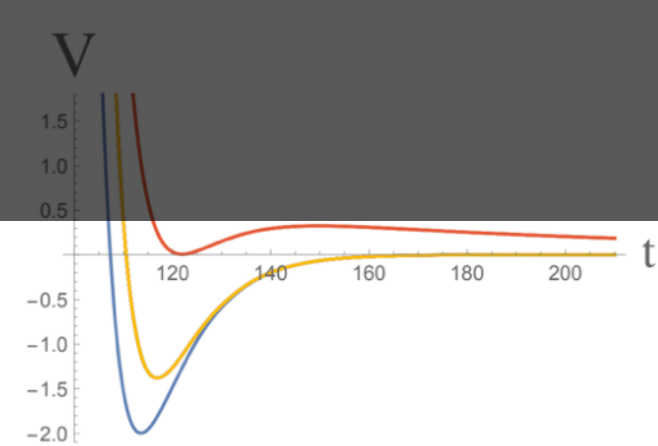

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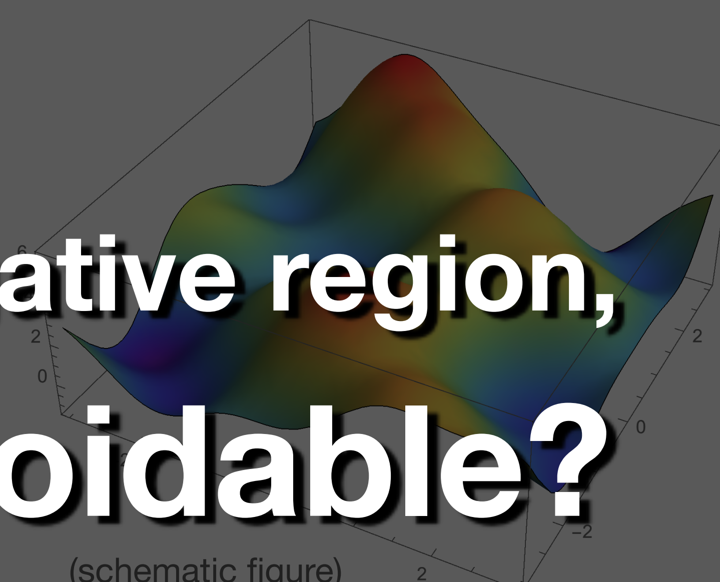

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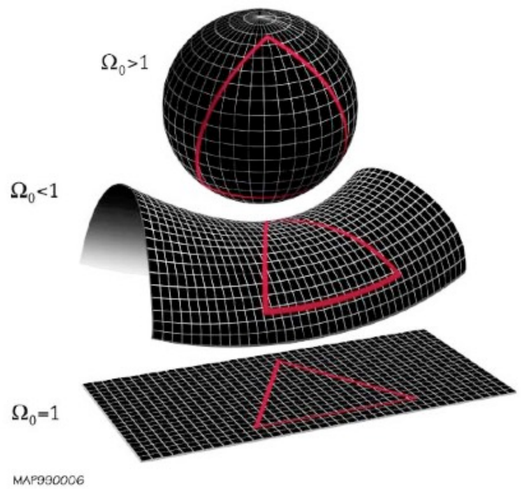
Once the universe enters the negative region,

is the Big Crunch unavoidable?

Spatial Curvature

Friedmann-Lemaitre-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\mathcal{K}r^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$



spatial curvature

positive (closed), zero (flat), or negative (open)

$$H^2 = \frac{\rho}{3} - \frac{\mathcal{K}}{a^2}$$

$$\dot{H} = -\frac{1}{2}(\rho + P) + \frac{\mathcal{K}}{a^2}$$

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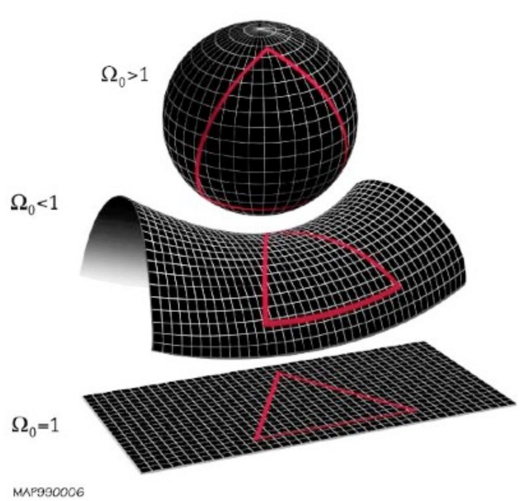

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Contraction → Expansion is possible.

simple example of a “bounce”

$\rho = -P = \Lambda$ (cosmological constant)

$\mathcal{K} > 0$

$$a(t) = \sqrt{\frac{3\mathcal{K}}{\Lambda}}\cosh\left(\sqrt{\frac{\Lambda}{3}}t\right)$$

Expansion, Contraction, and Expansion Again

Einstein Gravity & a Scalar Field

Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

Eqs. of motion

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{\mathcal{K}}{a^2}\,,$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 + \frac{\mathcal{K}}{a^2}\,,$$

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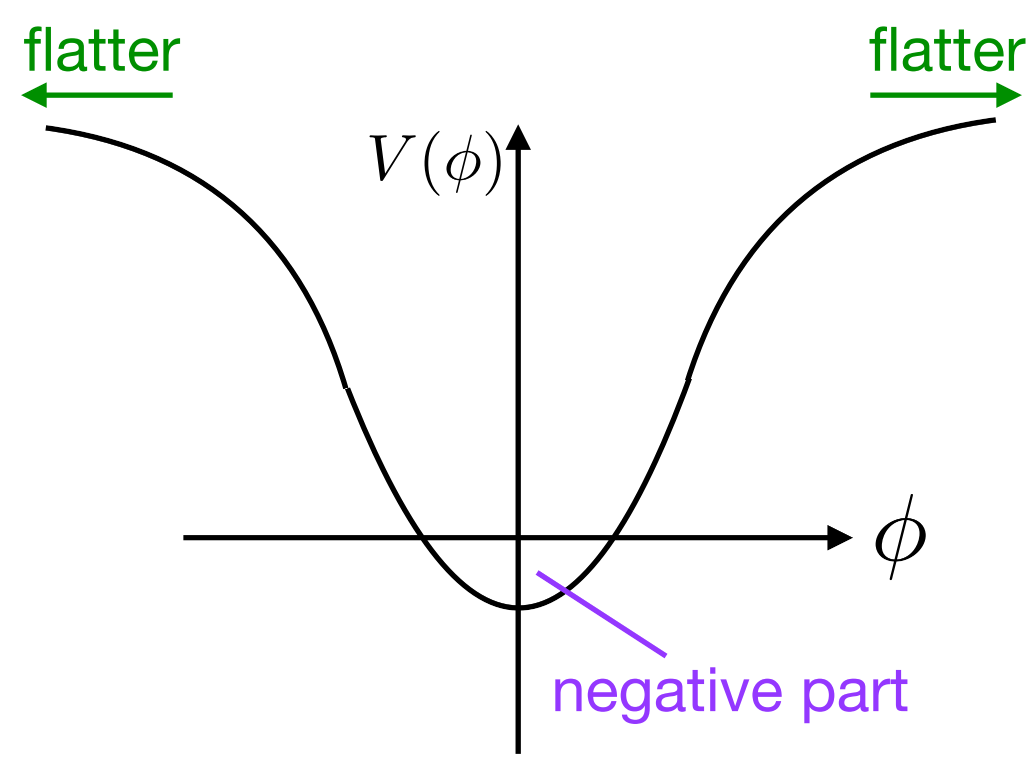
Scalar Potential



1. Initial Expansion Phase

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Suppose $\rho \gg \frac{\mathcal{K}}{a^2} > 0$ initially.

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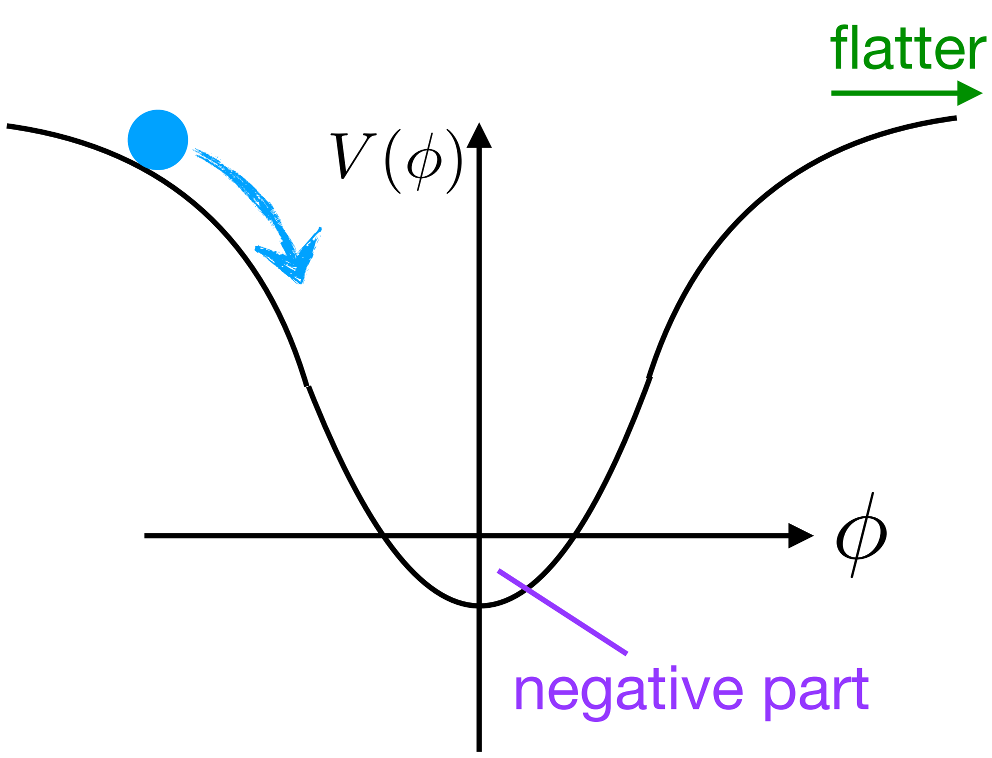

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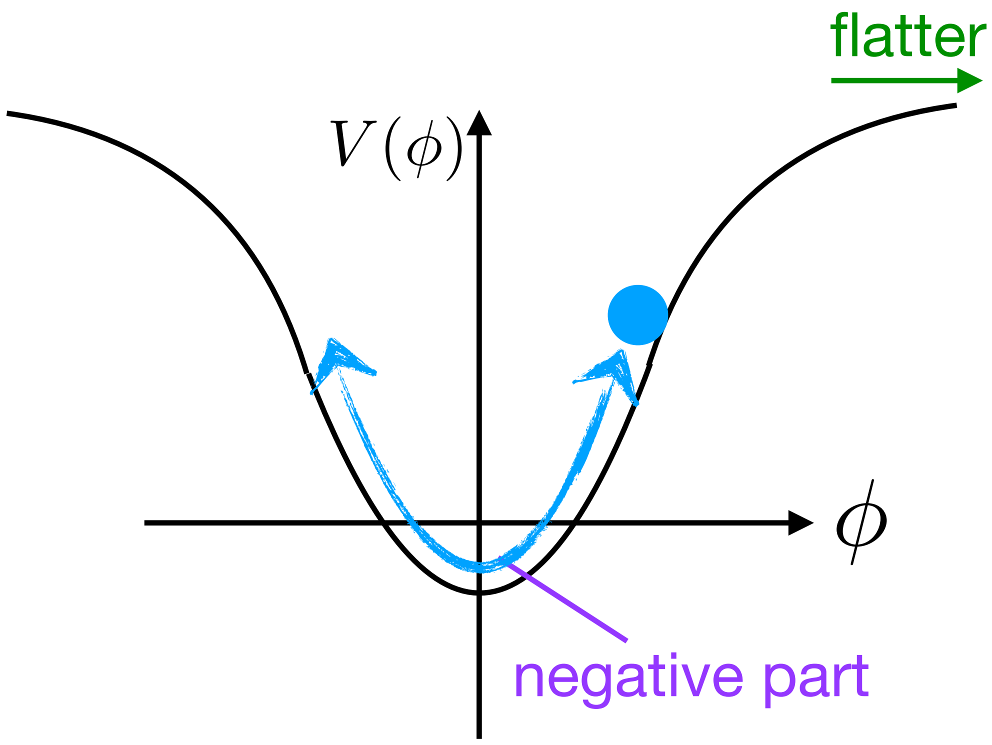

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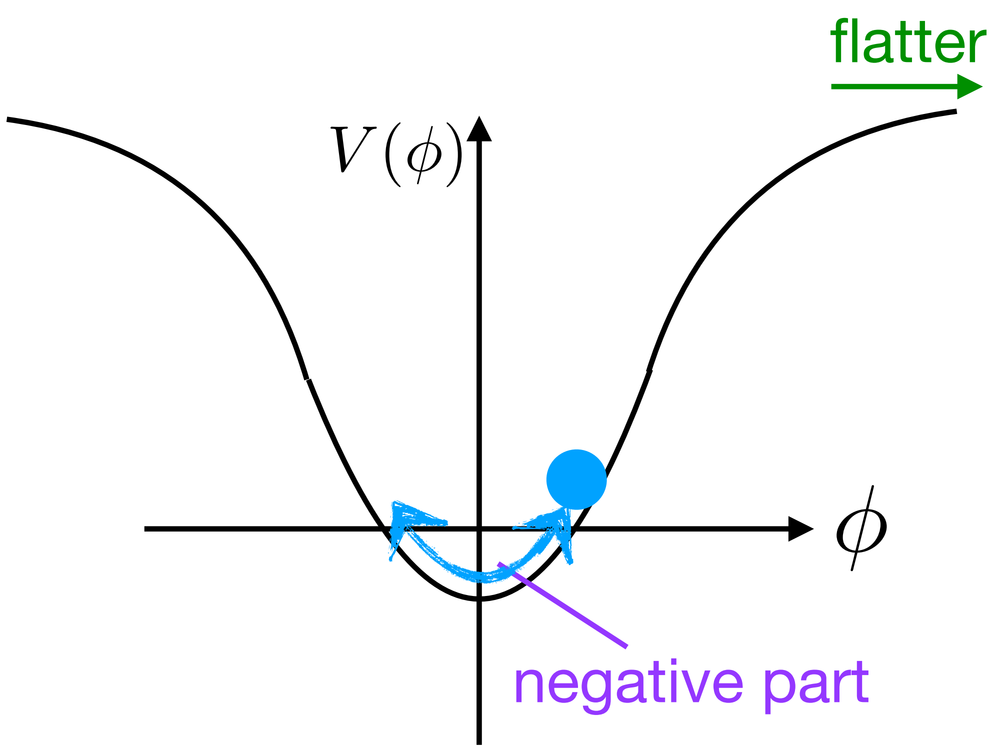

2. Contraction Phase

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During contraction, H < 0 works as ***anti-friction***.

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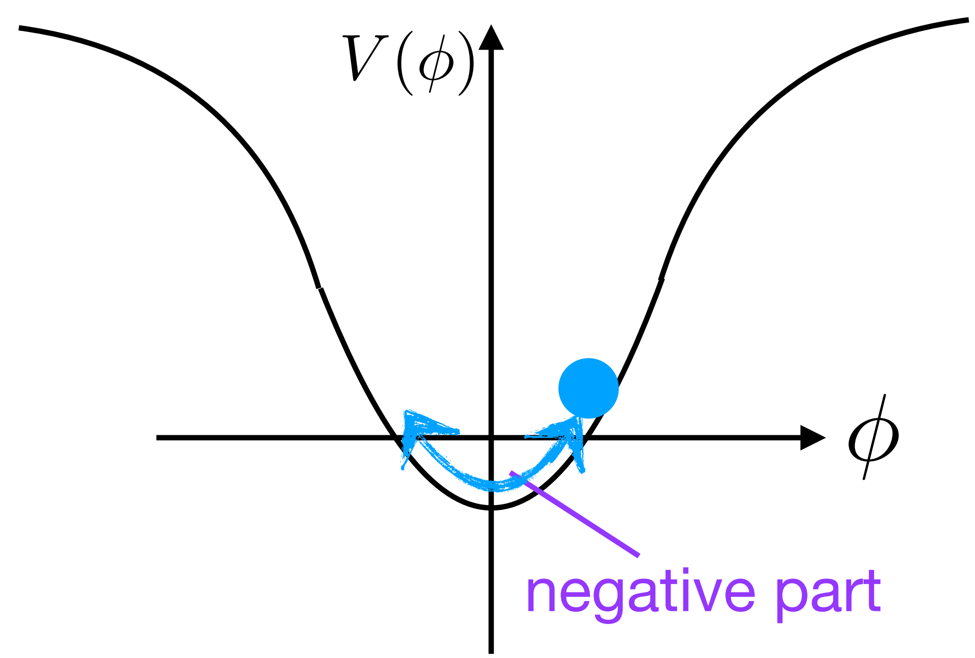

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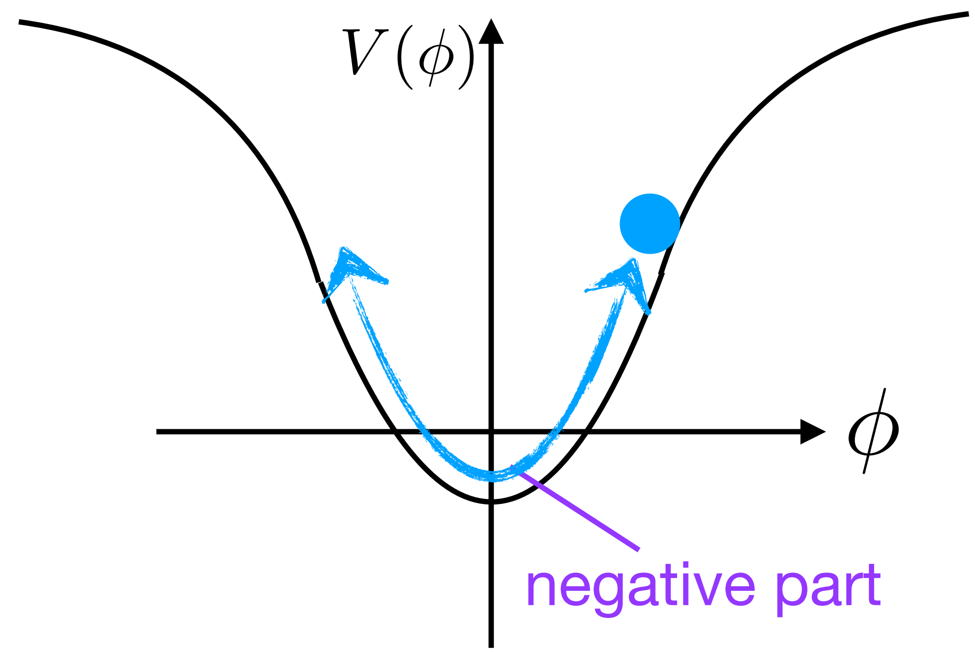

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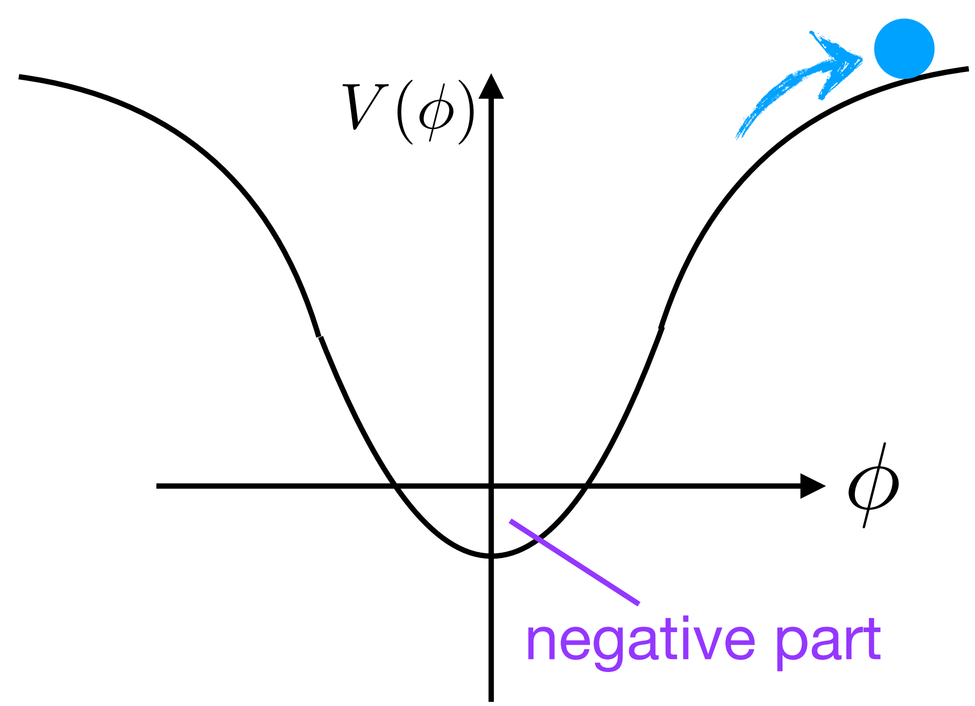

If the kinetic energy is sufficiently suppressed on the plateau, the positive curvature can make the universe expand again.

3. Second Expansion Phase

Eqs. of motion

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{\mathcal{K}}{a^2}\,,$$

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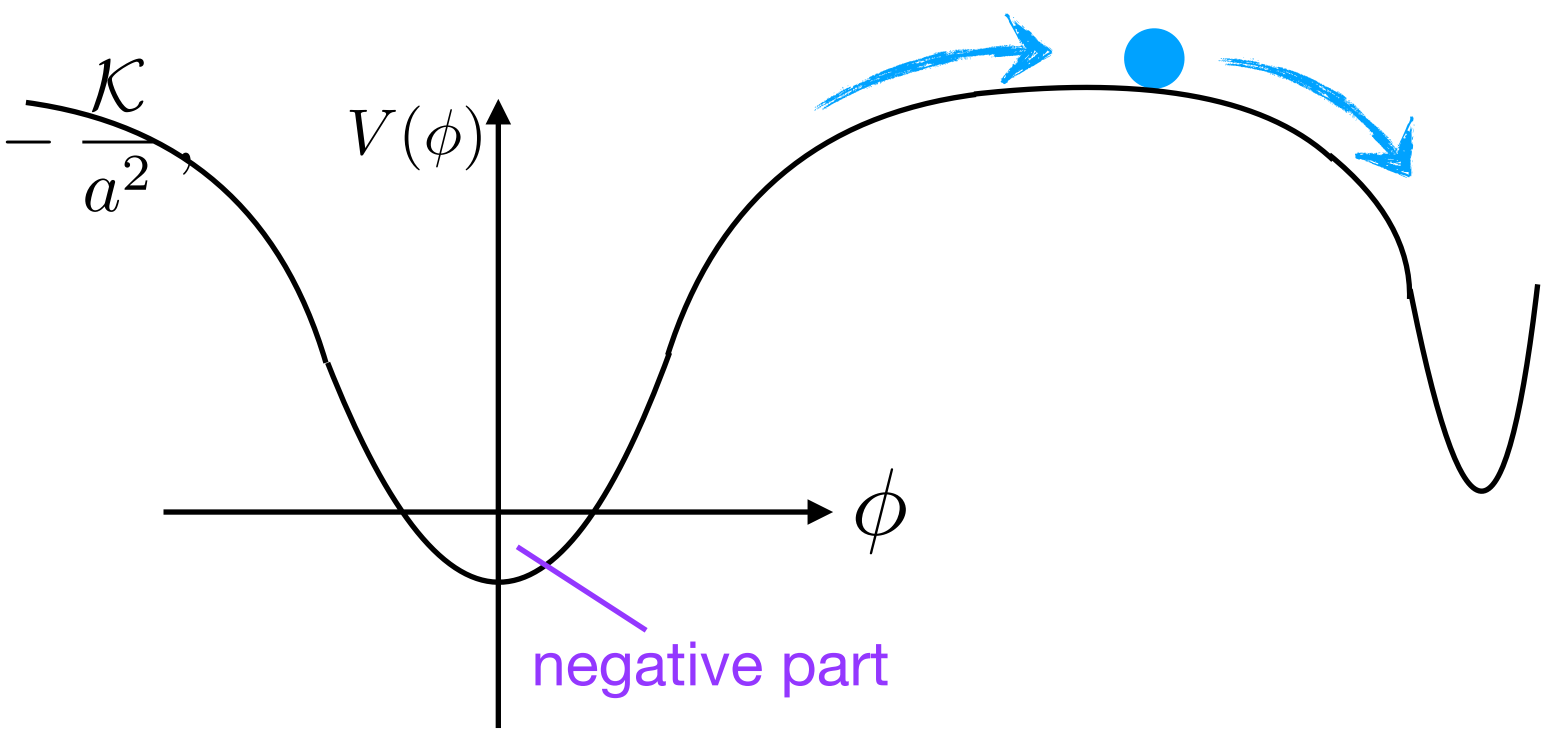


The flat part of the potential, *built in for the bounce purpose*, lets the universe naturally enter the **slow-roll inflation**ary regime.

The resultant cosmology can be **consistent with observation**.

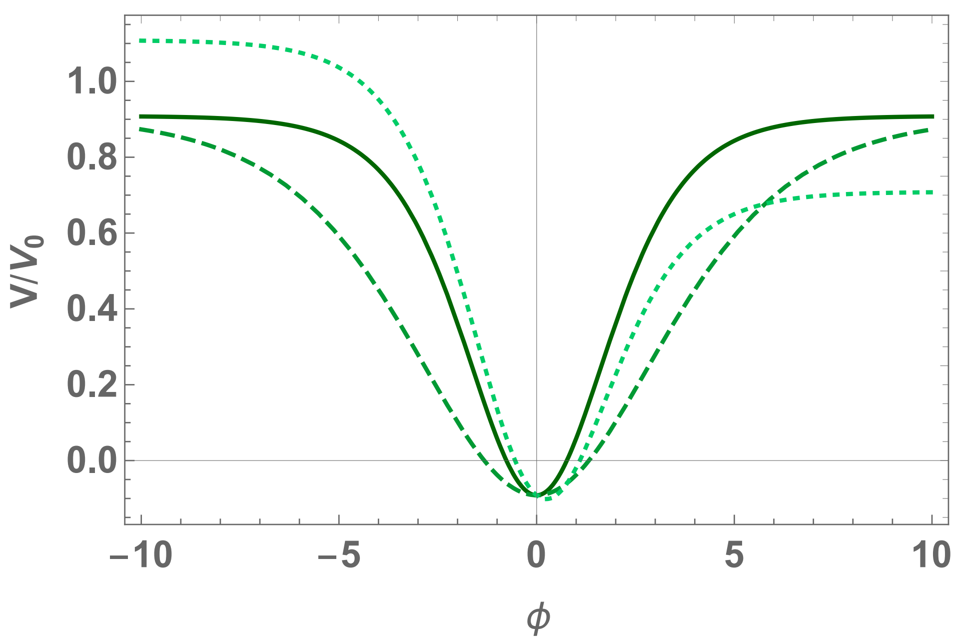
Example Model

Example Model

Scalar potential

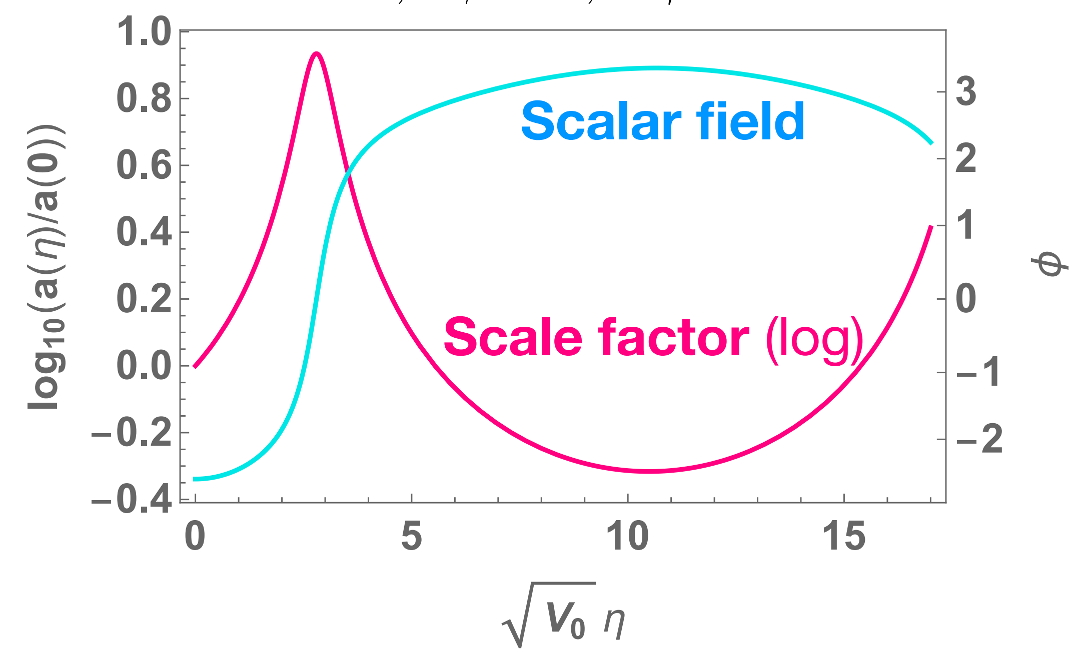
$$V(\phi) = V_0 \left(\tanh^2 \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \beta \tanh \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \gamma \right)$$

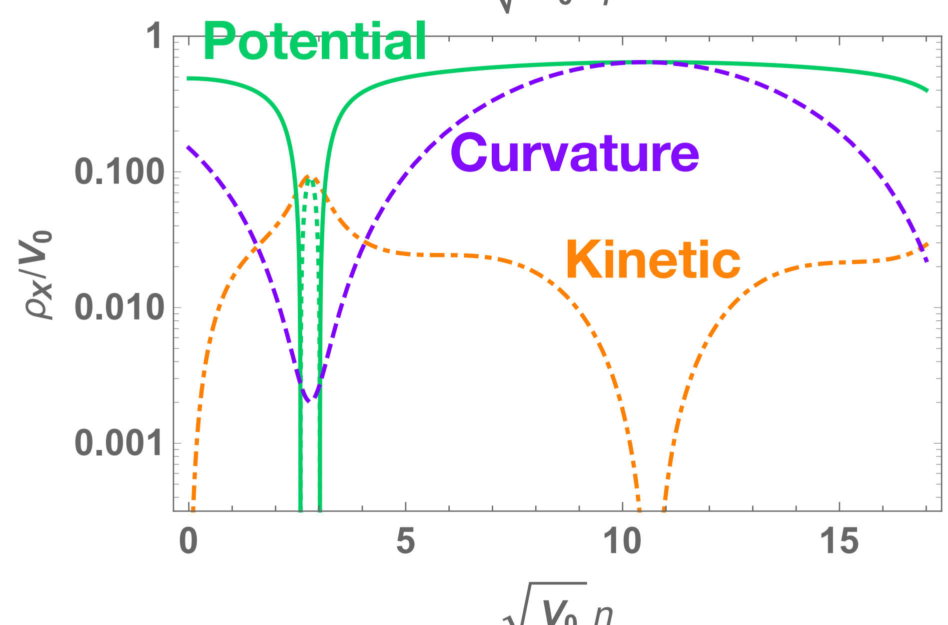
$$\alpha > 0,\ -1 < \beta < 1,\ -1 < \gamma \leq 0$$



Numerical results

$$\alpha = 1, \quad \beta = 0, \quad \gamma = -0.09143$$





Initial conditions for numerical calculation

$$\phi(0) = -\sqrt{6\alpha}, \qquad \dot{\phi}(0) = 0, \qquad \text{and} \qquad \frac{\mathcal{K}}{a(0)^2} = 0.05\sqrt{V_0}.$$

Possible origin of the positive curvature

Birth of Closed Universe

Mini-superspace approximation

Wave function of the universe: $\Psi[g_{\mu\nu}(t,x), \phi(t,x)] = \Psi[a(t), \phi(t)]$

Wheeler-De Whit eq.

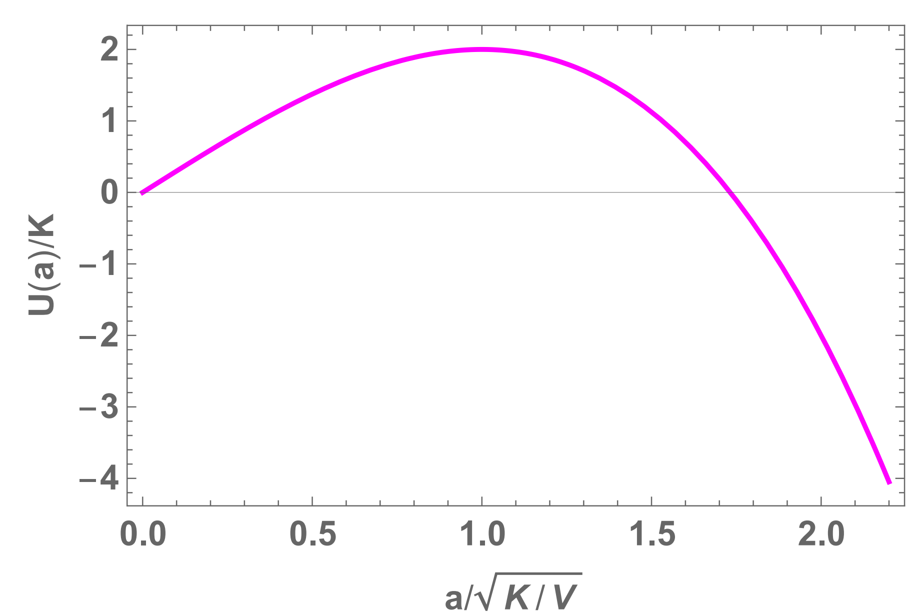
$$\mathcal{H}(a,\phi)\Psi(a,\phi) = 0$$

where

$$\mathcal{H}(a,\phi) = \frac{1}{12a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{1}{2a^3}\frac{\partial^2}{\partial \phi^2} - U(a,\phi)$$

[Kiefer, Sandhoefer, 0804.0672]

$$U(a,\phi) = a^3\left(\frac{3\mathcal{K}}{a^2} - V(\phi)\right)$$



Initial conditions [Vilenkin, PRD37, 888 (1988)]

$$a(0) = \sqrt{\frac{3\,\mathcal{K}}{V(\phi)}} \qquad \dot{a}(0) = 0$$

$$\phi(0) = \text{const.} \qquad \dot{\phi}(0) = 0$$

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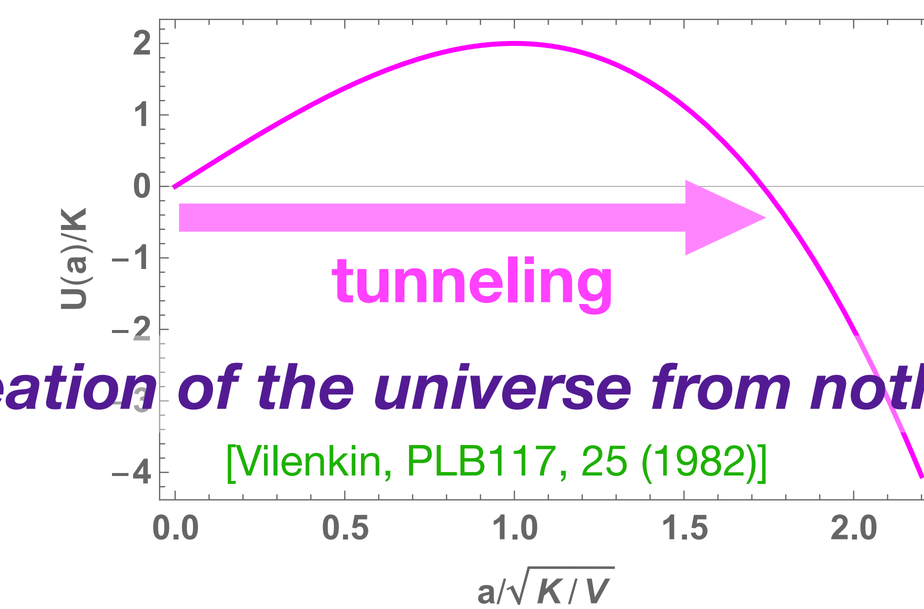

"Creation of the universe from nothing"

[Vilenkin, PLB117, 25 (1982)]

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- Our solutions open interesting possibilities
  for General Relativity and Our Universe (past and future).
- Many things to be explored.

Appendix

Dynamics of Scalar Field

approximation	regime	relevance
$\ddot{\phi} + \cancel{3H\dot{\phi}} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"No friction" e.g.) oscillation	relevant around H=0
$\cancel{\ddot{\phi}} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Slow-Roll regime" Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{\partial V(\phi)}{\partial \phi}} = 0$	"Ultra-Slow-Roll regime" Kinetic energy is important.	**attractor** solution **during contraction**
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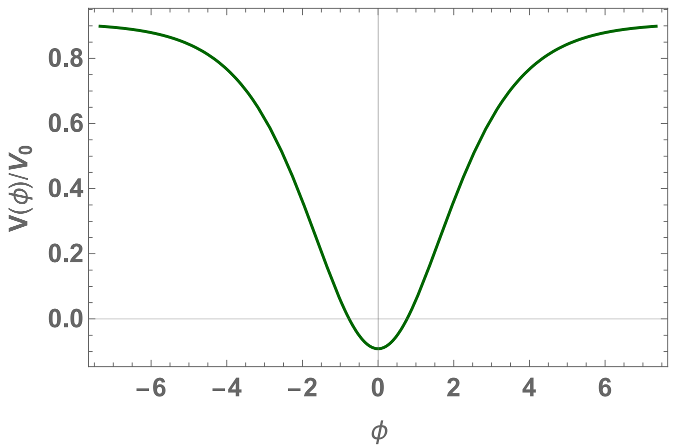
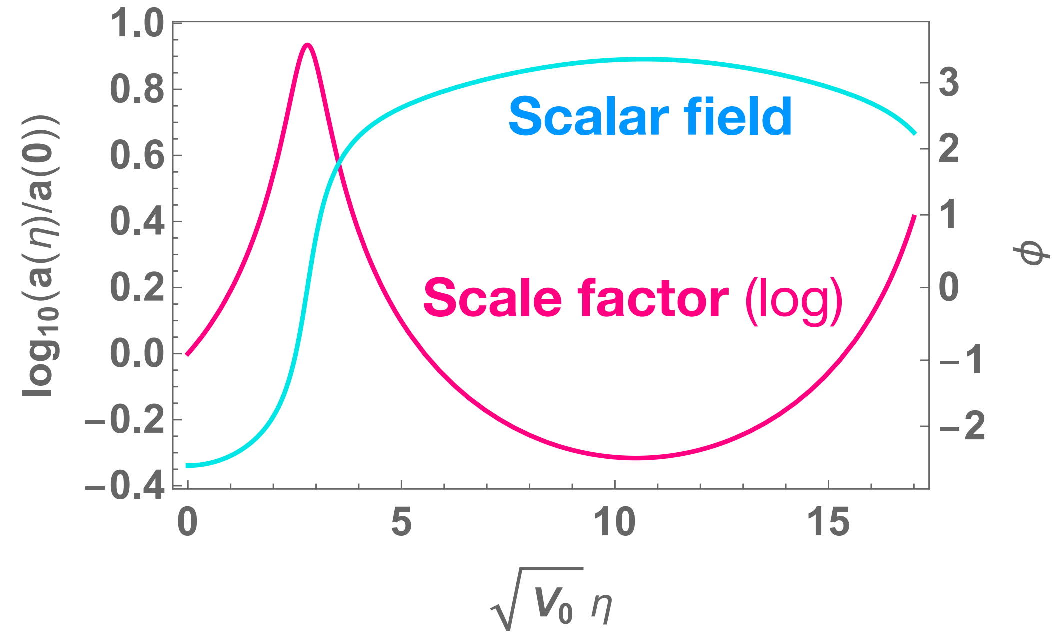
How to suppress the kinetic energy is the key for a successful bounce.

Tuning of the offset

$$\alpha = 1, \quad \beta = 0, \quad \gamma = -0.09143$$



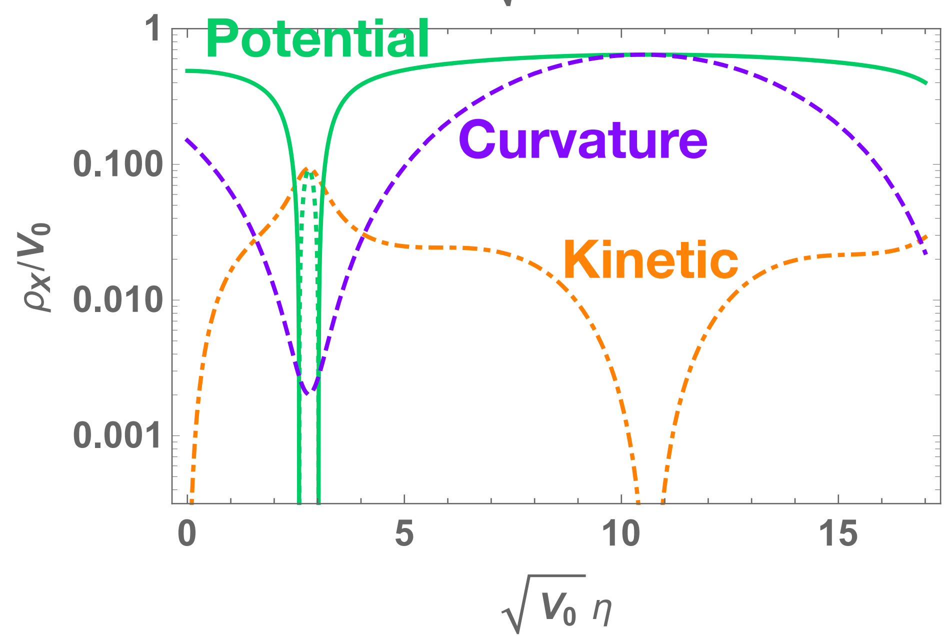


Potential energy

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V - \frac{3\mathcal{K}}{a^2}$$

Kinetic energy

"Curvature energy"

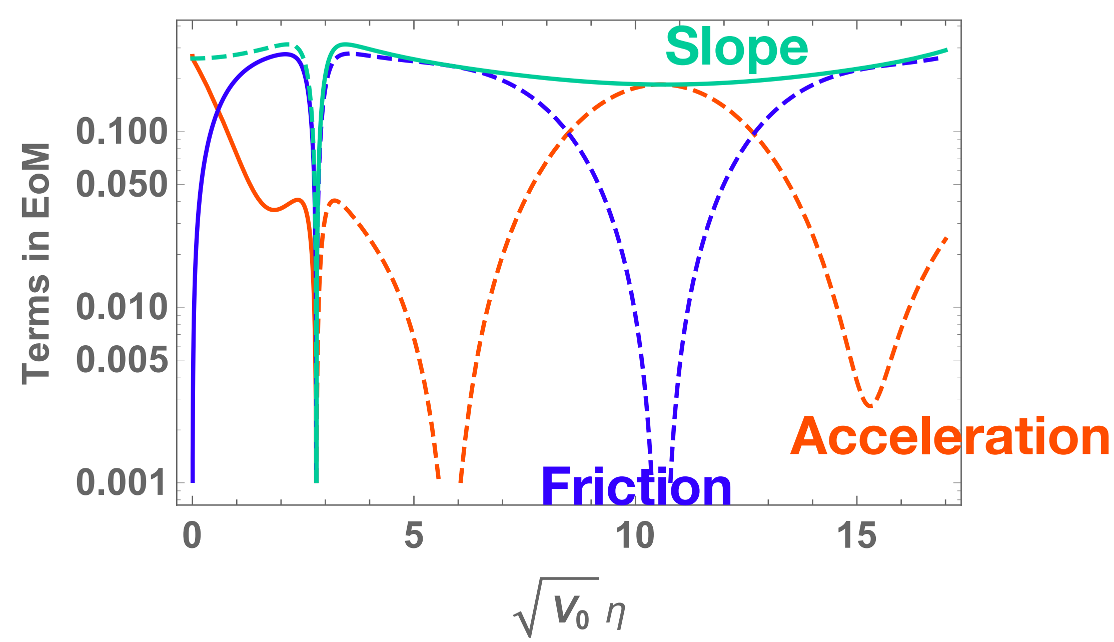


Slope

Acceleration

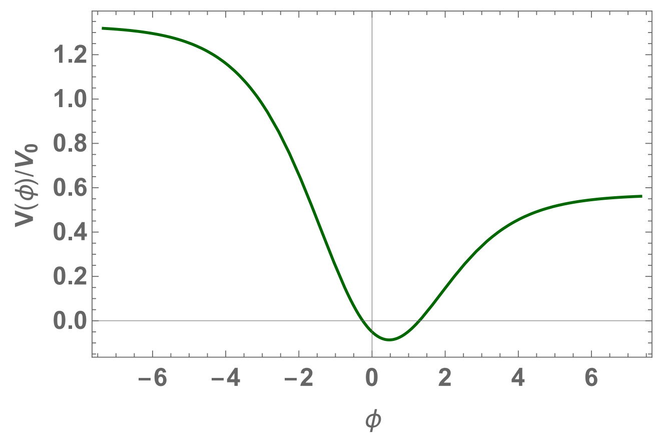
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Friction



Tuning of the left-right asymmetry

$$\alpha = 1, \quad \beta = -0.3805885, \quad \gamma = -0.05$$



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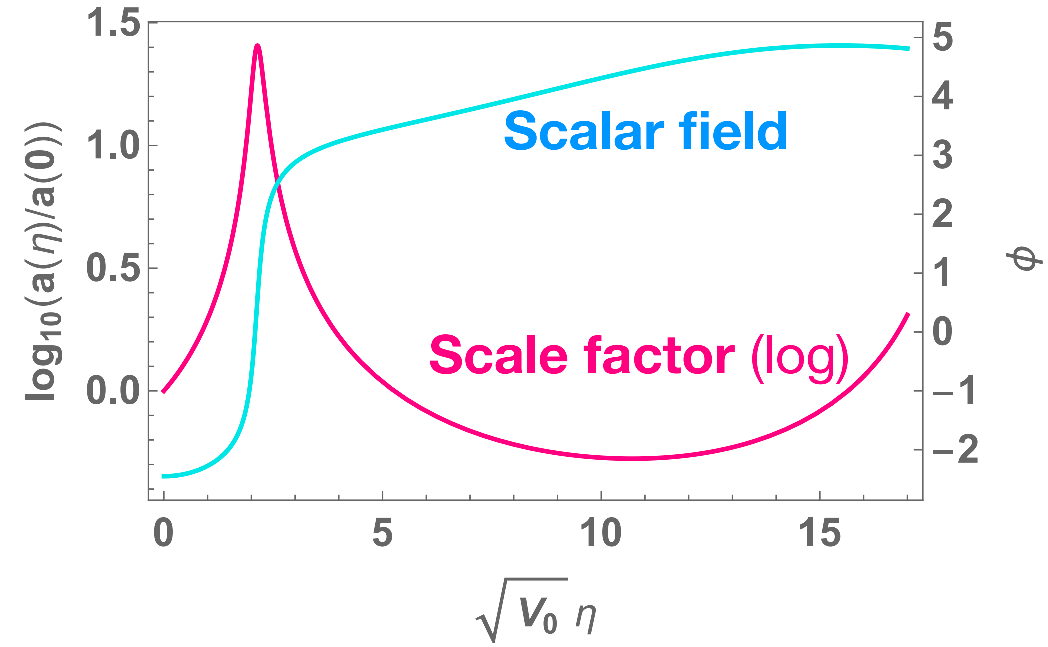
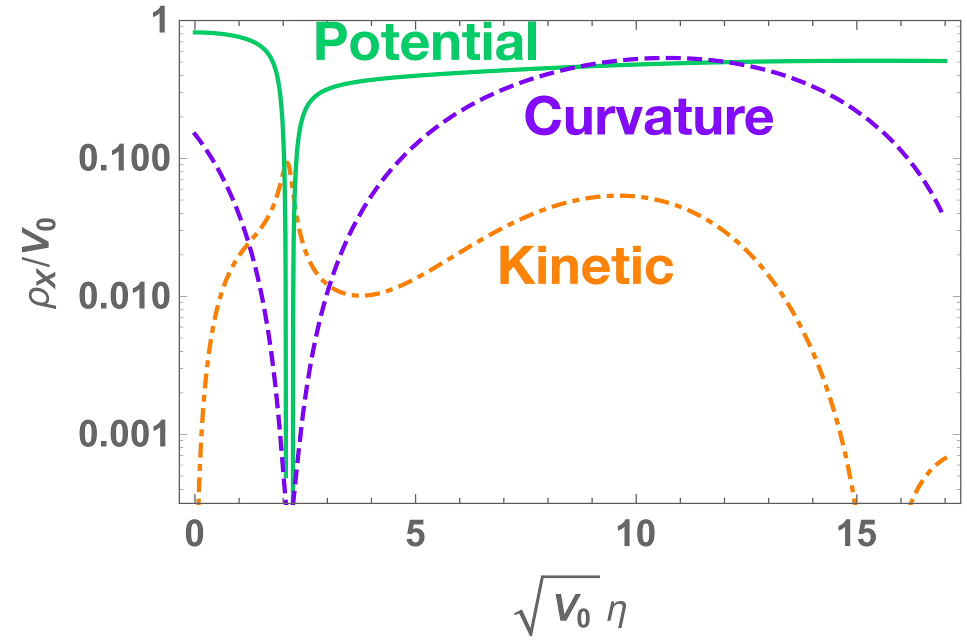
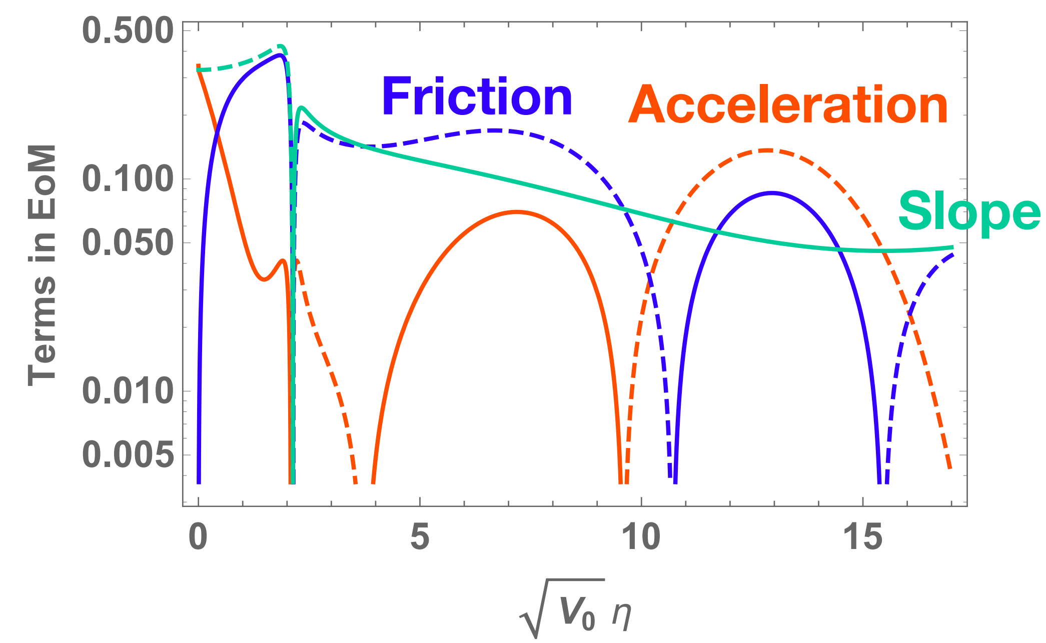
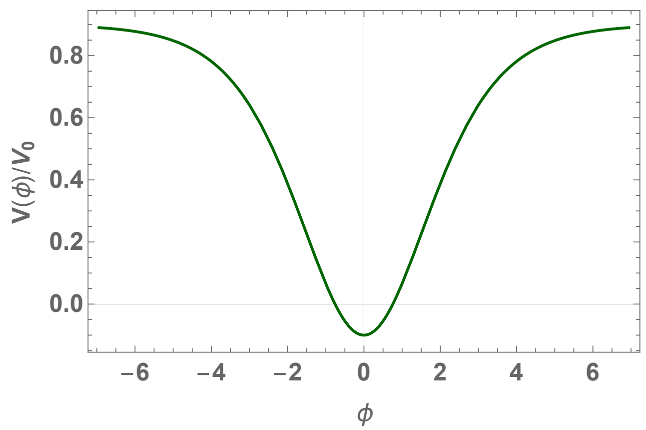






Tuning of the width

$$\alpha = 0.8924, \quad \beta = 0, \quad \gamma = -0.1$$



Potential energy

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"Curvature energy"

Acceleration

Slope

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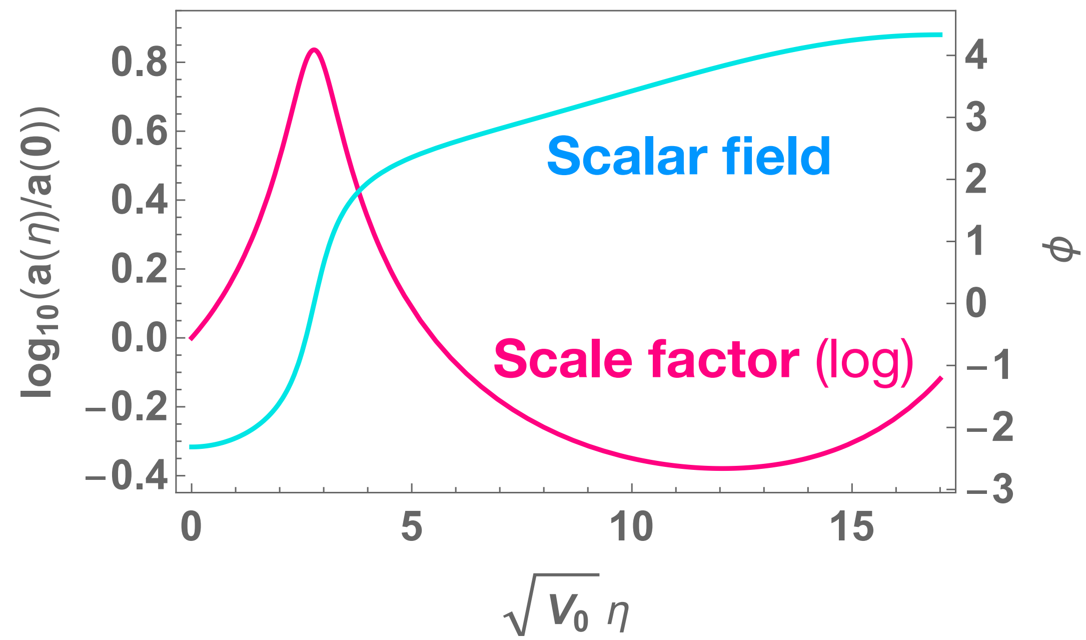
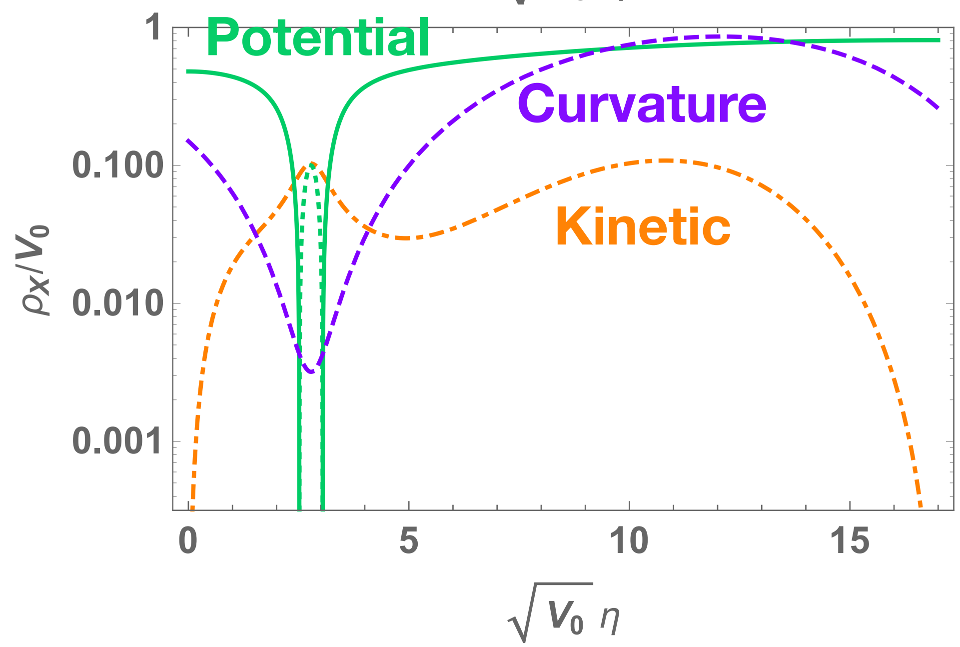
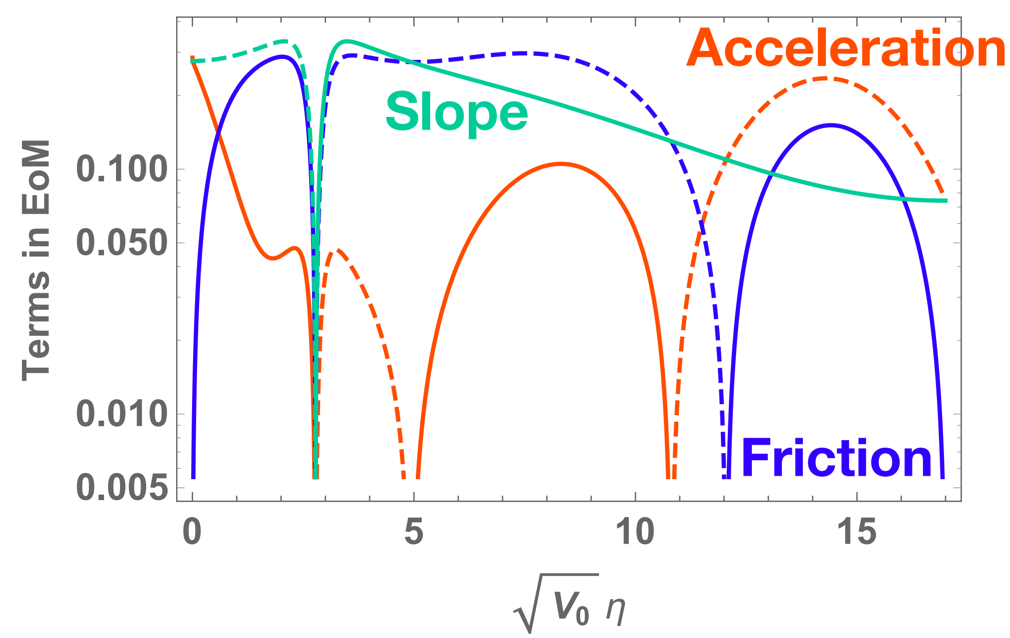
Friction







Birth of Closed Universe

Initial conditions [Vilenkin, PRD37, 888 (1988)]

$$a(0) = \sqrt{\frac{3\mathcal{K}}{V(\phi)}} \qquad \dot{a}(0) = 0$$

$$\phi(0) = \text{const.} \qquad \dot{\phi}(0) = 0$$

How this is determined is controversial.

Two proposals for nucleation probability [Vilenkin, PRD37, 888 (1988)]

$$\mathcal{P}(a,\phi) \propto \exp\left(\mp\frac{24\pi^2 M_{\mathrm{P}}^4}{V(\phi)}\right)$$

[Vilenkin, PRD30, 509 (1984)]

[Hartle, Hawking, PRD28, 2960 (1983)] *"no-boundary proposal"*

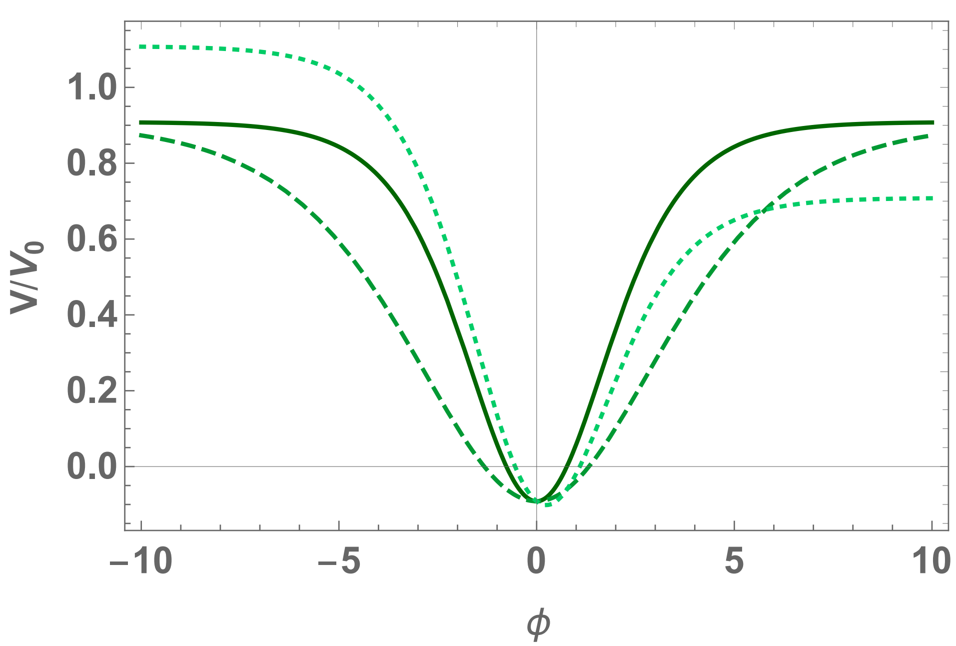
Even if the universe is born with tiny energy density by the Hartle-Hawking process, the energy density can increase in the contraction phase, which makes the scenario viable.

Cyclic solution

Scalar potential

$$V(\phi) = V_0 \left(\tanh^2 \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \beta \tanh \left[\frac{\phi}{\sqrt{6\alpha}} \right] + \gamma \right)$$

$$\alpha > 0, \ -1 < \beta < 1, \ -1 < \gamma \leq 0$$

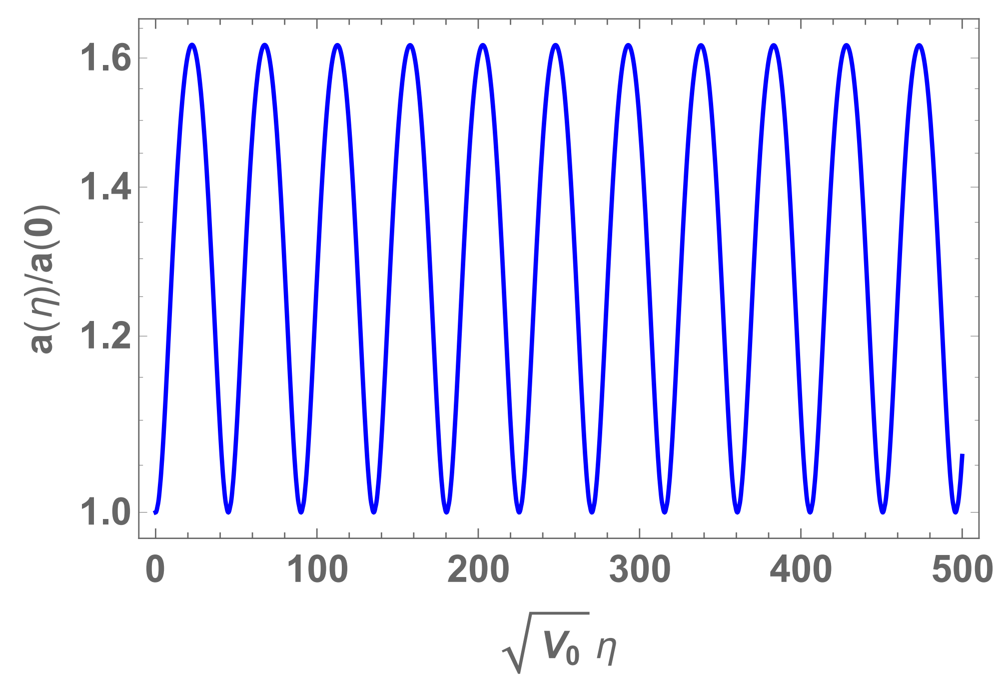


Initial conditions for numerical calculation

$$a(0) = \sqrt{\frac{3\mathcal{K}}{V(\phi)}} \qquad \dot{a}(0) = 0 \qquad \phi(0) = -3\sqrt{6\alpha} \qquad \dot{\phi}(0) = 0$$

Numerical results

$$\alpha = 5 \times 10^{-5}, \quad \beta = \gamma = 0, \quad \mathcal{K} = 0.05 V_0$$



(Details in the next slide)

Cyclic solution

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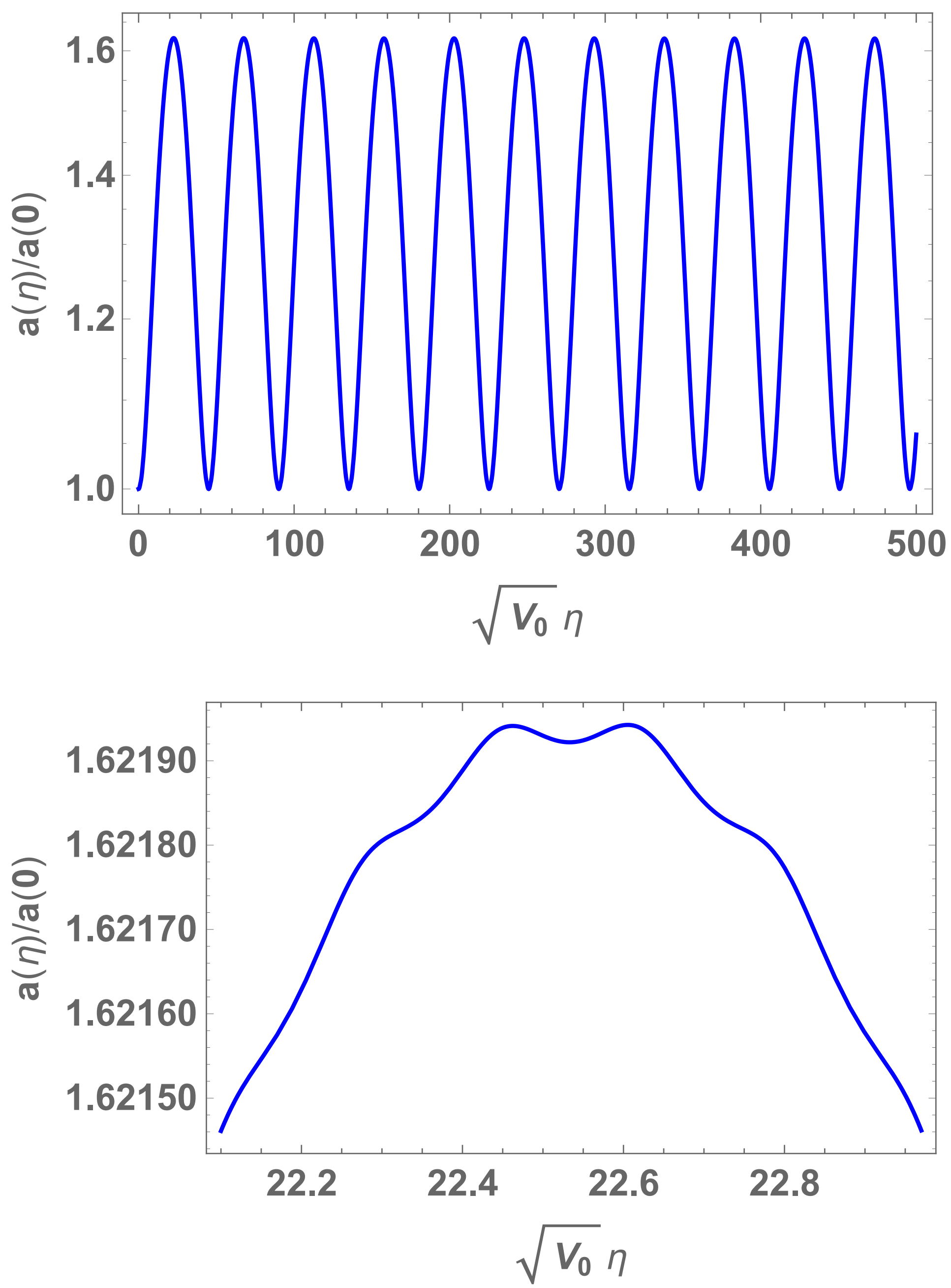
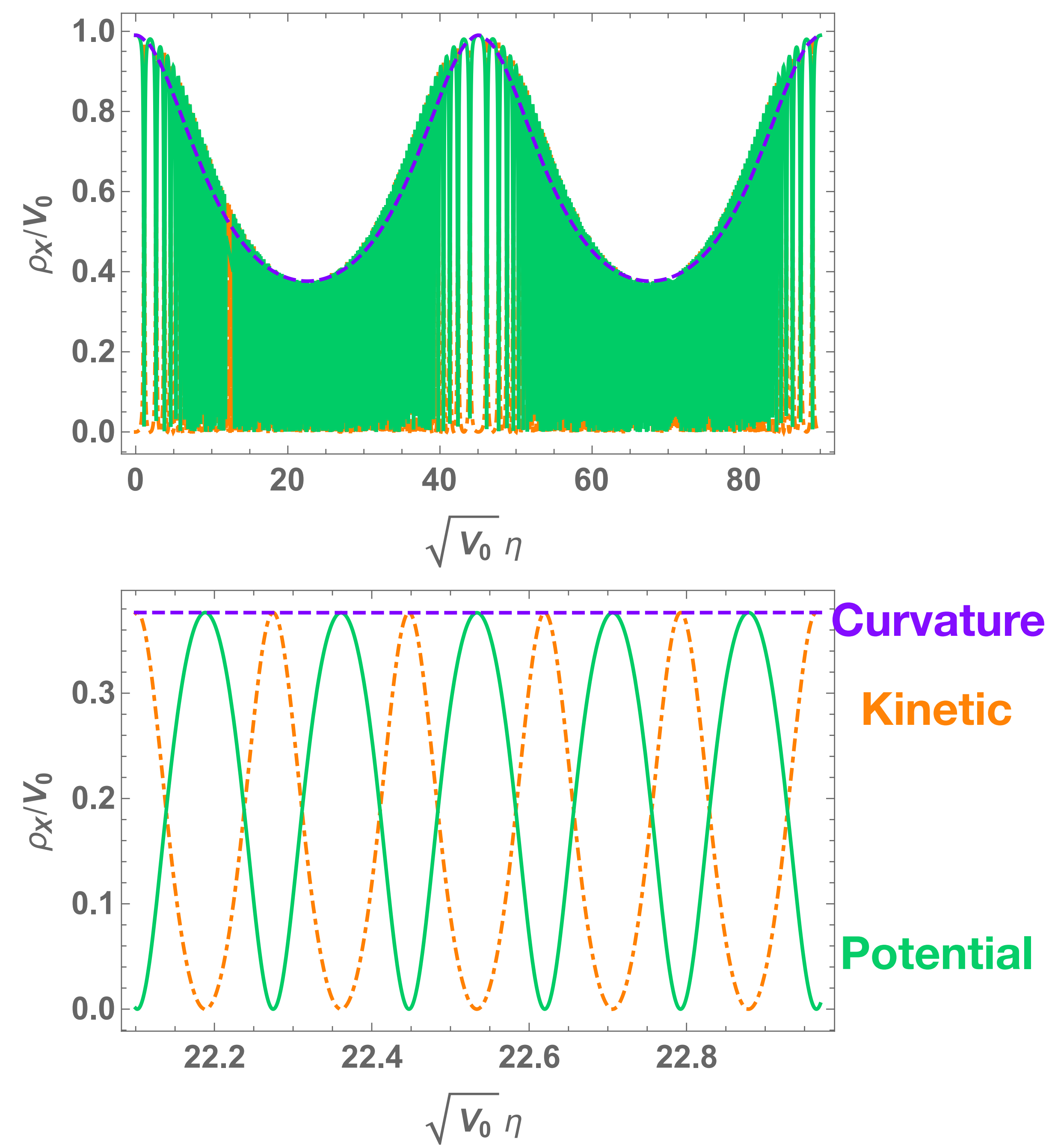
Scale factor



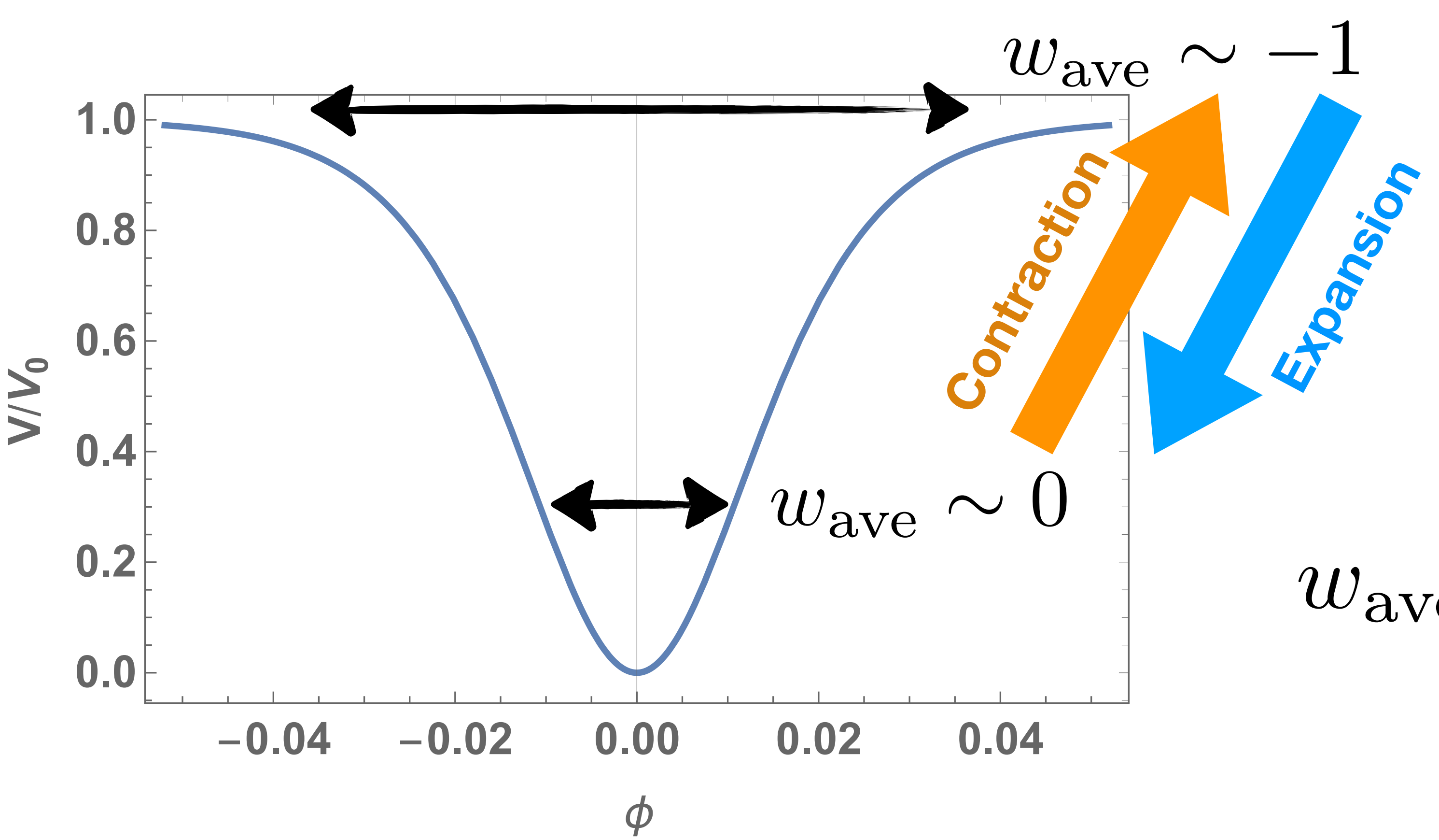
Energy densities



Mechanism of Cycles

Coarse graining oscillations of the scalar field

$$w_{\rm ave} = \langle w \rangle_{\rm osc}$$



$w_{\rm ave}$ **oscillates around -1/3.**

Discussion

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- Vacuum energy can change during the contraction phase.
- Bounce at an arbitrarily higher energy scale may be possible.

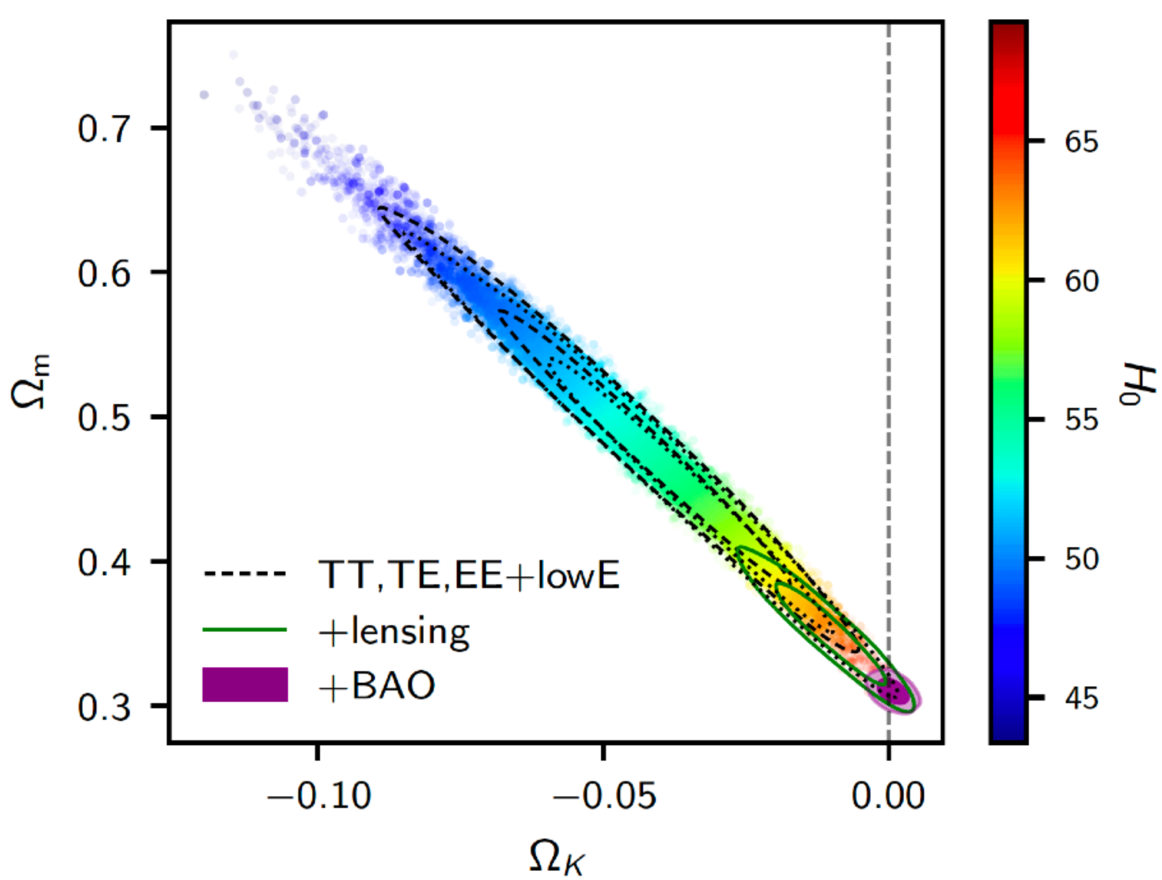
Constraints on Curvature

$$\Omega_{\mathcal{K}} = -0.056^{+0.028}_{-0.018}$$ (68%, Planck TT+lowE)

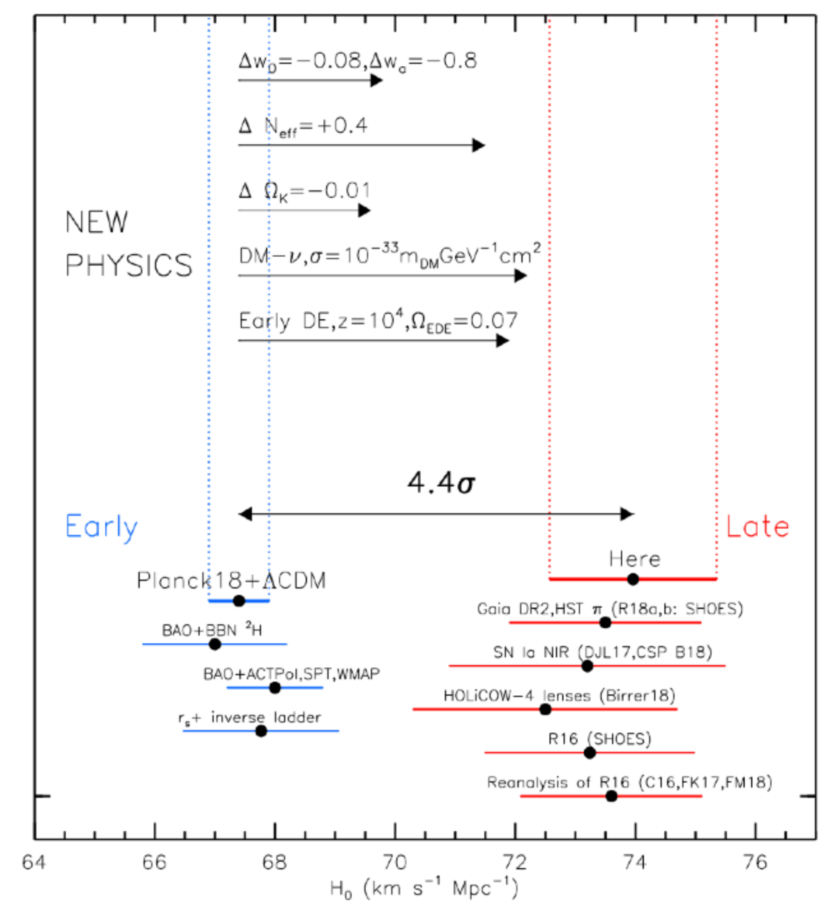
$$\Omega_{\mathcal{K}} = -0.044^{+0.018}_{-0.015}$$ (68%, Planck TT,TE,EE+lowE)

$$\Omega_{\mathcal{K}} = -0.0106 \pm 0.0065$$ (68%, Planck TT,TE,EE+lowE+lensing)

$$\Omega_{\mathcal{K}} = 0.0007 \pm 0.0019$$ (68%, Planck TT,TE,EE+lowE+lensing+BAO)

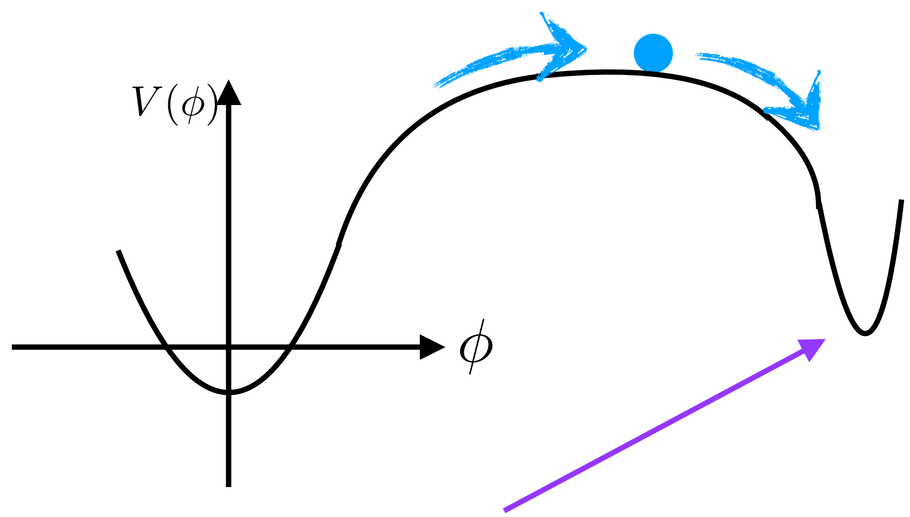


[Planck collaboration 2018: Cosmological parameters]



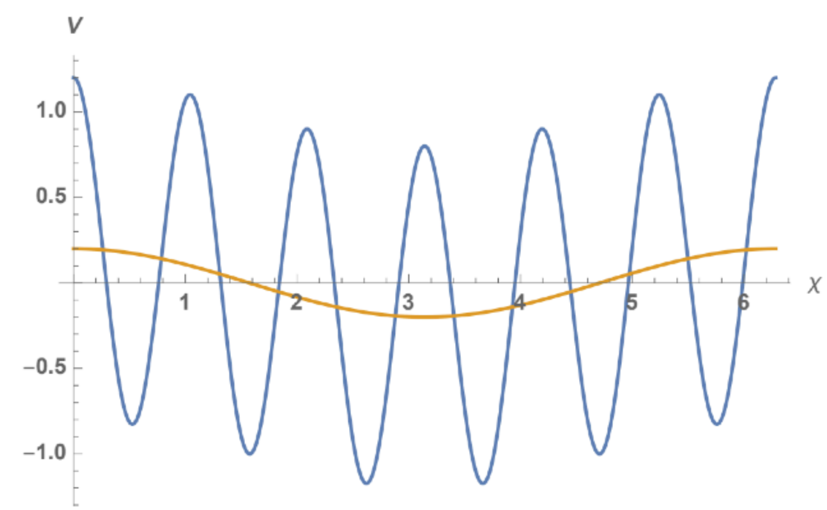
[Riess, Casertano, Yuan, Macri, Scolnic, 1903.07603]

Uplifting Vacuum Energy



Another minimum for ϕ may not be necessary.

Potential of light (axionic) field



[Graham, Kaplan, Rajendran, 1902.06793]

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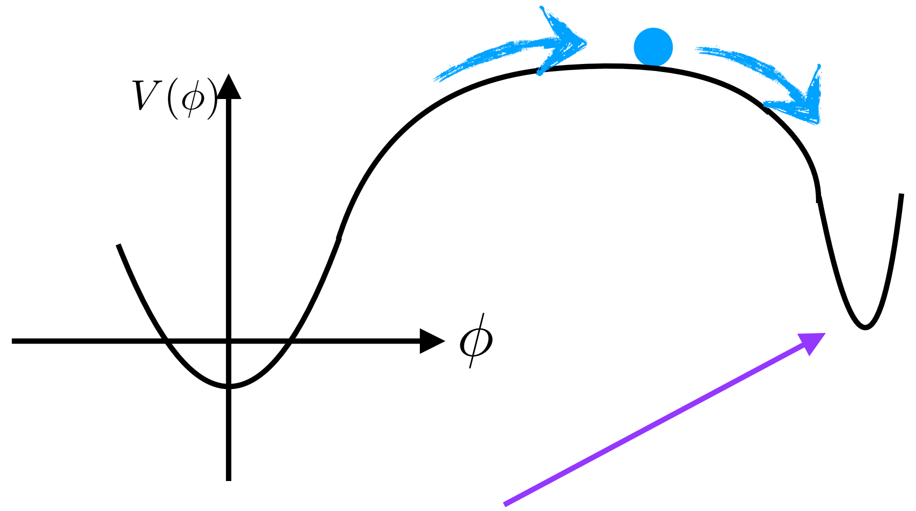

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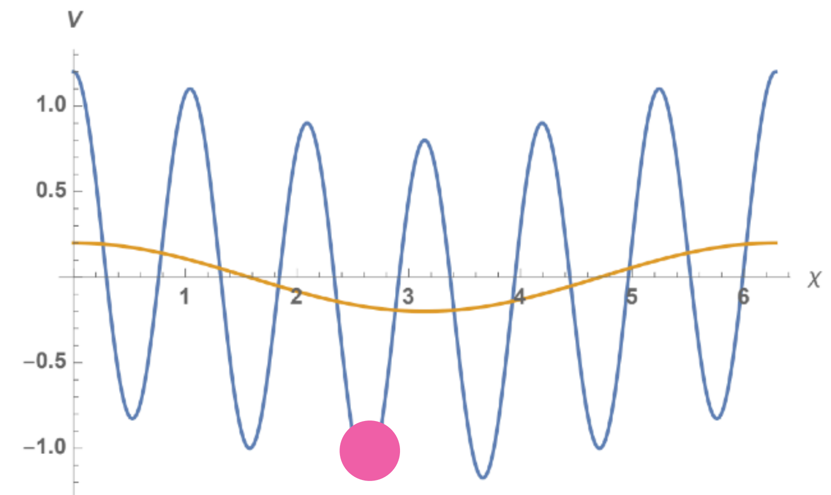

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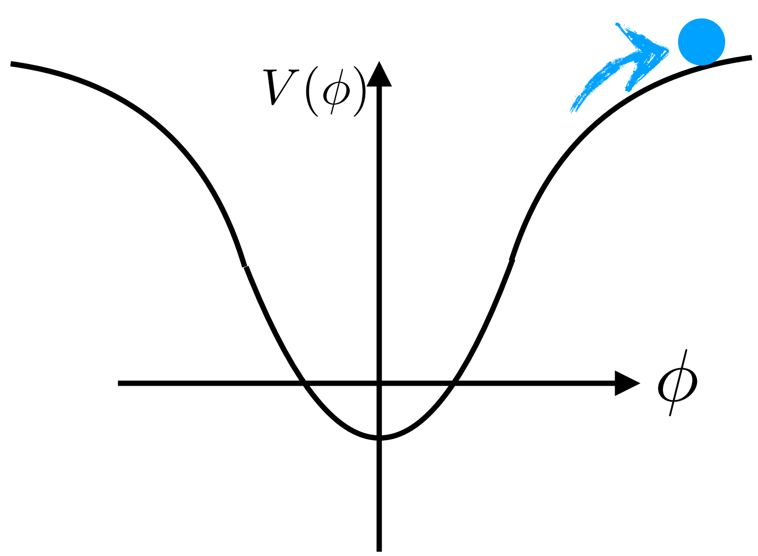

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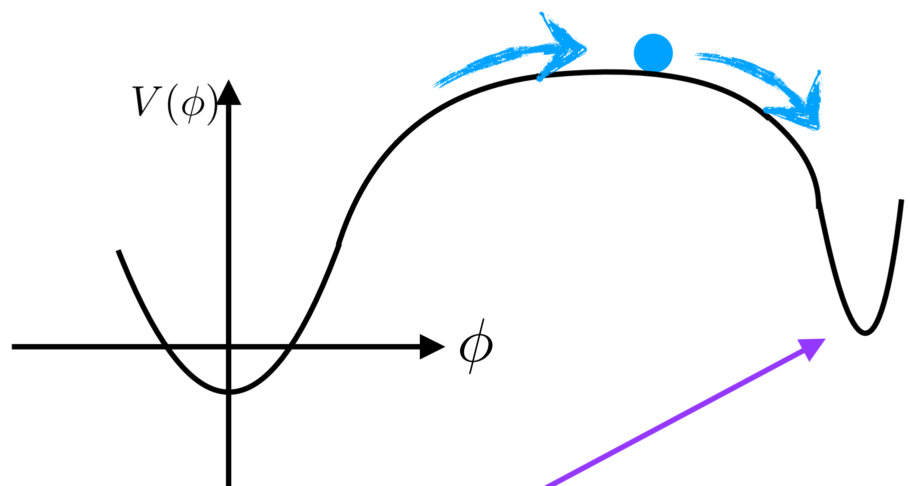

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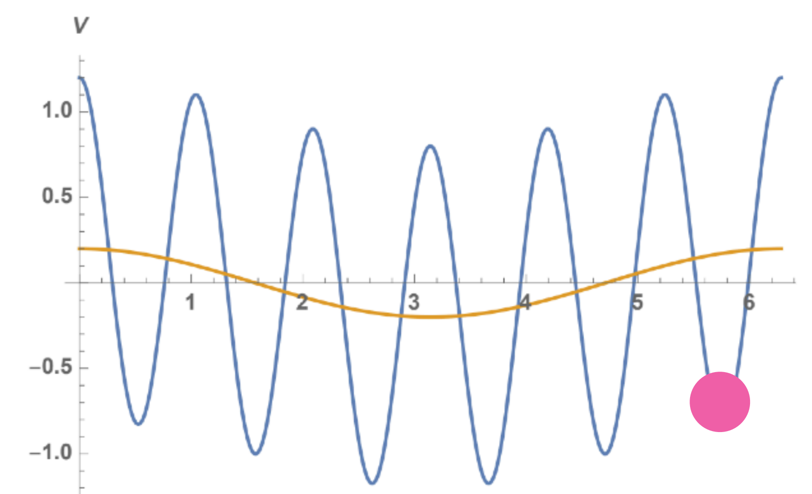

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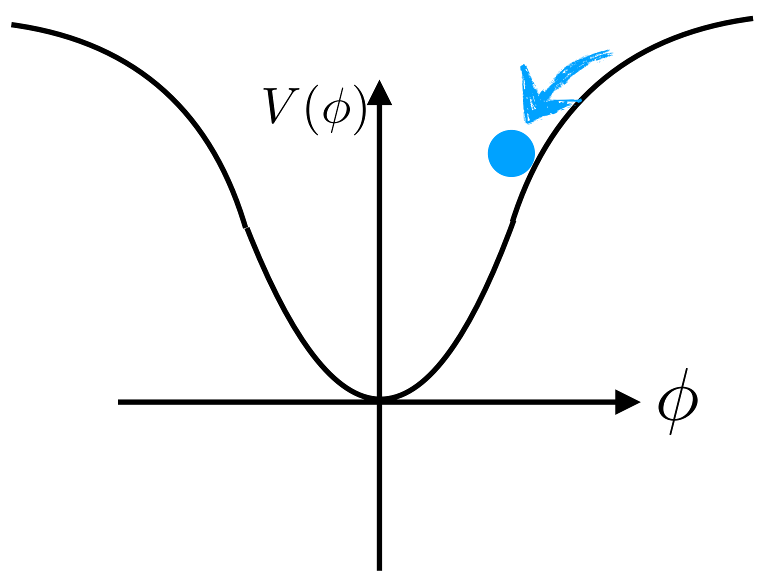
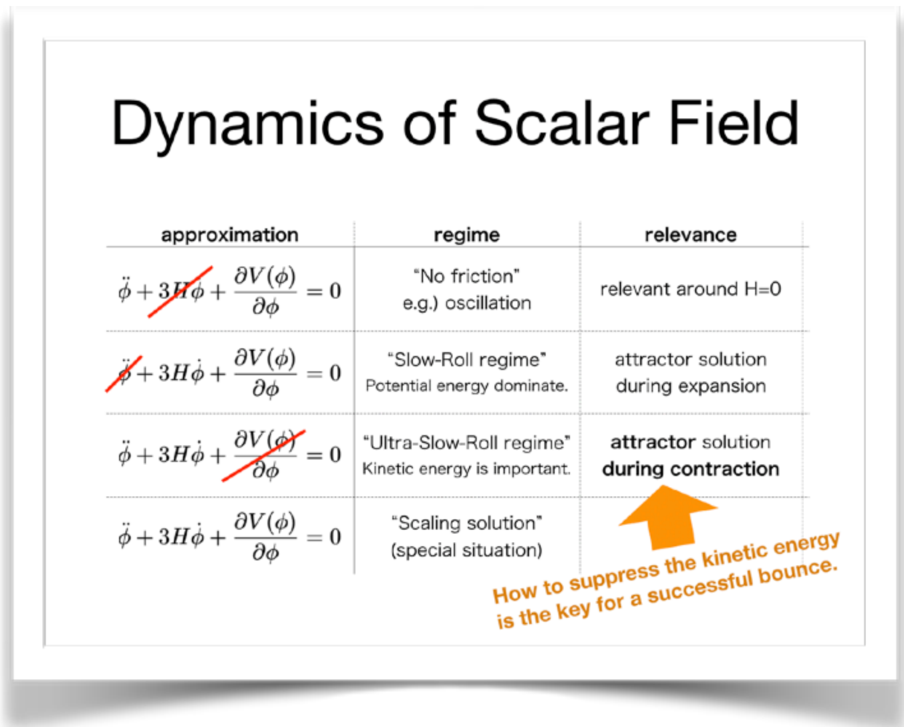


Uplift of the cosmological constant is possible.

Increase Energy before Bounce



Dynamics of Scalar Field

approximation	regime	relevance
$\ddot{\phi} + \cancel{3H\dot{\phi}} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"No friction" e.g.) oscillation	relevant around H=0
$\cancel{\ddot{\phi}} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Slow-Roll regime" Potential energy dominate.	attractor solution during expansion
$\ddot{\phi} + 3H\dot{\phi} + \cancel{\frac{\partial V(\phi)}{\partial \phi}} = 0$	"Ultra-Slow-Roll regime" Kinetic energy is important.	**attractor** solution **during contraction**
$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$	"Scaling solution" (special situation)	

Kinetic-potential scaling solution

$$\frac{1}{2}\dot{\phi}^2(t) \propto V(\phi(t))$$

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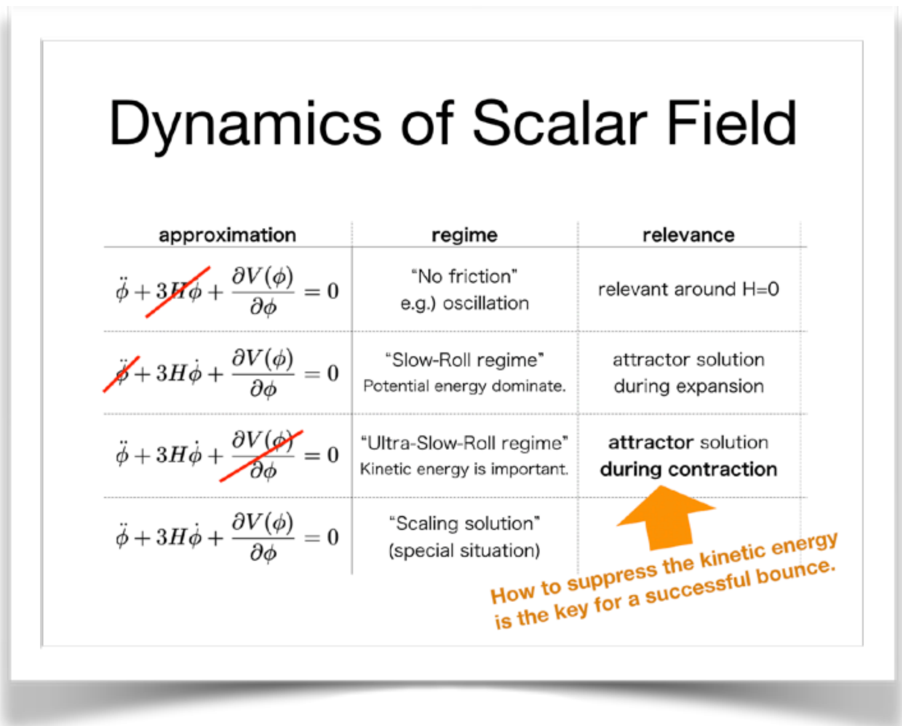

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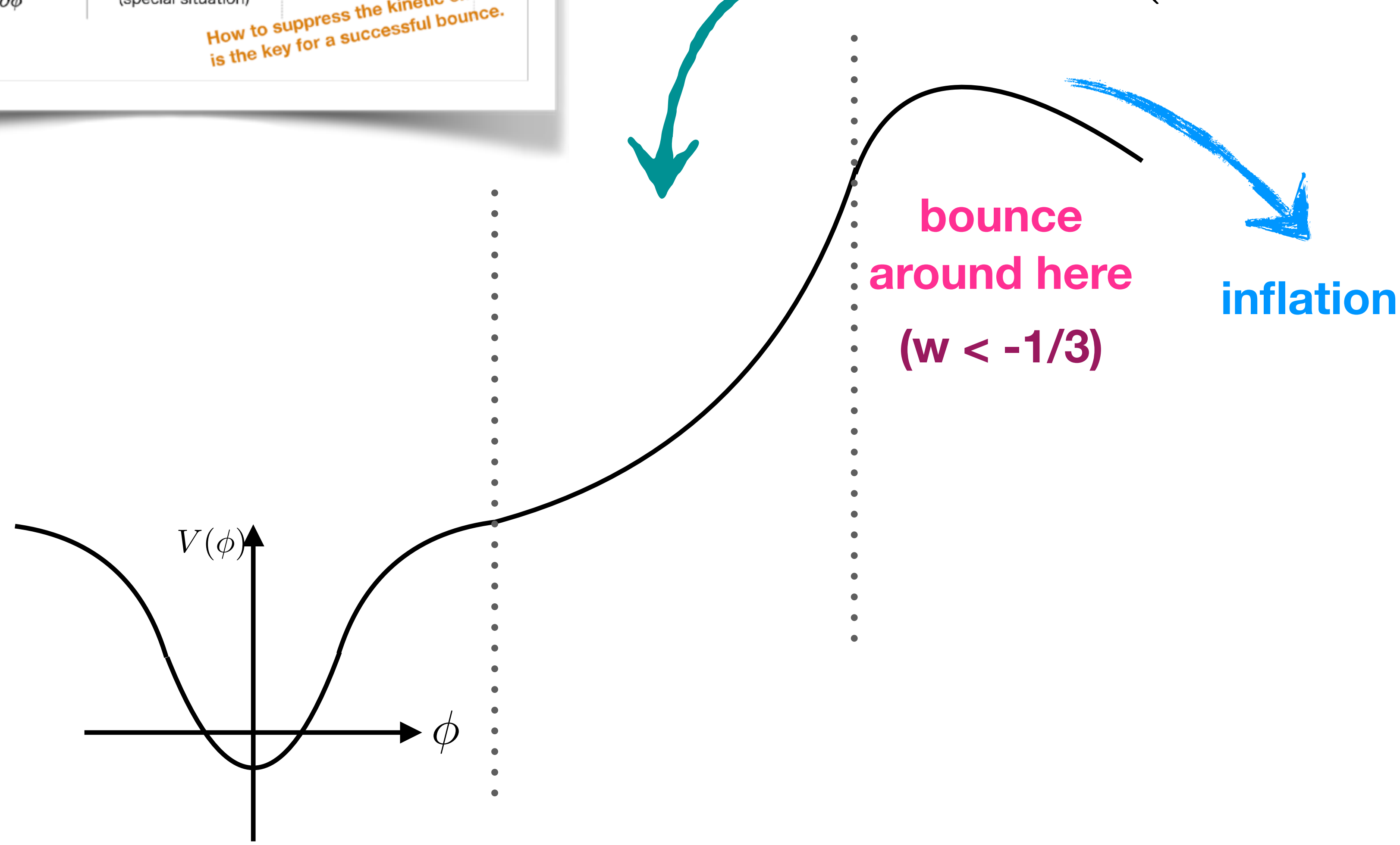

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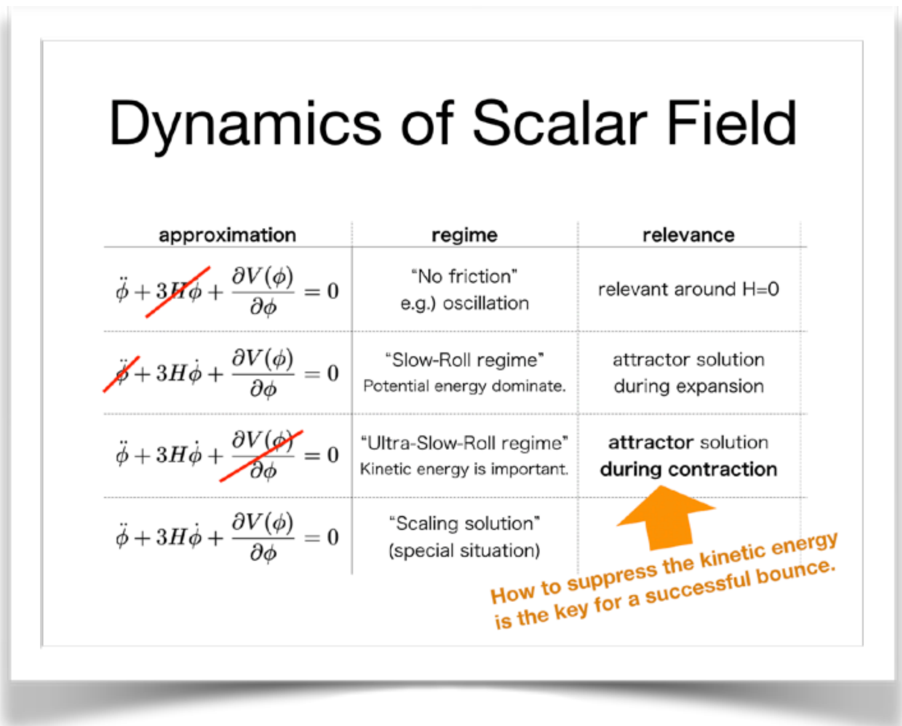

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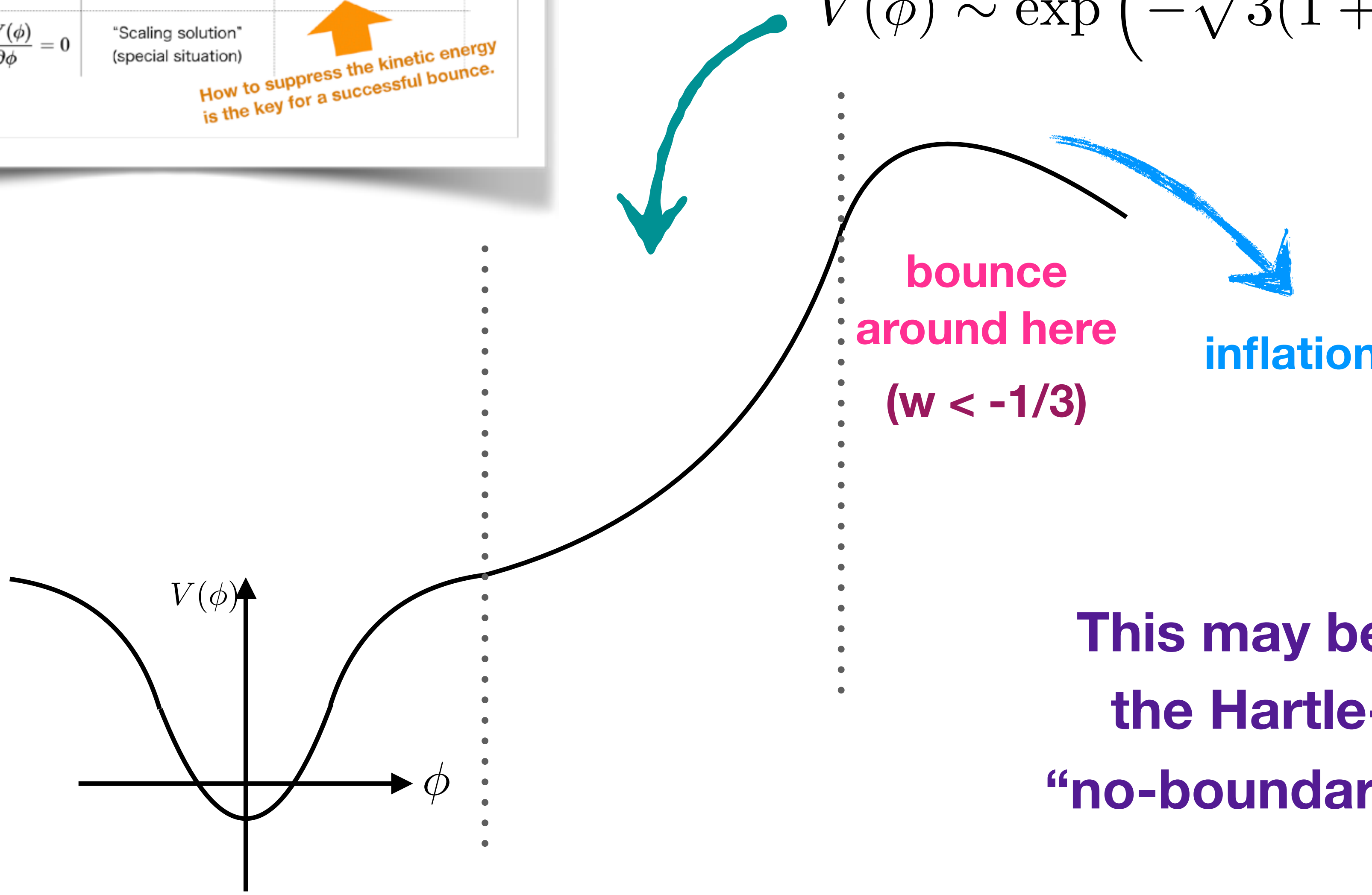

This may be useful for the Hartle-Hawking "no-boundary" scenario.

Related literature

Our scenario *as a whole* gives us a new interesting possibility,
but *each part* has been studied well in the literature.

Note: We do NOT violate the null energy condition.
We consider only NON-singular bounce.

Contraction by a negative potential

[Linde, hep-th/0110195]

[Felder et al., hep-th/0202017]

Cyclic universe

[Kardashev, MNRAS 243, 252 (1990)]

[Dabrowski, gr-qc/9503017]

[Graham, Horn, Kachru, Rajendran, Torroba, 1109.0282]

[Graham, Horn, Rajendran, Torroba, 1405.0282]

See also

[Biswas, 0801.1315]

[Biswas, Alexander, 0812.3182]

[Barrow, Ganguly, 1703.05969]

[Ganguly, Barrow, 1710.00747]

Bounce with positive curvature

[Martin, Peter, hep-th/0307077]

[Gordon, Turok, hep-th/0206138]

[Falciano, Lilley, Peter, 0802.1196]

[Haro, 1511.05048]

[Parker, Fulling, PRD7, 2357 (1973)]

[Starobinsky, SAL4, 82 (1978)]

[Barrow, Matzner, PRD21, 336 (1980)]

[Hawking, Les Houches 1983]

[Page, CQG 1, 417 (1984)]

[Schmidt, gr-qc/0108087]

[Cornish, Shellard, gr-qc/9708046]

What is new?

To best of our knowledge, our scenarios are the first bouncing/cyclic scenarios satisfying the following conditions.

<u>Conditions (common)</u>

(A) 4d Einstein gravity,
FLRW universe with positive spatial curvature,
A single real canonical scalar

(B) No violation of null energy condition,
No singularity

<u>conditions for “*N*-shaped” bouncing scenario</u>

(1) Expansion → Contraction → Expansion

(2) The last expansion can be an arbitrarily long inflation phase

<u>conditions for cyclic scenario</u>

(i) No need of negative potential

(ii) No need of fine tuning